Quadratic Functions and Equations in One Variable

You will learn

► Quadratic Functions and Equations

Pulau Warisan is located in Kuala Terengganu. The island becomes a new tourist attraction because it is a man-made island connected with a bridge. This bridge is similar to the one in Putrajaya.

Do you know that the shape of this bridge has special mathematics characteristics?

Why Study This Chapter?

Quadratic functions and equations are widely used in science, business, sports and others. In sports, quadratic functions are important in sports events such as shot put, discus and javelin. In architecture, we often see curved structures in the shape of parabola which are related to the mastery of quadratic concepts.

Walking Through Time

Al-Khawarizmi
(780 AD – 850 AD)

Al-Khawarizmi is well-known as the Father of Algebra. He was the founder of a few mathematics concepts. His work in algebra was outstanding. He was responsible for initiating the systematic and logical approach in solving linear and quadratic equations.

http://bt.sasbadi.com/m4001

WORD BANK

• quadratic function	• *fungsi kuadratik*
• axis of symmetry	• *paksi simetri*
• variable	• *pemboleh ubah*
• root	• *punca*
• maximum point	• *titik maksimum*
• minimum point	• *titik minimum*

1.1 Quadratic Functions and Equations

What is a quadratic expression in one variable?

Learning Standard

Identify and describe the characteristics of quadratic expressions in one variable.

Have you ever sketched the movement of a ball kicked by a football player, as shown in the picture?

The shape of this movement is a parabola.

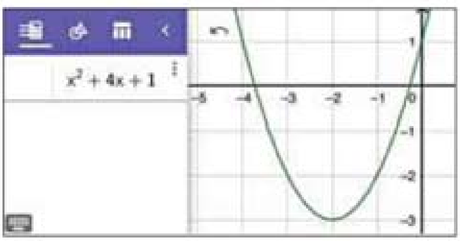
Do you know that this parabola has its own equation, just like a straight line which has its own equation?

Mind Stimulation 1

Aim: To identify and describe the characteristics of quadratic expressions in one variable.

Steps:

1. Based on the table in Step 3, insert all the expressions one by one in the dynamic geometry software as shown below.



Scan the QR Code to carry out this activity. *https://www.geogebra.org/graphing*

2. Observe the graph obtained.
3. Complete the table below.

Expression		Characteristic	
		Shape of graph	Coordinates of the lowest or highest point (if any)
(a)	$x^2 + 4x + 1$		
(b)	$x^{-2} - 1$		
(c)	$-2x^2 - 2x + 5$		
(d)	$5x + 4$		
(e)	$3x^2 - 2$		
(f)	$-2x^2 + 4x$		
(g)	$x^3 + 1$		

Discussion:

The graph of a quadratic expression is either $\cup$ or $\cap$ and has the highest point or the lowest point. Which expression is a quadratic expression? Justify your answer.

From the activity in Mind Stimulation 1, it is found that:

> A quadratic expression in one variable is an expression whereby the **highest power** for the variable is **two**.

In general,

> The general form of a quadratic expression is $ax^2 + bx + c$.

Where;
a, b and c are constants and $a \neq 0$,
x is a variable.

INTERACTIVE ZONE

Why is $a \neq 0$ in a quadratic expression? Discuss.

For example:

> $x^2 + 2x - 1$, $-y^2 + 3y$, $\frac{1}{3}m^2 - m + 4$ and $2n^2 + 5$ are quadratic expressions.

TIPS

Besides x, other letters can be used to represent variables.

Example 1

Determine whether each of the following expressions is a quadratic expression in one variable. If not, justify your answer.

(a) $2x^2 + 5$ (b) $x^3 - 6$

(c) $3x^2 + 2y + 1$ (d) $\frac{1}{2}m^2$

(e) $2x^2 - \frac{3}{x^2}$ (f) $4x^2 - x^{\frac{1}{2}}$

TIPS

The values of constants b and c can be zero.

MY MEMORY

$\frac{3}{x^2} = 3x^{-2}$

$x^{\frac{1}{2}} = \sqrt{x}$

Solution:

(a) $2x^2 + 5$ is a quadratic expression in one variable.

(b) $x^3 - 6$ is not a quadratic expression because the highest power of the variable x is 3.

(c) $3x^2 + 2y + 1$ is not a quadratic expression in one variable because there are two variables, x and y.

(d) $\frac{1}{2}m^2$ is a quadratic expression in one variable.

(e) $2x^2 - \frac{3}{x^2}$ is not a quadratic expression because there is a variable with a power which is not a whole number.

(f) $4x^2 - x^{\frac{1}{2}}$ is not a quadratic expression because there is a variable with a power which is not a whole number.

Mind Stimulation 2

Aim: To state the values of a, b and c in a quadratic expression.

Steps:

1. Observe (a) in the table below.
2. Determine the values of a, b and c for the subsequent quadratic expressions.

	Quadratic expression	Comparison
(a)	$2x^2 - 3x + 1$	$2x^2 - 3x + 1$ $ax^2 + bx + c$ $a =$ 2 $b =$ –3 $c =$ 1
(b)	$2x^2 - 4$	$\square x^2 + \square x + \square$ $ax^2 + bx + c$ $a = \square$ $b = \square$ $c = \square$
(c)	$\frac{1}{2}x^2 + 5x - \frac{3}{2}$	$a = \square$ $b = \square$ $c = \square$
(d)	$-x^2 + x$	$a = \square$ $b = \square$ $c = \square$
(e)	$-x^2 - 3x - 9$	$a = \square$ $b = \square$ $c = \square$
(f)	$\frac{1}{2}x^2$	$a = \square$ $b = \square$ $c = \square$

Discussion:

How do you determine the values of a, b and c?

From the activity in Mind Stimulation 2, it is found that:

All quadratic expressions can be written in the form of $ax^2 + bx + c$, where $a \neq 0$.

In a quadratic expression,

a is the coefficient of x^2,
b is the coefficient of x,
c is a constant.

INTERACTIVE ZONE

Why are a and b known as the coefficients and c the constant?

Self Practice 1.1a

1. Determine whether each of the following expressions is a quadratic expression in one variable. If not, justify your answer.

(a) $x^2 - 5$ (b) $2x^2 + x^{-2}$ (c) $3y^2 - 3x + 1$

(d) $-\frac{1}{2}m^2$ (e) $x^3 - x$ (f) $x^{\frac{1}{2}} + 2x - 1$

(g) $\frac{1}{x^2} + 4x - 1$ (h) $p^2 - \frac{1}{2}p + 3$ (i) $n(n - 2)$

2. Determine the values of a, b and c for each of the following quadratic expressions.

(a) $2x^2 - 5x + 1$ (b) $x^2 - 2x$ (c) $2y^2 + 1$

(d) $-\frac{1}{2}p^2 + 4p$ (e) $1 - x - 2x^2$ (f) $4x^2$

(g) $h^2 + \frac{3}{2}h - 4$ (h) $\frac{1}{3}k^2 - 2$ (i) $2r(r - 3)$

What is the relationship between a quadratic function and many-to-one relation?

Learning Standard

Recognise quadratic function as many-to-one relation, hence, describe the characteristics of quadratic functions.

What is the difference between a quadratic expression and a quadratic function?

MY MEMORY

Types of relation
- One-to-one relation
- One-to-many relation
- Many-to-one relation
- Many-to-many relation

A quadratic expression is written in the form of $ax^2 + bx + c$, whereas a quadratic function is written in the form of $f(x) = ax^2 + bx + c$.

INTERACTIVE ZONE

Discuss and give examples of many-to-one relation.

Mind Stimulation 3

Aim: To recognise quadratic functions as many-to-one relation.

Materials: Ruler, pencil

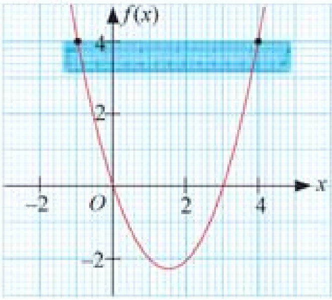
INFO ZONE

For a quadratic function, $y = f(x)$.

Steps:

1. Based on the graphs of functions $f(x)$ below, draw a line which is parallel to the x-axis on graphs (b) and (c), as in graph (a).
2. Mark the points of intersection between the graph of function $f(x)$ and the straight line.
3. State the number of points of intersection and the coordinates of the points of intersection.
4. Repeat Steps 1 to 3 by placing the ruler at different values of $f(x)$. Ensure the straight lines drawn are parallel to the x-axis.

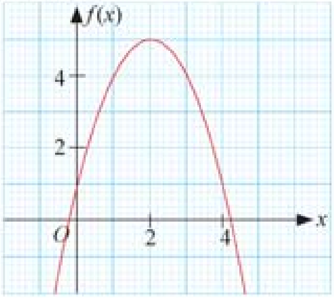
(a) $f(x) = x^2 - 3x$



Number of points of intersection = 2

Points of intersection = (4, 4), (–1, 4)

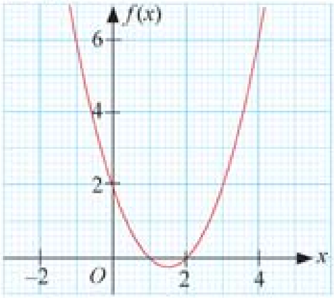
(b) $f(x) = -x^2 + 4x + 1$



Number of points of intersection =

Points of intersection = (   ), (   )

(c) $f(x) = x^2 - 3x + 2$



Number of points of intersection =

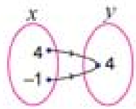
Points of intersection = (   ), (   )

Discussion:

1. What is the relationship between the x-coordinates and y-coordinates of both points of intersection for each function?
2. What is the type of relation of a quadratic function?

MY MEMORY

For a point on a Cartesian plane, the x-coordinate is the object and the y-coordinate is the image.



From the activity in Mind Stimulation 3, it is found that:

All quadratic functions have the same image for two different objects.

In general,

The type of relation of a quadratic function is a many-to-one relation.

Scan the QR Code to watch the vertical line test.
http://bt.sasbadi.com/m4006

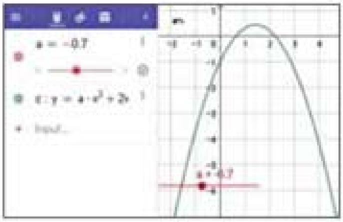
What is the shape of the graph of a quadratic function?

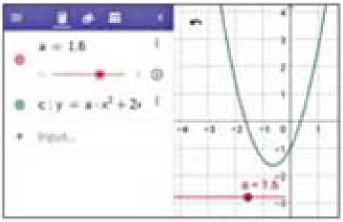
Mind Stimulation 4

Aim: To identify and describe the relationship between the value of a and the shape of the graph of a quadratic function.

Steps:

1. Drag the slider slowly from left to right. Observe the shape of the graph.





Scan the QR Code to carry out this activity. *https://www.geogebra.org/graphing/t5az2zwm*

2. Sketch at least two graphs for positive values of a and two graphs for negative values of a.

Discussion:

What is the relationship between the value of a and the shape of a graph?

From the activity in Mind Stimulation 4, it is found that:

For a graph of $f(x) = ax^2 + bx + c$, $a \neq 0$

(a) there are only two shapes of the graphs,

(b) the value of a determines the shape of the graph.

$a > 0$

$a < 0$

TIPS

The curved shape of the graph of a quadratic function is called a parabola.

What is the maximum or minimum point of a quadratic function?

Each sketch of the graph of a quadratic function has the highest or lowest value of y-coordinate based on the shape of the sketch.

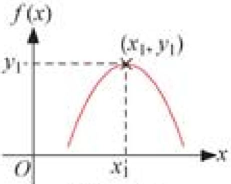


Diagram 1

For the sketch of the graph of a quadratic function with $a < 0$, y_1 is the highest value of y-coordinate and x_1 is the corresponding value for y_1. The point (x_1, y_1) is known as the **maximum point**.

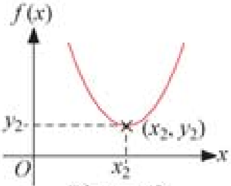


Diagram 2

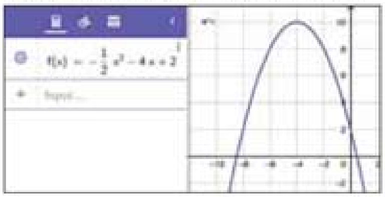
For the sketch of the graph of a quadratic function with $a > 0$, y_2 is the lowest value of y-coordinate and x_2 is the corresponding value for y_2. The point (x_2, y_2) is known as the **minimum point**.

Mind Stimulation 5

Aim: To explore the maximum or minimum point of a quadratic function.

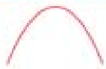
Steps:

1. Based on the table in Step 2, insert the quadratic functions in the dynamic geometry software.



Scan the QR Code to carry out this activity.
https://www.geogebra.org/graphing

2. Complete the table below as in (a).

	Quadratic function	Value of a	Shape of graph	Maximum / Minimum point and coordinates
(a)	$f(x) = -\frac{1}{2}x^2 - 4x + 2$	$a = -\frac{1}{2}$		Maximum point Coordinates = (–4, 10)
(b)	$f(x) = x^2 - 4x + 3$			point Coordinates =
(c)	$f(x) = -2x^2 - 4x + 1$			point Coordinates =

3. Repeat Steps 1 and 2 for various quadratic functions.

Discussion:

What is the relationship between the value of a and the maximum or minimum point?

From the activity in Mind Stimulation 5, it is found that:

> For a quadratic function $f(x) = ax^2 + bx + c$,
> the maximum point is obtained when $a < 0$,
> the minimum point is obtained when $a > 0$.

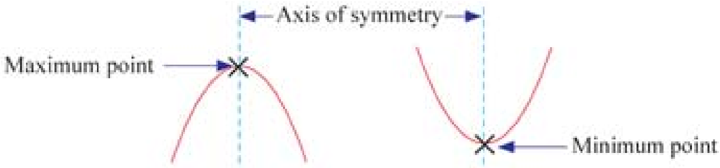
INFO ZONE

The maximum or minimum point is also called a stationary point or a turning point.

What is the axis of symmetry of the graph of a quadratic function?

The axis of symmetry of the graph of a quadratic function is a straight line that is parallel to the y-axis and divides the graph into two parts of the same size and shape.

The axis of symmetry will pass through the maximum or minimum point of the graph of the function as shown in the diagram below.



MY MEMORY

An axis of symmetry is a straight line that divides a geometrical shape or an object into two parts of the same size and shape.

Smart Mind

The equation of the axis of symmetry for a quadratic function is $x = -\frac{b}{2a}$.

Mind Stimulation 6

Aim: To draw and recognise the axis of symmetry of the graph of a quadratic function.

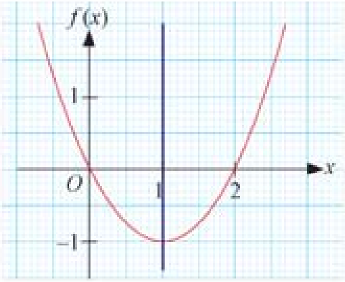
MY MEMORY

The equation of a straight line which is parallel to the y-axis is $x = h$.

Steps:

1. Using a ruler, draw the axis of symmetry for each graph of quadratic function below.
2. Write the equation of the axis of symmetry as in (a).

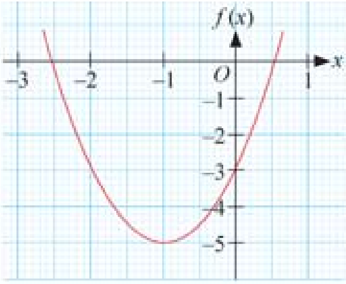
(a) $f(x) = x^2 - 2x$



Equation of axis of symmetry

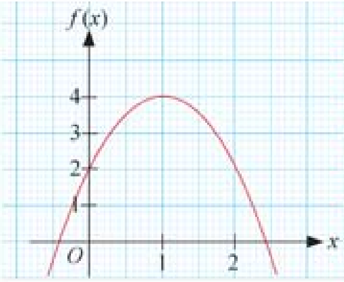
$x = 1$

(b) $f(x) = 2x^2 + 4x - 3$



Equation of axis of symmetry

(c) $f(x) = -2x^2 + 4x + 2$



Equation of axis of symmetry

Discussion:

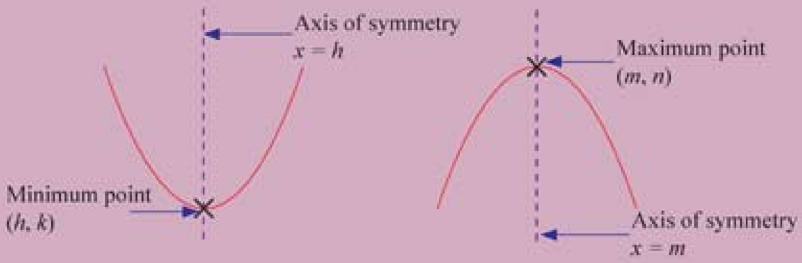
1. What is the relationship between the axis of symmetry of the graph of a quadratic function and the y-axis?
2. What is the relationship between the axis of symmetry of the graph of a quadratic function and the maximum or minimum point?

From the activity in Mind Stimulation 6, it is found that:

The axis of symmetry of the graph of a quadratic function is parallel to the y-axis and passes through the maximum or minimum point.

In general,

Each graph of quadratic function has one axis of symmetry which passes through the maximum or minimum point.

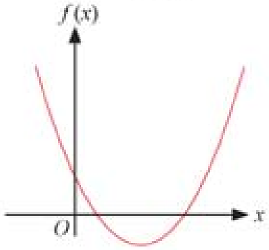


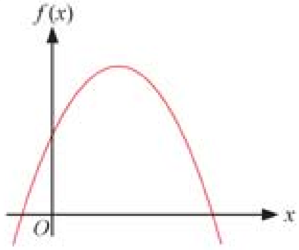
CHAPTER 1

Self Practice 1.1b

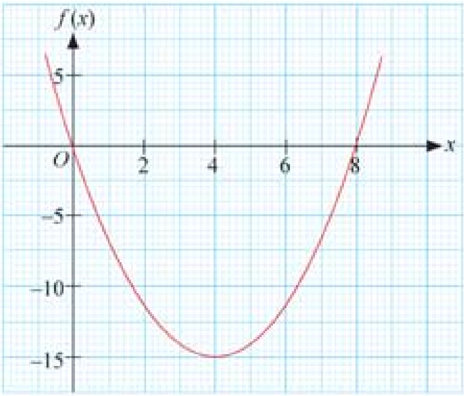
1. Determine whether the shapes of the following graphs of quadratic functions is ∪ or ∩.
   (a) $f(x) = x^2 - 4x + 1$
   (b) $g(x) = -x^2 + 2x - 4$

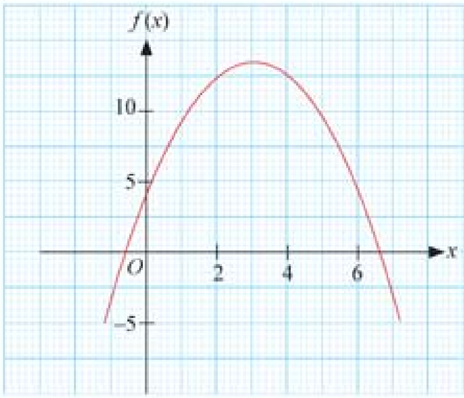
2. For each graph of quadratic function $f(x) = ax^2 + bx + c$ below, state the range of value of a and state whether the graph has a maximum or minimum point.

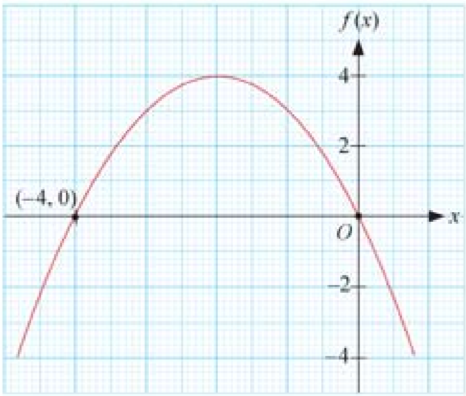
   (a)


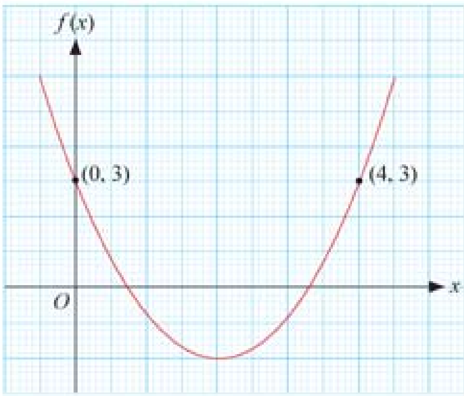
   (b)


3. Determine the maximum or minimum point and state the equation of the axis of symmetry for each graph of quadratic function below.

   (a)


   (b)


   (c)


   (d)


What are the effects of changing the values of a, b and c on graphs of quadratic functions, $f(x) = ax^2 + bx + c$?

Learning Standard

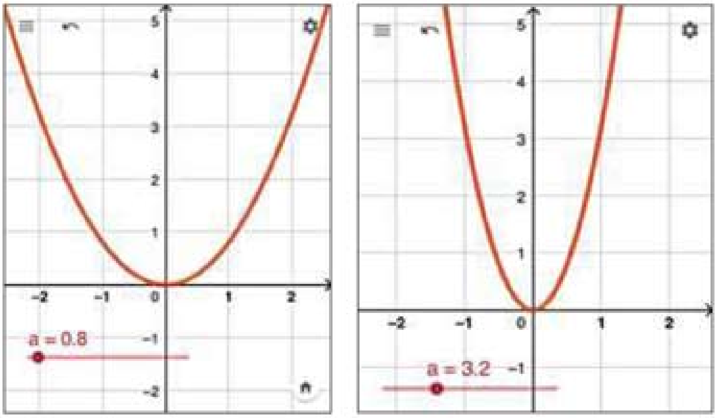
Investigate and make generalisation about the effects of changing the values of a, b and c on graphs of quadratic functions, $f(x) = ax^2 + bx + c$.

Mind Stimulation 7

Aim: To identify the effects of changing the values of a on graphs of quadratic functions $f(x) = ax^2 + bx + c$.

Steps:

1. Drag the slider from left to right.



Scan the QR Code to carry out this activity.
https://www.geogebra.org/graphing/nhxfjgy3

2. Observe the shape of the graph as the value of a changes.

Discussion:

What are the effects of changing the values of a to the graphs of quadratic functions?

From the activity in Mind Stimulation 7, it is found that:

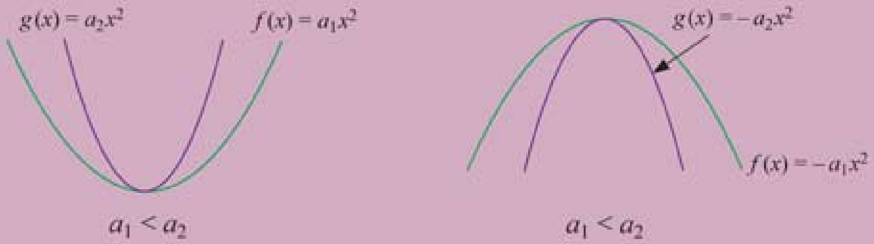
The value of a determines the shape of the graph.

INTERACTIVE ZONE

Discuss the effects on the curve of the graphs of quadratic functions when $a < 0$.

In general,

For the graph of a quadratic function $f(x) = ax^2 + bx + c$, the smaller the value of a, the wider the curved shape of the graph and vice versa.

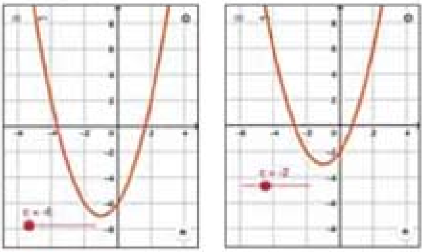


Mind Stimulation 9

Aim: To identify the effects of changing the values of c on graphs of quadratic functions $f(x) = ax^2 + bx + c$.

Steps:

1. Drag the slider from left to right.



2. Observe the position of the y-intercept as the value of c changes.

Scan the QR Code to carry out this activity.
https://www.geogebra.org/graphing/rv7njx84

MY MEMORY

The y-intercept is a point of a graph that intersects the y-axis.

Discussion:

What are the effects of changing the values of c to the graphs of quadratic functions $f(x) = ax^2 + bx + c$?

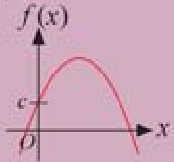
From the activity in Mind Stimulation 9, it is found that:

The value of c determines the position of the y-intercept.

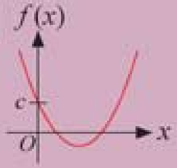
In general,

For the graph of a quadratic function $f(x) = ax^2 + bx + c$, the value of c determines the y-intercept of the graph.

$a < 0$



$a > 0$



Example 2

The quadratic function $f(x) = x^2 - 3x + c$ passes through a point A as given below. Calculate the value of c for each of the following cases.

(a) $A(0, 4)$ (b) $A(-1, 3)$

Solution:

(a) The point $A(0, 4)$ lies on the y-axis, thus $c = 4$.

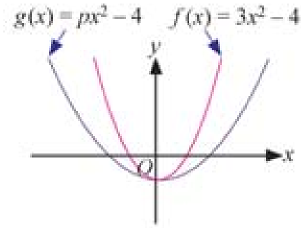
(b) $f(x) = x^2 - 3x + c$
Substitute the values of $x = -1$ and $f(x) = 3$ into the quadratic function.
$3 = (-1)^2 - 3(-1) + c$
$c = -1$

TIPS

c is the y-intercept of a quadratic function $f(x) = ax^2 + bx + c$.

Example 3

The diagram shows two graphs of quadratic functions, $y = f(x)$ and $y = g(x)$, drawn on the same axes. State the range of the values of p. Explain your answer.



Solution:

$0 < p < 3$.

Since the curve of the graph $g(x)$ is wider, thus $p < 3$.

For a graph with the shape $\smile$, $p > 0$.

Thus, $0 < p < 3$.

Self Practice 1.1c

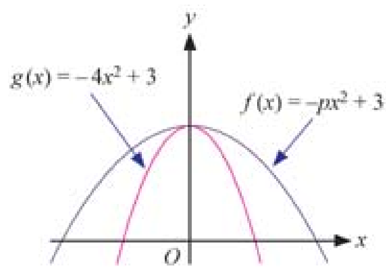
1. The quadratic functions below pass through the points stated. Calculate the value of c for each case.

(a) $f(x) = x^2 + 7x + c$, passes through point (0, 5).

(b) $f(x) = 2x^2 - 4x + c$, passes through point (2, –3).

(c) $f(x) = -2x^2 + x + c$, y-intercept = 4.

2. The diagram on the right shows two graphs of quadratic functions, $y = f(x)$ and $y = g(x)$, drawn on the same axes. State the range of the values of p. Explain your answer.

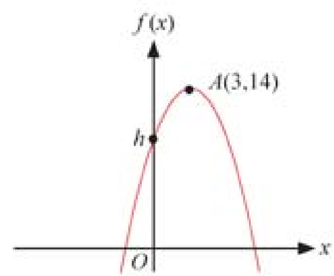


3. The diagram on the right shows the graph of a quadratic function $f(x) = kx^2 + 6x + h$. Point $A(3, 14)$ is the maximum point of the graph of quadratic function.

(a) Given k is an integer where $-2 < k < 2$, state the value of k.

(b) Using the value of k from (a), calculate the value of h.

(c) State the equation of the quadratic function formed when the graph is reflected in the x-axis. Give your answer in the form of $f(x) = ax^2 + bx + c$.

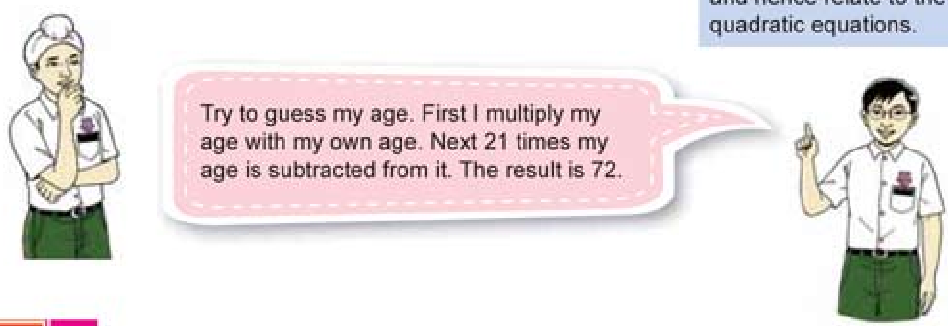


How do you form a quadratic equation based on a situation?

Learning Standard

Form quadratic functions based on situations, and hence relate to the quadratic equations.

A quadratic function is written in the form of $f(x) = ax^2 + bx + c$ while a quadratic equation is written in the general form, $ax^2 + bx + c = 0$.



Example 4

Mr Ganesan plans to make two different types of cards for Mathematics Club activities. The measurements of the cards are shown in the diagram below.

(a) Form a quadratic expression for the total area of the two cards, A cm^2, in terms of x.

(b) The total area of the two cards is 114 cm^2. Form a quadratic equation in terms of x.

Solution:

(a) $A = x^2 + x(2x + 1)$
$= x^2 + 2x^2 + x$
$= 3x^2 + x$

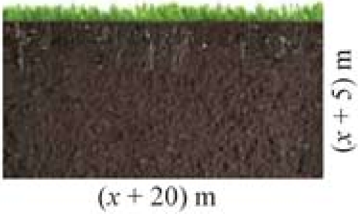
(b) $3x^2 + x = 114$
$3x^2 + x - 114 = 0$

Self Practice 1.1d

1. The diagram on the right shows a piece of land with a length of $(x + 20)$ m and a width of $(x + 5)$ m.

(a) Write a function for the area, A m^2, of the land.

(b) If the area of the land is 250 m^2, write a quadratic equation in terms of x. Give your answer in the form of $ax^2 + bx + c = 0$.



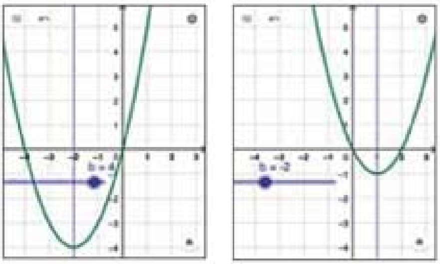
2. Aiman is 4 years older than his younger brother. The product of Aiman and his younger brother's ages is equal to their father's age. The father is 48 years old and Aiman's younger brother is p years old. Write a quadratic equation in terms of p.

Mind Stimulation 8

Aim: To identify the effects of changing the values of b on graphs of quadratic functions $f(x) = ax^2 + bx + c$.

Steps:

1. Drag the slider from left to right.



Scan the QR Code to carry out this activity.
https://www.geogebra.org/graphing/vpzgvwba

2. Observe the position of the axis of symmetry as the value of b changes.

Discussion:
What are the effects of changing the values of b to the graphs of quadratic functions?

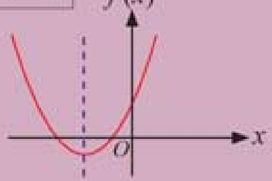
From the activity in Mind Stimulation 8, it is found that:

The value of b determines the position of the axis of symmetry.

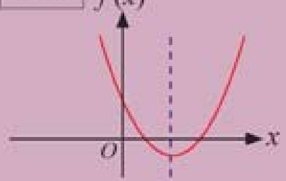
In general,

For the graph of a quadratic function $f(x) = ax^2 + bx + c$
if $a > 0$; $b > 0$, then the axis of symmetry lies on the left of the y-axis.
$b < 0$, then the axis of symmetry lies on the right of the y-axis.
$b = 0$, then the axis of symmetry is the y-axis.

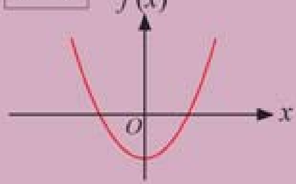
$b > 0$



$b < 0$

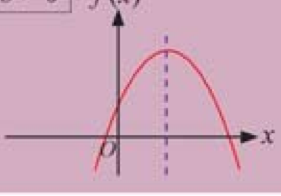


$b = 0$

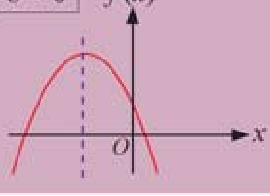


if $a < 0$; $b > 0$, then the axis of symmetry lies on the right of the y-axis.
$b < 0$, then the axis of symmetry lies on the left of the y-axis.
$b = 0$, then the axis of symmetry is the y-axis.

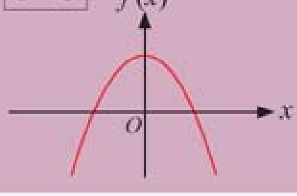
$b > 0$



$b < 0$



$b = 0$



What do you understand about the roots of a quadratic equation?

The roots of a quadratic equation $ax^2 + bx + c = 0$ are the values of the variable, x, which satisfy the equation.

Do you know how the roots of a quadratic equation are determined?

Learning Standard

Explain the meaning of roots of a quadratic equation.

INTERACTIVE ZONE

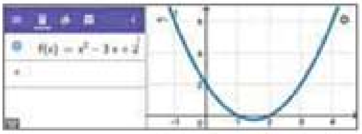
What is the meaning of "satisfy an equation"? Discuss.

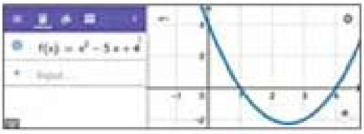
Mind Stimulation 10

Aim: To determine the values of a variable that satisfy a quadratic equation.

Steps:

1. Divide the class into two groups, A and B.
2. Group A will complete the table below without using the dynamic geometry software.
3. Group B will carry out this activity using the dynamic geometry software. Type each quadratic expression into the software. For each graph, determine the value of the quadratic expression for each given value of x.





Scan the QR Code to carry out this activity.
https://www.geogebra.org/graphing

4. Complete and determine the values of x that satisfy the quadratic equation in the table below.

$x^2 - 3x + 2 = 0$	
Value of x	Value of $x^2 - 3x + 2$
0	$0^2 - 3(0) + 2 = 2$
1	0
2	0
3	2
4	6
x are 1, 2	

$x^2 - 5x + 4 = 0$	
Value of x	Value of $x^2 - 5x + 4$
0	
1	
2	
3	
4	
x are	

$x^2 - 2x + 1 = 0$	
Value of x	Value of $x^2 - 2x + 1$
–2	
–1	
0	
1	
2	
x are	

$x^2 + x - 2 = 0$	
Value of x	Value of $x^2 + x - 2$
–2	
–1	
0	
1	
2	
x are	

$x^2 - 4x + 5 = 2$	
Value of x	Value of $x^2 - 4x + 5$
0	
1	
2	
3	
4	
x are	

$x^2 + 2x - 2 = 1$	
Value of x	Value of $x^2 + 2x - 2$
–3	
–2	
–1	
0	
1	
x are	

Discussion:
How can you determine the values of the variable that satisfy a quadratic equation?

From the activity in Mind Stimulation 10, it is found that:

(a) There are one or two values of the variable that satisfy a quadratic equation.
(b) The values of the variable that satisfy a quadratic equation are known as the roots of the quadratic equation.

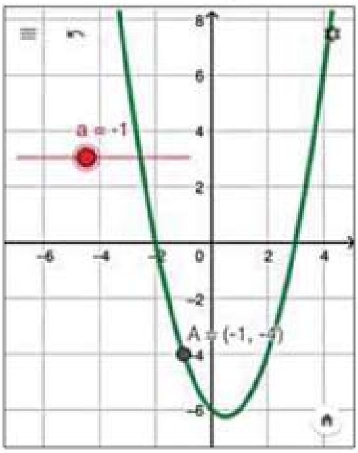
What is the relationship between the roots of a quadratic equation and the positions of the roots?

Mind Stimulation 11

Aim: To explore the positions of the roots of a quadratic equation on the graph of a quadratic function, $f(x) = 0$.

Steps:

1. Drag the slider to observe the changes of the x-coordinate and y-coordinate on the graph.
2. The roots of quadratic equation $x^2 - x - 6 = 0$ can be determined when $y = 0$. Drag the slider from left to right. Observe the coordinates of A.



Scan the QR Code to carry out this activity.
https://www.geogebra.org/graphing/bykrknjx

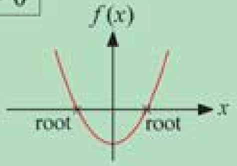
3. Determine the position of point A when y is 0.
4. Mark the point on the above diagram.

Discussion:
What do you notice about the positions of the roots of a quadratic equation on the graph of the quadratic function?

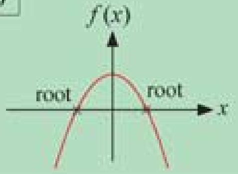
From the activity in Mind Stimulation 11, it is found that:

The roots of a quadratic equation $ax^2 + bx + c = 0$ are the points of intersection of the graph of the quadratic function $f(x) = ax^2 + bx + c$ and the x-axis, which are also known as the x-intercepts.

$a > 0$



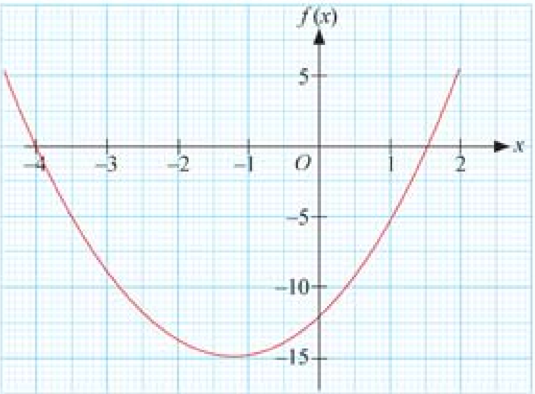
$a < 0$



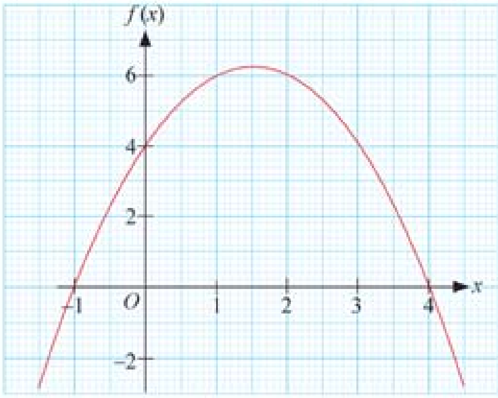
Example 5

For each graph of quadratic equation below, mark and state the roots of the given quadratic equation.

(a) $2x^2 + 5x - 12 = 0$

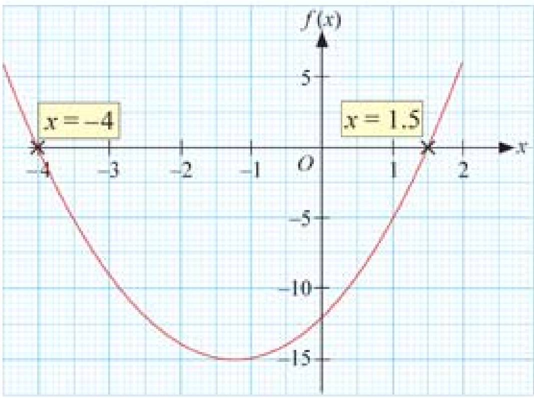


(b) $-x^2 + 3x + 4 = 0$



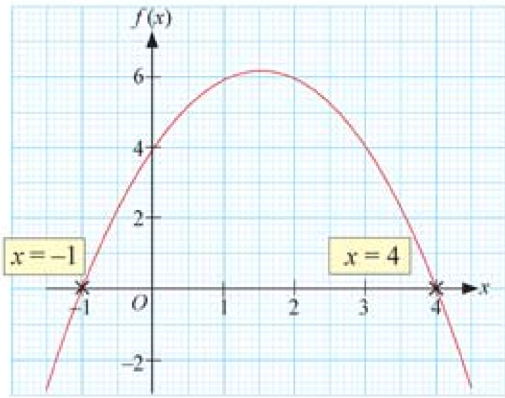
Solution:

(a) $2x^2 + 5x - 12 = 0$



The roots are –4 and 1.5.

(b) $-x^2 + 3x + 4 = 0$



The roots are –1 and 4.

Example 6

Determine whether each of the following values is a root of the given quadratic equation.

(a) $2x^2 - 7x + 3 = 0$; $x = 1$, $x = 3$

(b) $3x^2 - 7x + 5 = 3$; $x = 1$, $x = \frac{1}{3}$

Solution:

(a) $2x^2 - 7x + 3 = 0$

When $x = 1$,

Left:

$$2x^2 - 7x + 3 = 2(1)^2 - 7(1) + 3$$
$$= 2 - 7 + 3$$
$$= -2$$

Right: 0

-2 and 0: not the same

Thus, $x = 1$ is not a root of the equation $2x^2 - 7x + 3 = 0$.

When $x = 3$,

Left:

$$2x^2 - 7x + 3 = 2(3)^2 - 7(3) + 3$$
$$= 18 - 21 + 3$$
$$= 0$$

Right: 0

0 and 0: same

Thus, $x = 3$ is a root of the equation $2x^2 - 7x + 3 = 0$.

(b) $3x^2 - 7x + 5 = 3$

When $x = 1$,

Left:

$$3x^2 - 7x + 5 = 3(1)^2 - 7(1) + 5$$
$$= 3 - 7 + 5$$
$$= 1$$

Right: 3

1 and 3: not the same

Thus, $x = 1$ is not a root of the equation $3x^2 - 7x + 5 = 3$.

When $x = \frac{1}{3}$,

Left:

$$3x^2 - 7x + 5 = 3\left(\frac{1}{3}\right)^2 - 7\left(\frac{1}{3}\right) + 5$$
$$= \frac{1}{3} - \frac{7}{3} + 5$$
$$= 3$$

Right: 3

3 and 3: same

Thus, $x = \frac{1}{3}$ is a root of the equation $3x^2 - 7x + 5 = 3$.

MY MEMORY

The roots of a quadratic equation are the values of x that satisfy the equation.

Checking Answer

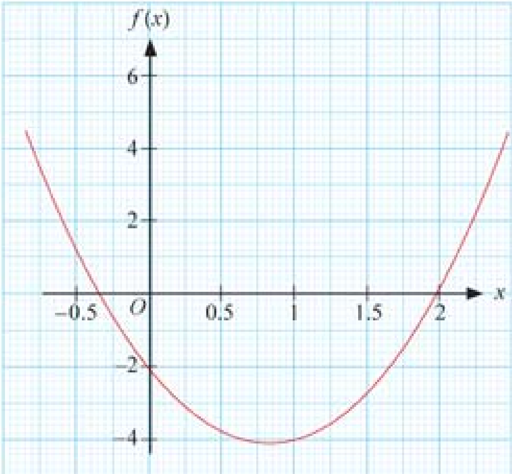
1. Press 2, Alpha, X, x^2, −, 7, Alpha, X, +, 3
2. Press CALC
   Display: x? 0.
3. Press 1 =
   Display: $2x^2 - 7x + 3$ −2.
4. Press CALC
   Display: x? 1.
5. Press 3 =
   Display: $2x^2 - 7x + 3$ 0.

CHAPTER 1

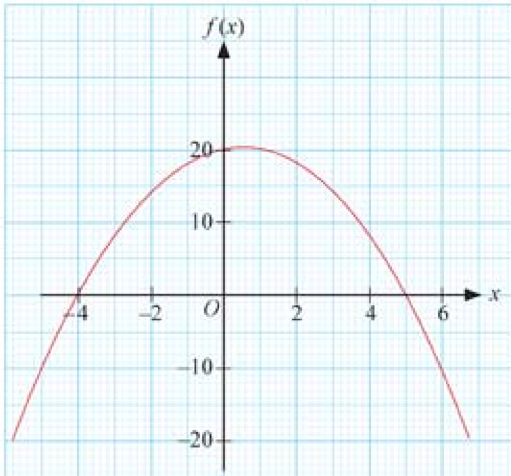
Self Practice 1.1e

1. For each graph of quadratic function below, state the roots of the given quadratic equation.

(a) $3x^2 - 5x - 2 = 0$



(b) $-x^2 + x + 20 = 0$



2. Determine whether each of the following values is a root of the given quadratic equation.

(a) $x^2 - 5x + 6 = 0; x = 3, x = 2$

(b) $2x^2 - x - 1 = 0; x = 1, x = \frac{1}{2}$

(c) $3x^2 - 5x - 2 = 0; x = -\frac{1}{3}, x = -2$

(d) $3x^2 + 4x + 2 = 6; x = 2, x = \frac{2}{3}$

3. Determine whether each of the following values is a root of the given quadratic equation.

(a) $(x - 1)(x + 4) = 0; x = -4, x = 2, x = 1$

(b) $2(x - 3)(x - 5) = 0; x = -3, x = 3, x = 5$

(c) $3(2 + x)(x - 4) = 0; x = -2, x = 2, x = 4$

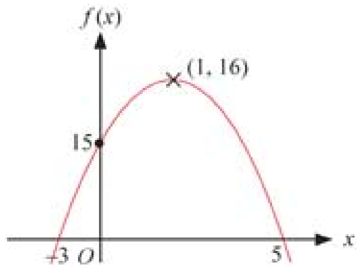
4. For the graph of quadratic function on the right, determine whether the given value of x is a root of the quadratic equation $f(x) = 0$.

(a) $x = 1$

(b) $x = -3$

(c) $x = 15$

(d) $x = 5$



How do you determine the roots of a quadratic equation by factorisation method?

Learning Standard

Determine the roots of a quadratic equation by factorisation method.

Factorisation method is one of the methods used to determine the roots of a quadratic equation.

A quadratic equation needs to be written in the form of $ax^2 + bx + c = 0$ before we carry out factorisation.

MY MEMORY

$2x^2 + 5x - 3$
$= (2x - 1)(x + 3)$

INFO ZONE

A quadratic equation can also be solved by using:
- method of completing the squares.
- formula
$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

Example 7

Determine the roots of the following quadratic equations by factorisation method.

(a) $x^2 - 5x + 6 = 0$

(b) $x^2 + \frac{7}{2}x = 2$

(c) $\frac{x}{2} = \frac{5x - 24}{x - 4}$

(d) $(y + 2)(y + 1) = 2(y + 11)$

Solution:

(a)
$$x^2 - 5x + 6 = 0$$
$$(x - 3)(x - 2) = 0$$
$$x = 3 \text{ or } x = 2$$

(b)
$$x^2 + \frac{7}{2}x = 2$$
$$2x^2 + 7x = 4$$
$$2x^2 + 7x - 4 = 0$$
$$(2x - 1)(x + 4) = 0$$
$$x = \frac{1}{2} \text{ or } x = -4$$

(c)
$$\frac{x}{2} = \frac{5x - 24}{x - 4}$$
$$x(x - 4) = 2(5x - 24)$$
$$x^2 - 4x = 10x - 48$$
$$x^2 - 14x + 48 = 0$$
$$(x - 8)(x - 6) = 0$$
$$x = 8 \text{ or } x = 6$$

(d)
$$(y + 2)(y + 1) = 2(y + 11)$$
$$y^2 + 3y + 2 = 2y + 22$$
$$y^2 + y - 20 = 0$$
$$(y + 5)(y - 4) = 0$$
$$y = -5 \text{ or } y = 4$$

Checking Answer

Steps to solve $x^2 - 5x + 6 = 0$.

1. Press [mode] 3 times until the following display is shown.
   [EQN] [MAT] [VCT]
   [1] [2] [3]
2. Press [1] to choose [EQN], which is equation.
3. Display shows [unknowns?] [2] [3] press [▷]
4. Display shows [Degree?] [2] [3] press [2], for power of 2
5. Display shows a? Enter the value 1, then press [=]
6. Display shows b? Enter the value –5, then press [=]
7. Display shows c? Enter the value 6, then press [=]
8. $x_1 = 3$ is displayed, press [=]
9. $x_2 = 2$ is displayed.

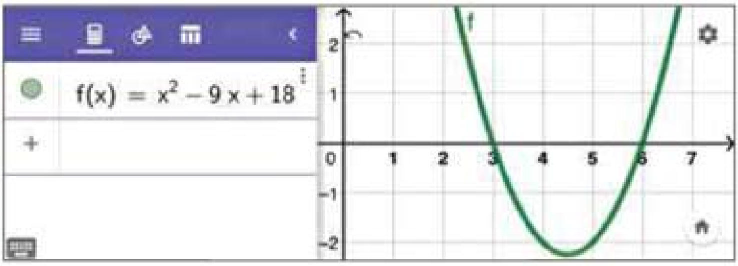
How do you determine the roots of a quadratic equation by the graphical method?

Mind Stimulation 12

Aim: To determine the roots of a quadratic equation on the graph of a quadratic function using the dynamic geometry software.

Steps:

1. Insert the quadratic equations in the dynamic geometry software.



Scan the QR Code to carry out this activity.
https://www.geogebra.org/graphing

2. Determine the roots of the quadratic equations and complete the following table.

Activity Sheet:

	Quadratic Equation	Roots
(a)	$x^2 - 9x + 18 = 0$	$x = 3, x = 6$
(b)	$4x^2 + 4x - 3 = 0$	
(c)	$-x^2 + 9x - 20 = 0$	
(d)	$-4x^2 - 11x + 3 = 0$	

TIPS

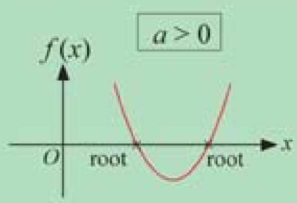
The root of a quadratic equation $ax^2 + bx + c = 0$ is the value of x which satisfies the quadratic equation.

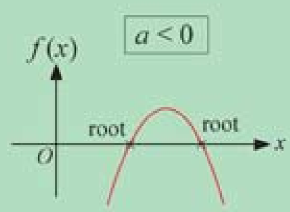
Discussion:

How do you determine the roots of a quadratic equation using the graphical method?

From the activity in Mind Stimulation 12, it is found that:

The roots of a quadratic equation $ax^2 + bx + c = 0$ can be obtained using the graphical method by reading the values of x which are the points of intersection of the graph of the quadratic function $f(x) = ax^2 + bx + c$ and the x-axis.





Self Practice 1.1f

1. Determine the roots of each of the following quadratic equations using the factorisation method.
   (a) $x^2 - 3x - 10 = 0$ (b) $x^2 - 10x + 16 = 0$ (c) $3x^2 - 5x + 2 = 0$
   (d) $2x^2 + 8x - 24 = 0$ (e) $2x^2 + 3x - 9 = 0$ (f) $4x^2 - 3x - 10 = 0$
   (g) $-3x^2 - x + 14 = 0$ (h) $x^2 - 5x = 0$ (i) $x^2 - 4 = 0$

2. Write each of the following quadratic equations in the general form. Hence, solve the quadratic equation.
   (a) $m(m + 2) = 3$ (b) $3p(11 - 2p) = 15$ (c) $\frac{1}{2}y^2 = 12 - y$
   (d) $a + \frac{5}{a} = 6$ (e) $\frac{8}{k} = 2 + k$ (f) $2h + \frac{6}{h} = 7$
   (g) $(h - 2)(h - 1) = 12$ (h) $(2x - 1)^2 = 3x - 2$ (i) $(r + 1)(r + 9) = 16r$

How do you sketch the graphs of quadratic functions?

Learning Standard

Sketch graphs of quadratic functions.

When sketching the graph of a quadratic function, the following characteristics should be shown on the graph.

1. The correct shape of the graph.
2. y-intercept.
3. x-intercept or one point that passes through the graph.

Case 1

The graph of a quadratic function intersects the x-axis.

MY MEMORY

$f(x) = x^2 - 4x + 3$
$a = 1, b = -4, c = 3$

Example 8

Sketch the following graphs of quadratic functions.

(a) $f(x) = x^2 - 4x + 3$
(b) $f(x) = x^2 - 6x + 9$
(c) $f(x) = -x^2 + 2x + 15$
(d) $f(x) = -2x^2 + 18$

MY MEMORY

The constant c of a quadratic function is the y-intercept of the graph of the quadratic function.

Solution:

(a) $f(x) = x^2 - 4x + 3$

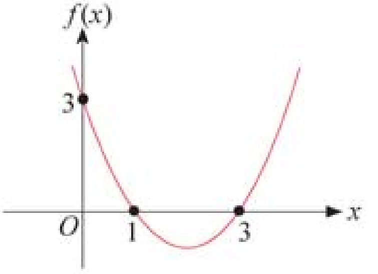
Value of $a = 1 > 0$, shape $\cup$

Value of $c = 3$, y-intercept $= 3$

When $f(x) = 0$, $x^2 - 4x + 3 = 0$

$(x - 3)(x - 1) = 0$

$x = 1$ or $x = 3$



(b) $f(x) = x^2 - 6x + 9$

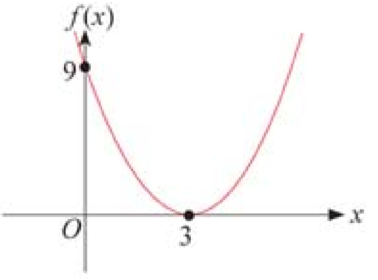
Value of $a = 1 > 0$, shape $\cup$

Value of $c = 9$, y-intercept $= 9$

When $f(x) = 0$, $x^2 - 6x + 9 = 0$

$(x - 3)(x - 3) = 0$

$x = 3$



(c) $f(x) = -x^2 + 2x + 15$

Value of $a = -1 < 0$, shape $\cap$

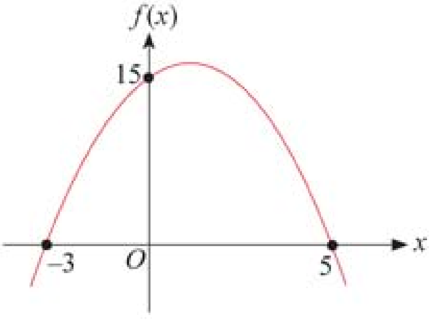
Value of $c = 15$, y-intercept $= 15$

When $f(x) = 0$, $-x^2 + 2x + 15 = 0$

$x^2 - 2x - 15 = 0$

$(x - 5)(x + 3) = 0$

$x = -3$ or $x = 5$



(d) $f(x) = -2x^2 + 18$

Value of $a = -2 < 0$, shape $\cap$

Value of $b = 0$, axis of symmetry is the y-axis

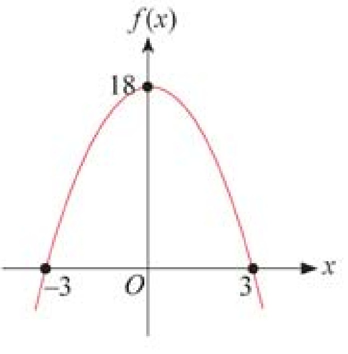
Value of $c = 18$, y-intercept $= 18$

When $f(x) = 0$, $-2x^2 + 18 = 0$

$x^2 - 9 = 0$

$(x + 3)(x - 3) = 0$

$x = -3$ or $x = 3$



Case 2

The graph of a quadratic function does not intersect the x-axis.

> **MY MEMORY**
> (a) $f(x) = x^2 + 1$
> $a = 1, b = 0, c = 1$
> (b) $f(x) = -x^2 - 3$
> $a = -1, b = 0, c = -3$

Example 9

Sketch each of the following graphs of quadratic functions.

(a) $f(x) = x^2 + 1$

(b) $f(x) = -x^2 - 3$

> **MY MEMORY**
> If $b = 0$ for a quadratic function, then the y-axis is the axis of symmetry of the graph of the quadratic function.

Solution:

(a) $f(x) = x^2 + 1$

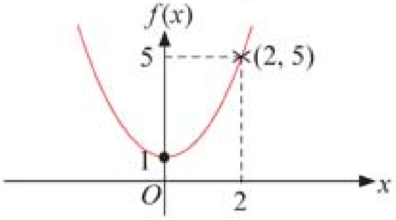
Value of $a = 1 > 0$, shape ∪

Value of $b = 0$, axis of symmetry is the y-axis

Value of $c = 1$, y-intercept is 1

thus the minimum point is $(0, 1)$

When $x = 2$, $f(2) = 2^2 + 1$
$= 5$



(b) $f(x) = -x^2 - 3$

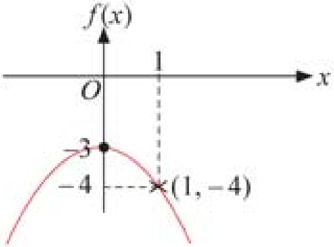
Value of $a = -1 < 0$, shape ∩

Value of $b = 0$, axis of symmetry is the y-axis

Value of $c = -3$, y-intercept is -3

thus the maximum point is $(0, -3)$

When $x = 1$, $f(1) = -(1)^2 - 3$
$= -4$



Self Practice 1.1g

1. Sketch each of the following graphs of quadratic functions.
   (a) $f(x) = 2x^2 + 2x - 24$
   (b) $f(x) = x^2 - 8x + 16$
   (c) $f(x) = -2x^2 + 2x + 40$
   (d) $f(x) = -2x^2 + 8$

2. Sketch each of the following graphs of quadratic functions.
   (a) $f(x) = x^2 + 5$
   (b) $f(x) = 2x^2 + 1$
   (c) $f(x) = -x^2 + 2$

How do you solve problems involving quadratic equations?

Learning Standard

Solve problems involving quadratic equations.

Example 10

Joseph wants to make the framework of a box in the shape of a cuboid using wooden rods. The price of the wooden rod is RM5 per metre. The base of the cuboid is a square. The height of the cuboid is 30 cm more than the length of its base. The total surface area of the box is 4 800 cm^2. Joseph's budget to build the frame of a box is RM15. Determine whether Joseph has enough budget.

Solution:

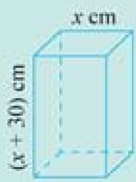
Understanding the problem

Length of the base = x cm

Height of the cuboid = $(x + 30)$ cm

Total surface area = 4 800 cm^2

Budget = RM15 for a box



Planning a strategy

- Determine the expression for the surface area of the cuboid.
- Form a quadratic equation.
- Solve the quadratic equation.
- Determine the measurements of the box and the actual cost.

Implementing the strategy

$$\begin{aligned}\text{Total surface area} &= 2(x)(x) + 4(x)(x + 30)\\ &= 2x^2 + 4x^2 + 120x\\ &= 6x^2 + 120x\end{aligned}$$

$$\begin{aligned}6x^2 + 120x &= 4\,800\\ 6x^2 + 120x - 4\,800 &= 0\\ x^2 + 20x - 800 &= 0\\ (x + 40)(x - 20) &= 0\\ x = -40 \text{ or } x &= 20\end{aligned}$$

$x = -40$ is not acceptable, thus $x = 20$ cm

The measurements of the box are 20 cm × 20 cm × 50 cm.

$$\begin{aligned}\text{Total length of the edges of the box} &= 8 \times 20 \text{ cm} + 4 \times 50 \text{ cm}\\ &= 160 \text{ cm} + 200 \text{ cm}\\ &= 360 \text{ cm}\\ &= 3.6 \text{ m}\end{aligned}$$

$$\begin{aligned}\text{Actual cost} &= 3.6 \times \text{RM}5\\ &= \text{RM}18\end{aligned}$$

The actual cost for a box is RM18.

INFO ZONE

The measurement of the length cannot be negative.

Checking Answer

When $x = 20$

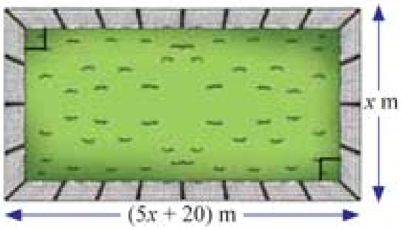
$$\begin{aligned}\text{Area} &= 6(20)^2 + 120(20)\\ &= 2\,400 + 2\,400\\ &= 4\,800\end{aligned}$$

Conclusion

Joseph does not have enough budget to build the framework of the box.

Self Practice 1.1h

1. A rectangular field needs to be fenced up using mesh wire. The length of the field is $(5x + 20)$ m and its width is x m.



   (a) Express the area of the field, A m^2, in terms of x.
   (b) Given the area of the field is 5 100 m^2, calculate the cost of fencing the field if the cost of the mesh wire used is RM20 per metre.

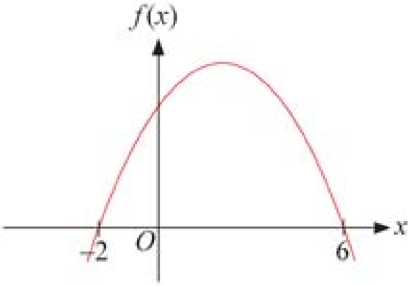
2. Encik Kamarul drove his car at an average speed of $(20t - 20)$ km h^{-1} for $(t - 3)$ hours along a highway. The distance travelled by Encik Kamarul was 225 km. The highway speed limit is 110 km h^{-1}. Did Encik Kamarul follow the highway speed limit?

Comprehensive Practice

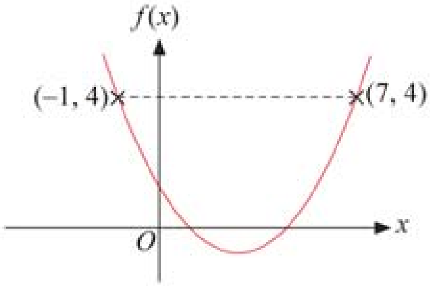
1. Determine whether each of the following expressions is a quadratic expression in one variable.
   (a) $p^2 - 4p + 1$ (b) $\frac{1}{2}y^2 - 4y + 9$ (c) $\frac{1}{3} - 2b + a^2$
   (d) $-m + 1$ (e) $b^2 + 2$ (f) $\frac{a^2 + 2a + 1}{3}$

2. State the equation of the axis of symmetry for each graph of quadratic function below.

(a)



(b)



3. Solve each of the following quadratic equations.
   (a) $4x^2 - 1 = 0$ (b) $x^2 - 81 = 0$ (c) $y^2 - 4y = 0$
   (d) $x^2 + 3x + 2 = 0$ (e) $2x^2 - x - 10 = 0$ (f) $(x - 2)^2 = 16$
   (g) $m^2 + 3m - 4 = 0$ (h) $2p^2 - 13p + 20 = 0$ (i) $(k - 4)(k - 1) = 18$
   (j) $\frac{h - 1}{3} = \frac{1}{h + 1}$ (k) $2(x - 2)^2 = 5x - 7$

4. Given one of the roots of the quadratic equation $x^2 + px - 18 = 0$ is 2, calculate the value of p.

5. Show that the quadratic equation $(m - 6)^2 = 12 - 2m$ can be written as $m^2 - 10m + 24 = 0$. Hence, solve the equation $(m - 6)^2 = 12 - 2m$.

6. Determine the coordinates of the minimum point from the graph of the quadratic function $f(x) = x^2 - 6x + 5$.

7. Given $x = 4$ is the axis of symmetry of the graph of the quadratic function $f(x) = 7 + 8x - x^2$, determine the coordinates of the maximum point.

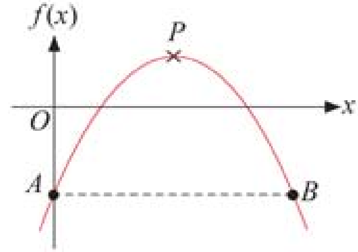
8. The diagram shows part of the graph of the quadratic function $f(x) = -x^2 + 6x - 5$. The straight line AB is parallel to the x-axis. Determine

(a) the coordinates of point A,

(b) the equation of the axis of symmetry,

(c) the coordinates of point B,

(d) the coordinates of the maximum point P.



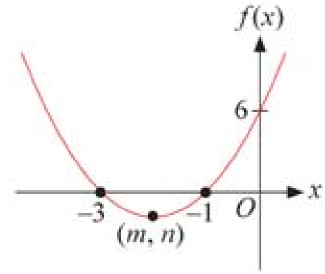
9. The diagram shows the graph of the quadratic function $f(x) = ax^2 + 8x + c$. Calculate the value of each of the following.

(a) c

(b) m

(c) a

(d) n



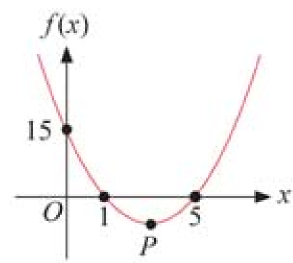
10. The diagram shows part of the graph of the quadratic function $f(x) = a(x - h)(x - k)$ where $h < k$. Point P is the minimum point of the graph of the quadratic function.

(a) Calculate the value of

(i) h, (ii) k, (iii) a.

(b) Determine the equation of the axis of symmetry.

(c) State the coordinates of point P.



11. The length of a rectangle is $(x + 1)$ cm and its width is 5 cm less than its length.

(a) Express the area of the rectangle, A cm^2, in terms of x.

(b) The area of the rectangle is 24 cm^2. Calculate the length and width of the rectangle.

12. Diagram 1 shows an isosceles triangle with a base of $4y$ cm and a height of $(y + 5)$ cm. Diagram 2 shows a square with sides of y cm.
The area of the triangle is more than the area of the square by 39 cm^2. Calculate the difference in perimeter between both shapes.

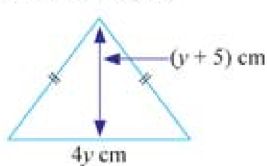


Diagram 1

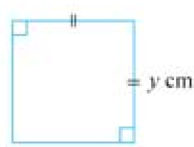


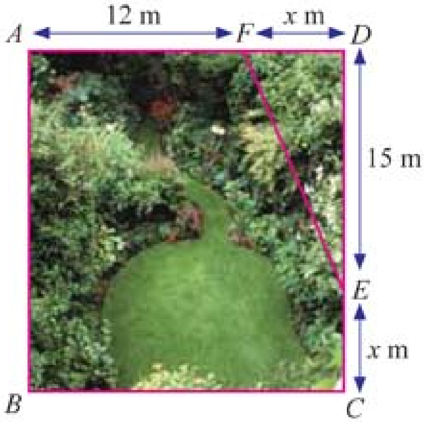
Diagram 2

13. The diagram shows a rectangular garden $ABCD$. E and F are two points on CD and AD respectively such that $CE = DF = x$ m. The lengths of $AF = 12$ m and $DE = 15$ m.

(a) Form an expression for the area of the rectangle, A m^2, in terms of x.

(b) The area of the rectangle is 460 m^2. Calculate the value of x.

(c) Aiman wants to build a small footpath from point E to point F with tiles which costs RM50 per metre. Aiman has a budget of RM1 000, determine whether Aiman has enough budget to build the footpath.

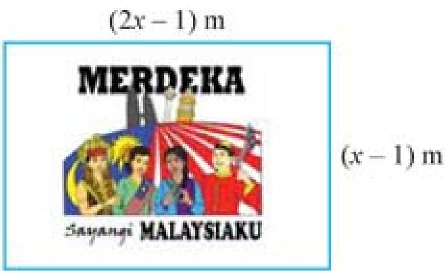


14. The History Club of SMK Seri Jaya has drawn two rectangular murals in conjunction with Malaysia's Independence Day.

(a) Express the difference in area between the two murals, A m^2, in terms of x.

(b) The difference in area between the two murals is 10 m^2. Calculate the value of x.

(c) Calculate the perimeter of the smaller mural.

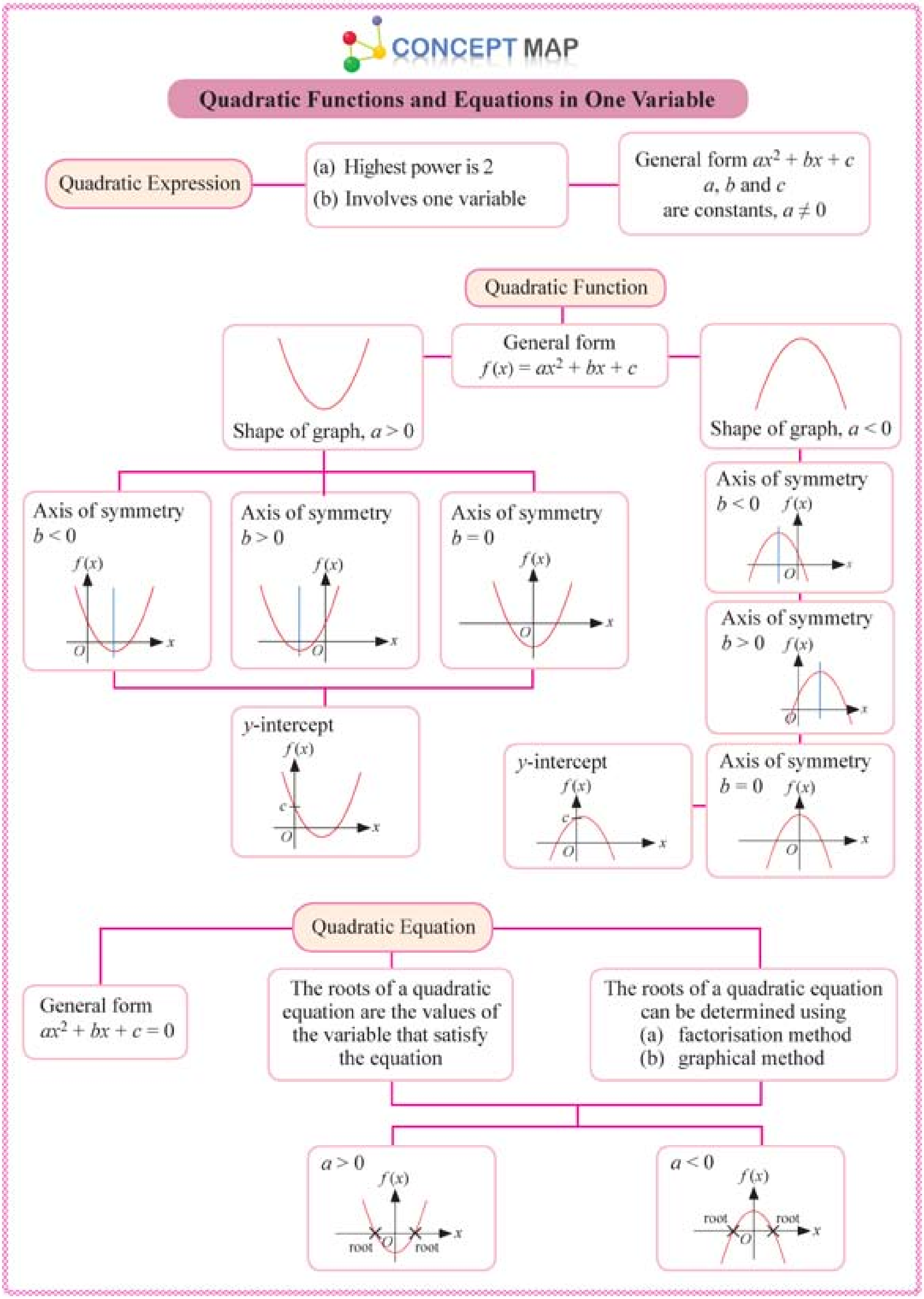


PROJECT

Use your creativity to build different shapes based on the examples below. Display your work at the Mathematics Corner.

Materials:

1. Graph paper/blank paper.
2. Protractor, a pair of compasses.
3. Coloured pens.


CONCEPT MAP
Quadratic Functions and Equations in One Variable
Quadratic Expression
(a) Highest power is 2
(b) Involves one variable
General form $ax^2 + bx + c$
a, b and c
are constants, $a \neq 0$
Quadratic Function
General form
$f(x) = ax^2 + bx + c$
Shape of graph, $a > 0$
Shape of graph, $a < 0$
Axis of symmetry
$b < 0$
Axis of symmetry
$b > 0$
Axis of symmetry
$b = 0$
y-intercept
Axis of symmetry
$b < 0$
Axis of symmetry
$b > 0$
Axis of symmetry
$b = 0$
y-intercept
Quadratic Equation
General form
$ax^2 + bx + c = 0$
The roots of a quadratic equation are the values of the variable that satisfy the equation
The roots of a quadratic equation can be determined using
(a) factorisation method
(b) graphical method
$a > 0$
root
root
$a < 0$
root
root

Self Reflection

Across

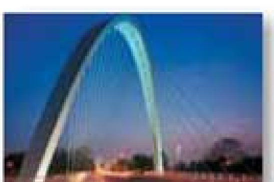
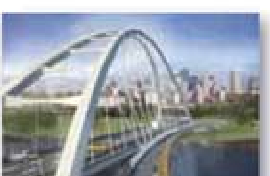
2. The shape of the graph of a quadratic function.
3. The highest point of the graph of a quadratic function.
4. The lowest point of the graph of a quadratic function.
5. A function which its highest power is two.

Down

1. The vertical axis that passes through the maximum or minimum point of the graph of a quadratic function.
2. A method used to determine the roots of a quadratic equation.
6. The values of variable that satisfy a quadratic equation.

Mathematics Exploration

The shape of the graph of a quadratic function is one of the most common shapes found in our daily life. Observe the following photos.

Use your creativity to draw a quadratic structure.