3

Logical Reasoning

You will learn ✏️

- ▶ Statements
- ▶ Arguments

The Parliament is the legislative body of the Federal government. All acts of law must be debated and approved by the House of Representatives and the Senate before the approval of the Yang di-Pertuan Agong.

Do you know why every act of law must be debated in the Parliament?

Why Study This Chapter? 🔍

Apart from debate, logical reasoning is often applied in the electrical engineering field especially in electronic circuit. It also plays an important role in computer programming and computer hardware design. In our daily lives, logical reasoning is applied in the problem solving process because it enhances our critical and analytical thinking skills.

WORD BANK

• converse	• *akas*
• deductive	• *deduktif*
• argument	• *hujah*
• implication	• *implikasi*
• inductive	• *induktif*
• contrapositive	• *kontrapositif*
• negation	• *penafian*
• quantifier	• *pengkuantiti*
• statement	• *pernyataan*
• compound statement	• *pernyataan majmuk*
• inverse	• *songsangan*

📷 **Walking Through** Time

Aristotle
(384 BC – 322 BC)

According to Aristotle, logic is not a part of Philosophy but it is just an instrument used by philosophers and scientists. Aristotle used logic as a technique for debating and linguistics knowledge.
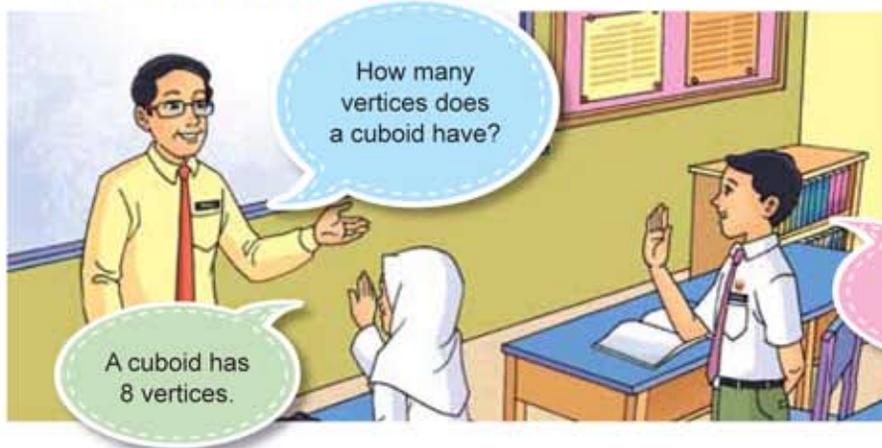
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3.1 Statements

Q What does a statement mean and how do you determine the truth value of a statement?

How many vertices does a cuboid have?

A cuboid has 12 vertices.

A cuboid has 8 vertices.

Most of the mathematical results involve statements.

> A statement is a sentence of which the truth value can be determined, that is either true or false, but not both.

Question, exclamation and command sentences are not statements. The truth values of these sentences cannot be determined.

Both answers given in the above dialogue are statements. The statement of "A cuboid has 8 vertices" is a true statement while the statement of "A cuboid has 12 vertices" is a false statement. Statements can be divided into true statements and false statements.

Example 1

Determine whether each sentence below is a statement or not a statement. Justify your answers.

(a) Please send the workbook.
(b) Kuala Lumpur Tower is the tallest tower in Malaysia.
(c) How do you come to school?
(d) $x + 3 = 5$.
(e) $-6 < -8$.

Malaysiaku

The Kuala Lumpur Tower is located at the peak of Bukit Nanas. This tower with a height of 421 metres is the 7th tallest communication tower in the world and the tallest in Southeast Asia.

Solution:

(a) Not a statement because the truth value cannot be determined.

(b) A statement because it is true.

(c) Not a statement because the truth value cannot be determined.

(d) Not a statement because the truth value cannot be determined.

(e) A statement because it is false.

Mind Stimulation 1

Aim: To determine the truth values of the given statements.

Steps:

1. Divide the class into groups.
2. State whether the mathematical sentences provided in the Activity Sheet are true statements or false statements with Round Robin.
3. Discuss and draw conclusions from the findings of this activity.

Activity Sheet:

	Mathematical sentence	Truth value
(a)	$28 + 12 = 40$	
(b)	$3^2 + 4^2 = 7^2$	
(c)	$(2 + 3)(2 - 3) = 2^2 - 3^2$	
(d)	$\sqrt{729} = 81^{\frac{3}{2}}$	
(e)	$(x - y)^2 = x^2 - 2xy + y^2$	
(f)	$\{a, b\}$ has 4 subsets.	
(g)	5 is a factor of 400.	
(h)	The lowest common multiple of the numbers 4 and 18 is 36.	

Discussion:

Are all the mathematical statements true? Discuss your reasons.

From the activity in Mind Stimulation 1, it is found that:

> Not all the mathematical statements are true. The truth values of the mathematical statements can be determined.

Example 2

Determine whether the following statements are true or false. If it is false, prove it.

(a) $(x + y)^2 = x^2 + 2xy - y^2$, $x \neq 0$, $y \neq 0$
(b) $(x + 5)^2 < 0$, $x \in \mathbf{R}$
(c) $2 + 6(4) > 4 + 6(2)$
(d) $2 \in \{\text{Factors of } 8\}$
(e) $\{2, 5, 7\} \cup \{\text{Prime numbers}\} = \{2, 5, 7\}$

INFO ZONE

$x \in \mathbf{R}$ means x is an element of the real numbers.

- Real numbers can be defined as any rational numbers or irrational numbers.

Solution:

(a) False Let $x = 2$ and $y = 3$.
$(x + y)^2 = (2 + 3)^2$
$= 25$
$x^2 + 2xy - y^2 = 2^2 + 2(2)(3) - 3^2$
$= 7$
Hence, $(x + y)^2 \neq x^2 + 2xy - y^2$

(b) False $(2 + 5)^2 = 49 > 0$
(c) True
(d) True
(e) False
$\{2, 5, 7\} \cup \{2, 3, 5, 7, 11, 13, ...\} = \{\text{Prime numbers}\}$

Example 3

Determine whether the following mathematics statements are true or false. Explain if the statement is false.

(a) All polygons have diagonals.

(b) Some perfect squares are whole numbers.

(c) All even numbers have prime factors.

(d) Some straight lines intersect the y-axis.

Solution:

(a) False. Triangles do not have diagonals.

(b) False. All perfect squares are whole numbers.

(c) True

(d) True

Self Practice 3.1a

1. Determine whether each sentence below is a statement or not a statement. Justify your answers.

   (a) Let's play in the field.

   (b) Malaysia is located in Asian continent.

   (c) Is $3 + 2 = 8$?

   (d) $x + 3 > x - 8$

   (e) $3x + 5 = -7$

Asian Continent

2. Construct a true statement by using the given digits and symbols.

   (a) 23, +, 9, 40, >

   (b) {3, 6, 9}, {3}, ⊂

   (c) $\dfrac{5}{6}$, $\dfrac{1}{4}$, ×, $\dfrac{10}{3}$, =

   (d) $x^2 + 3$, ⩽, $(x + 3)^2$

   (e) $\sqrt[3]{}$, 9, 27, 12, =, +

3. Determine whether the following statements are true or false.

   (a) All quadrilaterals have right angles.

   (b) Some rhombuses have four equal sides.

   (c) All triangles have equal sides.

   (d) Some polygons have five sides.

   (e) All circles can be divided equally into eight sectors.

Q How do you negate a statement?

We use the word "no" or "not" to negate a statement. The negation of statement p is written as ~p.

Indicator

'~p' is read as 'tilde p'.

Example 4

Form a negation (~p) for each of the following statements (p) by using the word "no" or "not".

(a) 12 is a multiple of 5.

(b) 41 is a prime number.

(c) All multiples of 5 are multiples of 10.

(d) 0.4 m is equal to 400 mm.

Solution:

(a) 12 is not a multiple of 5.

(b) 41 is not a prime number.

(c) Not all multiples of 5 are multiples of 10.

(d) 0.4 m is not equal to 400 mm.

Mind Stimulation 2

Aim: To determine the truth value of a statement after the negation.

Steps:

1. Divide the class into groups.

2. Determine the truth values of the following statements.

> (a) All even numbers are divisible by 2.
> (b) All factors of 6 are factors of 8.
> (c) 64 is a perfect cube number.
> (d) 50% of 600 is 30.
> (e) $\{a, b, c\}$ is a subset of $\{a, b\} \cap \{b, c\}$.

3. Form a negation for each statement in Step 2 by using the word "no" or "not".

4. Determine the truth values of the negation statements in Step 3.

Discussion:

What can you say about the truth values of the statements in Step 2 before and after the negation?

From the activity in Mind Stimulation 2, it is found that:

> The truth values change from true to false or vice versa through the process of negation.

Self Practice 3.1b

Form a negation (~p) for each of the following statements (p) by using the word "no" or "not". Then, determine the truth values of the negations.

1. 819 is a multiple of 9.

2. A kite has two axes of symmetry.

3. A cone has one curved surface.

4. Two parallel lines have the same gradient.

5. All quadratic equations have two equal roots.

🔍 How do you determine the truth value of a compound statement?

A compound statement is a combination of two or more statements by using the word "and" or "or".

Learning Standard

Determine the truth value of a compound statement.

Example 5

Combine the following statements, p and q, by using the words
(i) and,   (ii) or.

    (a) p : A pentagon has two diagonals.
        q : A heptagon has four diagonals.

    (b) p : A pyramid has five planes.
        q : A pyramid has five vertices.

    (c) p : -4 is an integer.
        q : 2 is an integer.

Solution:

(a) (i)   A pentagon has two diagonals and a heptagon has four diagonals.
    (ii) A pentagon has two diagonals or a heptagon has four diagonals.

(b) (i)   A pyramid has five planes and five vertices.
    (ii) A pyramid has five planes or five vertices.

(c) (i) -4 and 2 are integers.
    (ii) -4 or 2 is an integer.

Example 6

Determine the two statements, p and q, in the following compound statements.

(a) $5 + 3 > 5$ and $5 - 3 < 5$.

(b) 9 and 91 are prime numbers.

(c) $2^2 = 4$ or $2^3 = 8$.

(d) $-9 < 10$ or $9 < 10$.

Solution:

(a) $p : 5 + 3 > 5.$
$\quad q : 5 - 3 < 5.$

(b) $p :$ 9 is a prime number.
$\quad q :$ 91 is a prime number.

(c) $p : 2^2 = 4.$
$\quad q : 2^3 = 8.$

(d) $p : -9 < 10.$
$\quad q : 9 < 10.$

The word "and" in a mathematical statement means both while the word "or" means one of them or both.

Based on the picture on the right, three statements p, q and r are formed.

p : Rashid is running.

q : Kok Keong is running.

r : Melinda is running.

You can combine two statements by using the word "and" or "or".

p and q : Rashid and Kok Keong are running.

p and r : Rashid and Melinda are running.

q or r : Kok Keong or Melinda is running.

From the three compound statements above, we notice that the statement "p and q" is true because both Rashid and Kok Keong are running but "p and r" is false because not both Rashid and Melinda are running. However, the compound statement "q or r" is true because part of the statement is true.
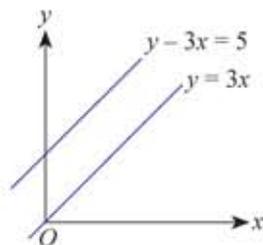
Hence, the truth value of a compound statement can be concluded as shown in the truth table below.

p	q	p and q	p or q
True	True	True	True
True	False	False	True
False	True	False	True
False	False	False	False

Example | **7**

Determine the truth values of the following compound statements.

(a) 2 and –5 are greater than 4.

(b) $x + 3 < x - 5$ and 99 is an odd number.

(c) 81 is a perfect square and 6 is a factor of 18.

(d) $y = 3x$ is parallel to $y - 3x = 5$ and the y-intercept of the straight line $y - 3x = 5$ is 3.

(e) $2 + 3 = 23$ or $2 \times 3 = 23$.

(f) $5 \times 5 \times 5 = 5^3$ or $\sqrt{125} = 5$.

(g) The sum of interior angles of a triangle or a quadrilateral is 360°.

(h) $4 - (-7) = 11$ or $4 + 7 = 11$.

Solution:

		Statement	Truth value
(a)	p	2 is greater than 4.	False
	q	–5 is greater than 4.	False
	p and q	2 and –5 are greater than 4.	False
(b)	p	$x + 3 < x - 5$.	False
	q	99 is an odd number.	True
	p and q	$x + 3 < x - 5$ and 99 is an odd number.	False
(c)	p	81 is a perfect square.	True
	q	6 is a factor of 18.	True
	p and q	81 is a perfect square and 6 is a factor of 18.	True
(d)	p	$y = 3x$ is parallel to $y - 3x = 5$.	True
	q	The y-intercept of the straight line $y - 3x = 5$ is 3.	False
	p and q	$y = 3x$ is parallel to $y - 3x = 5$ and the y-intercept of the straight line $y - 3x = 5$ is 3.	False
(e)	p	$2 + 3 = 23$.	False
	q	$2 \times 3 = 23$.	False
	p or q	$2 + 3 = 23$ or $2 \times 3 = 23$.	False
(f)	p	$5 \times 5 \times 5 = 5^3$.	True
	q	$\sqrt{125} = 5$.	False
	p or q	$5 \times 5 \times 5 = 5^3$ or $\sqrt{125} = 5$.	True
(g)	p	The sum of interior angles of a triangle is 360°.	False
	q	The sum of interior angles of a quadrilateral is 360°.	True
	p or q	The sum of interior angles of a triangle or a quadrilateral is 360°.	True
(h)	p	$4 - (-7) = 11$.	True
	q	$4 + 7 = 11$.	True
	p or q	$4 - (-7) = 11$ or $4 + 7 = 11$.	True

CHAPTER 3

Self Practice 3.1c

1. Combine the following statements p and q by using the words given in the brackets to form compound statements.

   (a) p : 2 is a prime factor of 6.   (or)
      q : 3 is a prime factor of 6.

   (b) p : A cone has one vertex.   (and)
      q : A cone has one plane.

   (c) p : A rhombus is a parallelogram.   (and)
      q : A trapezium is a parallelogram.

2. Determine the truth values of the following compound statements.

   (a) 49 is a multiple of 7 and a perfect square.
   (b) 3 hours = 120 minutes and 4 minutes = 240 seconds.
   (c) The coefficient of $9x$ is 9 and $9^0 = 0$.
   (d) $3 \in \{1, 2, 5\}$ and $\{8, 9\} \subset \{6, 7, 8\}$.
   (e) $\frac{2}{9}$ can be expressed as a recurring decimal or less than 1.
   (f) $\frac{4}{5}$ or $\frac{5}{4}$ is a proper fraction.
   (g) 6 or 8 is an odd number.
   (h) $\sqrt[4]{64} = 2$ or $2^3 = 8$.

How do you construct a statement in the form of an implication?

Learning Standard

Construct statement in the form of implication
(i) if p, then q
(ii) p if and only if q

Teacher, can I play football on the field?

If you can finish answering all the questions, then you can play.

Implication "If p, then q"

A statement "if p, then q" is known as an **implication** where
- p is denoted as the **antecedent**.
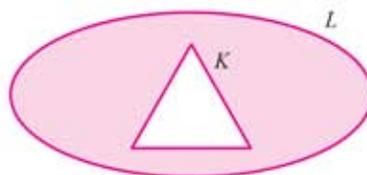- q is denoted as the **consequent**.

Example 8

Form an implication "if p, then q" with the given antecedent and consequent.

(a) Antecedent : k is divisible by 5.
Consequent : k is a multiple of 5.

(b) Antecedent : Set K is a subset of set L.
Consequent : $n(K) \leqslant n(L)$.

Solution:

(a) If k is divisible by 5, then k is a multiple of 5.

(b) If set K is a subset of set L, then $n(K) \leqslant n(L)$.

Example 9

Determine the antecedent and consequent for the following implications "if p, then q".

(a) If x is a factor of 16, then x is a factor of 64.

(b) If $x - y > 0$, then $x > y$.

Solution:

(a) Antecedent : x is a factor of 16.
Consequent : x is a factor of 64.

(b) Antecedent : $x - y > 0$.
Consequent : $x > y$.

Implication "p if and only if q"

Apart from the implication "if p, then q", the implication "p if and only if q" is also used frequently in logical reasoning.

> An implication "p if and only if q" consists of the following two implications:
> - if p, then q
> - if q, then p

Example 10

Form an implication "p if and only if q" with the following implications.

(a) If k is a prime number, then k has only two factors.
If k has only two factors, then k is a prime number.

(b) If $y = ax^n + b$ is a linear equation, then $n = 1$.
If $n = 1$, then $y = ax^n + b$ is a linear equation.

Solution:

(a) k is a prime number if and only if k has only two factors.

(b) $y = ax^n + b$ is a linear equation if and only if $n = 1$.

Example 11

Write two implications based on the implication "p if and only if q" given below.

(a) $\sqrt{r} = 15$ if and only if $r = 225$.

(b) $x < 6$ if and only if $x + 10 < 16$.

Solution:

(a) Implication 1: If $\sqrt{r} = 15$, then $r = 225$.
Implication 2: If $r = 225$, then $\sqrt{r} = 15$.

(b) Implication 1: If $x < 6$, then $x + 10 < 16$.
Implication 2: If $x + 10 < 16$, then $x < 6$.

Self Practice 3.1d

1. Form an implication "if p, then q" with the given antecedent and consequent.

   (a) Antecedent : $x = 3$
   Consequent : $x^4 = 81$

   (b) Antecedent : $ax^3 + bx^2 + cx + d = 0$ is a cubic equation.
   Consequent : $a \neq 0$

   (c) Antecedent : $n - 5 > 2n$
   Consequent : $n < -5$

   (d) Antecedent : $\dfrac{m}{n} > 1$
   Consequent : $m^2 > n^2$

2. Determine the antecedent and consequent for the following implications "if p, then q".

   (a) If x is an even number, then x^2 is an even number.

   (b) If set $K = \phi$, then $n(K) = 0$.

   (c) If x is a whole number, then $2x$ is an even number.

   (d) If a straight line AB is a tangent to a circle P, then the straight line AB touches the circle P at one point only.

3. Form an implication "p if and only if q" with the following implications.

   (a) If k is a perfect square, then $\sqrt{k}$ is a whole number.
   If $\sqrt{k}$ is a whole number, then k is a perfect square.

   (b) If $P \cap Q = P$, then $P \subset Q$.
   If $P \subset Q$, then $P \cap Q = P$.

   (c) If $pq = 1$, then $p = q^{-1}$ and $q = p^{-1}$.
   If $p = q^{-1}$ and $q = p^{-1}$, then $pq = 1$.

   (d) If $k^2 = 4$, then $(k + 2)(k - 2) = 0$.
   If $(k + 2)(k - 2) = 0$, then $k^2 = 4$.

4. Write two implications based on the implication "p if and only if q" given below.

   (a) PQR is a regular polygon if and only if $PQ = QR = PR$.

   (b) $\dfrac{m}{n}$ is an improper fraction if and only if $m > n$.

   (c) 9 is the y-intercept of a straight line $y = mx + c$ if and only if $c = 9$.

   (d) $f(x) = ax^2 + bx + c$ has a maximum point if and only if $a < 0$.

How do you construct and compare the truth value of converse, inverse and contrapositive of an implication?

Before comparing the truth value of converse, inverse and contrapositive of an implication, observe and state the differences between four statements in the following example.

Statement : If Ali is a prefect, then Ali is a disciplined person.
Converse : If Ali is a disciplined person, then Ali is a prefect.
Inverse : If Ali is not a prefect, then Ali is not a disciplined person.
Contrapositive : If Ali is not a disciplined person, then Ali is not a prefect.

In general,

Statement : If p, then q.
Converse : If q, then p.
Inverse : If $\sim p$, then $\sim q$.
Contrapositive : If $\sim q$, then $\sim p$.

Example 12

Write the converse, inverse and contrapositive of the following implications.

(a) If x is a positive number, then x is greater than 0.

(b) If $p^2 - q^2 > 0$, then $(p + q)(p - q) > 0$.

(c) If $x = 5$, then $x + 1 = 6$.

Solution:

(a) Statement : If x is a positive number, then x is greater than 0.
Converse : If x is greater than 0, then x is a positive number.
Inverse : If x is not a positive number, then x is not greater than 0.
Contrapositive : If x is not greater than 0, then x is not a positive number.

(b) Statement : If $p^2 - q^2 > 0$, then $(p + q)(p - q) > 0$.
Converse : If $(p + q)(p - q) > 0$, then $p^2 - q^2 > 0$.
Inverse : If $p^2 - q^2 \leqslant 0$, then $(p + q)(p - q) \leqslant 0$.
Contrapositive : If $(p + q)(p - q) \leqslant 0$, then $p^2 - q^2 \leqslant 0$.

TIPS
$\sim p$ is a complement of p. Then, the complement of $p^2 - q^2 > 0$ is $p^2 - q^2 \leqslant 0$.

(c) Statement : If $x = 5$, then $x + 1 = 6$.
Converse : If $x + 1 = 6$, then $x = 5$.
Inverse : If $x \neq 5$, then $x + 1 \neq 6$.
Contrapositive : If $x + 1 \neq 6$, then $x \neq 5$.

After identifying the converse, inverse and contrapositive of an implication, you will evaluate the truth of the statements mentioned earlier.

Mind Stimulation 3

Aim: To compare the truth values of converse, inverse and contrapositive of an implication.

Steps:

1. Determine the truth values for the statements p and q of each implication in the Activity Sheet.

2. Write the converse, inverse and contrapositive of an implication "if p, then q". Then, determine the truth values of the statements.

Activity Sheet

			Truth value
(a)	p:	12 is an even number.	True
	q:	12 is divisible by 2.	True
	Statement:	If 12 is an even number, then 12 is divisible by 2.	True
	Converse:	☐	☐
	Inverse:	☐	☐
	Contrapositive:	☐	☐
(b)	p:	50 is a multiple of 10.	☐
	q:	50 is a multiple of 20.	☐
	Statement:	☐	☐
	Converse:	If 50 is a multiple of 20, then 50 is a multiple of 10.	True
	Inverse:	☐	☐
	Contrapositive:	☐	☐
(c)	p:	The sum of interior angles in pentagon $PQRST$ is 360°.	☐
	q:	Pentagon $PQRST$ is a quadrilateral.	☐
	Statement:	☐	☐
	Converse:	☐	☐
	Inverse:	If the sum of interior angles in pentagon $PQRST$ is not 360°, then pentagon $PQRST$ is not a quadrilateral.	True
	Contrapositive:	☐	
(d)	p:	$x^2 < 0$.	☐
	q:	$x^2 + 3 > 0$.	☐
	Statement:	☐	☐
	Converse:	☐	☐
	Inverse:	☐	
	Contrapositive:	If $x^2 + 3 \leqslant 0$, then $x^2 \geqslant 0$.	True

Discussion:

1. Compare the truth value of a contrapositive and the truth value of an implication "if p, then q".
2. What is the relationship between the converse and inverse of an implication "if p, then q"?
3. What is the difference of the truth values between the converse and inverse of an implication "if p, then q"?

From the activity in Mind Stimulation 3, it is found that:

> 1. The truth value of contrapositive is the same as the truth value of an implication "if p, then q".
> 2. The converse and inverse are contrapositive to each other.
> 3. The converse and inverse also have the same truth value.

Hence, you can list the truth values of an implication "if p, then q", and its corresponding converse, inverse and contrapositive in the table below.

| p | q | Statement | Converse | Inverse | Contrapositive |
		If p, then q.	If q, then p.	If ~p, then ~q.	If ~q, then ~p.
True	True	True	True	True	True
True	False	False	True	True	False
False	True	True	False	False	True
False	False	True	True	True	True

In conclusion,

> The truth value of an implication "if p, then q" is always true except when p is true and q is false happen at the same time. If an antecedent is false, then the implication "if p, then q" is always true without depending on the truth value of the consequent.

Example 13

Determine the truth values of statement, converse, inverse and contrapositive of the implication
"If $2 \times 3 = 6$, then $8 - 2 \times 3 = 18$."

Solution:

		Antecedent	Consequent	Truth value
Statement:	If $2 \times 3 = 6$, then $8 - 2 \times 3 = 18$.	True	False	False
Converse:	If $8 - 2 \times 3 = 18$, then $2 \times 3 = 6$.	False	True	True
Inverse:	If $2 \times 3 \neq 6$, then $8 - 2 \times 3 \neq 18$.	False	True	True
Contrapositive:	If $8 - 2 \times 3 \neq 18$, then $2 \times 3 \neq 6$.	True	False	False

Self Practice 3.1e

1. Write the converse, inverse and contrapositive of the following implications.

   (a) If $x + 3 > 2$, then $x > -1$.

   (b) If $(k - 3)(k + 4) = 0$, then $k = 3$ or $k = -4$.

   (c) If $ABCD$ is a parallelogram, then AB is parallel to CD.

2. Determine the truth values of implication, converse, inverse and contrapositive for each of the following statements.

   (a) If 2 and 5 are factors of 10, then 2×5 is 10.

   (b) If 4 is a root of $x^2 - 16 = 0$, then 4 is not a root of $(x + 4)(x - 4) = 0$.

   (c) If a rectangle has four axes of symmetry, then the rectangle has four sides.

   (d) If $55 + 55 = 4 \times 5$, then $666 + 666 = 6 \times 6$.

🔍 How do you determine a counter-example to negate the truth of a particular statement?

For a false statement, at least one counter-example can be given to negate the truth of that statement. For example, the statement "All polygons have two or more diagonals." is false as a triangle does not have a diagonal. The triangle is a counter-example to support the false value.

Learning Standard

Determine a counter-example to negate the truth of a particular statement.

Example 14

Determine the truth value of the following mathematical statements. If it is false, give one counter-example to support your answer.

(a) The sum of interior angles of all polygons is $180°$.

(b) Some prime numbers are even numbers.

(c) 4 and 8 are the factors of 20.

(d) 6 or 36 is a multiple of 9.

Solution:

(a) False because the sum of interior angles of a pentagon is $540°$.

(b) True.

(c) False because 8 is not a factor of 20.

(d) True.

Example 15

Write the mathematical statement requested in the brackets for each of the following. Then, determine the truth value of each statement written. If it is false, give one counter-example to support your answer.

(a) $6 \in \{3, 6, 9\}$. (Negation)

(b) All multiples of 10 are multiples of 2. (Negation)

(c) If $x > 5$, then $x > 3$. (Converse)

(d) If x is a root of $x^3 - 1 = 0$, then $x = 1$. (Inverse)

(e) If $k^2 > 0$, then $k > 0$. (Contrapositive)

Solution:

(a) Negation: $6 \notin \{3, 6, 9\}$. False because 6 is an element of $\{3, 6, 9\}$.

(b) Negation: Not all multiples of 10 are multiples of 2. False because all the multiples of 10 are divisible by 2.

(c) Converse: If $x > 3$, then $x > 5$. False because $4 > 3$ but $4 < 5$.

(d) Inverse: If x is not a root of $x^3 - 1 = 0$, then $x \neq 1$. True.

(e) Contrapositive: If $k \leqslant 0$, then $k^2 \leqslant 0$. False because $-2 < 0$ but $(-2)^2 = 4 > 0$.

Self Practice 3.1f

1. Determine the truth values of the following mathematical statements. If it is false, give one counter-example to support your answer.

   (a) All rectangles are squares.

   (b) Some perfect squares are divisible by 5.

   (c) 5 or 9 have two factors.

   (d) 36 is a multiple of 4 and 14.

2. Write the mathematical statement requested in the brackets for each of the following. Then, determine the truth value of each statement written. If it is false, give one counter-example to support your answer.

   (a) $100_8 - 77_8 = 1_8$ (Negation)

   (b) A cuboid has four uniform cross sections. (Negation)

   (c) If $y = 2x$ is parallel to $y = 2x - 1$, then $y = 2x$ and $y = 2x - 1$ have the same gradient. (Converse)

   (d) If a triangle ABC has a right angle at C, then $c^2 = a^2 + b^2$. (Inverse)

   (e) If $w < 7$, then $w < 5$. (Contrapositive)