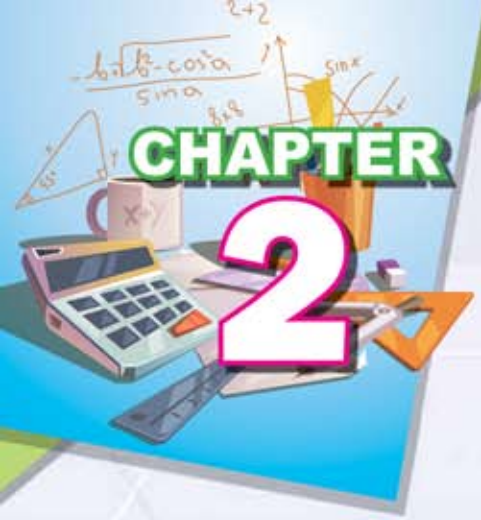


CHAPTER 2

# Number Bases

**You will learn**

- Number Bases

Malaysia has become a major focus of various technological developments which are capable of transforming the people's lifestyle in this 21st century. These advances in technology enable Malaysians to enjoy fast download rates, hologram technology in education, medicine, industries, self-driving cars and more. A society that is proficient in information technology and telecommunication needs to be well versed in number bases as they have become the basis of all technologies.

Do you know the relationship between the number bases and technology?

## Why Study This Chapter?

Number bases are the keys to all calculations in daily life. Among the fields involved are computer science and other areas that use information technology as the basis of research and development such as biotechnology, design technology, aerospace design, pharmacy and others.

## Walking Through Time

**Brahmagupta**
(598 AD – 668 AD)

Brahmagupta was an astronomer from the state of Rajasthan in the north-west of India. He introduced the digit 0 to the number system which has become the basis for all the number bases used in olden times and today.

http://bt.sasbadi.com/m4033

## WORD BANK

| | |
|---|---|
| • number base | • *asas nombor* |
| • binary | • *binari* |
| • index | • *indeks* |
| • place value | • *nilai tempat* |
| • digit value | • *nilai digit* |
| • number system | • *sistem nombor* |

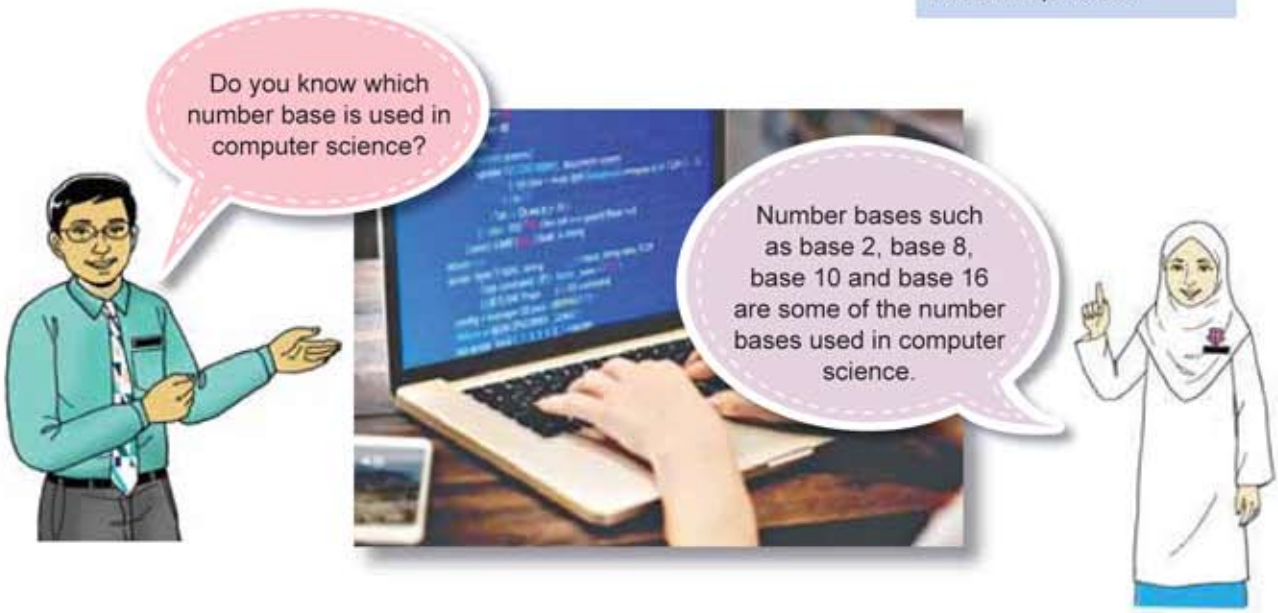
# 2.1 Number Bases

**How do you represent and explain numbers in various bases in terms of numerals, place values, digit values and number values based on the collection process?**

**Learning Standard**

Represent and explain numbers in various bases in terms of numerals, place values, digit values and number values based on the collection process.

Number bases are number systems consisting of digits from 0 to 9. The number systems are made up of numbers with various bases. Base ten is a decimal number system used widely in daily life.




The table below shows the digits used in base two up to base ten.

| Number base | Digit |
|---|---|
| Base 2 | 0, 1 |
| Base 3 | 0, 1, 2 |
| Base 4 | 0, 1, 2, 3 |
| Base 5 | 0, 1, 2, 3, 4 |
| Base 6 | 0, 1, 2, 3, 4, 5 |
| Base 7 | 0, 1, 2, 3, 4, 5, 6 |
| Base 8 | 0, 1, 2, 3, 4, 5, 6, 7 |
| Base 9 | 0, 1, 2, 3, 4, 5, 6, 7, 8 |
| Base 10 | 0, 1, 2, 3, 4, 5, 6, 7, 8, 9 |

**INFO ZONE**

Digits are the symbols used or combined to form a number in the number system. 0, 1, 2, 3, 4, 5, 6, 7, 8, 9 are the ten digits used in the decimal number system.
For example, 2 145 has 4 digits.

**Example 1**

Give two examples of numbers that represent numbers in base two up to base ten.

***Solution:***

| Number base | Number | |
|---|---|---|
| 2 | $10_2$ | $1001_2$ |
| 3 | $21_3$ | $1201_3$ |
| 4 | $23_4$ | $213_4$ |
| 5 | $41_5$ | $342_5$ |
| 6 | $35_6$ | $4510_6$ |
| 7 | $64_7$ | $463_7$ |
| 8 | $17_8$ | $472_8$ |
| 9 | $78_9$ | $385_9$ |
| 10 | $69_{10}$ | $2893_{10}$ |

**TIPS**

Each base has digits from 0 to a digit which is less than its base. For example, base two has only digits 0 and 1.

**INFO ZONE**

number → $32_5$ ← base

is read as

"Three two base five"

## What are the place values involved in numbers in base two up to base ten?

Each base has place values according to each respective base. The place values of a base are the repeated multiplication of that base. Let's say $a$ is a base, then its place values start with $a^0$, $a^1$, $a^2$, …, $a^n$ as shown in the table below.

**MY MEMORY**

$a$ - base
$n$ - power
$a^4 = a \times a \times a \times a$

$a^n$

| Number base | $a^n$ | Place value | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| | | $a^7$ | $a^6$ | $a^5$ | $a^4$ | $a^3$ | $a^2$ | $a^1$ | $a^0$ |
| Base 2 | $2^n$ | 128 | 64 | 32 | 16 | 8 | 4 | 2 | 1 |
| Base 3 | $3^n$ | 2187 | 729 | 243 | 81 | 27 | 9 | 3 | 1 |
| Base 4 | $4^n$ | 16384 | 4096 | 1024 | 256 | 64 | 16 | 4 | 1 |
| Base 5 | $5^n$ | 78125 | 15625 | 3125 | 625 | 125 | 25 | 5 | 1 |
| Base 6 | $6^n$ | 279936 | 46656 | 7776 | 1296 | 216 | 36 | 6 | 1 |
| Base 7 | $7^n$ | 823543 | 117649 | 16807 | 2401 | 343 | 49 | 7 | 1 |
| Base 8 | $8^n$ | 2097152 | 262144 | 32768 | 4096 | 512 | 64 | 8 | 1 |
| Base 9 | $9^n$ | 4782969 | 531441 | 59049 | 6561 | 729 | 81 | 9 | 1 |
| Base 10 | $10^n$ | 10000000 | 1000000 | 100000 | 10000 | 1000 | 100 | 10 | 1 |

**Example 2**

State the place value of each digit in the numbers below.
(a) $6231_8$ (b) $111101_2$

***Solution:***

(a)

| Number in base 8 | 6 | 2 | 3 | 1 |
|---|---|---|---|---|
| Place value | $8^3$ | $8^2$ | $8^1$ | $8^0$ |

(b)

| Number in base 2 | 1 | 1 | 1 | 1 | 0 | 1 |
|---|---|---|---|---|---|---|
| Place value | $2^5$ | $2^4$ | $2^3$ | $2^2$ | $2^1$ | $2^0$ |

**Walking Through Time**

Konrad Zuse (1910 – 1995) was the inventor and pioneer of modern computers from Germany. He was the founder of the programmable computer. He designed the first high-level programming language known as *Plankalkuel*.

## How do you state the value of a particular digit in a number in various bases?

The value of a particular digit in a number is the multiplication of a digit and the place value that represents the digit.

**Multiplication of digit and place value**

$10\underline{1}0_2$

| Number | 1 | 0 | $\underline{1}$ | 0 |
|---|---|---|---|---|
| Place value | $2^3$ | $2^2$ | $2^1$ | $2^0$ |
| Digit value | | | $1 \times 2^1 = 2$ | |

$\underline{2}012_3$

| Number | $\underline{2}$ | 0 | 1 | 2 |
|---|---|---|---|---|
| Place value | $3^3$ | $3^2$ | $3^1$ | $3^0$ |
| Digit value | $2 \times 3^3 = 54$ | | | |

$\underline{4}432_5$

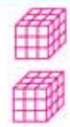
| Number | $\underline{4}$ | 4 | 3 | 2 |
|---|---|---|---|---|
| Place value | $5^3$ | $5^2$ | $5^1$ | $5^0$ |
| Digit value | $4 \times 5^3 = 500$ | | | |

**Use of place value block**

$10\underline{1}0_2$

| Number | 1 | 0 | $\underline{1}$ | 0 |
|---|---|---|---|---|
| Place value | $2^3$ | $2^2$ | $2^1$ | $2^0$ |
| Digit value | | |  2 | |

$\underline{2}012_3$

| Number | $\underline{2}$ | 0 | 1 | 2 |
|---|---|---|---|---|
| Place value | $3^3$ | $3^2$ | $3^1$ | $3^0$ |
| Digit value |  54 | | | |

$\underline{4}432_5$

| Number | $\underline{4}$ | 4 | 3 | 2 |
|---|---|---|---|---|
| Place value | $5^3$ | $5^2$ | $5^1$ | $5^0$ |
| Digit value |  500 | | | |

**Multiplication of digit and place value**

$\underline{2}71_8$

| Number | $\underline{2}$ | 7 | 1 |
|---|---|---|---|
| Place value | $8^2$ | $8^1$ | $8^0$ |
| Digit value | $2 \times 8^2 = 128$ | | |

**Use of place value block**

$\underline{2}71_8$

| Number | $\underline{2}$ | 7 | 1 |
|---|---|---|---|
| Place value | $8^2$ | $8^1$ | $8^0$ |
| Digit value |  128 | | |



**Example 3**

State the value of the underlined digit in each of the following numbers.

(a) $\underline{3}41_8$ (b) $\underline{5}037_9$ (c) $3\underline{5}01_6$ (d) $2134_{\underline{5}}$

***Solution:***

(a) $\underline{3}41_8$

| $8^2$ | $8^1$ | $8^0$ |
|---|---|---|
| $\underline{3}$ | 4 | 1 |

$3 \times 8^2 = 192$

(b) $\underline{5}037_9$

| $9^3$ | $9^2$ | $9^1$ | $9^0$ |
|---|---|---|---|
| $\underline{5}$ | 0 | 3 | 7 |

$5 \times 9^3 = 3645$

(c) $3\underline{5}01_6$

| $6^3$ | $6^2$ | $6^1$ | $6^0$ |
|---|---|---|---|
| 3 | $\underline{5}$ | 0 | 1 |

$5 \times 6^2 = 180$

(d) $2134_{\underline{5}}$

| $5^3$ | $5^2$ | $5^1$ | $5^0$ |
|---|---|---|---|
| 2 | 1 | 3 | $\underline{4}$ |

$4 \times 5^0 = 4$

## How do you state the numerical value of a number in various bases?

The numerical value of a number in various bases can be determined by calculating the sum of the digit values of the number.


Numbers in base two have only digits 0 and 1.

(a) Determine the value of a number in base two.

**Collection process**

| Number | 1 | 1 | 0 | 0 | 1 |
|---|---|---|---|---|---|
| Place value | $2^4$ | $2^3$ | $2^2$ | $2^1$ | $2^0$ |
| Digit value | $1 \times 2^4 = 16$ | $1 \times 2^3 = 8$ | $0 \times 2^2 = 0$ | $0 \times 2^1 = 0$ | $1 \times 2^0 = 1$ |
| Number value | $16 + 8 + 0 + 0 + 1 = 25_{10}$ | | | | |

**Adding digit values using blocks**

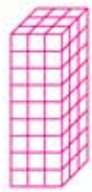
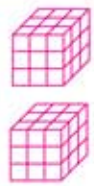
| Number | 1 | 1 | 0 | 0 | 1 |
|---|---|---|---|---|---|
| Place value | $2^4$ | $2^3$ | $2^2$ | $2^1$ | $2^0$ |
| Digit value |  16 |  8 | | |  1 |
| Number value | $16 + 8 + 0 + 0 + 1 = 25_{10}$ | | | | |

(b) Determine the value of a number in base three.

**Collection process**

| Number | 1 | 2 | 0 | 2 | 1 |
|---|---|---|---|---|---|
| Place value | $3^4$ | $3^3$ | $3^2$ | $3^1$ | $3^0$ |
| Digit value | $1 \times 3^4 = 81$ | $2 \times 3^3 = 54$ | $0 \times 3^2 = 0$ | $2 \times 3^1 = 6$ | $1 \times 3^0 = 1$ |
| Number value | $81 + 54 + 0 + 6 + 1 = 142_{10}$ | | | | |

**Adding digit values using blocks**

| Number | 1 | 2 | 0 | 2 | 1 |
|---|---|---|---|---|---|
| Place value | $3^4$ | $3^3$ | $3^2$ | $3^1$ | $3^0$ |
| Digit value |  81 |  54 | |  6 |  1 |
| Number value | $81 + 54 + 0 + 6 + 1 = 142_{10}$ | | | | |

(c) Determine the value of a number in base four.

**Collection process**

| Number | 3 | 0 | 2 | 1 |
|---|---|---|---|---|
| Place value | $4^3$ | $4^2$ | $4^1$ | $4^0$ |
| Digit value | $3 \times 4^3 = 192$ | $0 \times 4^2 = 0$ | $2 \times 4^1 = 8$ | $1 \times 4^0 = 1$ |
| Number value | $192 + 0 + 8 + 1 = 201_{10}$ | | | |

**INTERACTIVE ZONE**

Is the value of $243_8$ equal to $243_5$? Discuss.

**INFO ZONE**

Writing the base sign for a number in base 10 is optional, which can be written or left out.

**Adding digit values using blocks**

| Number | 3 | 0 | 2 | 1 |
|---|---|---|---|---|
| Place value | $4^3$ | $4^2$ | $4^1$ | $4^0$ |
| Digit value |  192 | |  8 |  1 |
| Number value | $192 + 0 + 8 + 1 = 201_{10}$ | | | |

**Smart Mind**

State two numbers in different bases with the same value.



**Example 4**

Determine the values of the following numbers.

(a) $340_5$ (b) $341_7$ (c) $1506_8$

**Smart Mind**

Convert your year of birth to a number base that you prefer.

***Solution:***

(a) $340_5$

| Number | 3 | 4 | 0 |
|---|---|---|---|
| Place value | $5^2$ | $5^1$ | $5^0$ |
| Number value | $(3 \times 5^2) + (4 \times 5^1) + (0 \times 5^0)$<br>$= 75 + 20 + 0$<br>$= 95_{10}$ | | |

**INTERACTIVE ZONE**

What will happen if a number in a base higher than 10 is used? Discuss.

(b) $341_7$

| Number | 3 | 4 | 1 |
|---|---|---|---|
| Place value | $7^2$ | $7^1$ | $7^0$ |
| Number value | $(3 \times 7^2) + (4 \times 7^1) + (1 \times 7^0)$<br>$= 147 + 28 + 1$<br>$= 176_{10}$ | | |

(c) $1506_8$

| Number | 1 | 5 | 0 | 6 |
|---|---|---|---|---|
| Place value | $8^3$ | $8^2$ | $8^1$ | $8^0$ |
| Number value | $(1 \times 8^3) + (5 \times 8^2) + (0 \times 8^1) + (6 \times 8^0)$<br>$= 512 + 320 + 0 + 6$<br>$= 838_{10}$ | | | |

**Checking Answer**

1. Press the MODE key 2 times until
   SD REG BASE
   1 2 3
   appears on the screen.
2. Press 3 to choose BASE.
3. Press OCT.
4. Press 1506 then press =.
5. Press DEC, the answer 838 is displayed.

## Self Practice 2.1a

1. Write three numbers to represent numbers in base two up to base nine.

2. Circle three numbers which do not represent numbers in base six.

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| 245 | 332 | 461 | 212 | 371 | 829 | 345 | 123 |

3.

| | | | |
|---|---|---|---|
| 234 | 673 | 336 | 281 |

Based on the four numbers above, identify and list all the numbers with the following bases.

(a) Base five (b) Base seven (c) Base eight (d) Base nine

4. Determine the place value of the underlined digit in each of the following numbers.

(a) $1\underline{1}0010_2$ (b) $\underline{2}14_5$ (c) $600\underline{1}_7$ (d) $5114\underline{0}_6$ (e) $\underline{1}200_3$

(f) $6\underline{8}3_9$ (g) $\underline{2}331_4$ (h) $\underline{7}321_8$ (i) $\underline{5}241_6$ (j) $3221\underline{5}$

5. Determine the value of the underlined digit in each of the following numbers.

(a) $\underline{1}110_2$ (b) $3\underline{2}4_5$ (c) $87\underline{3}_9$ (d) $\underline{2}35_6$ (e) $\underline{2}100_3$

(f) $1\underline{6}623_7$ (g) $\underline{1}101_2$ (h) $1\underline{7}76_8$ (i) $2\underline{3}1_4$ (j) $111\underline{1}01_2$

6. Determine the values of the following numbers in base ten.

(a) $23_6$ (b) $425_8$ (c) $110101_2$ (d) $338_9$ (e) $364_7$

(f) $33_4$ (g) $123_5$ (h) $1217_8$ (i) $515_6$ (j) $1121_3$

7. Determine the values of $p$ and $q$.

(a) $1101_2 = (1 \times 2^p) + (1 \times q) + (1 \times 2^0)$

(b) $375_8 = (3 \times 8^p) + (q \times 8^1) + (5 \times 8^0)$

(c) $1321_4 = (1 \times p^q) + (3 \times 4^2) + (2 \times 4^1) + (1 \times 4^0)$

8. Calculate the sum of the values of digit 8 and digit 3 in $1823_9$.

9. Rearrange the following numbers in ascending order.

(a) $110_2, 1101_2, 111_2, 1110_2$ (b) $1123_4, 132_4, 231_4, 112_4$ (c) $324_5, 124_5, 241_5, 231_5$

10. Rearrange the following numbers in descending order.

(a) $111101_2, 1213_4, 81_9$ (b) $123_4, 73_8, 313_5$ (c) $253_6, 161_7, 222_3$

11. Calculate the difference between the values of digit 5 in $1576_8$ and $125_7$.

## How do you convert numbers from one base to another base using various methods?

**Learning Standard**

Convert numbers from one base to another using various methods.

A number can be converted to other bases by using various methods, such as the division using place value and the division using base value. These processes involve converting

(a) a number in base ten to another base.
(b) a number in a certain base to base ten and then to another base.
(c) a number in base two directly to base eight.
(d) a number in base eight directly to base two.

### How do you convert a number in base ten to another base?

A number in base ten can be converted to another base by dividing the number using the place value or the base value required. The number $58_{10}$ can be converted to base two by

(a) dividing 58 using the place value in base two.
(b) dividing 58 by two.

**Example 5**

Rajang River which is the longest river in Malaysia is 563 kilometres.
Convert $563_{10}$ to a number in

(a) base five. (b) base eight.

***Solution:***

(a) Base five

**Division using place value**

| Place value | 625 | 125 | 25 | 5 | 1 |
|---|---|---|---|---|---|
| Step | The value of 625 is greater than 563 | 4<br>125) 563<br>− 500<br>63 | 2<br>25) 63<br>− 50<br>13 | 2<br>5) 13<br>− 10<br>3 | 3<br>1) 3<br>− 3<br>0 |
| Base 5 | 0 | 4 | 2 | 2 | 3 |
| Answer | $4223_5$ | | | | |

563 is divided by the place value of 125. Its remainder is transferred to the previous place value for the next division until a zero remainder is obtained.

**Alternative Method**

**Division using base value**

| | | |
|---|---|---|
| 5 | 563 | Remainder |
| 5 | 112 | −3 |
| 5 | 22 | −2 |
| 5 | 4 | −2 |
| | 0 | −4 |

The digits are read from the bottom upwards.

The division is continued until digit zero is obtained.

$563_{10} = 4223_5$

(b) Base eight

**Division using place value**

| Place value | 4096 | 512 | 64 | 8 | 1 |
|---|---|---|---|---|---|
| Step | The value of 4096 is greater than 563 | 1<br>512) 563<br>− 512<br>51 | The value of 64 is greater than 51 | 6<br>8) 51<br>− 48<br>3 | 3<br>1) 3<br>− 3<br>0 |
| Base 8 | 0 | 1 | 0 | 6 | 3 |
| Answer | $1063_8$ | | | | |

563 is divided by the place value of 512. Its remainder is transferred to the previous place value for the next division until a zero remainder is obtained.

**Alternative Method**

**Division using base value**

Remainder

8 | 563
8 | 70 − 3
8 | 8 − 6
8 | 1 − 0
0 − 1

The digits are read from the bottom upwards.

The division is continued until digit zero is obtained.

$563_{10} = 1063_8$

**How do you convert a number in a certain base to base ten and then to another base?**

A number in base $p$ can be converted to base ten and then to base $q$. In the process of converting a number in base two to base nine, the number in base two is converted to base ten and then to base nine.

Base $p$ ⟶ Base 10 ⟶ Base $q$

**Example 6**

Convert $253_6$ to a number in base nine.

***Solution:***

**Step 1**

Convert the number in base six to base ten.

| Place value | $6^2$ | $6^1$ | $6^0$ |
|---|---|---|---|
| Number in base 6 | 2 | 5 | 3 |
| Value of number in base 10 | $(2 \times 6^2) + (5 \times 6^1) + (3 \times 6^0)$<br>$= 105_{10}$ | | |

**Step 2**

Convert the number in base ten to base nine.

9 | 105
9 | 11 − 6
9 | 1 − 2
0 − 1

$253_6 = 105_{10} = 126_9$

## Example 7

Convert $334_5$ to a number in base two.

***Solution:***

**Step 1**

Convert the number in base five to base ten.

| Place value | $5^2$ | $5^1$ | $5^0$ |
|---|---|---|---|
| Number in base 5 | 3 | 3 | 4 |
| Value of number in base 10 | $(3 \times 5^2) + (3 \times 5^1) + (4 \times 5^0)$ $= 94_{10}$ | | |

**Step 2**

Convert the number in base ten to base two.

| | | |
|---|---|---|
| 2 | 94 | |
| 2 | 47 | – 0 |
| 2 | 23 | – 1 |
| 2 | 11 | – 1 |
| 2 | 5 | – 1 |
| 2 | 2 | – 1 |
| 2 | 1 | – 0 |
| | 0 | – 1 |

$334_5 = 94_{10} = 1011110_2$

### How do you convert a number in base two to base eight?

A number in base two can be converted directly to base eight. Each digit in base eight is equivalent to three digits in base two.

1. Separate each of the three digits of a number in base two from the right to the left.
2. Determine the sum of the digit values for the combined three digits in base two.
3. Combine the number in base eight.

## Example 8

Convert the numbers in base two to numbers in base eight.

(a) $110111_2$ (b) $1101101_2$

***Solution:***

(a) $110111_2$

| Number in base 2 | 1 | 1 | 0 | 1 | 1 | 1 |
|---|---|---|---|---|---|---|
| Place value | $2^2$ | $2^1$ | $2^0$ | $2^2$ | $2^1$ | $2^0$ |
| Digit value | 4 | 2 | 0 | 4 | 2 | 1 |
| Base 8 | $4 + 2 + 0$ $= 6$ | | | $4 + 2 + 1$ $= 7$ | | |
| | $67_8$ | | | | | |

$110111_2 = 67_8$

**INFO ZONE**

| Base 2 | Base 8 |
|---|---|
| 000 | 0 |
| 001 | 1 |
| 010 | 2 |
| 011 | 3 |
| 100 | 4 |
| 101 | 5 |
| 110 | 6 |
| 111 | 7 |

**Alternative Method**

$\underbrace{110}_{6}\ \underbrace{111}_{7}$

$110111_2 = 67_8$

By referring to the table in INFO ZONE, you can easily convert a number in base two to base eight.

(b) $1101101_2$

| Number in base 2 | | | 1 | 1 | 0 | 1 | 1 | 0 | 1 |
|---|---|---|---|---|---|---|---|---|---|
| Place value | $2^2$ | $2^1$ | $2^0$ | $2^2$ | $2^1$ | $2^0$ | $2^2$ | $2^1$ | $2^0$ |
| Digit value | 0 | 0 | 1 | 4 | 0 | 1 | 4 | 0 | 1 |
| Base 8 | $0+0+1$ $=1$ | | | $4+0+1$ $=5$ | | | $4+0+1$ $=5$ | | |
| | $155_8$ | | | | | | | | |

$1101101_2 = 155_8$

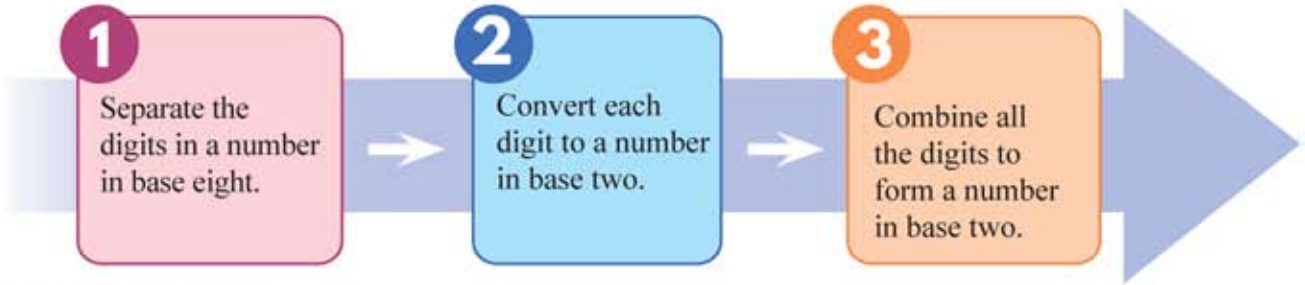
**Alternative Method**

$\underbrace{1}_{1}\ \underbrace{101}_{5}\ \underbrace{101}_{5}$

$1101101_2 = 155_8$

**How do you convert a number in base eight to base two?**

A number in base eight can be converted directly to base two. Each digit in base eight is equivalent to three digits in base two.




**Example 9**

Convert the numbers in base eight to numbers in base two.

(a) $517_8$ (b) $725_8$

***Solution:***

(a) $517_8$

| Base 8 | 5 | | | 1 | | | 7 | | |
|---|---|---|---|---|---|---|---|---|---|
| | $4+1$ | | | 1 | | | $4+2+1$ | | |
| Place value | $2^2$ | $2^1$ | $2^0$ | $2^2$ | $2^1$ | $2^0$ | $2^2$ | $2^1$ | $2^0$ |
| Base 2 | 1 | 0 | 1 | 0 | 0 | 1 | 1 | 1 | 1 |
| | $101001111_2$ | | | | | | | | |

$517_8 = 101001111_2$

(b) $725_8$

| Base 8 | 7 | | | 2 | | | 5 | | |
|---|---|---|---|---|---|---|---|---|---|
| | $4+2+1$ | | | 2 | | | $4+1$ | | |
| Place value | $2^2$ | $2^1$ | $2^0$ | $2^2$ | $2^1$ | $2^0$ | $2^2$ | $2^1$ | $2^0$ |
| Base 2 | 1 | 1 | 1 | 0 | 1 | 0 | 1 | 0 | 1 |
| | $111010101_2$ | | | | | | | | |

$725_8 = 111010101_2$

**INFO ZONE**

| Base 2 | Base 10 | Base 16 |
|---|---|---|
| $0000_2$ | 0 | 0 |
| $0001_2$ | 1 | 1 |
| $0010_2$ | 2 | 2 |
| $0011_2$ | 3 | 3 |
| $0100_2$ | 4 | 4 |
| $0101_2$ | 5 | 5 |
| $0110_2$ | 6 | 6 |
| $0111_2$ | 7 | 7 |
| $1000_2$ | 8 | 8 |
| $1001_2$ | 9 | 9 |
| $1010_2$ | 10 | A |
| $1011_2$ | 11 | B |
| $1100_2$ | 12 | C |
| $1101_2$ | 13 | D |
| $1110_2$ | 14 | E |
| $1111_2$ | 15 | F |

## Self Practice 2.1b

1. Convert $494_{10}$ to numbers in the following bases.
   (a) Base two (b) Base four (c) Base five (d) Base eight (e) Base nine

2. Convert each of the following numbers to the number base given in brackets.
   (a) $43_8$ (base 3) (b) $112_3$ (base 5) (c) $526_7$ (base 2)
   (d) $1213_4$ (base 6) (e) $1134_5$ (base 8) (f) $321_9$ (base 4)

3. Convert the value of digit 5 in the number $154_6$ to a number in base 3.

4. Convert each of the following numbers to a number in base eight.
   (a) $111101_2$ (b) $1110_2$ (c) $11110111_2$
   (d) $101010_2$ (e) $111000_2$ (f) $111010101_2$

5. Convert each of the following numbers to a number in base two.
   (a) $43_8$ (b) $112_8$ (c) $57_8$
   (d) $1217_8$ (e) $635_8$ (f) $243_8$

## How do you perform calculations involving addition and subtraction of numbers in various bases?

**Learning Standard**

Perform calculations involving addition and subtraction of numbers in various bases.

Addition and subtraction in number bases can be carried out in the following two methods:
(a) Using vertical form, that is to write the numbers vertically when performing addition and subtraction.
(b) Conversion of numbers in a certain base to base ten.

### Addition of numbers in various bases

**Vertical Form**

1. Add the given digits starting from the right to the left.
2. The sum of the digits in base ten is converted to the given base to be written in the answer space.
3. This process is repeated until all the digits in the number are added up.

**Conversion of Base**

1. Convert a number to base ten and perform addition.
2. Convert the answer in base ten to the required base.

## Example 10

Calculate each of the following.

(a) $110_2 + 111_2$ (b) $673_8 + 175_8$ (c) $1837_9 + 765_9$

***Solution:***

### Vertical Form

(a) $110_2 + 111_2$

$$\begin{array}{r} {\scriptstyle 1\ 1\ \ \ } \\ 1\ 1\ 0_2 \\ +\ 1\ 1\ 1_2 \\ \hline 1\ 1\ 0\ 1_2 \end{array}$$

**TIPS**

Perform addition as usual and convert the values in base ten to base two.

$0 + 1 = 1_{10} = 1_2$

$1 + 1 = 2_{10} = 10_2$ ← Write 0 in the answer space. 1 is carried forward to the next place value.

$1 + 1 + 1 = 3_{10} = 11_2$

$110_2 + 111_2 = 1101_2$

(b) $673_8 + 175_8$

$$\begin{array}{r} {\scriptstyle 1\ 1\ \ } \\ 6\ 7\ 3_8 \\ +\ 1\ 7\ 5_8 \\ \hline 1\ 0\ 7\ 0_8 \end{array}$$

**TIPS**

Perform addition as usual and convert the values in base ten to base eight.

$3 + 5 = 8_{10} = 10_8$ ← Write 0 in the answer space. 1 is carried forward to the next place value.

$1 + 7 + 7 = 15_{10} = 17_8$

$1 + 6 + 1 = 8_{10} = 10_8$ ← Write 7 in the answer space. 1 is carried forward to the next place value.

$673_8 + 175_8 = 1070_8$

(c) $1837_9 + 765_9$

$$\begin{array}{r} {\scriptstyle 1\ 1\ 1\ \ } \\ 1\ 8\ 3\ 7_9 \\ +\ \ 7\ 6\ 5_9 \\ \hline 2\ 7\ 1\ 3_9 \end{array}$$

**TIPS**

Perform addition as usual and convert the values in base ten to base nine.

$7 + 5 = 12_{10} = 13_9$ ← Write 3 in the answer space. 1 is carried forward to the next place value.

$1 + 3 + 6 = 10_{10} = 11_9$

$1 + 8 + 7 = 16_{10} = 17_9$ ← Write 1 in the answer space. The number 1 on the left is carried forward to the next place value.

$1 + 1 = 2_{10} = 2_9$ ← Write 7 in the answer space. 1 is carried forward to the next place value.

$1837_9 + 765_9 = 2713_9$

### Conversion of Base

(a) $110_2 + 111_2$

$110_2 \rightarrow 6_{10}$

$111_2 \rightarrow +7_{10}$

$13_{10}$

| | | |
|---|---|---|
| 2 | 13 | |
| 2 | 6 | −1 |
| 2 | 3 | −0 |
| 2 | 1 | −1 |
| | 0 | −1 |

$110_2 + 111_2 = 1101_2$

(b) $673_8 + 175_8$

$673_8 \rightarrow 443_{10}$

$175_8 \rightarrow +125_{10}$

$568_{10}$

| | | |
|---|---|---|
| 8 | 568 | |
| 8 | 71 | −0 |
| 8 | 8 | −7 |
| 8 | 1 | −0 |
| | 0 | −1 |

$673_8 + 175_8 = 1070_8$

(c) $1837_9 + 765_9$

$1837_9 \rightarrow 1411_{10}$

$765_9 \rightarrow +626_{10}$

$2037_{10}$

| | | |
|---|---|---|
| 9 | 2037 | |
| 9 | 226 | −3 |
| 9 | 25 | −1 |
| 9 | 2 | −7 |
| | 0 | −2 |

$1837_9 + 765_9 = 2713_9$

## Subtraction of numbers in various bases

### Vertical Form

1. Subtract the given digits starting from the right to the left.
2. The difference is written in the answer space. The difference is always less than the given base and its value is equal to base ten.
3. This process is repeated until all the digits in the number are subtracted.



### Conversion of Base

1. Convert a number to base ten and perform subtraction.
2. Convert the answer in base ten to the required base.

**Example 11**

Calculate each of the following.

(a) $4005_6 - 325_6$

(b) $6241_7 - 613_7$

(c) $372_8 - 77_8$

(d) $1827_9 - 65_9$

***Solution:***

**Vertical Form**

(a) $4005_6 - 325_6$

$$\begin{array}{r} {\scriptstyle 5}\\ {\scriptstyle 3\ \not{8}\ 6}\\ \not{4}\,0\,0\,5_6\\ -\ \ 3\,2\,5_6\\ \hline 3\,2\,4\,0_6 \end{array}$$

Transfer numbers according to base six. Use 6 to perform subtraction. Subtract as usual.

$5 - 5 = 0_6$

$6 - 2 = 4_6$

$5 - 3 = 2_6$

$4005_6 - 325_6 = 3240_6$

**Conversion of Base**

$4005_6 \rightarrow 869_{10}$

$325_6 \rightarrow -\,125_{10}$

$744_{10}$

| | | |
|---|---|---|
| 6 | 744 | |
| 6 | 124 | − 0 |
| 6 | 20 | − 4 |
| 6 | 3 | − 2 |
| | 0 | − 3 |

$4005_6 - 325_6 = 3240_6$

(b) $6241_7 - 613_7$

$$\begin{array}{r} {\scriptstyle 5\ 7\ 3\ 7} \\ 6\not{2}\not{4}1_7 \\ -\ \ 6\ 1\ 3_7 \\ \hline 5\ 3\ 2\ 5_7 \end{array}$$

Transfer numbers according to base seven. Use 7 to perform subtraction. Subtract as usual.

$7 + 1 - 3 = 5_7$

$3 - 1 = 2_7$

$7 + 2 - 6 = 3_7$

$6241_7 - 613_7 = 5325_7$

$6241_7 \rightarrow 2\ 1\ 8\ 5_{10}$

$613_7 \rightarrow -\ 3\ 0\ 4_{10}$

$1\ 8\ 8\ 1_{10}$

| | | |
|---|---|---|
| 7 | 1881 | |
| 7 | 268 | $-5$ |
| 7 | 38 | $-2$ |
| 7 | 5 | $-3$ |
| | 0 | $-5$ |

$6241_7 - 613_7 = 5325_7$

(c) $372_8 - 77_8$

$$\begin{array}{r} {\scriptstyle 2\ 6\ 8} \\ 3\not{7}2_8 \\ -\ \ 7\ 7_8 \\ \hline 2\ 7\ 3_8 \end{array}$$

Transfer numbers according to base eight. Use 8 to perform subtraction. Subtract as usual.

$8 + 2 - 7 = 3_8$

$8 + 6 - 7 = 7_8$

$372_8 - 77_8 = 273_8$

$372_8 \rightarrow 2\ 5\ 0_{10}$

$77_8 \rightarrow -\ 6\ 3_{10}$

$1\ 8\ 7_{10}$

| | | |
|---|---|---|
| 8 | 187 | |
| 8 | 23 | $-3$ |
| 8 | 2 | $-7$ |
| | 0 | $-2$ |

$372_8 - 77_8 = 273_8$

(d) $1827_9 - 65_9$

$$\begin{array}{r} {\scriptstyle 7\ 9} \\ 1\not{8}2\ 7_9 \\ -\ \ 6\ 5_9 \\ \hline 1\ 7\ 5\ 2_9 \end{array}$$

Transfer numbers according to base nine. Use 9 to perform subtraction. Subtract as usual.

$7 - 5 = 2_9$

$9 + 2 - 6 = 5_9$

$1827_9 - 65_9 = 1752_9$

$1827_9 \rightarrow 1\ 4\ 0\ 2_{10}$

$65_9 \rightarrow -\ 5\ 9_{10}$

$1\ 3\ 4\ 3_{10}$

| | | |
|---|---|---|
| 9 | 1343 | |
| 9 | 149 | $-2$ |
| 9 | 16 | $-5$ |
| 9 | 1 | $-7$ |
| | 0 | $-1$ |

$1827_9 - 65_9 = 1752_9$

## Self Practice 2.1c

**1.** Calculate the value of the following.

(a) $11_2 + 10_2$ (b) $11011_2 + 11110_2$ (c) $210_3 + 121_3$ (d) $1112_3 + 101_3$

(e) $13_4 + 10_4$ (f) $1330_4 + 1120_4$ (g) $423_5 + 130_5$ (h) $3244_5 + 203_5$

(i) $351_6 + 122_6$ (j) $123_6 + 50_6$ (k) $166_7 + 253_7$ (l) $633_7 + 150_7$

(m) $1713_8 + 105_8$ (n) $453_8 + 262_8$ (o) $183_9 + 17_9$ (p) $5703_9 + 750_9$

**2.** Calculate the value of the following.

(a) $1111_2 - 10_2$ (b) $1011_2 - 101_2$ (c) $2210_3 - 211_3$ (d) $1012_3 - 121_3$

(e) $131_4 - 121_4$ (f) $1030_4 - 122_4$ (g) $423_5 - 100_5$ (h) $3204_5 - 2013_5$

(i) $3531_6 - 114_6$ (j) $1253_6 - 150_6$ (k) $6026_7 - 243_7$ (l) $6503_7 - 160_7$

(m) $1753_8 - 1005_8$ (n) $4403_8 - 202_8$ (o) $1853_9 - 1207_9$ (p) $8703_9 - 7250_9$

## How do you solve problems involving number bases?

**Example 12**

**Learning Standard**

Solve problems involving number bases.



| Mode of transportation | Percentage (%) |
|---|---|
| Bus | 25 |
| Car | 40 |
| Walking | 17 |
| Bicycle | 10 |
| Motorcycle | 8 |

The table above shows a study of various ways of 200 pupils going to school every day.

(a) State the number of pupils who go to school by bus and by car in base four.
(b) Calculate the total number of pupils who go to school by bus and by car in base four.
(c) Calculate the difference in number of pupils, in base seven, between those who walk to school and those who go to school by motorcycle.

### Understanding the problem

Total number of pupils = 200

The given percentages need to be converted to number of pupils.

### Planning a strategy

(a) Convert the number of pupils who go to school by bus and by car to a number in base four.
(b) Add the answer in (a).
(c) Subtract the number of pupils who walk by those who go to school by motorcycle. Then, convert the answer to a number in base seven.

### Implementing the strategy

(a) Bus = $302_4$

$\frac{25}{100} \times 200 = 50_{10}$

$4 \mid 50$
$4 \mid 12 - 2$
$4 \mid 3 - 0$
$0 - 3$

Car = $1100_4$

$\frac{40}{100} \times 200 = 80_{10}$

$4 \mid 80$
$4 \mid 20 - 0$
$4 \mid 5 - 0$
$4 \mid 1 - 1$
$0 - 1$

(b)
$$\begin{array}{r} 1\,1\,0\,0_4 \\ +\quad 3\,0\,2_4 \\ \hline 2\,0\,0\,2_4 \end{array}$$

$1100_4 + 302_4 = 2002_4$

(c) $34 - 16 = 18_{10}$

$\frac{17}{100} \times 200 = 34_{10}$

$\frac{8}{100} \times 200 = 16_{10}$

$7 \mid 18$
$7 \mid 2 - 4$
$0 - 2 \quad = 24_7$

### Conclusion

(a) Bus = $302_4$
Car = $1100_4$
(b) $1100_4 + 302_4 = 2002_4$
(c) Difference in number of pupils between those who walk to school and those who go to school by motorcycle = $24_7$

### Checking Answer

$80 + 50 = 130_{10}$

$4 \mid 130$
$4 \mid 32 - 2$
$4 \mid 8 - 0$
$4 \mid 2 - 0$
$0 - 2 \quad = 2002_4$

## Self Practice 2.1d

1. A piece of rope is cut into 4 equal parts. Calculate the value of $x$.

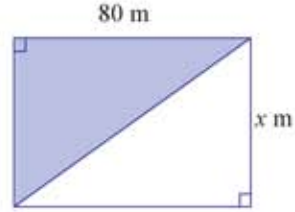

2. Alex Wong's marks in History, Mathematics and Bahasa Melayu are $82_9$, $234_5$ and $11001_2$ respectively.
   (a) What is the total mark obtained by Alex Wong in base ten?
   (b) What is the difference in marks between Bahasa Melayu and Mathematics? Write the answer in base five.

3. A watch store gives discounts for several brands of ladies' wrist watches in conjunction with Mother's Day. After the discount, Madam Santhi buys a watch of brand $M$ which costs $RM134_5$ and Puan Aminah buys a watch of brand $N$ which costs $RM50_7$. Who gets a higher discount?

Brand $M$ RM80

Brand $N$ RM70

4. Ashri Satem, a residential developer, wants to build a rectangular playground. The playground is divided into two sections, the coloured section is the children's playground and the blank section is the exercise area for adults. If the perimeter of the playground is $3300_4$ metres, calculate the area of the children's playground.




## Comprehensive Practice

1. Complete the following number sequences in ascending order.
   (a) $234_5$, ____, ____, ____.
   (b) $101_2$, ____, ____, ____.
   (c) $30_7$, ____, ____, ____.

2. State the value of digit 2 in $3\underline{2}40_4$.

3. (a) Convert $111001110_2$ to base eight.
   (b) Convert $367_8$ to base two.

4. Convert $287_9$ to numbers in the following bases.
   (a) Base two (b) Base five
   (c) Base seven (d) Base eight

5. Calculate the value of the following.
   (a) $111_2 + 1110_2$ (b) $140_7 + 302_7$ (c) $275_9 - 218_9$

**6.** Determine whether each of the following statements is true or false.

(a) $123_4 > 23_8$ (b) $72_9 < 341_5$ (c) $452_6 > 334_8$

**7.** Calculate the value of $110_2 + 278_9 + 132_4$ in base ten.

**8.** Calculate the value of $231_4 - 110_2$ in base ten.

**9.** Given $1122_3 + y_7 = 118_9$, calculate the value of $y$.

**10.** Fill in the blanks with numbers in ascending order according to the given bases.

(a) $63_8$, $110100_2$, $\square\square_8$, $\square\square\square\square\square\square_2$, $67_8$.

(b) $124_{10}$, $148_9$, $\square\square\square_8$, $241_7$, $\square\square\square\square_5$.

**11.** The total number of visitors to Zoo Negara in January is $20202_5$. What is the average number of visitors to Zoo Negara each day in January, if it opens every day? Give the answer in base five.


**12.** Azizan buys a bag filled with 100 small balls for his son. $\frac{1}{5}$ of the balls are green, $\frac{1}{4}$ are yellow, $\frac{1}{10}$ are pink and the remainder is blue. What is the number of blue balls in base eight?

**13.** Kishendran buys a batik shirt at a discount of 25%. The original price of the shirt is $RM220_4$. What is the price of the shirt after discount in base seven?

**PROJECT**

Do you know that besides numbers in base two up to base ten, there are other number bases such as numbers in base sixteen, base thirty-two and base sixty-four? By surfing the Internet, explain the usage of these bases in the field of science and information technology by giving suitable examples.

## CONCEPT MAP

**Number Bases**

### Conversion of base

Convert a number in base ten to another base and vice versa.

| Place value | $6^2$ | $6^1$ | $6^0$ |
|---|---|---|---|
| Number in base 6 | 2 | 5 | 3 |
| Value of number in base 10 | $(2 \times 6^2) + (5 \times 6^1) + (3 \times 6^0)$ $= 105_{10}$ | | |

$$\begin{array}{r|l} 9 & 105 \\ 9 & 11 \quad -6 \\ 9 & 1 \quad -2 \\ & 0 \quad -1 \end{array}$$

$253_6 = 105_{10} = 126_9$

Convert a number in base two to base eight and vice versa.

| Number in base 2 | 1 | 1 | 0 | 1 | 1 | 1 |
|---|---|---|---|---|---|---|
| Place value | $2^2$ | $2^1$ | $2^0$ | $2^2$ | $2^1$ | $2^0$ |
| Digit value | 4 | 2 | 0 | 4 | 2 | 1 |
| Base 8 | $4 + 2 + 0 = 6$ | | | $4 + 2 + 1 = 7$ | | |
| | $67_8$ | | | | | |

$110111_2 = 67_8$

| Base 8 | 5 | | | 1 | | | 7 | | |
|---|---|---|---|---|---|---|---|---|---|
| | $4 + 1$ | | | 1 | | | $4 + 2 + 1$ | | |
| Place value | $2^2$ | $2^1$ | $2^0$ | $2^2$ | $2^1$ | $2^0$ | $2^2$ | $2^1$ | $2^0$ |
| Base 2 | 1 | 0 | 1 | 0 | 0 | 1 | 1 | 1 | 1 |
| | $101001111_2$ | | | | | | | | |

$517_8 = 101001111_2$

### Computation involving addition and subtraction

Vertical Form

$$\begin{array}{r} {\scriptstyle 1} \\ 1\,1\,0_2 \\ +\ 1\,1\,1_2 \\ \hline 1\,1\,0\,1_2 \end{array}$$

$0 + 1 = 1_{10} = 1_2$
$1 + 1 = 2_{10} = 10_2$
$1 + 1 + 1 = 3_{10} = 11_2$

Conversion of Base

$110_2 \rightarrow 6_{10}$
$111_2 \rightarrow +\ 7_{10}$
$13_{10}$

$$\begin{array}{r|l} 2 & 13 \\ 2 & 6 \quad -1 \\ 2 & 3 \quad -0 \\ 2 & 1 \quad -1 \\ & 0 \quad -1 \end{array}$$

$110_2 + 111_2 = 1101_2$

Vertical Form

$$\begin{array}{r} {\scriptstyle 5\ 7\ 3\ 7} \\ 6\,2\,\not{4}\,1_7 \\ -\ \ 6\,1\,3_7 \\ \hline 5\,3\,2\,5_7 \end{array}$$

$7 + 1 - 3 = 5_7$
$3 - 1 = 2_7$
$7 + 2 - 6 = 3_7$

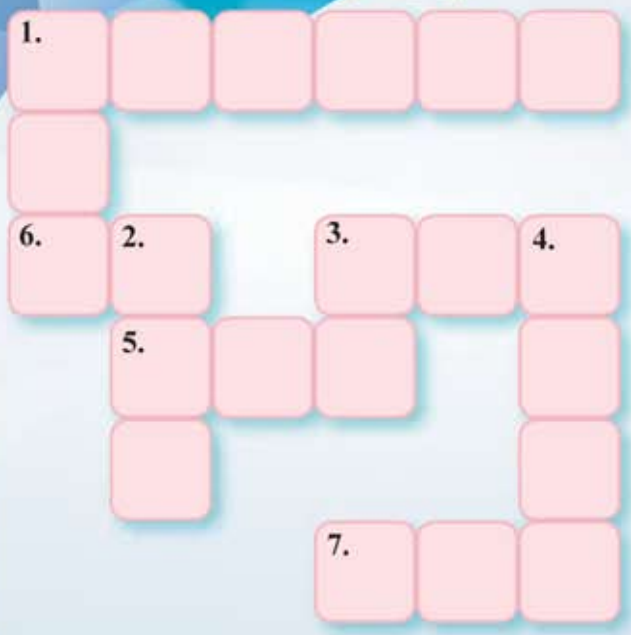
Conversion of Base

$6241_7 \rightarrow 2185_{10}$
$613_7 \rightarrow -\ 304_{10}$
$1881_{10}$

$$\begin{array}{r|l} 7 & 1881 \\ 7 & 268 \quad -5 \\ 7 & 38 \quad -2 \\ 7 & 5 \quad -3 \\ & 0 \quad -5 \end{array}$$

$6241_7 - 613_7 = 5325_7$

## Self Reflection




**Across**

1. Convert $65_8$ to base two.
3. Convert $43_{10}$ to base four.
5. Convert $633_7$ to base ten.
6. Calculate $132_4 + 110_2$ and give the answer in base ten.
7. Calculate $2220_3 + 211_3$ and give the answer in base ten.

**Down**

1. Calculate $461_7 - 141_5$ and give the answer in base ten.
2. Convert $110011110_2$ to base eight.
3. Calculate $10101_2 + 111_2$ and give the answer in base ten.
4. Convert $400_{10}$ to base five.

Scan the QR Code to carry out this activity.
*http://bt.sasbadi.com/m4053*

## Mathematics Exploration

Each number in base ten can be written as a number in another base. Prepare a chart of various number bases as shown below.
What is the relationship between the number bases given?

| Base 10 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | ... | 50 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Base 2 | | | | | | | | | | | | | | | | | | | | |
| Base 4 | | | | | | | | | | | | | | | | | | | | |
| Base 8 | | | | | | | | | | | | | | | | | | | | |
| Base 16 | | | | | | | | | | | | | | | | | | | | |
| Base 32 | | | | | | | | | | | | | | | | | | | | |