CHAPTER 4

Ratios, Rates and Proportions

What will you learn?

- Ratios
- Rates
- Proportions
- Ratios, Rates and Proportions
- Relationship between Ratios, Rates and Proportions, with Percentages, Fractions and Decimals

Why study this chapter?

Ratios, rates and proportions are involved in the fields that require the concept of comparisons. An astronomer measures the distance in the solar system by comparing each distance in the solar system with the distance from the Earth to the Sun. A nurse uses the concept of rates when taking a patient's pulse. Discuss other fields which involve the concept of comparisons.

The *Hippocampus kuda* is a type of seahorse found in river estuaries. These seahorses are facing extinction and must be conserved and appreciated. To estimate the seahorse population at a certain habitat, marine biologists will tag the first sample of the seahorses and then release them back to the original sites where they were collected from.

After a certain period of time, the second sample is collected. The marine biologists will record the number of tagged seahorses. How do the marine biologists estimate the population of the seahorses in the river estuaries by using this method?

Walking through Time

Vitruvian Man

The Vitruvian Man, is a drawing by Leonardo da Vinci around the year 1490 which depicts a human body inscribed in a circle and a square. In this drawing, the human body is said to be drawn based on certain ratios and proportions which were suggested by a Roman architect called Vitruvius.

For more information:

https://goo.gl/VNOYjx

Word Link

• rate	• *kadar*
• proportion	• *kadaran*
• ratio	• *nisbah*
• equivalent ratio	• *nisbah setara*
• percentage	• *peratusan*

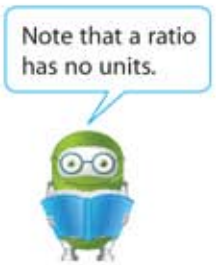
Open the folder downloaded from page vii for the audio of Word Link.

4.1 Ratios

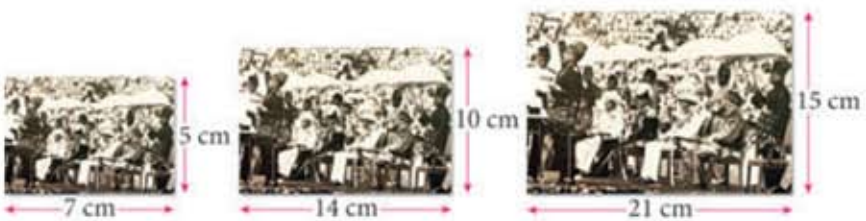
Ratio is used to compare two or more quantities of the same kind that are measured in the same unit. For example, the ratio of 5000 g to 9 kg can be represented as

$$5000\text{ g} : 9\text{ kg} = 5\text{ kg} : 9\text{ kg}$$
$$= 5 : 9$$

The ratio of a to b is written as $a : b$.



How do you represent the relationship of three quantities?



LEARNING STANDARDS

Represent the relation between three quantities in the form of $a : b : c$.

The picture above shows three photos of different sizes.
How do you represent the relationship of the sizes of these three photos in a ratio?

Example 1

Represent the ratio of 0.02 m to 3 cm to 4.6 cm in the form of $a : b : c$.

Solution

$$0.02\text{ m} : 3\text{ cm} : 4.6\text{ cm} = 2\text{ cm} : 3\text{ cm} : 4.6\text{ cm}$$
$$= 2 : 3 : 4.6$$
$$= 20 : 30 : 46$$
$$= 10 : 15 : 23$$

Self Practice 4.1a

1. Represent the relationship of the following three quantities in the form of $a : b : c$.
   (a) 2 weeks to 16 days to 1 week
   (b) 0.1 kg to 50 g to 0.25 kg
   (c) 4 minutes to 120 seconds to 1.6 hours
   (d) $3\frac{1}{5}$ m to 480 cm to 6400 mm

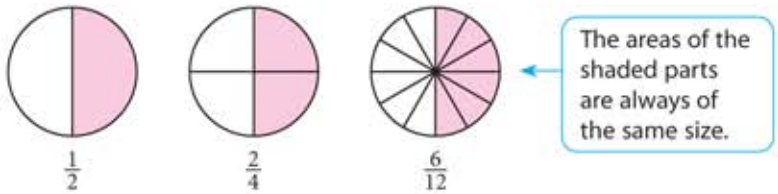
2. Tahir pays RM5.60 for a plate of *nasi beriani*, RM1.20 for a glass of tea and 30 sen for a piece of *kuih*. Represent the relationship of the prices of *nasi beriani*, tea and *kuih* in the form of $a : b : c$.

What are equivalent ratios?

LEARNING STANDARDS

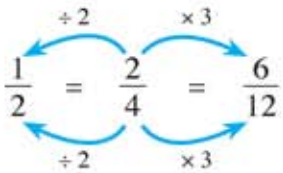
Identify and determine the equivalent ratios in numerical, geometrical or daily situation contexts.

Observe the equivalent fractions that represent the shaded parts in the diagram below.



$\frac{1}{2}$ $\frac{2}{4}$ $\frac{6}{12}$

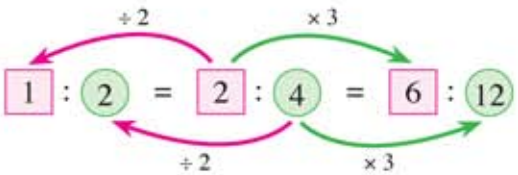
The areas of the shaded parts are always of the same size.



SMART TIPS

Equivalent ratios can be found by writing the ratios as equivalent fractions.

These fractions can be expressed in ratios as follows.



1 : 2, 2 : 4 and 6 : 12 are known as **equivalent ratios**. To find an equivalent ratio, multiply or divide each part of the ratio by the same whole number.

Think Smart

Are 23 : 46 and $\frac{1}{6} : \frac{1}{3}$ equivalent to 1 : 2?

Example 2

Which of the following ratios are equivalent to 27 : 45?

9 : 15 5 : 3 54 : 90 $\frac{1}{2} : \frac{5}{6}$

Solution

$27 : 45 = 27 \times 2 : 45 \times 2$
$= 54 : 90$

$27 : 45 = 27 \div 9 : 45 \div 9$
$= 3 : 5$

$27 : 45 = 27 \div 3 : 45 \div 3$
$= 9 : 15$

$27 : 45 = 27 \times \frac{1}{54} : 45 \times \frac{1}{54}$
$= \frac{1}{2} : \frac{5}{6}$

Thus, ratios which are equivalent to 27 : 45 are 9 : 15, 54 : 90 and $\frac{1}{2} : \frac{5}{6}$.

SMART TIPS

Ratio 3 : 5 ≠ 5 : 3.

Alternative Method

$\frac{1}{2} : \frac{5}{6} = \frac{1}{2} \times 54 : \frac{5}{6} \times 54$
$= 27 : 45$

Example 3

Puan Habibah mixed 4 cups of vinegar with 8 cups of water to obtain a natural cleaning solution to clean the glass window of her house. Identify and determine two possible equivalent ratios for this cleaning solution.

Solution

$4 : 8 = 4 \times 2 : 8 \times 2$ $\qquad$ $4 : 8 = 4 \div 2 : 8 \div 2$

$= 8 : 16$ $\qquad$ $= 2 : 4$

The two possible equivalent ratios of 4 : 8 are 8 : 16 and 2 : 4.

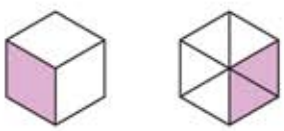
Self Practice 4.1b

1. Which of the following ratios are equivalent to 18 : 24 : 45?

$3 : 4 : 9$ $\qquad$ $36 : 48 : 90$ $\qquad$ $0.6 : 0.8 : 1.5$ $\qquad$ $\frac{2}{5} : \frac{8}{15} : 1$

2. Identify and determine the equivalent ratios of the shaded parts in each of the following diagrams.

(a)



(b)

3. At a construction site, a worker prepares a concrete mixture which acts as the basic solid foundation of a building. The concrete mixture is obtained by adding 10 parts of cement to 20 parts of sand and 30 parts of gravel. Identify and determine two possible equivalent ratios for this mixture.

How do you express ratios in their simplest form?

To express a ratio in its simplest form, divide the quantities by the Highest Common Factor (HCF) or multiply the quantities by the Lowest Common Multiple (LCM).

LEARNING STANDARDS

Express ratios of two and three quantities in simplest form.

Example 4

Express each of the following ratios in its simplest form.

(a) 800 g : 1.8 kg $\qquad$ (b) 32 : 24 : 20

(c) $\frac{3}{5} : \frac{7}{10}$ $\qquad$ (d) 0.04 : 0.12 : 0.56

Solution

(a) $800 \text{ g} : 1.8 \text{ kg} = 800 \text{ g} : 1\,800 \text{ g}$ ← Convert to the same unit.

$= \frac{800}{200} : \frac{1\,800}{200}$ ← Divide both parts by 200.

$= 4 : 9$

SMART TIPS

A ratio of $a : b$ is said to be in its simplest form if a and b are integers with no common factors other than 1.

(b) $32 : 24 : 20 = \frac{32}{4} : \frac{24}{4} : \frac{20}{4}$ ← Divide the three parts by 4, that is, the HCF of 32, 24 and 20.

$= 8 : 6 : 5$

(c) $\frac{3}{5} : \frac{7}{10} = \frac{3}{5} \times 10 : \frac{7}{10} \times 10$ ← Multiply both parts by 10, that is, the LCM of 5 and 10.

$= 6 : 7$

(d) $0.04 : 0.12 : 0.56 = 0.04 \times 100 : 0.12 \times 100 : 0.56 \times 100$ ← Multiply the three parts by 100.

$= 4 : 12 : 56$

$= \frac{4}{4} : \frac{12}{4} : \frac{56}{4}$ ← Divide the three parts by 4, that is, the HCF of 4, 12 and 56.

$= 1 : 3 : 14$

Self Practice 4.1c

1. Express each of the following ratios in its simplest form.

(a) 240 g : 1.6 kg

(b) 30 : 42 : 48

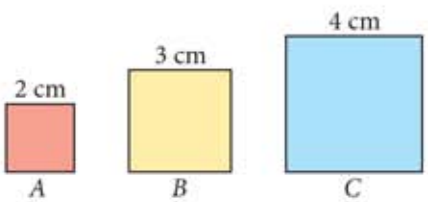
(c) $\frac{2}{5} : \frac{8}{9}$

(d) 0.09 : 0.12 : 0.24

Mastery Q 4.1

Open the folder downloaded from page vii for extra questions of Mastery Q 4.1.

1. The diagram shows three squares, A, B and C.



(a) Represent each of the following in the form of $a : b : c$.

(i) The length of square A to the length of square B to the length of square C.

(ii) The perimeter of square A to the perimeter of square B to the perimeter of square C.

(iii) The area of square A to the area of square B to the area of square C.

(b) Write the equivalent ratios based on your answers in (a).

(c) What is the relationship between the numbers in the ratio of areas and the numbers in the ratio of length?

2. In a workshop, a mechanic uses a jack with a force of 120 pounds to lift a car with a mass of 1 350 kg. State the ratio of the mass of the car to the force required to lift the car. Express the ratio in its simplest form. (1 pound = 0.45 kg)

3. The table shows the number of boys and girls in three Form 1 classes.

Students	Class 1 Amanah	Class 1 Bestari	Class 1 Cekap
Boys	12	9	9
Girls	16	20	12

(a) Which classes have equivalent ratios of the number of girls to the number of boys?

(b) Write the ratio of the number of girls in class 1 Amanah to the number of girls in class 1 Bestari to the number of girls in class 1 Cekap in the form of $a : b : c$.

(c) Class 1 Amanah and class 1 Bestari are combined during the Physical Education and Health Education lesson. Find the ratio of the number of girls to the number of boys in the combined class. Express your answer in its simplest form.

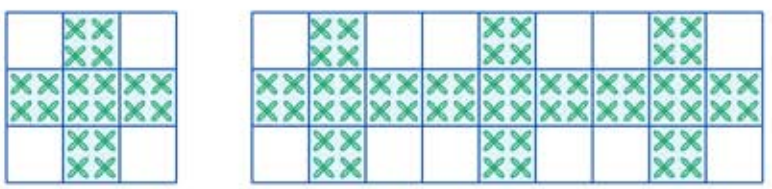
4. Identify and determine the possible equivalent ratios for each of the following.

is equivalent to

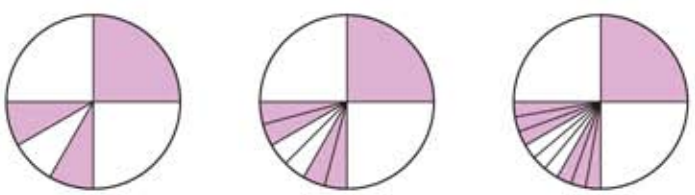
1 : 5 as 4 : 9 : 2 as 37 : 74 as 6 : 1 as 0.3 : 1.2 : 0.5

5. Identify and determine the equivalent ratios of the shaded parts in each of the following diagrams.

(a)



(b)



6. A farmer prepares a soil medium by mixing 3 parts of loamy soil, 2 parts of peat and 1 part of river sand. Identify and determine two possible equivalent ratios for this soil medium.

4.2 Rates

What is the relationship between ratios and rates?

LEARNING STANDARDS

Determine the relationship between ratios and rates.

Exploration Activity 1

Individual

Aim: To determine the relationship between ratios and rates.
Instruction: Open the folder downloaded from page vii.

1. Open the file *ratio and rates.pdf* and print the file.
2. State the ratio of two quantities for the measurements involved in each of the situations given.
3. State the quantities involved and also their units of measurement.

Situation	Ratio in the form $\frac{a}{b}$	Quantities involved	Units of measurement
A car travels 285 km in 3 hours.	$\frac{285 \text{ km}}{3 \text{ hours}}$	Distance and time	km and hour
A plant grows 24 cm in 4 months.			
A baby's mass increases by 1.3 kg in 60 days.			
Karim's pulse rate is 75 beats per minute.			
The force acting on the surface area of 1 square metre is 2 Newtons.			

In Exploration Activity 1, we compare two quantities measured in different units. For example, in the ratio $\frac{285 \text{ km}}{3 \text{ hours}}$, we compare the distance travelled in km with the time taken in hours. The ratio $\frac{285 \text{ km}}{3 \text{ hours}}$ is known as a **rate**.

Rates show how two quantities with different units are related to each other.

SMART TIPS

Rate is a special ratio that compares two quantities with different units of measurement.

Let's Discuss

Whether $\frac{1.5 \text{ kg}}{3 \text{ kg}}$ or $\frac{1.5 \text{ kg}}{3 \text{ g}}$ is a rate? Discuss.

Example 5

State the rate and the two quantities (including the units) involved in each of the following situations.

(a) Fatin buys 2 kg of mangoes at the total price of RM10.

(b) A car consumes 1 litre of petrol for a distance of 12 km.

Solution

(a) $\text{Rate} = \dfrac{\text{RM10}}{\text{2 kg}}$

The two quantities involved are mass (kg) and total amount of money (RM).

(b) $\text{Rate} = \dfrac{\text{12 km}}{\text{1 litre}}$

The two quantities involved are distance (km) and volume (litre).

Conversion of units of rates

Two stalls sell cherry tomatoes from Cameron Highlands. Which stall sells cherry tomatoes at a cheaper price?
To compare the price rates, convert the units first.

Stall *A*

Stall *B*

$$\text{The price of cherry tomatoes at stall } B = \frac{\text{RM8}}{\text{500 g}}$$
$$= \frac{8 \times 2}{500 \times 2}$$
$$= \frac{16}{1\,000}$$ ← 1 000 g = 1 kg
$$= \text{RM16 per kg}$$

Thus, the price of cherry tomatoes at stall *A* is cheaper than the price of cherry tomatoes at stall *B*.

Example 6

(a) Rajan is riding his bicycle at a speed of 5 m/s. Convert 5 m/s to km/h.

(b) The density of a type of metal is 2 700 kg per m^3.
State the density of this metal in g per cm^3.

Solution

(a) 5 m/s
$$= \frac{5 \text{ m}}{1 \text{ s}}$$
$$= 5 \text{ m} \div 1 \text{ s}$$
$$= \frac{5}{1\,000} \text{ km} \div \frac{1}{60 \times 60} \text{ h}$$
$$= \frac{5}{1\,000} \times \frac{60 \times 60}{1}$$
$$= 18 \text{ km/h}$$

(b) Density
$$= \frac{2\,700 \text{ kg}}{1 \text{ m}^3}$$
$$= \frac{2\,700 \times 1\,000}{100 \times 100 \times 100}$$ ← 1 kg = 1 000 g
$$= 2.7 \text{ g/cm}^3$$
$$= 2.7 \text{ g per cm}^3$$

1 m^3
$= 1 \text{ m} \times 1 \text{ m} \times 1 \text{ m}$
$= 100 \text{ cm} \times 100 \text{ cm} \times 100 \text{ cm}$

SMART TIPS

m/s means metre per second.

Self Practice 4.2a

1. State the rate and the two quantities (including the units) involved in each of the following situations.
   (a) The train fare for two adult passengers travelling from Johor Bahru to Kuala Lumpur is RM154.
   (b) 20 litres of water flow out of a water tank each time the water is pumped.
   (c) The tuition fee paid by Haruiri is RM240 for 4 subjects.
   (d) The price of the fertiliser used on a 10 hectare farm is RM500.
   (e) The shaft of an engine makes 600 revolutions in 3 seconds.

2. The table shows the speeds of two objects, A and B. Convert the units of measurement to determine which object moves faster.

Object	Speed
A	25 m per second
B	8 km per hour

3. The mass per unit area of a type of metal sheet is 3 kg per m^2. State the rate in g per 100 cm^2.

4. An oil palm plantation uses fertiliser at a rate of 350 kg per hectare. State the rate of fertiliser consumption in g per m^2. [1 hectare = 10 000 m^2]

Mastery Q 4.2

Open the folder downloaded from page vii for extra questions of Mastery Q 4.2.

1. The mass of a piece of 5 cm^3 aluminium is 13.5 g. Find the density of the aluminium in g per cm^3.

2.

Orang-utan	Mass at birth (kg)	Mass after 60 days (kg)
Borneo	0.3	6.3
Sumatera	0.7	7.7

   (a) Based on the information in the table above, state
      (i) the ratios of the mass at birth to the mass after 60 days for the two species of orang-utan,
      (ii) the rate of growth in kg per 60 days for the two species of orang-utan.
   (b) State the two quantities involved for the rate obtained in (a)(ii).

3. Halim wants to buy milk. The cartons of milk are sold in three different sizes as shown in the diagram.
   (a) Write the price rate for each carton of milk.
   (b) Determine the price per litre for the 500 m*l* carton of milk.
   (c) Which carton of milk offers the most affordable price? Justify your answer.

RM2.25 RM4.00 RM7.50

4.3 Proportions

What is the relationship between ratios and proportions?

LEARNING STANDARDS

Determine the relationship between ratios and proportions.

Exploration Activity 2

Pair

Aim: To determine the relationship between ratios and proportions.
Instruction: Perform the activity in pairs.

1. Select a page from a novel to copy. The time allocated for copying is 5 minutes.
2. Your friend acts as a timekeeper to tell you the time to begin and the time to stop copying.
3. Copy at a comfortable rate to minimise spelling mistakes.
4. Stop copying when the time is up. Mark the place you have stopped copying.
5. Count the total number of words copied in 5 minutes.
6. Exchange roles and repeat Steps 1 to 5 to obtain data for your friend. Use the same page when copying.
7. Copy and record your findings in the following table.

Name of student		
Number of words		
Time (minutes)	5	5

8. Based on the results in the table, answer the following questions:
   (a) What is the rate of copying in 5 minutes for you and your friend?
   (b) Convert the rate of copying in 5 minutes to the number of words per minute.
   (c) If both of you continued copying at the same rate, how many words could each person copy in 1 hour?

In Exploration Activity 2, you have used proportion to find the number of words that you and your friend copied in 1 hour.
For example,

$$\frac{45 \text{ words}}{5 \text{ minutes}} \overset{\times 12}{=} \frac{540 \text{ words}}{60 \text{ minutes}}$$

Let's Discuss

Based on Exploration Activity 2, discuss the similarities and differences among ratios, rates and proportions. Give an example for each one.

Proportion is a relationship that states that the two ratios or two rates are equal. Proportion can be expressed in the form of fraction.

Example 7

Write a proportion for each of the following situations:

(a) If 10 beans have a mass of 17 g, then 30 beans have a mass of 51 g.

(b) The Le Tour De Langkawi cycling race covers a distance of 1 180 km which starts from Kedah and ends in Melaka. On the map, this distance is 23.6 cm where 1 cm represents 50 km.

Solution

(a) $\frac{17\text{ g}}{10\text{ beans}} = \frac{51\text{ g}}{30\text{ beans}}$ (b) $\frac{50\text{ km}}{1\text{ cm}} = \frac{1\,180\text{ km}}{23.6\text{ cm}}$

Self Practice 4.3a

1. Write a proportion for each of the following situations:

(a) If the price of 3 balls is RM5, then the price of 12 balls is RM20.

(b) The height of four wooden blocks stacked vertically on top of one another is 24 cm. When Raju stacks 13 wooden blocks on top of one another, the height of the stacked blocks is 78 cm.

(c) There are 13 boys and 15 girls in each classroom at Ria Kindergarten. If there are 65 boys in the kindergarten, then there are 75 girls.

How do you determine the unknown value in a proportion?

LEARNING STANDARDS

Determine an unknown value in a proportion.

Example 8

Electricity costs 43.6 sen for 2 kilowatt-hour (kWh). How much does 30 kWh cost?

Solution

Unitary method

The cost of electricity for 2 kWh
= 43.6 sen

The cost of electricity for 1 kWh
$= \frac{43.6\text{ sen}}{2}$
= 21.8 sen

The cost of electricity for 30 kWh
= 30 × 21.8
= 654 sen

Proportion method

Let the cost of electricity for 30 kWh be x sen.

Then,

$$\frac{43.6\text{ sen}}{2\text{ kWh}} \overset{\times 15}{=} \frac{x\text{ sen}}{30\text{ kWh}}$$

$x = 43.6 \times 15$
$= 654$

Cross multiplication method

Let the cost of electricity for 30 kWh be x sen.

Then,

$$\frac{43.6}{2} \quad \frac{x}{30}$$

$$2 \times x = 43.6 \times 30$$

$$x = \frac{43.6 \times 30}{2}$$

$$= 654$$

Thus, the cost of electricity consumption for 30 kWh is RM6.54.

SMART TIPS

If $\frac{a}{b} = \frac{c}{d}$, then $a \times d = b \times c$.

Self Practice 4.3b

1. In a sports carnival, there are 200 players in 8 rugby teams. Determine the number of players in 2 teams if each team has the same number of players.
2. During the *SEGAK* test in the Physical Education lesson, Amir is able to do 60 push-ups in 3 minutes. How many push-ups can Amir do in 5 minutes? (Assume that Amir does the push-ups at the same rate.)
3. A farmer plants three chilli plants per 0.5 m^2. How many chilli plants can he plant in an area of 85 m^2?

Mastery Q 4.3

Open the folder downloaded from page vii for extra questions of Mastery Q 4.3.

1. Write a proportion for each of the following situations. Use a suitable variable to represent the required information.
   (a) Puan Jamilah mixes 175 m*l* of olive oil and 50 m*l* of vinegar to make a marinade. What is the volume of vinegar required if Puan Jamilah uses 300 m*l* of olive oil to make the marinade?
   (b) A meteorologist used a rain gauge to measure the amount of rain in a city. He found out that the total rainfall in the city was 7.8 mm within 3 hours. If the rainfall continues at the same rate, how long will it take for the rainfall to reach 11.7 mm?

2. A gear is a simple machine consisting of wheels with teeth. When a large gear rotates 4 times, a small gear will rotate 18 times at the same time. How many times will the large gear rotate if the small gear rotates 54 times?
3. A gardener spends half an hour to mow and weed the lawn which has a measurement of 20 m × 15 m. He is paid RM30 per hour. How much is he paid for a lawn which has a measurement of 40 m × 30 m?

4.4 Ratios, Rates and Proportions

How do you determine the ratio of three quantities when two or more ratios of two quantities are given?

Example 9

LEARNING STANDARDS

Determine the ratio of three quantities, given two or more ratios of two quantities.

If $p : q = 7 : 3$ and $q : r = 3 : 5$, find the ratio of $p : q : r$.

Solution

$p : q = 7 : \textcircled{3}$ $\quad$ $q : r = \textcircled{3} : 5$

same

Thus, $p : q : r = 7 : 3 : 5$.

Example 10

At the reading corner of Class 1 Jujur, the ratio of the number of storybooks to the number of reference books is 2 : 5. The ratio of the number of reference books to the number of magazines is 3 : 2. Find the ratio of the number of storybooks to the number of reference books to the number of magazines.

Solution

Let x = number of storybooks
y = number of reference books
z = number of magazines

$x : y = 2 : 5$ $\quad$ $y : z = 3 : 2$

$= 2 \times 3 : 5 \times 3$ $\quad$ $= 3 \times 5 : 2 \times 5$

$= 6 : \textcircled{15}$ $\quad$ $= \textcircled{15} : 10$

same

Change the value of y in both ratios to the same number by determining the LCM of 5 and 3.

Thus, $x : y : z = 6 : 15 : 10$, that is, the ratio of the number of storybooks to the number of reference books to the number of magazines is 6 : 15 : 10.

Self Practice 4.4a

1. If $p : q = 2 : 9$ and $q : r = 9 : 7$, find the ratio of $p : q : r$.
2. A certain amount of money was donated to three charity organisations P, Q and R. The ratio of the amount of donation received by P to Q is 2 : 3. The ratio of the amount of donation received by Q to R is 4 : 1. Find the ratio of the amount of donation received by organisation P to organisation Q to organisation R.

How do you determine the ratio or the related value?

LEARNING STANDARDS

Determine the ratio or the related value given
(i) the ratio of two quantities and the value of one quantity.
(ii) the ratio of three quantities and the value of one quantity.

Example 11

Nurin's mother tries a bread recipe by mixing flour with water. The ratio of the flour to the water is 5 : 3. If Nurin's mother has 480 g of flour, what is the mass of water, in g, that is needed?

Solution

Unitary method

Flour	:	Water
5	:	3
480 g	:	

5 parts of flour = 480 g

$$1 \text{ part of flour} = \frac{480}{5}$$
$$= 96 \text{ g}$$

Mass of water needed = 3 × 96 g
= 288 g

Cross multiplication method

Let m = mass of water needed

Flour	:	Water
5	:	3
480 g	:	m g

Thus, $\frac{m}{3} = \frac{480}{5}$

$$5 \times m = 3 \times 480$$
$$m = \frac{3 \times 480}{5}$$
$$= 288$$

Example 12

In funding a bicycle shelter project, the ratio of the donations contributed by the school canteen operator, Jaya Bookstore and PIBG is 2 : 6 : 5. If the PIBG had donated RM900, find the amounts donated by the school canteen operator and Jaya Bookstore respectively.

Solution

Unitary method

School canteen operator	:	Jaya Bookstore	:	PIBG
2	:	6	:	5
	:		:	RM900

5 parts = RM900

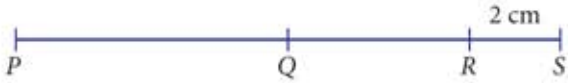
$$1 \text{ part} = \frac{900}{5}$$
$$= \text{RM}180$$

School canteen operator donated 2 parts.
Amount donated = 2 × RM180
= RM360

Jaya Bookstore donated 6 parts.
Amount donated = 6 × RM180
= RM1 080

Try This *Use cross multiplication method to solve Example 12.*

Example 13



A piece of wire of length 12 cm is divided into three parts. Given that $PQ : QR = 3 : 2$ and $RS = 2$ cm, determine $PQ : QR : RS$.

Solution

$PR = 12 - 2$
$= 10$ cm

The total ratio of PQ and $QR = 3 + 2$
$= 5$

5 parts $= 10$ cm

1 part $= \frac{10}{5}$
$= 2$ cm

$PQ = 3 \times 2$ cm
$= 6$ cm

$QR = 2 \times 2$ cm
$= 4$ cm

Thus, $PQ : QR : RS = 6 : 4 : 2$
$= 3 : 2 : 1$

Self Practice 4.4b

1. The ratio of the price of a *baju kebaya* to the price of a *baju kurung* is 7 : 4. If the price of the *baju kebaya* is RM84, find the price of the *baju kurung*.
2. The ratio of Encik Arif's mass to his son's mass is 3 : 2. If his son's mass is 42 kg, find Encik Arif's mass.
3. Zanariah, Rusita and Hanifah shared the cost of a birthday present for their mother in the ratio 5 : 3 : 2. Hanifah paid RM50 for the present. Calculate the total amount of money Zanariah and Rusita paid.
4. Kadir, Chandran and Ping Wei participated in a Science quiz. The number of questions they had answered was in the ratio 4 : 6 : 3. Chandran had answered 30 questions. Calculate the total number of questions Kadir and Ping Wei had answered.
5. 42 students in Class 1 Dedikasi have registered as members of the Computer Club, Robotics Club and Taekwondo Club. Each student is allowed to register for one club only. It is given that the number of students who had registered as members of the Robotics Club is one third of the number of students who had registered as members of the Computer Club and the number of students who had registered as members of the Taekwondo Club is 14. Determine the ratio of the number of members of the Computer Club to the number of members of the Robotics Club to the number of members of the Taekwondo Club.

How do you determine the value related to a rate?

LEARNING STANDARDS
Determine the value related to a rate.

Example 14

Mr Tan jogs at a steady rate on a treadmill and his heart beats 420 times in 4 minutes. Find the number of times his heart will beat if he jogs on the treadmill at the same rate for 12 minutes.

Solution

$$\text{Heart rate} = \frac{420 \text{ times}}{4 \text{ minutes}}$$

Let the number of heartbeats be x times in 12 minutes.

Proportion method

$$\frac{420 \text{ times}}{4 \text{ minutes}} = \frac{x \text{ times}}{12 \text{ minutes}}$$

$$\frac{420 \text{ times}}{4 \text{ minutes}} \overset{\times 3}{=} \frac{x \text{ times}}{12 \text{ minutes}}$$

$$x = 1\,260$$

Cross multiplication method

$$\frac{420}{4} \times \frac{x}{12}$$

$$4 \times x = 420 \times 12$$

$$x = \frac{420 \times 12}{4}$$

$$x = 1\,260$$

Mr Tan's heart will beat 1 260 times in 12 minutes.

Try This *Use unitary method to solve Example 14.*

Did You Know

In real-life situations, the heart rate per minute is not constant. This is because the rate varies according to an individual's condition at different times.

Self Practice 4.4c

1. A person who walks for exercise burns 2.9 calories of energy per minute. How many calories does that person burn during a 20-minute walk?
2. Six cartons of lime juice are sold at RM12.25. Determine the price for 24 similar cartons of lime juice.
3. On Monday, Anis used 8 cups of flour to make 60 pieces of buttermilk biscuits. On Tuesday, Anis plans to make 15 pieces of buttermilk biscuits. If the rate of the flour used to make the biscuits remains the same, how many cups of flour should she use?
4. Suppose the exchange rate of USD1 is equivalent to RM3.90. Ahmad goes to the money changer to change RM200 to US Dollars at this rate. The money changer gives Ahmad back a change of RM5 together with the US Dollars. How much US Dollars does Ahmad receive?

How do you solve problems?

LEARNING STANDARDS

Solve problems involving ratios, rates and proportions, including making estimations.

Example 15

A farmer uses the capture, mark, release and recapture method to estimate the population of snails in his vegetable farm. He captures 24 snails and marks on the shell of each of them. Then, he releases them back into the farm. Two weeks later, he captures randomly 30 snails and finds that 5 of them are marked. He uses the following proportion to estimate the population of snails in the vegetable farm.

$$\frac{\text{Number of marked snails}}{\text{Number of recaptured snails}} = \frac{\text{Number of snails which are captured and marked}}{\text{Population of snails in the vegetable farm}}$$

Estimate the population of snails in the vegetable farm.

Solution

Let the population of snails in the farm = x

$$\frac{\text{Number of marked snails}}{\text{Number of recaptured snails}} = \frac{\text{Number of snails which are captured and marked}}{\text{Population of snails in the vegetable farm}}$$

$$\frac{5}{30} = \frac{24}{x}$$

$$\frac{{}^{1}\cancel{5}}{\cancel{30}_{6}} = \frac{24}{x}$$

$$\frac{1}{6} = \frac{24}{x} \quad (\times 24 \text{ on both sides})$$

$$x = 6 \times 24$$

$$= 144$$

Let's Discuss

How would you find the value of x if you do not change $\frac{5}{30}$ in its simplest form?

The farmer estimates that there are 144 snails in the vegetable farm.

Did You Know

By knowing the pest population in a certain habitat, scientists can estimate the optimum use of pesticides so that the ecological balance can be maintained without affecting the survival of other living organisms.

Self Practice 4.4d

1. The table shows the results of a study to estimate the population of a type of freshwater fish that lives in a lake.

Capture	Number of fish caught	
	Marked	Unmarked
First	60	
Second (After a week)	5	20

Estimate the population of the freshwater fish that live in the lake.

2. A football team played 28 games and won 4 out of every 7 games contested. There were no tie games.
   (a) How many games did this football team lose?
   (b) Calculate the team's win-loss ratio.
   (c) If this trend continues, estimate the number of losses of the team once they have won 20 games.

Mastery Q 4.4

Open the folder downloaded from page vii for extra questions of Mastery Q 4.4.

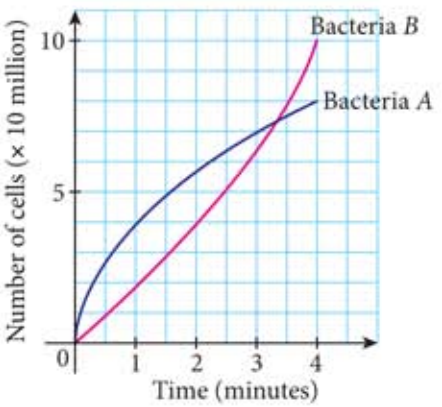
1. The scouts are having an annual camp. Every 12 scouts are under the supervision of a teacher. How many teachers will be involved if there are 90 scouts attending the annual camp?

2. Akmal, Bakri and Cadin share their mother's medical expenses which cost RM4 200. Cadin pays RM2 100 while Akmal pays three quarters of Bakri's amount. Find the ratio of the expenses shared by Akmal to Bakri to Cadin.

3. The table shows the prices of four types of fish. Siti wants to buy 1 kg each of two types of fish and the total amount she is willing to spend is less than RM20. What are the possible combinations of fish that Siti can buy? Show your calculations.

Type of fish	Price
Pomfret (*Bawal*)	RM1.50 per 100 g
Hardtail scad (*Cencaru*)	RM3.20 per 500 g
One finlet scad (*Selar*)	RM2.70 per 300 g
Indian mackerel (*Kembung*)	RM5.40 per 400 g

4. The graph shows the growth rate of bacteria *A* and bacteria *B* in a culture.
   (a) Which bacteria shows the fastest growth rate in the first minute? State this growth rate.
   (b) What is the difference in the number of cells between bacteria *A* and bacteria *B* in the 4th minute?
   (c) Estimate the time when there are equal numbers of the two types of bacteria.



5. A 30 kg bag of fertiliser is labelled 15-20-10. These numbers mean that the contents in this bag of fertiliser contain 15% nitrogen, 20% phosphorus and 10% potassium by mass. The remaining 55% is made up of micronutrients and other fillers.
   (a) Find the ratio of nitrogen to phosphorus to potassium.
   (b) Calculate the mass, in kg, of the contents of nitrogen, phosphorus and potassium in the bag.

4.5 Relationship between Ratios, Rates and Proportions, with Percentages, Fractions and Decimals

What is the relationship between percentages and ratios?

LEARNING STANDARDS

Determine the relationship between percentages and ratios.

Flashback

A percentage is a fraction with a denominator of 100.

Exploration Activity 3

Individual

Aim: To determine the relationship between percentages and ratios.

1. Copy and complete the table below.

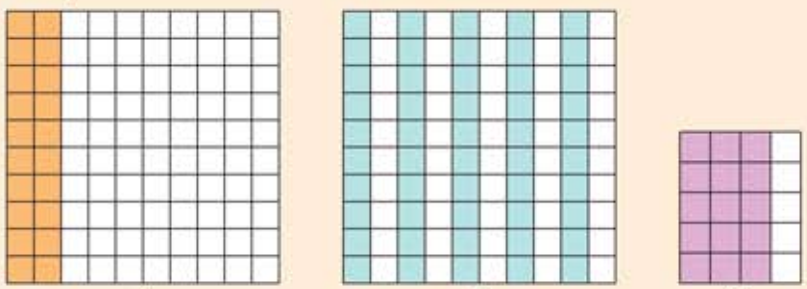


Diagram	The ratio of the number of shaded squares to the number of unshaded squares	The percentage of the shaded squares	The percentage of the unshaded squares
A			
B			
C			

2. Based on the results in the table above, discuss the relationship between percentages and ratios.

From Exploration Activity 3, it is found that a percentage is a ratio that describes a part of 100.

For example, $20\% = \frac{20}{100}$

Let's Discuss

The diagram shows 10 marbles.
What does the ratio 3 : 7 represent in the diagram?
What is the percentage of black marbles in the diagram?

Did You Know

The ratio that compares the number of parts to 100 parts can be expressed in the form of fraction, decimal and percentage.

Example 16

In a class, the ratio of the number of girls to the number of boys is 3 : 2. Find the percentage of the girls in the class.

Solution

The ratio of the number of girls to the total number of students = 3 : 5

$$= \frac{3}{5}$$

$$\frac{3}{5} = \frac{3 \times 20}{5 \times 20}$$ ← Change to a fraction with a denominator of 100.

$$= \frac{60}{100}$$

Thus, the percentage of the girls in the class is 60%.

Alternative Method

The percentage of the girls
$= \frac{3}{5} \times 100\%$
$= 60\%$

Example 17

Zakaria is thrifty. He saves 30% of his daily pocket money. Find the ratio of his daily savings to his total daily pocket money received.

Solution

The percentage of Zakaria's daily savings = 30%

$$= \frac{30}{100}$$

The ratio of his daily savings to his total daily pocket money received = 30 : 100

= 3 : 10

Self Practice 4.5a

1. In a class, the ratio of the number of students who wear spectacles to the number of students who do not wear spectacles is 1 : 3. Find the percentage of the students who do not wear spectacles in the class.
2. It is known that 35% of the hard disk of a computer has been filled with data. Find the ratio of the capacity that has been filled with data to the capacity that has not been filled with data.

How do you determine the percentage of a quantity by applying the concept of proportions?

Flashback

A proportion is a relationship that states that two ratios are equal.

LEARNING STANDARDS

Determine the percentage of a quantity by applying the concept of proportions.

Example 18

Determine the percentage of each of the following quantities by applying the concept of proportions.

(a) A box contains 8 ribbons. Two of the ribbons are blue. What is the percentage of blue ribbons in the box?

(b) Puan Jorana has RM300. She spends RM15 for her transport fee. What is the percentage of the amount of money she spends?

Solution

(a) Let the percentage of blue ribbons in the box be x.

$$\frac{x}{100} = \frac{\text{Number of blue ribbons}}{\text{Total number of ribbons}}$$ ← Write a proportion

$$\frac{x}{100} = \frac{2}{8}$$

$$8 \times x = 2 \times 100$$

$$x = \frac{2 \times 100}{8}$$

$$= 25$$

Thus, 25% of the ribbons in the box are blue.

(b) Let the percentage of the amount of money spent be y.

$$\frac{\text{Amount of money spent}}{\text{Total amount of money}} = \frac{y}{100}$$

$$\frac{15}{300} \overset{\div 3}{=} \frac{y}{100}$$

$$y = 5$$

Puan Jorana spends 5% of the total amount of money.

Example 19

At a sale carnival, Encik Rosli chooses a shirt from a rack which displays '45% price reduction'. The original price of the shirt is RM85.00. When Encik Rosli scans the price tag of the shirt, the scanner shows that the price is RM57.80. By applying the concept of proportions, determine whether this percentage discount corresponds to the percentage reduction displayed. Give a reason for your answer.

Solution

Let p be the percentage discount of the original price.

Thus, $$\frac{p}{100} = \frac{\text{Selling price}}{\text{Original price}}$$

$$\frac{p}{100} = \frac{57.80}{85.00}$$

$$85p = 5780$$

$$p = \frac{5780}{85}$$

$$= 68$$

The selling price is 68% of the original price.

Percentage discount obtained = 100% – 68%
= 32%

The percentage discount obtained by Encik Rosli is less than the percentage reduction displayed.

Self Practice 4.5b

1. Determine the percentage of each of the following quantities by applying the concept of proportions.
   (a) 14 out of 56 students completed their homework in 1 hour. What is the percentage of the students who had completed their homework in 1 hour?
   (b) A class has 45 students. 18 of them are girls. What is the percentage of girls in the class?
   (c) The bee hummingbird is the smallest bird in the world. It can weigh as little as 2 grams. The ostrich is the largest bird in the world and it can weigh as much as 150 kilograms. What percent of the mass of a bee hummingbird is the mass of an ostrich?

2. A shoe factory outlet is having a sales promotion in conjunction with the school holidays. A pair of shoes with an original price of RM45 is sold at a 25% discount. By applying the concept of proportions, determine the amount a consumer could save when he buys this pair of shoes during the sales promotion.

How do you solve problems?

MATHEMATICS APPLICATION TEST

LEARNING STANDARDS

Solve problems involving relationship between ratios, rates and proportions with percentages, fractions and decimals.

The table shows the monthly rental rates and the deposits required when renting apartments in Taman Bukit Damai.

Type of apartment	Two-room	Three-room
Rental rate (RM per month)	450	550
Deposit	1 month rent	2 months rent

(a) Ben and Farid rent the two-room apartment and the three-room apartment respectively. Find the ratio of the initial payment made by Ben to the initial payment made by Farid.

(b) The monthly salaries of Ben and Farid are RM3 750 and RM5 000 respectively. By applying the concept of proportions, determine the percentage of the monthly rental spent to the monthly salary of each of them.

Solution

(a) Initial payment made by Ben $= 450 + 450$
$= \text{RM}900$

Initial payment made by Farid $= 550 + (2 \times 550)$
$= \text{RM}1\,650$

Ratio of initial payment made by Ben to Farid $= 900 : 1\,650$
$= 6 : 11$

(b) Let x be the percentage of the monthly rental spent to the monthly salary of Ben.

$$\frac{x}{100} = \frac{450}{3750}$$

$$x = \frac{450}{3750} \times 100$$

$$= 12$$

Ben spends 12% of his monthly salary on rental.

Career in Mathematics

Chemists use proportions to calculate the amount of each pigment required to produce the appropriate colour of paint.

Let y be the percentage of the monthly rental spent to the monthly salary of Farid.

$$\frac{y}{100} = \frac{550}{5000}$$

$$y = \frac{550}{5000} \times 100$$

$$= 11$$

Farid spends 11% of his monthly salary on rental.

Self Practice 4.5c

1. There are 40 passengers on a bus. At the next bus stop, 8 passengers get off and 18 passengers get on the bus.

(a) By applying the concept of proportions, determine the percentage of the passengers who get off the bus compared to the number of passengers originally on the bus.

(b) What is the ratio of the passengers who get on the bus at the bus stop compared to the new total number of passengers on the bus?

Mastery Q 4.5

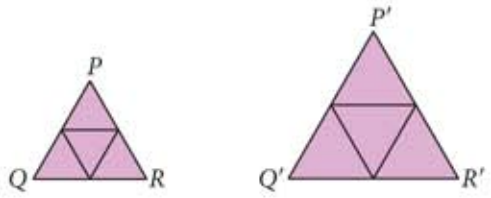
Open the folder downloaded from page vii for extra questions of Mastery Q 4.5.

1. In a basket, the ratio of the number of red apples to the number of green apples is 3 : 5.

(a) Write the fraction of the number of red apples over the total number of apples. Hence, express this fraction in decimal.

(b) What is the percentage of red apples in the basket?

2.



Original After enlargement

Karim uses a photocopy machine to enlarge the diagram of PQR to 150% of its original size.

(a) Write the ratio of the length of $P'Q'$ to the length of PQ.

(b) Is the ratio of the length of $P'R'$ to the length of PR same as the ratio of the length of $P'Q'$ to the length of PQ?

(c) Use your knowledge on ratios and percentages to explain the meaning of 'enlarge diagram PQR to 150% of its original size'.

3. A book with an original price of RM25 is sold at a 30% discount in a bookshop. If the same book is sold online at the original price of RM20, the customers are given a 15% discount. By applying the concept of proportions, determine which one is the better choice for buying the book. Justify your choice.

SUMMARY

Ratio	Rate	Proportion
Compares two or three quantities measured in the same unit.	Compares two quantities measured in different units.	A relationship that states that two ratios or two rates are equal.
Written in the form of $a : b$ or $a : b : c$.	Can be expressed as a fraction involving different units. For example: Speed is measured in km/h.	Can be expressed as a fraction. For example: $\frac{a_1}{a_2} = \frac{b_1}{b_2}$ Thus, a is proportional to b.
Can be expressed as a percentage, a fraction and a decimal.	Cannot be expressed as a percentage.	Can be solved using the unitary method, the proportion method or the cross multiplication method.

At the end of this chapter, I can...	Very good	Work harder
represent the relation between three quantities in the form of $a : b : c$.		
identify and determine the equivalent ratios in numerical, geometrical or daily situation contexts.		
express the ratios of two and three quantities in simplest form.		
determine the relationship between ratios and rates, ratios and proportions.		
determine an unknown value in a proportion.		
determine the ratio of three quantities, given two or more ratios of two quantities.		
determine the ratio or the related value given (i) the ratio of two quantities and the value of one quantity. (ii) the ratio of three quantities and the value of one quantity.		
determine the value related to a rate.		
solve problems involving ratios, rates and proportions, including making estimations.		
determine the relationship between percentages and ratios.		
determine the percentage of a quantity by applying the concept of proportions.		
solve problems involving relationship between ratios, rates and proportions with percentages, fractions and decimals.		

Test Yourself

1. Copy and match the following pairs of equivalent ratios.

6 : 9 •	• 1 : 15
6 : 90 •	• 2 : 3
9 : 60 •	• 3 : 20

2.

Aquarium *A*

Aquarium *B*

Aquarium *A* contains 16 goldfish and aquarium *B* contains 20 goldfish.

(a) State the ratio of the number of goldfish in aquarium *A* to the number of goldfish in aquarium *B*.

(b) A few goldfish are subsequently added into each aquarium such that the ratio in (a) remains the same. Find the minimum number of goldfish added into each of the aquariums.

3. The information below shows the ratios of the scores obtained by Group *P* and Group *Q* in a quiz.

Ratio of the scores obtained by Group *P*				
Chong	:	Rahim	:	Hassan
5	:	4	:	7

Ratio of the scores obtained by Group *Q*				
Nurin	:	Bala	:	Shanthy
3	:	2	:	1

The total score obtained by Group *P* is 144 and the total score obtained by Group *Q* is 168.

(a) Find Rahim's score.

(b) State the ratio of Hassan's score to Bala's score.

(c) Who obtained the highest score? What is the score?

Self Mastery

4.

Anis goes to a grocery store to buy her favourite brand of orange juice. She finds that the orange juice is sold in different volumes and prices. Which bottle of orange juice offers the most affordable price?

5. Puan Kavitha has a water tank which measures 120 cm in length, 60 cm in width and 50 cm in height. Water is filled into the tank at a rate of 2.4 litres per minute. If Puan Kavitha starts to fill the empty tank with water at 0630 hours, at what time will the water tank be full? ($1\,l = 1\,000\ cm^3$)

6. Atong uses the following proportion to estimate the height of a tree.

$$\frac{\text{Height of the tree}}{\text{Length of the tree's shadow}} = \frac{\text{Height of the student}}{\text{Length of the student's shadow}}$$

Atong's height is 1.55 m and one afternoon, he finds that his shadow is 0.93 m on the ground. Estimate the height of the tree that casts a 6-m shadow on the ground.

Challenge Yourself

7. The table shows the prices of RON 95 petrol on two different days.

Date	29 February 2016	6 March 2016
Price of RON 95 petrol	RM1.75 per litre	RM1.60 per litre

(a) If Lai Huat pays RM15 every time he fills up his motorcycle fuel tank with RON 95 petrol, what is the difference in volume, in litres, of the petrol filled on 29 February 2016 compared to 6 March 2016?

(b) Lai Huat suggests moving to his new house on 8 March 2016.

	Monthly rental (RM)	**Commuting distance (km)**
Current house	300	24
New house	340	18

Lai Huat fills up his motorcycle fuel tank with RON 95 petrol to travel from his house to the workplace. If the fuel consumption rate for his motorcycle is 20 km per litre, should Lai Huat move to his new house? Give a reason and show your calculations. (Assume that Lai Huat works an average of 20 days per month.)

8. The table shows the parking rates at a car park.

Parking rate	**Monday – Friday (8:00 a.m. – 5:00 p.m.)**
First 2 hours or a part thereof	RM1.60
Every subsequent hour or a part thereof	RM1.00
After 5:00 p.m. to 10:00 p.m. (per entry)	RM2.00
Lost ticket	RM20.00

(a) On Wednesday, Puan Zaiton parks her car from 1030 hours to 1400 hours. How much does Puan Zaiton need to pay?

(b) On Thursday, Mr Ong parks his car at 0800 hours. Upon his return to the car park at 2000 hours, he finds that he has left his parking ticket in the office on the 16th floor. Should Mr Ong go back to his office to take the parking ticket or pay the penalty by reporting the lost ticket? Justify your answer.

ASSIGNMENT

Black glutinous rice porridge is a popular dessert in Malaysia. Look at the black glutinous rice porridge recipe below. If this recipe serves 6 people, calculate the quantity of each ingredient that is required to serve all the students in your class or all your family members at home. Use your knowledge on ratios and rates to prepare this recipe. Try this recipe in school or at home with the help of your teacher or family members.

Black Glutinous Rice Porridge

Ingredients:

- 200 g of black glutinous rice (soaked overnight in water)
- 125 g of palm sugar (*gula melaka*)
- 270 m*l* of coconut milk (add more if you prefer a richer taste)
- 6 cups of water
- 3 pieces of *pandan* leaves, halved and knotted.

Method:

1. Wash and drain the black glutinous rice thoroughly.
2. Place the glutinous rice in a pot filled with 5 cups of water and the knotted *pandan* leaves.
3. Bring the glutinous rice to a boil and then lower the heat to a simmer until the rice is cooked and soft. Stir the glutinous rice occasionally.
4. Meanwhile, add the palm sugar (*gula melaka*) and a cup of water to another pot. Heat up the pot and stir the mixture on low heat until a thick syrup is formed.
5. Add the palm sugar (*gula melaka*) mixture together with coconut milk to the cooked glutinous rice. Stir until the mixture is well blended.
6. Turn off the heat. The black glutinous rice porridge is ready to be served warm or chilled.

Exploring MATHEMATICS

Leonardo da Vinci (famous for the painting of Mona Lisa), Michelangelo (an artist, architect and poet of the Italian Renaissance) and a few other artists studied the relationship of mathematical ratios on the human body. One of the ratios they used was $\frac{\text{height of head}}{\text{height of the whole body}} = \frac{2}{5}$.

Find pictures or photos of people in magazines or newspapers. Measure their entire body to determine the ratio. Is the ratio always $\frac{2}{5}$?