CHAPTER 13 The Pythagoras' Theorem

What will you learn?

- The Pythagoras' Theorem
- The Converse of Pythagoras' Theorem

Why study this chapter?

As the fundamental knowledge for solving problems involving right-angled triangles. Discuss the fields that involve solving problems related to right-angled triangles.

Right angles exist in many objects around us. In the construction of buildings, how does a civil engineer ensure that the corners of the walls of the building are built at right angles?

Walking through Time

Pythagoras

Pythagoras (569 B.C. – 475 B.C.) was a mathematician and also a philosopher who had contributed substantially to the development of mathematics today. He was the first person who proved the Pythagoras' theorem.

For more information:

http://goo.gl/r4JZ

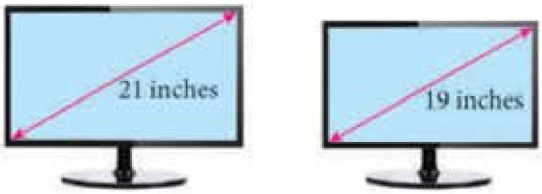
Word Link

• converse of Pythagoras' theorem	• *akas teorem Pythagoras*
• hypotenuse	• *hipotenus*
• Pythagoras' theorem	• *teorem Pythagoras*

Open the folder downloaded from page vii for the audio of Word Link.

13.1 The Pythagoras' Theorem

What is a hypotenuse?



LEARNING STANDARDS

Identify and define the hypotenuse of a right-angled triangle.

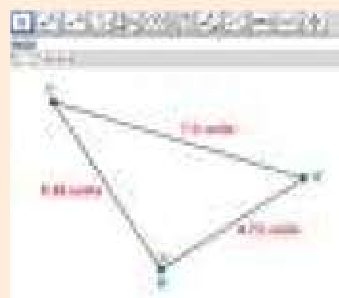
We are often told about the size of the monitor screen of a computer as 19 inches, 21 inches or 24 inches and so forth. The size is measured according to the length of the monitor diagonally. What is the relationship between the size of the monitor screen and the length and width of the monitor screen?

Exploration Activity 1

Group

Aim: To identify the hypotenuse of a right-angled triangle.

Instruction:
- Perform the activity in groups of four.
- Open the folder downloaded from page vii.

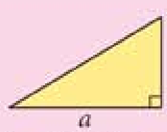


1. Open the file *Hypotenuse.ggb* using *GeoGebra*. The screen displayed shows a right-angled triangle with the length of each side.
2. Identify and record the longest side.
3. Click and drag points *A*, *B* or *C* to change the shape of the triangle and repeat the exploration in Step **2**.
4. Discuss your findings with your friends.

From the results of Exploration Activity 1, it is found that the longest side of a right-angled triangle is always the side opposite to the right angle.

The longest side opposite to the right angle is known as the **hypotenuse** of the right-angled triangle.

Let's Discuss

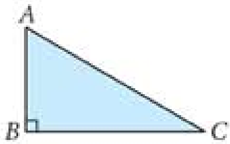


Discuss with your friends and explain why the side labelled as *a* is not the hypotenuse.

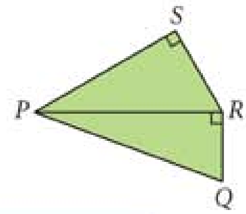
Example 1

For each of the following, identify the hypotenuse.

(a)



(b)



Solution

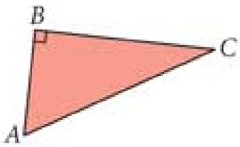
(a) AC is the hypotenuse. ← The side opposite to the right angle.

(b) PR is the hypotenuse of triangle PSR.
PQ is the hypotenuse of triangle PRQ.

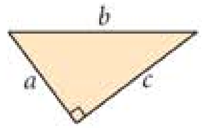
Self Practice 13.1a

1. For each of the following, identify the hypotenuse.

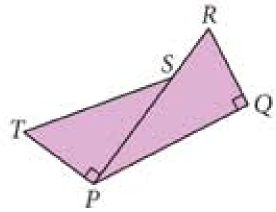
(a)



(b)



(c)



What is the relationship between the sides of a right-angled triangle?

LEARNING STANDARDS

Determine the relationship between the sides of a right-angled triangle. Hence, explain the Pythagoras' theorem by referring to the relationship.

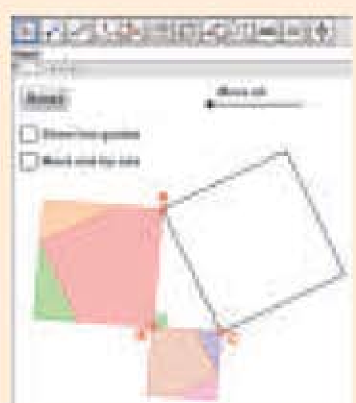
Exploration Activity 2

Flip Classroom

Aim: To explore and explain the Pythagoras' theorem.

Instruction:
- Explore by yourself before the lesson begins and discuss in groups of four during the lesson.
- Open the folder downloaded from page vii.

1. Open the file *Pythagoras.ggb* using *GeoGebra*. The screen displayed shows a right-angled triangle ABC with a square drawn on each side of the triangle.
2. Click and drag the coloured shapes in the square on sides AB and BC, and move them into the square on side AC. Do all the coloured shapes fill up the square on side AC perfectly?
3. Click and drag the slider 'Move all' or click at the checkboxes 'Show line guides' and 'Move one by one' for help.



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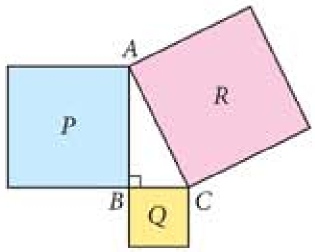
4. Click and drag points A, B and C to change the shape of the right-angled triangle and repeat your exploration.
5. Discuss your findings with your friends.
6. Consider the area of a square, state the relationship between sides AB, BC and AC.

From the results of Exploration Activity 2, it is found that the area of the square on the hypotenuse is equal to the total area of the squares on the other two sides.



Area of R = Area of P + Area of Q

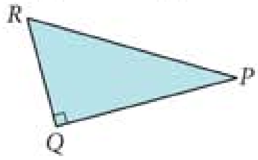
$$AC^2 = AB^2 + BC^2$$

This relationship is known as the **Pythagoras' theorem**.

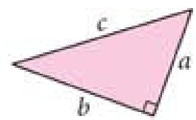
Example 2

For each of the following, state the relationship between the lengths of sides of the given right-angled triangle.

(a)



(b)



Solution

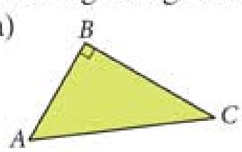
(a) $PR^2 = QR^2 + PQ^2$

(b) $c^2 = a^2 + b^2$

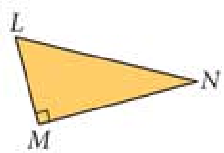
Self Practice 13.1b

1. For each of the following, state the relationship between the lengths of sides of the given right-angled triangle.

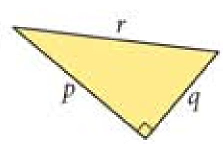
(a)



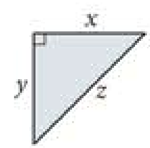
(b)



(c)



(d)



How do you determine the length of the unknown side of a right-angled triangle?

Pythagoras' theorem can be used to determine the length of an unknown side in a right-angled triangle if the lengths of two other sides are given.

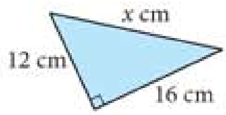
LEARNING STANDARDS

Determine the lengths of the unknown side of
(i) a right-angled triangle.
(ii) combined geometric shapes.

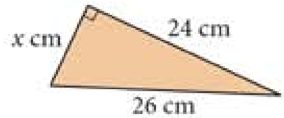
Example 3

For each of the following, calculate the value of x.

(a)



(b)



Solution

(a)
$$\begin{aligned} x^2 &= 12^2 + 16^2 \\ &= 144 + 256 \\ &= 400 \\ x &= \sqrt{400} \\ &= 20 \end{aligned}$$

(b)
$$\begin{aligned} 26^2 &= x^2 + 24^2 \\ x^2 &= 26^2 - 24^2 \\ &= 676 - 576 \\ &= 100 \\ x &= \sqrt{100} \\ &= 10 \end{aligned}$$

Smart

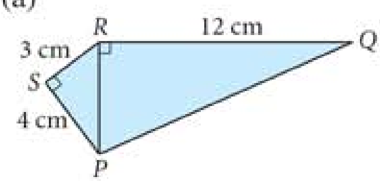
The length of the hypotenuse can be calculated using the Pol function. For instance, Example 3(a),

press $\boxed{\text{Pol(}}$ $\boxed{1}$ $\boxed{2}$ $\boxed{,}$ $\boxed{1}$ $\boxed{6}$ $\boxed{)}$ $\boxed{=}$

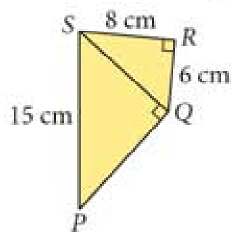
Example 4

Calculate the length of PQ in each of the following diagrams.

(a)



(b)



Did You Know

Three numbers a, b and c that satisfy $c^2 = a^2 + b^2$ are known as a Pythagorean triples (or triplets). For example, (3, 4, 5), (5, 12, 13), (7, 24, 25) and so forth.

Solution

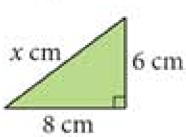
(a)
$$\begin{aligned} PR^2 &= 3^2 + 4^2 \\ &= 9 + 16 \\ &= 25 \\ PQ^2 &= PR^2 + RQ^2 \\ &= 25 + 12^2 \\ &= 169 \\ PQ &= \sqrt{169} \\ &= 13 \text{ cm} \end{aligned}$$

(b)
$$\begin{aligned} QS^2 &= 6^2 + 8^2 \\ &= 36 + 64 \\ &= 100 \\ PS^2 &= QS^2 + PQ^2 \\ PQ^2 &= PS^2 - QS^2 \\ &= 15^2 - 100 \\ &= 125 \\ PQ &= \sqrt{125} \\ &= 11.18 \text{ cm (2 decimal places)} \end{aligned}$$

Self Practice 13.1c

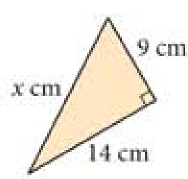
1. For each of the following, calculate the value of x. Give your answer correct to two decimal places if necessary.

(a)



(b)

(c)



(d)

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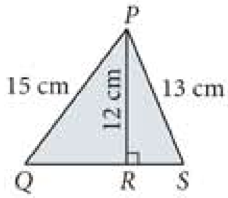
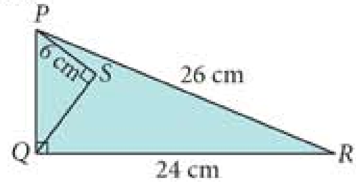
2. Calculate the length of QS in each of the following diagrams. Give your answer correct to two decimal places if necessary.

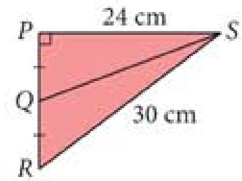
(a)



(b)



(c)



How do you solve problems?

LEARNING STANDARDS

Solve problems involving the Pythagoras' theorem.

MATHEMATICS APPLICATION ZONE

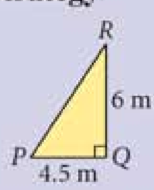
A fireman climbs up a ladder to save a child who is trapped on the third floor as shown in the diagram. The third floor is 6 m high from the horizontal ground. The base of the ladder is 4.5 m away from the wall of the building. How long is the ladder?

Solution

Understanding the problem
- Distance of the third floor from the horizontal ground = 6 m
- Distance of the base of the ladder from the building = 4.5 m
- Find the length of the ladder.

Devising a plan
- Draw a right-angled triangle PQR to represent the given information.
- Use Pythagoras' theorem.

Implementing the strategy



$$PR^2 = PQ^2 + QR^2$$
$$= 4.5^2 + 6^2$$
$$= 20.25 + 36$$
$$= 56.25$$
$$PR = 7.5 \text{ m}$$

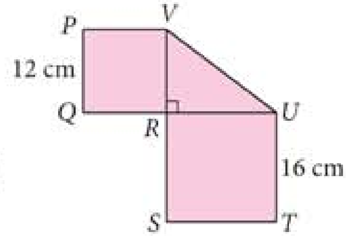
Thus, the length of the ladder is 7.5 m.

Doing reflection

$$7.5^2 = 56.25$$
$$4.5^2 + 6^2 = 56.25$$

Example 5

In the diagram, $PVRQ$ and $RUTS$ are squares. Calculate the perimeter of the whole diagram.



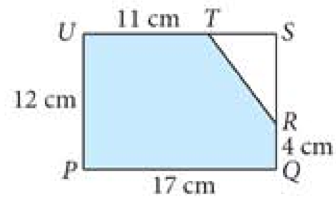
Solution

$$\begin{aligned} VU^2 &= VR^2 + RU^2 \\ &= 12^2 + 16^2 \\ &= 400 \\ VU &= \sqrt{400} \\ &= 20 \text{ cm} \end{aligned}$$

Perimeter of the whole diagram
$= 20 + 16 + 16 + 16 + 12 + 12 + 12$
$= 104$ cm

Self Practice 13.1d

1. In the diagram, $PQSU$ is a rectangle. Calculate the perimeter of the shaded region.

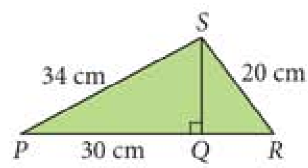


2. Ship A is at 34 km north of ship B. Ship C is at 10 km west of ship A. Calculate the distance between ship B and ship C, correct to two decimal places.

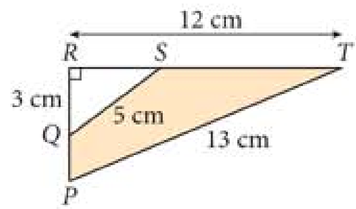
Mastery Q 13.1

Open the folder downloaded from page vii for extra questions of Mastery Q 13.1.

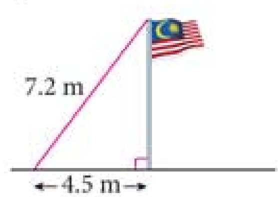
1. In the diagram, PQR is a straight line. Calculate the length of QR.



2. In the diagram, PQR and RST are straight lines. Calculate the perimeter of the shaded region.



3. One end of a rope of length 7.2 m is tied to the tip of a flag pole. The other end of the rope is tied to a spot on the horizontal ground 4.5 m away from the base of the flag pole. Calculate the height of the flag pole and give your answer correct to two decimal places.



4. A ship departs from point O and sails towards southwest for a distance of 300 km and then towards northwest for a distance of 450 km. Calculate the final distance of the ship from point O and give your answer correct to two decimal places.

13.2 The Converse of Pythagoras' Theorem

LEARNING STANDARDS

Determine whether a triangle is a right-angled triangle and give justification based on the converse of the Pythagoras' theorem.

How do you determine whether a triangle is a right-angled triangle?

Exploration Activity 3

Aim: To explore the converse of Pythagoras' theorem.

Instruction:
- Explore by yourself before the lesson begins and discuss in groups of four during the lesson.
- Open the folder downloaded from page vii.

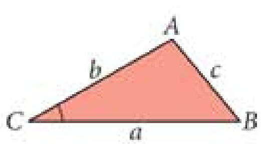
1. Open the file *Converse of Pythagoras.ggb* using *GeoGebra*. The screen displayed shows a triangle ABC with angles at vertex B and vertex C.
2. Click and drag point A towards left or right and observe the change in the information displayed in red colour. Copy and record your observations in the following table for a few sets of values. Click and drag point B or C to change the shape of the triangle if necessary.

Comparative value (red)	Size of angle (red)
$AB^2 > AC^2 + BC^2$	
$AB^2 < AC^2 + BC^2$	
$AB^2 = AC^2 + BC^2$	

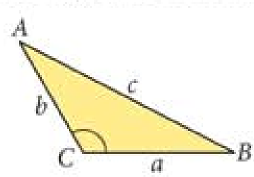
3. Repeat Step **2** for the information displayed in blue colour.
4. Discuss your findings with your friends.
5. What are the conclusions that can be made?

Flashback
- An acute angle is an angle less than 90°.
- An obtuse angle is an angle more than 90° but less than 180°.

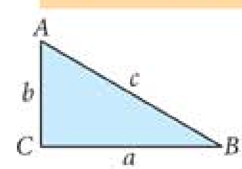
From the results of Exploration Activity 3, it is found that



If $c^2 < a^2 + b^2$, then the angle opposite to side c is an **acute angle**.



If $c^2 > a^2 + b^2$, then the angle opposite to side c is an **obtuse angle**.



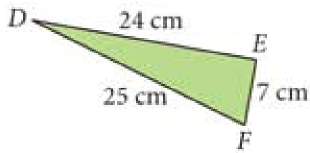
If $c^2 = a^2 + b^2$, then the angle opposite to side c is a **right angle**.

The **converse of Pythagoras' theorem** states that:
If $\boldsymbol{c^2 = a^2 + b^2}$, then the angle opposite to side c is a **right angle**.

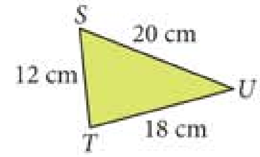
Example 6

Determine whether each of the following triangles is a right-angled triangle.

(a)



(b)



Solution

(a) The longest side = 25 cm

Thus, $25^2 = 625$

$24^2 + 7^2 = 576 + 49$

$= 625$

Hence, DEF is a right-angled triangle.

(b) The longest side = 20 cm

Thus, $20^2 = 400$

$18^2 + 12^2 = 324 + 144$

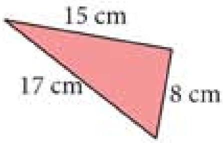
$= 468$

Hence, STU is not a right-angled triangle.

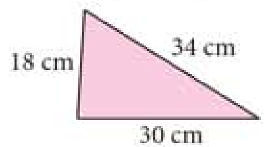
Self Practice 13.2a

1. Determine whether each of the following triangles is a right-angled triangle.

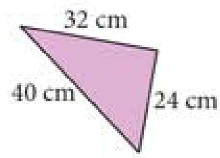
(a)



(b)



(c)



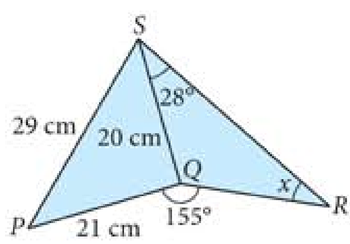
How do you solve problems?

LEARNING STANDARDS

Solve problems involving the converse of the Pythagoras' theorem.

Example 7

In the diagram, calculate the value of x.



Solution

20 cm, 21 cm and 29 cm satisfy $c^2 = a^2 + b^2$.

Thus, $\angle PQS = 90°$

$\angle SQR = 360° - 90° - 155°$

$= 115°$

$x = 180° - 115° - 28°$

$= 37°$

Career in Mathematics

A housing contractor and a civil engineer will use the Pythagoras' theorem to solve problems involving right angles in the construction of buildings.

CHAPTER 13

Example 8

Sheila is given three straws to form a frame in the shape of a right-angled triangle. The straws are 15 cm, 20 cm and 25 cm long respectively. Will she be able to form the frame in the shape of a right-angled triangle?

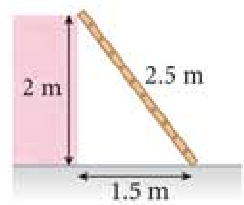
Solution

$$
\begin{aligned}
15^2 + 20^2 &= 225 + 400 \\
&= 625 \\
25^2 &= 625 \\
15^2 + 20^2 &= 25^2
\end{aligned}
$$

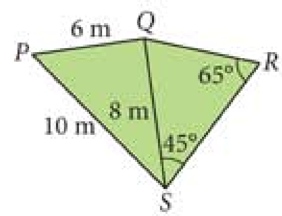
Therefore, Sheila will be able to form the frame in the shape of a right-angled triangle.

Self Practice 13.2b

1. A 2.5 m long ladder leans against the wall of a building. The base of the ladder is 1.5 m away from the wall. Explain how you would determine whether the wall is vertical.



2. In the diagram, find $\angle PQR$.



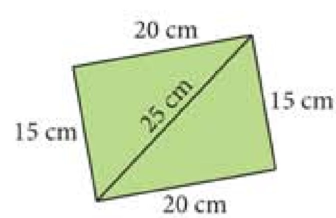
Mastery Q 13.2

Open the folder downloaded from page vii for extra questions of Mastery Q 13.2.

1. Explain whether the lengths of sides in each of the following can form a right-angled triangle:
   (a) 9 cm, 40 cm, 41 cm
   (b) 27 m, 45 m, 35 m
   (c) 2.5 cm, 6 cm, 6.5 cm
   (d) 13 m, 84 m, 85 m

2. A carpenter wishes to fix a triangular piece of wood measuring 12 cm, 16 cm and 20 cm onto a L-shaped wooden structure, as shown in the diagram. Explain whether the triangular piece of wood can be fixed perfectly onto the L-shaped structure.

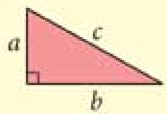
3. Kanang draws a quadrilateral with measurements as shown in the diagram. What is the name of the quadrilateral he has drawn? Explain.



SUMMARY

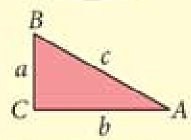
Book

Website



- c is the hypotenuse.
- c is the longest side opposite to the right angle.
- $c^2 = a^2 + b^2$

Pythagoras' theorem



If $c^2 = a^2 + b^2$,
then, $\angle ACB = 90°$

Discussion

Teacher

At the end of this chapter, I can...	Very good	Work harder
identify and define the hypotenuse of a right-angled triangle.		
determine the relationship between the sides of a right-angled triangle. Hence, explain the Pythagoras' theorem by referring to the relationship.		
determine the lengths of the unknown side of (i) a right-angled triangle. (ii) combined geometric shapes.		
solve problems involving the Pythagoras' theorem.		
determine whether a triangle is a right-angled triangle and give justification based on the converse of the Pythagoras' theorem.		
solve problems involving the converse of the Pythagoras' theorem.		

CHAPTER 13

Let's PRACTISE

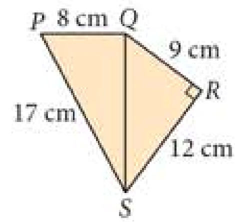
Test Yourself

1. In the diagram, calculate the length of
   (a) PR (b) SR

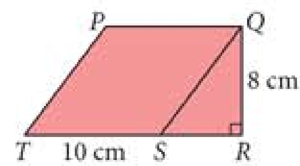
2. A car moves up a slope from P to Q. When the car reaches Q, the horizontal distance and the vertical distance it has covered are 150 m and 2 m respectively. Explain how you would calculate the actual distance the car has moved, correct to two decimal places.

Self Mastery

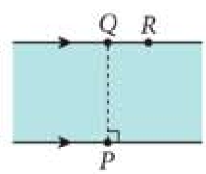
3. Based on the diagram,
   (a) calculate the length of QS.
   (b) explain whether PQS is a right-angled triangle.



4. In the diagram, $PQST$ is a rhombus and TSR is a straight line. Calculate the area of the whole diagram.

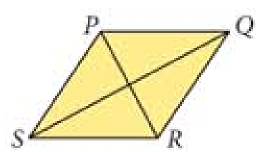


5. The width of a river is 18 m. Imran swims across the river from point P to point Q, as shown in the diagram. Due to strong water currents, Imran eventually lands at point R which is 6 m away from Q. Explain how you would calculate the actual distance Imran has swum, correct to two decimal places.

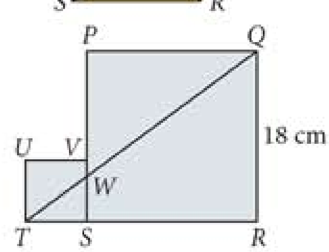


Challenge Yourself

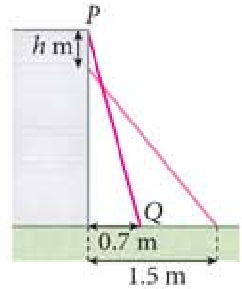
6. In the diagram, $PQRS$ is a rhombus. PR and SQ are 16 cm and 30 cm long respectively. Explain how you would calculate the length of side SR.



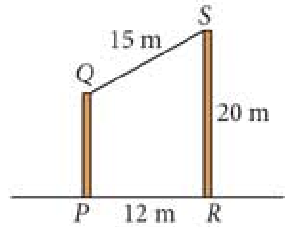
7. In the diagram, $PQRS$ and $UVST$ are squares. Given $TQ = 30$ cm, find the area of $UVST$.



8. The diagram shows a ladder PQ leaning against the wall. The ladder is 2.5 m long and the base of the ladder is 0.7 m away from the wall. When the top of the ladder slides down a distance of h m, the base of the ladder becomes 1.5 m away from the wall. Find the value of h.

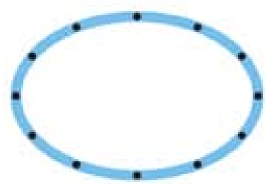


9. The distance between two poles, PQ and RS, on a horizontal ground is 12 m. A piece of 15 m long string is tied to both tips of the poles. If the height of pole RS is 20 m, explain how you would find the height of pole PQ.



10. Ali wishes to saw a piece of wood into the shape of a right-angled triangle. The hypotenuse of the wood must be 35 cm long and the lengths of the two other sides of the wood must be in the ratio 3 : 4. Explain how you would find the lengths of the other two sides and hence help Ali saw the wood.

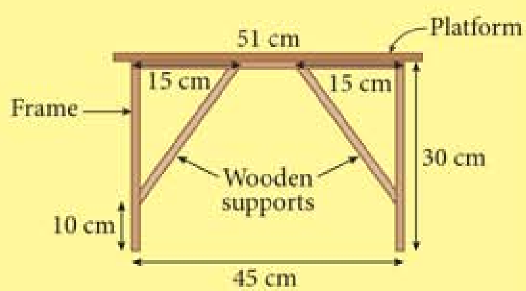
11. A loop of thread is marked with 12 points so that the adjacent points are of the same distance from one another. Explain how you would form a right-angled triangle using the loop of thread.



ASSIGNMENT

Making a wooden stool

A carpenter wishes to make a wooden stool according to the design shown in the diagram.



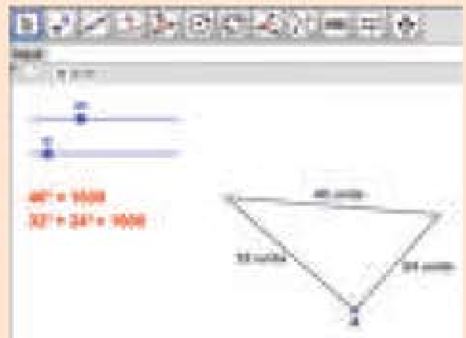
The wooden stool he wants to make consists of three parts. Part 1 is the frame; part 2 is the platform top for sitting on while part 3 are the supports for the corners of the frame. Each end of the supports is fixed at 15 cm from the corners of the frame, and the other end is fixed at 10 cm from the legs of the wooden stool. The platform is 51 cm long. Explain how you would use the Pythagoras' theorem to help the carpenter make the wooden stool.

Exploring Mathematics

A The lengths of the three sides of a right-angled triangle such as (3, 4, 5) and (8, 15, 17) are known as Pythagorean triples.
You can explore the Pythagorean triples through the following activity.

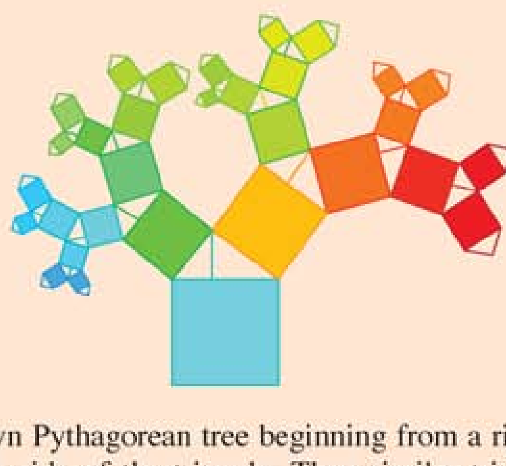
Open the folder downloaded from page vii for this activity.

1. Open the file *Triples.ggb* using *GeoGebra*.
2. Is the triangle shown a right-angled triangle?
3. Click and drag the slider *m* and the slider *n* and observe the changes on the screen displayed.
4. Do the lengths of sides shown constitute a set of triples?
5. Click and drag the slider *m* and the slider *n* for other combinations.
6. Explain what you have observed.
7. Present your findings in class.



B **Constructing a Pythagorean Tree**

A Pythagorean tree is a pattern constructed based on right-angled triangles and squares related to Pythagoras' theorem. The Pythagorean tree was invented by a Dutch mathematics teacher in 1942.



Construct your own Pythagorean tree beginning from a right-angled triangle with squares drawn on each side of the triangle. Then similar triangles are drawn on the sides of the squares. Then squares are drawn on the sides of the new triangles and so on.