CHAPTER 3

Squares, Square Roots, Cubes and Cube Roots

What will you learn?

- Squares and Square Roots
- Cubes and Cube Roots

Why study this chapter?

As the basic knowledge in fields that require the concepts of the area of squares and the volume of cubes. Discuss the fields that involve both the concepts mentioned.

The national silat squad had won three gold medals in the 2015 SEA Games. The squad had shown incredible performances when they defeated the competitors from the other countries. In the silat competition, our team performed on a square court with an area of 100 m^2. How do you determine the length of the sides of the court?

Scientists use the idea of volume to explain the structure of some cube-shaped crystals. What is the relationship between the length of the sides of a salt crystal and its volume?

Walking through Time

Christoff Rudolff

René Descartes

In the year 1637, the symbols of squares and cubes were used by a French mathematician, René Descartes, in his book, *Geometrie*. The symbols of square roots and cube roots were introduced by a German mathematician, Christoff Rudolff, in the year 1525 in his book *Die Coss*.

For more information:

http://goo.gl/fBrPNI http://goo.gl/9flVlm

Word Link

• estimation	• *anggaran*
• square	• *kuasa dua*
• perfect square	• *kuasa dua sempurna*
• cube	• *kuasa tiga*
• perfect cube	• *kuasa tiga sempurna*
• square root	• *punca kuasa dua*
• cube root	• *punca kuasa tiga*

Open the folder downloaded from page vii for the audio of Word Link.

3.1 Squares and Square Roots

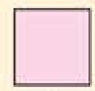
What are squares and perfect squares?

About 2 500 years ago, a group of academics created patterns of numbers in the shape of squares by arranging pebbles in equal number of rows and columns. Are you able to determine the next pattern of square?

LEARNING STANDARDS

Explain the meaning of squares and perfect squares.

Flashback



The area of a square with sides of 1 unit is $1 \times 1 = 1$ unit^2.

Exploration Activity 1

Aim: To explore the formation of squares.

Instruction:
- Explore by yourself before the lesson begins and discuss in groups of four during the lesson.
- Open the folder downloaded from page vii.

1. Open the file *squares.ggb* using *GeoGebra*. The screen displayed shows a square with sides of 1 unit.

2. Click and drag the blue slider on the screen displayed to change the length of the sides of the square and determine the corresponding area of each square.
3. Copy and complete the table below for the area of the squares.

Length of the sides of a square (unit)	Area of the square in the form of repeated multiplication (unit^2)	Area (unit^2)
1	1×1	

4. What is the relationship between the area of the square and the length of the sides of the square?

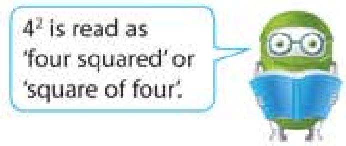
From the results of Exploration Activity 1, it is found that a square with

a side length of (unit)	1,	2,	3,	4,	…
has an area of (unit^2)	1,	4,	9,	16,	…

For example, for a square with a side length of 4 units,

$$\begin{aligned} \text{area} &= 4 \times 4 \\ &= 16 \text{ unit}^2 \end{aligned}$$

We can state that the square of 4 is 16.
The square of 4 is written as 4^2.
Thus, we write $4^2 = 16$.



Exploration Activity 2

Group

Aim: To explain the meaning of perfect squares.

Instruction:
- Perform the activity in groups of four.
- Open the folder downloaded from page vii.

1. Open the file *grid.pdf* and print the file on a piece of paper.
2. Cut the grid into small pieces of paper with size 1 unit × 1 unit.
3. Arrange the pieces of paper starting from one piece, two pieces, three pieces and so on to form a square (if possible).
4. Copy and complete the table below.

Number of pieces of paper with size 1 unit × 1 unit	1	2	3	4	5	6	7	8	9	10
Can the arrangement form a square? (Mark ✓ or ✗)	✓	✗	✗							

5. Write the numbers that represent the number of pieces of paper with size 1 unit × 1 unit that can be arranged to form a square.
6. What is the relationship between the numbers that represent the number of pieces of paper and the formation of a square?

From the results of Exploration Activity 2, it is found that only a certain number of pieces of paper with size 1 unit × 1 unit can be arranged to form a square.

The numbers of pieces of paper that can form a square in this activity are 1, 4, 9, 16, …

The numbers 1, 4, 9, 16, … are known as **perfect squares**.

Let's Discuss

Open and print the file *multiplication table.xls* from the folder downloaded from page vii. Circle all the perfect squares. Discuss how the multiplication table can be used to identify other perfect squares.

Try This *State the subsequent perfect squares.*

How do you determine whether a number is a perfect square?

We can use the method of prime factorisation to determine whether a number is a perfect square.

In this method, if the prime factors can be grouped into two identical groups, then the number is a perfect square.

LEARNING STANDARDS

Determine whether a number is a perfect square.

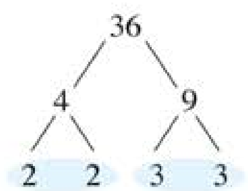
Example 1

Determine whether each of the following numbers is a perfect square.

(a) 36 (b) 54

Solution

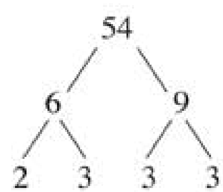
(a)



$36 = (2 \times 3) \times (2 \times 3)$ ← These prime factors can be grouped into two identical groups.

Thus, 36 is a perfect square.

(b)



$54 = 2 \times 3 \times 3 \times 3$ ← These prime factors cannot be grouped into two identical groups.

Thus, 54 is not a perfect square.

SMART TIPS

Perfect square can be written as a product of two equal factors.
For example,
$225 = 15 \times 15$ or 15^2
225 is a perfect square.

Let's Discuss

Discuss why the prime factors of a perfect square must be grouped into two identical groups.

Self Practice 3.1a

1. Determine whether each of the following numbers is a perfect square.
(a) 45 (b) 100 (c) 214 (d) 324

What is the relationship between squares and square roots?

Exploration Activity 3

LEARNING STANDARDS
State the relationship between squares and square roots.

Aim: To state the relationship between squares and square roots.

Instruction:
- Explore by yourself before the lesson begins and discuss in groups of four during the lesson.
- Open the folder downloaded from page vii.

1. Open the file *squares and square roots.ggb* using *GeoGebra*. The screen displayed shows some squares with different areas.
2. Click and drag the squares to the scale shown to determine the length of the sides of the corresponding squares.

3. Copy and complete the table below.

Area (unit^2)	1	4	9	16	25	36
Length of the sides (unit)						

4. Based on the results obtained from the table, discuss with your friends the relationship between the area of each square and the length of its sides.

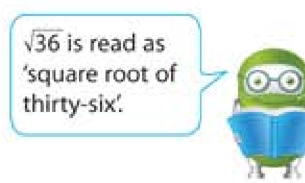
From the results of Exploration Activity 3, it is found that
a square with an area of (unit^2) 1, 4, 9, 16, 25, 36
has a side length of (unit) 1, 2, 3, 4, 5, 6
that is, the area of each square is the square of its side length.

SMART TIPS
Finding the square root of the area of a square is equivalent to finding the length of the sides of the square.

For example, for a square with an area of 36 unit^2, the length of its side is 6 units,

$$\begin{aligned} \text{area (unit}^2) &= 36 \\ &= 6 \times 6 \\ &= 6^2 \end{aligned}$$

We can state that the square of 6 is 36.
Thus, the square root of 36 is 6.
By using the symbol of square root, $\sqrt{\ }$, we write $\sqrt{36} = 6$.



Finding the square and finding the square root are inverse operations.

Example 2

Complete each of the following:

(a) $9 \times 9 = 81$
Thus $\sqrt{81} = \sqrt{\square \times \square}$
$= \square$

(b) $32^2 = 1\,024$
Thus $\sqrt{1\,024} = \sqrt{\square^{\square}}$
$= \square$

Solution

(a) $\sqrt{81} = \sqrt{9 \times 9}$
$= 9$

(b) $\sqrt{1\,024} = \sqrt{32^2}$
$= 32$

Let's Discuss

The square root of a number could be a positive or a negative value.

$(-5) \times (-5) = 25$
Is $\sqrt{25} = -5$ true?
Discuss the statement above.

Self Practice 3.1b

1. Copy and complete each of the following:
(a) $5 \times 5 = 25$
Thus,
$\sqrt{25} = \sqrt{\square \times \square}$
$= \square$
(b) $8 \times 8 = 64$
Thus,
$\sqrt{64} = \sqrt{\square \times \square}$
$= \square$
(c) $24^2 = 576$
Thus,
$\sqrt{576} = \sqrt{\square^{\square}}$
$= \square$

How do you determine the square of a number?

We can determine the square of a number by multiplying the number by itself.

LEARNING STANDARDS

Determine the square of a number with and without using technological tools.

Example 3

Find the value of each of the following without using a calculator.

(a) 6^2
(b) $\left(\frac{3}{4}\right)^2$
(c) $(-0.5)^2$

Solution

(a) $6^2 = 6 \times 6$
$= 36$

(b) $\left(\frac{3}{4}\right)^2 = \frac{3}{4} \times \frac{3}{4}$
$= \frac{9}{16}$

(c) $(-0.5)^2 = (-0.5) \times (-0.5)$
$= 0.25$

Example 4

Find the value of each of the following by using a calculator.

(a) 43^2
(b) $\left(-\frac{7}{13}\right)^2$
(c) 2.96^2

Solution

(a) $43^2 = 1\,849$ Press [4] [3] [x^2] [=]

(b) $\left(-\frac{7}{13}\right)^2 = \frac{49}{169}$ Press [(] [(–)] [7] [$a^b/_c$] [1] [3] [)] [x^2] [=]

(c) $2.96^2 = 8.7616$ Press [2] [·] [9] [6] [x^2] [=]

Think Smart

What is the largest square number that is less than 200?

Self Practice 3.1c

1. Find the value of each of the following without using a calculator.
   (a) 8^2 (b) $\left(-\frac{5}{6}\right)^2$ (c) 1.4^2

2. Find the value of each of the following by using a calculator.
   (a) 29^2 (b) $\left(\frac{9}{11}\right)^2$ (c) $(-15.3)^2$

How do you determine the square root of a number?

LEARNING STANDARDS

Determine the square roots of a number without using technological tools.

Example 5

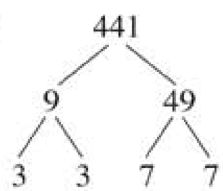
Find the value of each of the following without using a calculator.

(a) $\sqrt{64}$ (b) $\sqrt{441}$

Solution

(a) $\sqrt{64} = \sqrt{8 \times 8}$
$= 8$

(b)



$441 = 3 \times 3 \times 7 \times 7$
$= 3 \times 7 \times 3 \times 7$
$= 21 \times 21$
$\sqrt{441} = \sqrt{21 \times 21}$
$= 21$

Example 6

Find the value of each of the following without using a calculator.

(a) $\sqrt{\frac{4}{25}}$ (b) $\sqrt{2\frac{7}{9}}$ (c) $\sqrt{\frac{27}{48}}$ (d) $\sqrt{0.36}$

SMART TIPS

Prime factorisation is a more systematic method used to find the square root of a larger number.

Solution

(a) $\sqrt{\frac{4}{25}} = \sqrt{\frac{2}{5} \times \frac{2}{5}}$
$= \sqrt{\left(\frac{2}{5}\right)^2}$
$= \frac{2}{5}$

Alternative Method

(a) $\sqrt{\frac{4}{25}} = \frac{\sqrt{4}}{\sqrt{25}}$
$= \frac{\sqrt{2^2}}{\sqrt{5^2}}$
$= \frac{2}{5}$

(b) $\sqrt{2\frac{7}{9}} = \sqrt{\frac{25}{9}}$ ← Change to improper fraction first.
$= \sqrt{\left(\frac{5}{3}\right)^2}$
$= \frac{5}{3}$
$= 1\frac{2}{3}$

Think Smart

The square root of a number is the same as the number itself. What is that number?

CHAPTER 3

(c) $\sqrt{\frac{27}{48}} = \sqrt{\frac{\cancel{27}^{9}}{\cancel{48}_{16}}}$ ← Simplify the fraction first.

$= \sqrt{\frac{9}{16}}$

$= \sqrt{\left(\frac{3}{4}\right)^2}$

$= \frac{3}{4}$

(d) $\sqrt{0.36} = \sqrt{0.6^2}$ ← Express as a square in terms of other decimal number.

$= 0.6$

Alternative Method

(d) $\sqrt{0.36} = \sqrt{\frac{36}{100}}$

$= \sqrt{\left(\frac{6}{10}\right)^2}$

$= \frac{6}{10}$

$= 0.6$

Self Practice 3.1d

1. Find the value of each of the following without using a calculator.

(a) $\sqrt{81}$ (b) $\sqrt{49}$ (c) $\sqrt{121}$ (d) $\sqrt{900}$

(e) $\sqrt{\frac{49}{81}}$ (f) $\sqrt{7\frac{1}{9}}$ (g) $\sqrt{\frac{50}{128}}$ (h) $\sqrt{2.25}$

How do you determine the square root of a positive number using technological tools?

LEARNING STANDARDS

Determine the square roots of a positive number using technological tools.

Example 7

Calculate the value of each of the following by using a calculator and give your answer correct to two decimal places.

(a) $\sqrt{89}$ (b) $\sqrt{154.7}$ (c) $\sqrt{6\frac{2}{7}}$

Solution

(a) $\sqrt{89} = 9.43$ (2 d.p.) Press [√] [8] [9] [=]

(b) $\sqrt{154.7} = 12.44$ (2 d.p.) Press [√] [1] [5] [4] [·] [7] [=]

(c) $\sqrt{6\frac{2}{7}} = 2.51$ (2 d.p.) Press [√] [6] [$a\frac{b}{c}$] [2] [$a\frac{b}{c}$] [7] [=]

Self Practice 3.1e

1. Find the value of each of the following by using a calculator. Give your answer correct to two decimal places.

(a) $\sqrt{43}$ (b) $\sqrt{37.81}$ (c) $\sqrt{\frac{7}{15}}$ (d) $\sqrt{12\frac{5}{6}}$

How do you estimate the square and square root of a number?

Example 8

Estimate the value of
(a) 27.5^2 (b) $\sqrt{54}$

LEARNING STANDARDS

Estimate
(i) the square of a number,
(ii) the square roots of a number.

Solution

(a) 27.5 is between 20 and 30.
27.5^2 is between 20^2 and 30^2.
that is, 27.5^2 is between 400 and 900.
Thus, $27.5^2 \approx 900$

(b) 54 is between perfect squares 49 and 64.
$\sqrt{54}$ is between $\sqrt{49}$ and $\sqrt{64}$,
that is, $\sqrt{54}$ is between 7 and 8.
Thus, $\sqrt{54} \approx 7$

Scan the QR Code or visit **https://goo.gl/UxjF6c** to open the file *Example 8_pdf* to learn about the estimation using number line.

Self Practice 3.1f

1. Estimate the value of each of the following:
(a) 61^2 (b) 22.5^2 (c) 8.7^2 (d) $(-0.188)^2$
(e) $\sqrt{34}$ (f) $\sqrt{17.6}$ (g) $\sqrt{128}$ (h) $\sqrt{0.85}$

What generalisation can be made when two square roots are multiplied?

LEARNING STANDARDS

Make generalisation about multiplication involving:
(i) square roots of the same numbers,
(ii) square roots of different numbers.

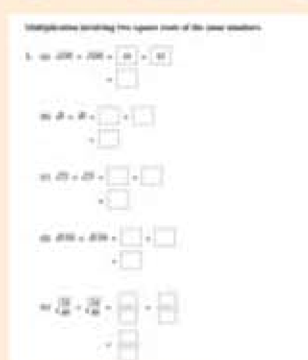
Exploration Activity 4

Pair

Aim: To make a generalisation about multiplication involving square roots.

Instruction:
- Perform the activity in pairs.
- Open the folder downloaded from page vii.

1. Open the file *multiplication of square roots.pdf* and print the file.
2. Complete the boxes for Questions 1 and 2.
3. What can you conclude from the products of multiplication obtained in Questions 1 and 2?



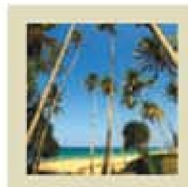
From the results of Exploration Activity 4, it is found that

- the product of two equal square root numbers is the number itself, that is, $\sqrt{a} \times \sqrt{a} = a$.
- the product of two different square root numbers is the square root of the product of the two numbers, that is, $\sqrt{a} \times \sqrt{b} = \sqrt{ab}$.

How do you solve problems?

MATHEMATICS APPLICATION

Maslina wishes to paste a photo on a piece of cardboard. Both the photo and the cardboard are squares. The length of the cardboard is 12 cm and the area of the photo is 90.25 cm². How should Maslina paste the photo such that the photo lies in the middle of the cardboard?



Solution

Length of photo $= \sqrt{90.25}$
$= \sqrt{9.5 \times 9.5}$
$= 9.5$ cm

The remaining length of side of the cardboard after the photo is pasted $= 12 - 9.5$
$= 2.5$ cm

Distance of the photo from the side of the cardboard
$= 2.5 \div 2$
$= 1.25$ cm

Thus, the photo has to be pasted at a distance of 1.25 cm from the sides of the cardboard such that the photo lies in the middle of the cardboard.

LEARNING STANDARDS

Pose and solve problems involving squares and square roots.

Let's Discuss

Given the area of a square cardboard is 156.25 cm² whereas the area of a square photo is less than 156.25 cm² but greater than 90.25 cm². Discuss the possible length of the photo used if the measurement of its length is a whole number.

Scan the QR Code or visit https://youtu.be/Nsb77HbXcCM to learn about the application of squares and square roots.

Self Practice 3.1g

1. Ai Ling has a piece of square cloth. The area of the cloth is between 6 400 cm² and 12 100 cm². She wants to use the cloth to sew a piece of square tablecloth to cover the surface of a square table. The length of each side of the table is 92 cm.
   (a) What is the length of the cloth, in cm, that Ai Ling should sew?
   [Assume that the length of the cloth is a whole number.]
   (b) Ai Ling plans to decorate the surrounding sides of the tablecloth with white lace that is 4.5 m long so that the tablecloth will look nice. Is the length of the white lace enough? Give a reason for your answer.

Mastery Q 3.1

Open the folder downloaded from page vii for extra questions of Mastery Q 3.1.

1. Determine whether each of the following numbers is a perfect square. Use the prime factorisation method to support your answer.
(a) 216 (b) 1 000 (c) 1 024

2. The prime factorisation of 100 is $2 \times 2 \times 5 \times 5$. Explain how you could find the square root of 100 by using the prime factorisation method.

3. Copy and complete each of the following based on the relationship between square and square root.

______ is equal to	6^2	as	10^2	as	14^2	as	19^2	as	22^2
	36		100		196		361		484
______ is equal to	$\sqrt{36}$	as	$\sqrt{100}$	as	$\sqrt{196}$	as	$\sqrt{361}$	as	$\sqrt{484}$
	☐		☐		☐		☐		☐

4. Find the value of each of the following without using a calculator.
(a) $(-6)^2$ (b) $\left(\frac{2}{7}\right)^2$ (c) $\left(-4\frac{1}{3}\right)^2$ (d) $(-8.1)^2$
(e) $\sqrt{361}$ (f) $\sqrt{\frac{9}{49}}$ (g) $\sqrt{2\frac{14}{25}}$ (h) $\sqrt{1.21}$

5. Calculate the value of each of the following by using a calculator. Give your answers correct to two decimal places for (e) to (h).
(a) 127^2 (b) $(-34.6)^2$ (c) 0.097^2 (d) $\left(-2\frac{5}{8}\right)^2$
(e) $\sqrt{76}$ (f) $\sqrt{108.4}$ (g) $\sqrt{\frac{11}{28}}$ (h) $\sqrt{2\frac{3}{5}}$

6. A pyramid has a square base area of 52 900 m^2. Find the length of each side of the base of the pyramid.

7. Estimate the value of each of the following:
(a) 297^2 (b) 51.9^2 (c) $(-0.038)^2$ (d) $(-8.12)^2$
(e) $\sqrt{14}$ (f) $\sqrt{220}$ (g) $\sqrt{8.3}$ (h) $\sqrt{0.5}$

8. A group of 100 members from a Cultural Club participated in the Malaysia Citrawarna Parade. They made various formations throughout the parade.
(a) When the members of the cultural group made a formation in the shape of a square, state the number of members in each row of the square.
(b) At a certain instance, the members of the group formed two squares simultaneously. Determine the number of members in each row of each square.

3.2 Cubes and Cube Roots

What are cubes and perfect cubes?

LEARNING STANDARDS

Explain the meaning of cubes and perfect cubes.

Exploration Activity 5

Pair

Aim: To explore the formation of cubes.
Instruction: Perform the activity in pairs.

1. The diagram shows three cubes which are made up of unit cubes. Observe the three cubes shown.

SMART TIPS

The number of unit cubes in a cube is the volume of the cube.

2. Copy and complete the table below.

Length of the sides of a cube (unit)	Volume of the cube in the form of repeated multiplication ($unit^3$)	Number of unit cubes ($unit^3$)
1	$1 \times 1 \times 1$	

3. Discuss with your friends and write the relationship between the number of unit cubes and the length of the sides of the cube.

From the results of Exploration Activity 5, it is found that a cube with

a side length of (unit)	1, 2, 3, …
has a number of unit cubes of ($unit^3$)	1, 8, 27, …

For example, for a cube with a side length of 2 units,

number of unit cubes $= 2 \times 2 \times 2$
$= 8\ unit^3$

We can state that the cube of 2 is 8.
The cube of 2 is written as 2^3.
Thus, we write $2^3 = 8$.

2^3 is read as 'two cubed' or 'cube of two'.

Exploration Activity 6

Group

Aim: To explain the meaning of perfect cubes.
Instruction: Perform the activity in groups of four.

1. Arrange unit blocks beginning with 1 unit block, followed by 2 unit blocks, 3 unit blocks and so on to form a cube (if possible).

2. Copy and complete the table below.

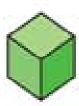
Number of unit blocks	1	2	3	4	5	6	7	8	9	10
Can the arrangement form a cube? (Mark ✓ or ✗)	✓	✗	✗							

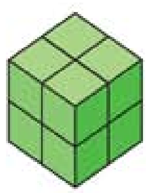
3. Write the numbers that represent the number of unit blocks that can be arranged to form a cube.

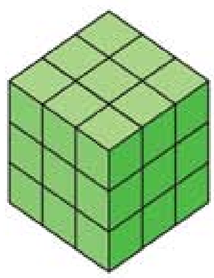
4. What is the relationship between the numbers that represent the number of unit blocks and the formation of a cube?

From the results of Exploration Activity 6, it is found that only a certain number of unit blocks can be arranged to form a cube.

For example,


1 unit block


8 unit blocks


27 unit blocks

The numbers of unit blocks that can form a cube are 1, 8 and 27.

The numbers 1, 8 and 27 are known as **perfect cubes**.

Try This *State the subsequent perfect cubes.*

Did You Know

The volume of a table salt crystal is a perfect cube.

Think Smart

Zaiton said,
"$2^3 = 2 \times 3 = 6$."
How do you explain to Zaiton that her statement is false?

How do you determine whether a number is a perfect cube?

LEARNING STANDARDS

Determine whether a number is a perfect cube.

We can also use the method of prime factorisation to determine whether a number is a perfect cube. In this method, if the prime factors can be grouped into three identical groups, then the number is a perfect cube.

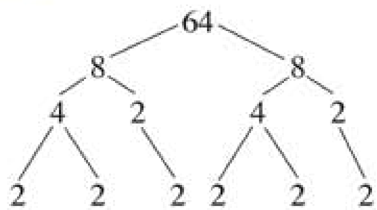
Example 9

Determine whether each of the following numbers is a perfect cube.

(a) 64 (b) 240

Solution

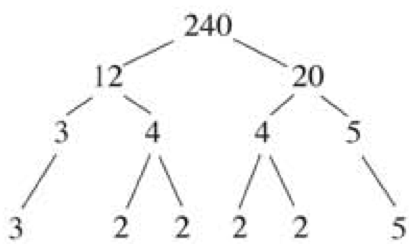
(a)



$64 = 2 \times 2 \times 2 \times 2 \times 2 \times 2$ ← These prime factors can be grouped into three identical groups.

Thus, 64 is a perfect cube.

(b)



$240 = 3 \times 2 \times 2 \times 2 \times 2 \times 5$ ← These prime factors cannot be grouped into three identical groups.

Thus, 240 is not a perfect cube.

SMART TIPS

Perfect cube can be written as a product of three equal factors.
For example,
$64 = 4 \times 4 \times 4$
64 is a perfect cube.

Think Smart

This number is a perfect square and also a perfect cube. What is this number?

Let's Discuss

Discuss why the prime factors of a perfect cube must be grouped into three identical groups.

Self Practice 3.2a

1. By using the prime factorisation method, determine whether each of the following numbers is a perfect cube.
(a) 27 (b) 45 (c) 215 (d) 343

What is the relationship between cubes and cube roots?

LEARNING STANDARDS

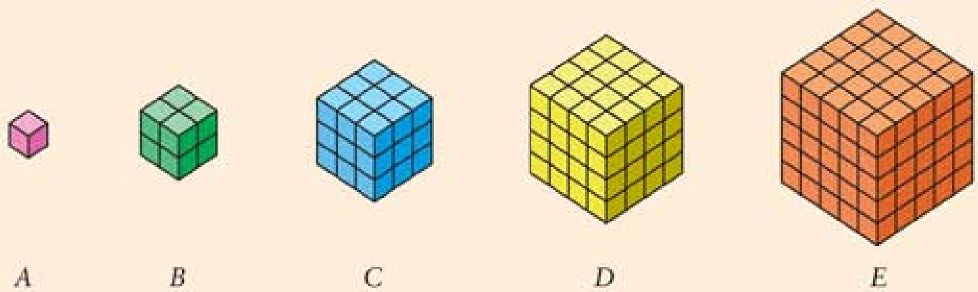
State the relationship between cubes and cube roots.

Exploration Activity 7

Group

Aim: To state the relationship between cubes and cube roots.
Instruction: Perform the activity in groups of four.

1. Observe cubes A to E in the diagram below.



2. Complete the following table for the length of the sides of each cube.

Cube	A	B	C	D	E
Volume (unit^3)	1	8	27	64	125
Length of the sides (unit)					

3. Based on the results from the table, discuss with your friends the relationship between the volume of each cube and the length of its sides.

From the results of Exploration Activity 7, it is found that
a cube with a volume of (unit^3) 1, 8, 27, 64, 125
has a side length of (unit) 1, 2, 3, 4, 5
that is, the volume of each cube is the cube of its side length.

SMART TIPS

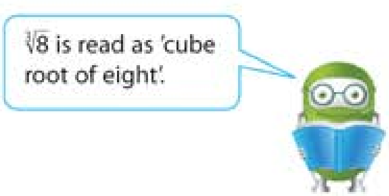
Finding the cube root of the volume of a cube is equivalent to finding the length of the sides of the cube.

For example, for a cube with a volume of 8 unit^3, the length of its side is 2 units,

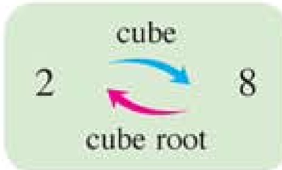
$$\text{volume (unit}^3) = 8 = 2 \times 2 \times 2 = 2^3$$

We can state that the cube of 2 is 8.
Thus, the cube root of 8 is 2.

By using the symbol of cube root, $\sqrt[3]{\ }$,
we write $\sqrt[3]{8} = 2$.



Finding the cube and finding the cube root are inverse operations.



Think Smart

The cube root of a number is the same as the number itself. What is the number?

Example 10

Complete each of the following:

(a) $4 \times 4 \times 4 = 64$

Thus $\sqrt[3]{64} = \sqrt[3]{\square \times \square \times \square}$

$= \square$

(b) $(-0.5) \times (-0.5) \times (-0.5) = -0.125$

Thus $\sqrt[3]{-0.125} = \sqrt[3]{\square \times \square \times \square}$

$= \square$

(c) $\left(\frac{1}{6}\right)^3 = \frac{1}{216}$

Thus $\sqrt[3]{\frac{1}{216}} = \sqrt[3]{\square^{\square}}$

$= \square$

Solution

(a) $\sqrt[3]{64} = \sqrt[3]{4 \times 4 \times 4}$

$= 4$

(b) $\sqrt[3]{-0.125} = \sqrt[3]{-0.5 \times (-0.5) \times (-0.5)}$

$= -0.5$

(c) $\sqrt[3]{\frac{1}{216}} = \sqrt[3]{\left(\frac{1}{6}\right)^3}$

$= \frac{1}{6}$

Self Practice 3.2b

1. Copy and complete each of the following:

(a) $8 \times 8 \times 8 = 512$

Thus, $\sqrt[3]{512} = \sqrt[3]{\square \times \square \times \square}$

$= \square$

(b) $0.3 \times 0.3 \times 0.3 = 0.027$

Thus, $\sqrt[3]{0.027} = \sqrt[3]{\square \times \square \times \square}$

$= \square$

(c) $\left(-\frac{1}{2}\right)^3 = -\frac{1}{8}$

Thus, $\sqrt[3]{-\frac{1}{8}} = \sqrt[3]{\square^{\square}}$

$= \square$

How do you determine the cube of a number?

LEARNING STANDARDS

Determine the cube of a number with and without using technological tools.

Example 11

Find the value of each of the following without using a calculator.

(a) 4^3 (b) 0.2^3 (c) $\left(-\frac{3}{5}\right)^3$

Solution

(a) $4^3 = 4 \times 4 \times 4$
$= 64$

(b) $0.2^3 = 0.2 \times 0.2 \times 0.2$
$= 0.008$

(c) $\left(-\frac{3}{5}\right)^3 = \left(-\frac{3}{5}\right) \times \left(-\frac{3}{5}\right) \times \left(-\frac{3}{5}\right)$
$= -\frac{27}{125}$

SMART TIPS

The cube of a positive number is always a positive value whereas the cube of a negative number is always a negative value.

Example 12

Find the value of each of the following using a calculator.

(a) 18^3 (b) $\left(-4\frac{1}{2}\right)^3$ (c) $(-6.3)^3$

Solution

(a) $18^3 = 5\ 832$ Press [1] [8] [x^3] [=]

(b) $\left(-4\frac{1}{2}\right)^3 = -91\frac{1}{8}$ Press [(] [(–)] [4] [$a\frac{b}{c}$] [1] [$a\frac{b}{c}$] [2] [)] [x^3] [=]

(c) $(-6.3)^3 = -250.047$ Press [(] [(–)] [6] [.] [3] [)] [x^3] [=]

Self Practice 3.2c

1. Determine the value of each of the following without using a calculator.

(a) 6^3 (b) $(-7)^3$ (c) $\left(-\frac{2}{9}\right)^3$ (d) $(-0.3)^3$ (e) $\left(2\frac{3}{5}\right)^3$

2. Find the value of each of the following using a calculator.

(a) 26^3 (b) $(-5.1)^3$ (c) $\left(\frac{3}{10}\right)^3$ (d) $\left(-1\frac{7}{11}\right)^3$ (e) $\left(4\frac{4}{5}\right)^3$

How do you determine the cube root of a number?

Example 13

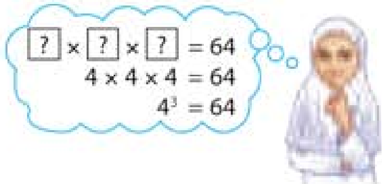
Find the value of each of the following without using a calculator.

(a) $\sqrt[3]{64}$ (b) $\sqrt[3]{216}$

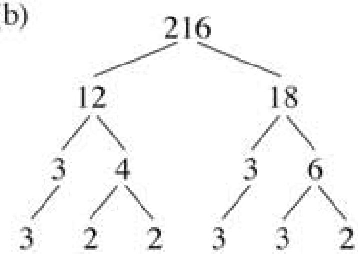
> **LEARNING STANDARDS**
> Determine the cube root of a number without using technological tools.

Solution

(a) $\sqrt[3]{64} = \sqrt[3]{4^3}$
$= 4$



(b)



$216 = 3 \times 2 \times 2 \times 3 \times 3 \times 2$
$= (3 \times 2) \times (3 \times 2) \times (3 \times 2)$
$= 6 \times 6 \times 6$

$\sqrt[3]{216} = \sqrt[3]{6 \times 6 \times 6}$
$= 6$

> **SMART TIPS**
> Prime factorisation is a more systematic method used to find the cube root of a larger number.

Example 14

Find the value of each of the following without using a calculator.

(a) $\sqrt[3]{\dfrac{8}{125}}$ (b) $\sqrt[3]{-\dfrac{81}{192}}$ (c) $\sqrt[3]{3\dfrac{3}{8}}$

Solution

(a) $\sqrt[3]{\dfrac{8}{125}}$

$= \sqrt[3]{\dfrac{2}{5} \times \dfrac{2}{5} \times \dfrac{2}{5}}$

$= \sqrt[3]{\left(\dfrac{2}{5}\right)^3}$

$= \dfrac{2}{5}$

(b) $\sqrt[3]{-\dfrac{81}{192}}$

$= \sqrt[3]{-\dfrac{\cancel{81}^{\,27}}{\cancel{192}_{\,64}}}$ ← Simplify the fraction first.

$= \sqrt[3]{-\dfrac{27}{64}}$

$= \sqrt[3]{\left(-\dfrac{3}{4}\right)^3}$

$= -\dfrac{3}{4}$

(c) $\sqrt[3]{3\dfrac{3}{8}} = \sqrt[3]{\dfrac{27}{8}}$ ← Convert to improper fraction first.

$= \sqrt[3]{\left(\dfrac{3}{2}\right)^3}$

$= \dfrac{3}{2}$

$= 1\dfrac{1}{2}$

> **Alternative Method**
> (a) $\sqrt[3]{\dfrac{8}{125}} = \dfrac{\sqrt[3]{8}}{\sqrt[3]{125}}$
> $= \dfrac{\sqrt[3]{2^3}}{\sqrt[3]{5^3}}$
> $= \dfrac{2}{5}$

> **SMART TIPS**
> The cube root of a positive number is always a positive value whereas the cube root of a negative number is always a negative value.

Example 15

Find the value of each of the following without using a calculator.

(a) $\sqrt[3]{0.027}$ (b) $\sqrt[3]{-0.008}$

Solution

(a) $\sqrt[3]{0.027}$
$= \sqrt[3]{0.3^3}$
$= 0.3$

(b) $\sqrt[3]{-0.008}$
$= \sqrt[3]{(-0.2)^3}$
$= -0.2$

Express as a cube in terms of other decimal number.

Alternative Method

(b) $\sqrt[3]{-0.008}$
$= \sqrt[3]{-\frac{8}{1000}}$
$= \sqrt[3]{\left(-\frac{2}{10}\right)^3}$
$= -\frac{2}{10}$
$= -0.2$

CHAPTER 3

Self Practice 3.2d

1. Given $9\,261 = 3^3 \times 7^3$, find $\sqrt[3]{9\,261}$ without using a calculator.
2. Find $\sqrt[3]{2\,744}$ by using the prime factorisation method.
3. Find the value of each of the following without using a calculator.
   (a) $\sqrt[3]{27}$ (b) $\sqrt[3]{-125}$ (c) $\sqrt[3]{343}$ (d) $\sqrt[3]{-1\,000}$
4. Find the value of each of the following without using a calculator.
   (a) $\sqrt[3]{\frac{8}{125}}$ (b) $\sqrt[3]{-\frac{1}{27}}$ (c) $\sqrt[3]{\frac{24}{81}}$ (d) $\sqrt[3]{1\frac{61}{64}}$
   (e) $\sqrt[3]{0.001}$ (f) $\sqrt[3]{-0.064}$ (g) $\sqrt[3]{-0.216}$ (h) $\sqrt[3]{0.000343}$

How do you determine the cube root of a number using technological tools?

LEARNING STANDARDS

Determine the cube root of a number using technological tools.

Example 16

Calculate the value of each of the following by using a calculator. Give your answer correct to two decimal places.

(a) $\sqrt[3]{24}$ (b) $\sqrt[3]{-104.8}$ (c) $\sqrt[3]{-1\frac{2}{9}}$

Solution

(a) $\sqrt[3]{24} = 2.88$ (2 d.p.) Press [∛] [2] [4] [=]

(b) $\sqrt[3]{-104.8} = -4.71$ (2 d.p.) Press [∛] [(–)] [1] [0] [4] [·] [8] [=]

(c) $\sqrt[3]{-1\frac{2}{9}} = -1.07$ (2 d.p.) Press [∛] [(–)] [1] [a b/c] [2] [a b/c] [9] [=]

Self Practice 3.2e

1. Find the value of each of the following using a calculator and give your answer correct to two decimal places.
   (a) $\sqrt[3]{15}$ (b) $\sqrt[3]{-74}$ (c) $\sqrt[3]{164.2}$ (d) $\sqrt[3]{\frac{7}{9}}$ (e) $\sqrt[3]{-1\frac{2}{5}}$

How do you estimate the cube and cube root of a number?

LEARNING STANDARDS

Estimate
(i) the cube of a number,
(ii) the cube root of a number.

Example 17

Estimate the value of
(a) 4.2^3 (b) $\sqrt[3]{180}$

Solution

(a) 4.2 is between 4 and 5.
4.2^3 is between 4^3 and 5^3,
that is, 4.2^3 is between 64 and 125.
Thus, $4.2^3 \approx 64$

(b) 180 is between perfect cubes 125 and 216.
$\sqrt[3]{180}$ is between $\sqrt[3]{125}$ and $\sqrt[3]{216}$,
that is, $\sqrt[3]{180}$ is between 5 and 6.
Thus, $\sqrt[3]{180} \approx 6$

Scan the QR Code or visit **https://goo.gl/UxjF6c** to open the file *Example 17_pdf* to learn about the estimation using number line.

Self Practice 3.2f

1. Estimate the value of each of the following:
   (a) 2.1^3 (b) $(-9.6)^3$ (c) 19.7^3 (d) $(-43.2)^3$
2. Estimate the value of each of the following:
   (a) $\sqrt[3]{7}$ (b) $\sqrt[3]{69}$ (c) $\sqrt[3]{-118}$ (d) $\sqrt[3]{-26.8}$

How do you solve problems?

MATHEMATICS APPLICATION

LEARNING STANDARDS

Solve problems involving cubes and cube roots.

A sculptor carves a small cube from a wooden cube block that has a side length of 6 cm. If the volume of the remaining wooden block is 189 cm^3, find the length of the small cube that is being removed.

Solution

Understanding the problem
- Length of the wooden block = 6 cm
- Volume of the remaining wooden block after a small cube is removed = 189 cm^3
- Find the length of the small cube.

Devising a plan
- Volume of the wooden block = Cube of the length of its side
- Volume of the small cube
$= \begin{pmatrix}\text{Volume of the}\\ \text{wooden block}\end{pmatrix} - \begin{pmatrix}\text{Volume of the remaining}\\ \text{wooden block}\end{pmatrix}$
- Length of the small cube
= Cube root of the volume of the small cube

Implementing the strategy

Volume of the wooden block $= 6^3$
$= 216 \text{ cm}^3$

Volume of the small cube
$= \begin{pmatrix}\text{Volume of the}\\ \text{wooden block}\end{pmatrix} - \begin{pmatrix}\text{Volume of the remaining}\\ \text{wooden block}\end{pmatrix}$
$= 216 - 189$
$= 27 \text{ cm}^3$

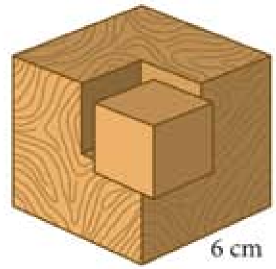
Length of the small cube $= \sqrt[3]{27}$
$= 3 \text{ cm}$

Doing reflection

Volume of the small cube $= 3^3$
$= 27 \text{ cm}^3$

Volume of the wooden block $= 27 + 189$
$= 216 \text{ cm}^3$

Thus, the length of the wooden block $= \sqrt[3]{216}$
$= 6 \text{ cm}$



Creative & Innovative

Material:
Manila card

Task:
Design a closed box to fill eight ping pong balls by using the least material.

Reflection:
What is the shape of the box that saves the most amount of material?

Think Smart

Amira shapes a cuboid measuring 5 cm, 2 cm and 5 cm from plasticine. How many identical cuboids are needed to form a big cube?

Self Practice 3.2g

1. Malik wants to make a cubic frame using a piece of wire with a length of 150 cm. If the volume of the cube is 2 197 cm^3, is the length of the wire sufficient? Give a reason for your answer.

Mastery Q 3.2

Open the folder downloaded from page vii for extra questions of Mastery Q 3.2.

1. Determine whether each of the following numbers is a perfect cube. Explain how you could support your answer by using the prime factorisation method.
(a) 128 (b) 343 (c) 1 000

2. The prime factorisation of 3 375 is $3 \times 3 \times 3 \times 5 \times 5 \times 5$.

Explain how you could find the cube root of 3 375 by using the prime factorisation method.

3. Find the value of each of the following without using a calculator.
(a) $(-5)^3$ (b) $\left(\frac{4}{5}\right)^3$ (c) $\left(-1\frac{1}{6}\right)^3$ (d) $(-3.2)^3$
(e) $\sqrt[3]{125}$ (f) $\sqrt[3]{-512}$ (g) $\sqrt[3]{729}$ (h) $\sqrt[3]{-27\,000}$
(i) $\sqrt[3]{\frac{8}{125}}$ (j) $\sqrt[3]{-\frac{64}{343}}$ (k) $\sqrt[3]{-0.512}$ (l) $\sqrt[3]{1.331}$

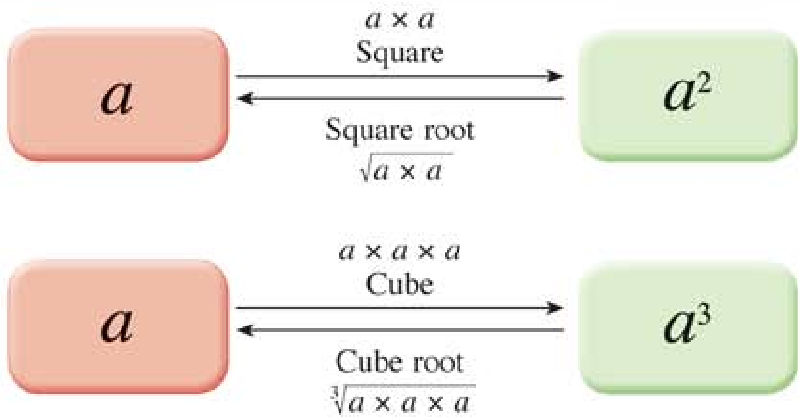
4. Calculate the value of each of the following using a calculator. Give your answers correct to two decimal places for (e) to (h).
(a) 202^3 (b) $(-17.6)^3$ (c) 0.041^3 (d) $\left(-2\frac{3}{7}\right)^3$
(e) $\sqrt[3]{34.8}$ (f) $\sqrt[3]{215.7}$ (g) $\sqrt[3]{-0.94}$ (h) $\sqrt[3]{-\frac{7}{11}}$

5. Estimate the value of each of the following:
(a) 2.9^3 (b) $(-10.12)^3$ (c) 14.87^3 (d) $(-0.88)^3$
(e) $\sqrt[3]{65}$ (f) $\sqrt[3]{344}$ (g) $\sqrt[3]{-728.9}$ (h) $\sqrt[3]{8\frac{1}{8}}$

6. The photo shows a decorative cubic box. The area of each face of the box is 2 500 mm^2.
(a) Find the length, in mm, of the decorative box.
(b) Write the volume of the decorative box in cubic notation.

7. Find the value of each of the following:
(a) $\sqrt[3]{8} + (-0.3)^2$ (b) $4^2 \times \sqrt[3]{-125}$
(c) $\sqrt{36} \div \left(2\frac{1}{2}\right)^2$ (d) $3^2 - \sqrt[3]{27} \div (-1)^3$
(e) $5^2 \times \sqrt[3]{-216} \div \sqrt{\frac{4}{9}}$ (f) $\sqrt[3]{-\frac{1}{343}} \times \left(2^3 - \sqrt{2\frac{7}{9}}\right)$

SUMMARY



At the end of this chapter, I can...	Very good	Work harder
explain the meaning of squares, perfect squares, cubes and perfect cubes.		
determine whether a number is a – perfect square. – perfect cube.		
state the relationship between squares, square roots, cubes and cube roots.		
determine the square and cube of a number with and without using technological tools.		
determine the square root and cube root of a number without using technological tools.		
determine the square root of a positive number and the cube root of a number using technological tools.		
estimate the square, square root, cube and cube root of a number.		
make generalisation about multiplication involving – square roots of the same numbers. – square roots of different numbers.		
pose and solve problems involving squares, square roots, cubes and cube roots.		
perform computations involving addition, subtraction, multiplication, division and the combination of those operations on squares, square roots, cubes and cube roots.		

How do you perform computations involving different operations on squares, square roots, cubes and cube roots?

LEARNING STANDARDS

Perform computations involving addition, subtraction, multiplication, division and the combination of those operations on squares, square roots, cubes and cube roots.

Example 18

Find the value of each of the following:

(a) $0.5^2 + \sqrt[3]{1\,000}$

(b) $(-3)^3 - \sqrt{64}$

(c) $\sqrt{25} + (-0.2)^2 \div \sqrt[3]{0.008}$

(d) $\sqrt[3]{-3\frac{3}{8}} \times (\sqrt{36} - 2^3)^2$

Solution

(a) $0.5^2 + \sqrt[3]{1\,000}$
$= 0.25 + 10$
$= 10.25$

(b) $(-3)^3 - \sqrt{64}$
$= -27 - 8$
$= -35$

(c) $\sqrt{25} + (-0.2)^2 \div \sqrt[3]{0.008}$
$= 5 + 0.04 \div 0.2$ ← Solve the ÷ operation first.
$= 5 + 0.2$
$= 5.2$

(d) $\sqrt[3]{-3\frac{3}{8}} \times (\sqrt{36} - 2^3)^2$
$= \sqrt[3]{-\frac{27}{8}} \times (6 - 8)^2$
$= -\frac{3}{2} \times (-2)^2$
$= -\frac{3}{2_1} \times 4^2$
$= -6$

SMART TIPS

Find the value of squares, square roots, cubes or cube roots.
↓
Solve the operation in the brackets.
↓
Solve the operations × and ÷ from left to right.
↓
Solve the operations + and – from left to right.

Self Practice 3.2h

1. Calculate the value of each of the following:

(a) $\sqrt{49} + 3^2$

(b) $\sqrt[3]{27} - 1.5^2$

(c) $\sqrt[3]{-64} \times 0.2^3$

(d) $(-2)^2 \div \sqrt{100}$

(e) $\sqrt{2\frac{1}{4}} - \sqrt[3]{15\frac{5}{8}}$

(f) $\sqrt[3]{\frac{1}{125}} \times 0.3^2$

(g) $\sqrt{\frac{12}{27}} \div \left(-\frac{2}{3}\right)^3$

(h) $(-5)^2 + \sqrt{2\frac{7}{9}} - 2^3$

(i) $(\sqrt{16} - 6)^2 \times \sqrt[3]{-\frac{54}{128}}$

Let's PRACTISE

Test Yourself

1. Mark (✓) for the number which is a perfect square.

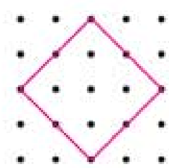
27 ( )	32 ( )	18 ( )	4 ( )
81 ( )	8 ( )	125 ( )	49 ( )

2. Copy and complete the operation steps below by filling up the empty boxes with suitable numbers.

$$\sqrt{1\frac{11}{25}} - (-0.1)^3 = \sqrt{\frac{\square}{25}} - (-0.1)^3$$
$$= \frac{\square}{5} - (\square)$$
$$= \square$$

Self Mastery

3. The diagram shows a square drawn by Siti. She claims that the length of the side of the square is $\sqrt{8}$ units. Show how you would validate Siti's answer.



4. **Application** **Aviation**

A helicopter landing pad is a square and has an area of 400 m^2. Use the prime factorisation method to find the length of the side of the landing pad.

5.

$$512 = (2 \times 2 \times 2) \times (2 \times 2 \times 2) \times (2 \times 2 \times 2)$$

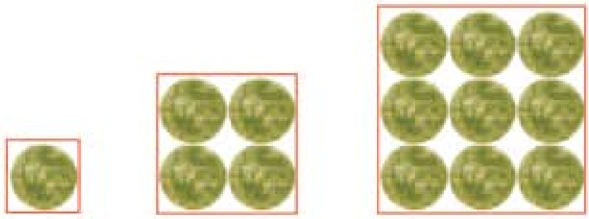
(a) Based on the mathematical statement above, Fong Yee states that 512 is a perfect cube. Explain how you would support Fong Yee's answer.
(b) Fong Yee also states that 512 is not a perfect square. Explain the reason Fong Yee said so.

6. Mohan used a can of paint to paint a square backdrop. A can of paint could cover the entire backdrop of 38 m^2. Estimate the length of the side of the backdrop.

7. A big cube with a side length of 30 cm is cut into 27 small cubes of the same size. Find
(a) the length of the side of each small cube,
(b) the area of the top surface of each small cube.

Challenge Yourself

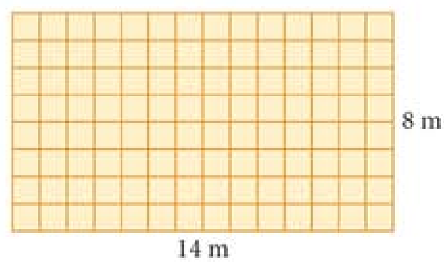
8. Amirul arranges 20 sen coins in the shapes of squares as shown in the diagram.



(a) Calculate the total value, in RM, of the
(i) fourth square,
(ii) 10th square.
(b) If Amirul has 20 sen coins with an amount of RM60, determine the arrangement of the 20 sen coins to form the largest square.

9. **Application** **Construction**

Stella wants to beautify the patio of her house by placing square patio stones on it. Each patio stone has an area of 1 m^2. She makes a sketch of the layout plan as shown in the diagram.



(a) How many patio stones are needed for the construction?
(b) If Stella suggests using the same number of patio stones as in (a) but changes the layout plan into the shape of a square, is it still possible for Stella to work? Explain your answer.

10. **Application** **History**

Hypatia was an Egyptian mathematician who was born in the year 370 A.D. In a finding, Hypatia posed the following problems:

- This number is the sum of two square numbers.
- The square of this number is also the sum of two square numbers.

Square numbers:
1, 4, 9, 16, ...
$5 = 1 + 4$
$5^2 = 9 + 16$

One of the numbers that satisfies the constraints posed by Hypatia is 5. Find three other numbers.

ASSIGNMENT

The photo shows a square boxing ring. Other than boxing, there are other sports which use square floors as surfaces.

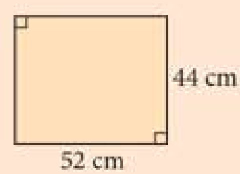
By conducting research via Internet, reference books or visiting a library, find out what other sports also use square floors as surfaces. Find out the length of the side and the area of this square surface. Relate the rules of this sports in describing the purpose of having a square floor as its surface.

Exploring MATHEMATICS

Chess is played on a square board. The board is made up of 32 light squares and 32 dark squares.

You have a piece of rectangular board with measurements of 44 cm × 52 cm. You decide to use the board to make your own chessboard such that

- each square grid on the board has a side length which is a whole number.
- each chess piece is placed on a square grid with an area of not less than 9 cm^2.



Determine all the possible measurements of the chessboard that you can create.