CHAPTER 5

Network in Graph Theory

You will learn

- Network

The transportation system in Malaysia which comprises land, water and air transportation are developed in line with the country's progress. The land transportation system, particularly public transport is developed at a fast pace based on the increase in the number of commuters in big cities. City rail services such as Light Rail Transit (LRT), Mass Rapid Transit (MRT), monorails and commuter trains in the cities are amongst the types of public transportation that are increasingly well accepted by the public.

Do you know that the transportation system is a type of network?

Why Study This Chapter?

A network is used to link objects in the same field based on its needs. Networks are widely used in transportation, computer, social, business, investigation, medicine, science, neuroscience, social science and gaming fields.

WORD BANK

• weighted	• *berpemberat*
• vertex	• *bucu*
• degree	• *darjah*
• discrete	• *diskret*
• loop	• *gelung*
• graph	• *graf*
• simple	• *mudah*
• tree	• *pokok*
• network	• *rangkaian*
• subgraph	• *subgraf*
• edge	• *tepi*
• directed	• *terarah*

Walking Through Time

Leonhard Euler
(1707 – 1783)

Leonhard Euler, a mathematician in the 18th century, was born in Basel, Switzerland. In the year 1735, Euler solved a mathematics and logic problem known as the Seven Bridges of Königsberg and developed a mathematical structure called graph – a diagram consisting of dots (vertices) which are linked by lines or arcs (edges).

http://bt.sasbadi.com/m4129

5.1 Network

What is the relationship between a network and a graph?

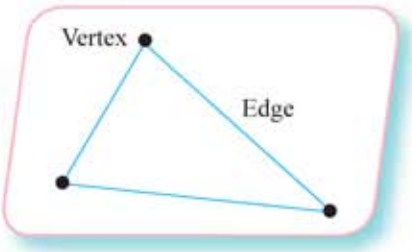
Learning Standard

To identify and explain a network as a graph.

In Year 5, you were introduced to systems of computer networks and the world of Internet through the subject of Information and Communication Technology.

The link between a group of computers and the associated devices, that is a computer network, enables information to be searched, used and shared easily. Do you know the relationship between a network and a graph?

A graph is used to represent data consisting of discrete objects and to show the relationship between these objects in a simple graphical manner. In the field of mathematics, graph theory in particular, a **graph** is interpreted as a series of dots which are either linked or not linked to one another by lines. Each dot is known as a **vertex** and the line joining two vertices is known as an **edge**.



A graph is usually used to represent a certain network. A **network** is part of a graph with the vertices and edges having their own characteristics. The structure of network data has a many-to-many relation. Examples of graphs that involve networks are as follows.

Land transport network

Vertex
Regions, towns, cities or certain buildings that are linked

Edge
Roads, highways or railway lines

Social network

Vertex
Individuals, groups or organisations

Edge
Types of relationships such as friends, colleagues or families

INFO ZONE

A Cold Chain System is a system used in the world of medicine. The function of this system is to transport, distribute and store vaccine and blood in a fixed temperature range from the source to the place they are used.

A graph is denoted by a set of ordered pairs $G = (V, E)$, where

- V is the set of dots or vertices.
  $V = \{v_1, v_2, v_3, \ldots v_n\}$
- E is the set of edges or lines linking each pair of vertices.
  $E = \{e_1, e_2, e_3, \ldots e_n\}$
  $E = \{(a_1, b_1), (a_2, b_2), \ldots (a_n, b_n)\}$; a and b are pairs of vertices.

INFO ZONE

G = Graph
v = Vertex or dot
e = Edge or line or arc
d = Degree
Σ = Sum

The **degree**, ***d***, is the number of edges that connect two vertices. The sum of degrees of a graph is twice the number of edges, that is

$$\Sigma d(v) = 2E; v \in V$$

MY MEMORY

$\in$: an element of

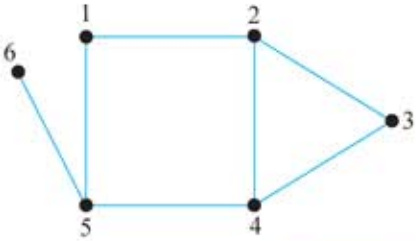
What do you understand about a simple graph?

A simple graph has no loops and no multiple edges. The sum of degrees of the graph is twice the number of edges.

Example 1

Based on the simple graph given, determine

(a) V and $n(V)$
(b) E and $n(E)$
(c) sum of degrees.



Solution:

(a) $V = \{1, 2, 3, 4, 5, 6\}$ ← Set of vertices
$n(V) = 6$ ← Number of vertices

(b) $E = \{(1, 2), (1, 5), (2, 3), (2, 4), (3, 4), (4, 5), (5, 6)\}$ ← Set of vertex pairs
$n(E) = 7$ ← Number of edges

(c) Sum of degrees
$\Sigma d(v) = 2(E)$
$= 2(7)$
$= 14$

Degree of vertex: $d(1) = 2$
$d(2) = 3$
$d(3) = 2$
$d(4) = 3$
$d(5) = 3$
$d(6) = 1$
Sum = 14

The degree of vertex 1 is two, that is the edges which connect vertex 1 to vertex 2 and vertex 1 to vertex 5.

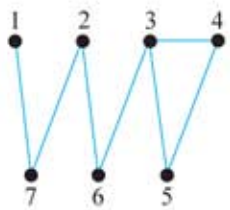
INFO ZONE

The edge for the vertex pair (1, 2) is also the edge for the vertex pair (2, 1).

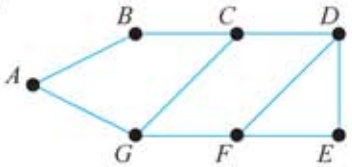
Example 2

State the number of vertices, edges and the sum of degrees for the following simple graphs.

(a)



(b)



Solution:

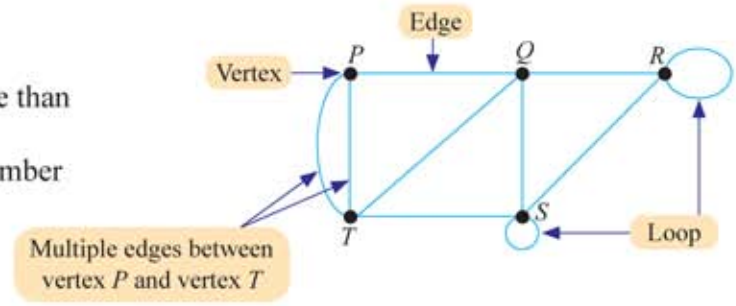
(a) Number of vertices = 7
Number of edges = 7
Sum of degrees = 2 × Number of edges
= 2 × 7
= 14

(b) Number of vertices = 7
Number of edges = 9
Sum of degrees = 2 × Number of edges
= 2 × 9
= 18

What is the meaning of multiple edges and loops of a graph?

Multiple edges

- Involve two vertices.
- The vertices are connected by more than one edge.
- The sum of degrees is twice the number of edges.



Loops

- Involve one vertex.
- The edge is in the form of an arc that starts and ends at the same vertex.
- Each loop adds 2 to the degree.

Let the graph given be denoted by a set of ordered pairs, $G(V, E)$, then

$V = \{A, B, C, D\}$

Multiple edges

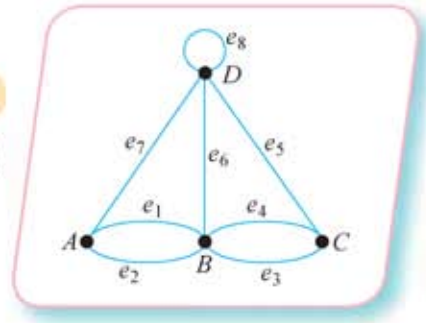
A loop

$E = \{\overbrace{(A, B)}, \overbrace{(A, B)}, \overbrace{(B, C)}, \overbrace{(B, C)}, (C, D), (B, D), (A, D), \overbrace{(D, D)}\}$

$E = \{e_1, e_2, e_3, e_4, e_5, e_6, e_7, e_8\}$

Second edge of AB

First edge of AB



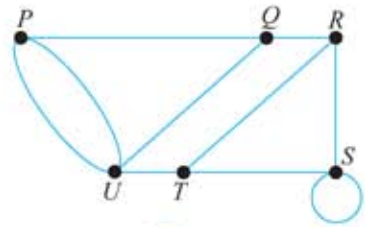
Example 3

The diagram on the right shows a graph with a loop and multiple edges. State

(a) V and $n(V)$

(b) E and $n(E)$

(c) sum of degrees.



MY MEMORY

V = Set of vertices
E = Set of edges

Solution:

(a) $V = \{P, Q, R, S, T, U\}$
$n(V) = 6$

(b) $E = \{(P, Q), (P, U), (P, U), (Q, R), (Q, U), (R, S), (R, T), (S, S), (S, T), (T, U)\}$
$n(E) = 10$

(c) Sum of degrees = 20

Degree of vertex P = 3
Degree of vertex Q = 3
Degree of vertex R = 3
Degree of vertex S = 4
Degree of vertex T = 3
Degree of vertex U = 4

The sum of degrees is 20.

TIPS

The degree of a vertex with a loop in an undirected graph is 2, one in clockwise direction and the other in anticlockwise direction.

Example 4

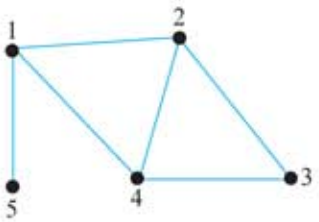
Draw a simple graph based on the given information.

(a) $V = \{1, 2, 3, 4, 5\}$
$E = \{(1, 2), (1, 4), (1, 5), (2, 3), (2, 4), (3, 4)\}$

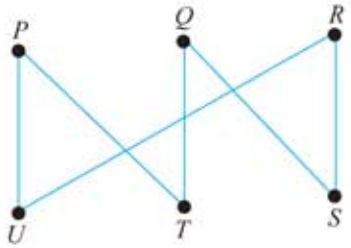
(b) $V = \{P, Q, R, S, T, U\}$
$E = \{(P, U), (P, T), (Q, T), (Q, S), (R, S), (R, U)\}$

Solution:

(a)



(b)



Example 5

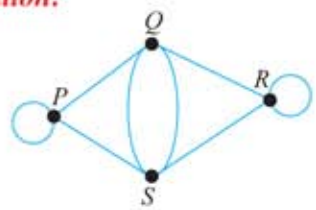
Draw a graph with multiple edges and loops based on the given information.

(a) $V = \{P, Q, R, S\}$
$E = \{(P, P), (P, Q), (P, S), (Q, S), (Q, S), (Q, R), (S, R), (R, R)\}$

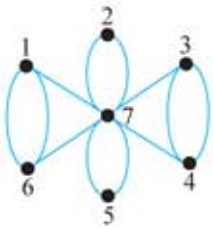
(b) $V = \{1, 2, 3, 4, 5, 6, 7\}$
$E = \{(1, 6), (1, 6), (2, 7), (2, 7), (5, 7), (5, 7), (3, 4), (3, 4), (1, 7), (6, 7), (3, 7), (4, 7)\}$

Solution:

(a)



(b)



TIPS

- The vertex pair for a loop is in the form (a, a).
- The vertex pairs for multiple edges are in the forms (a, b), (a, b).

Example 6

Determine whether a graph with the following degrees of vertices can be drawn.

(a) 3, 2, 2, 1, 3

(b) 2, 1, 1, 3, 3, 2

Solution:

(a) Sum of degrees = 3 + 2 + 2 + 1 + 3
= 11

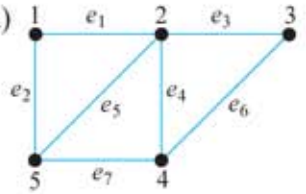
The graph cannot be drawn because the sum of degrees is odd.

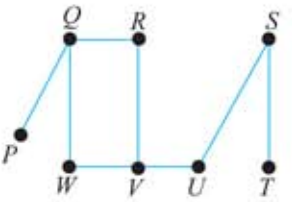
(b) Sum of degrees = 2 + 1 + 1 + 3 + 3 + 2
= 12

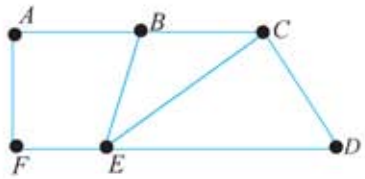
The graph can be drawn because the sum of degrees is even.

Self Practice 5.1a

1. Three simple graphs are given below.

(a) 

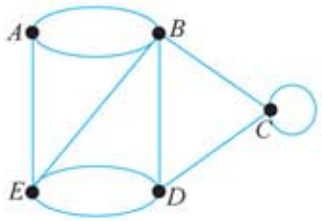
(b) 

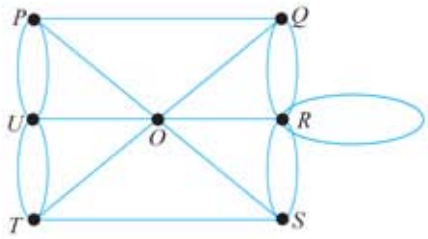
(c) 

For each graph, determine

(i) V and $n(V)$ (ii) E and $n(E)$ (iii) sum of degrees

2. Two graphs with multiple edges and loops are given below.

(a) 

(b) 

For each graph, determine

(i) V and $n(V)$. (ii) E and $n(E)$. (iii) sum of degrees.

3. Draw a simple graph based on the given information.

(a) $V = \{1, 2, 3, 4, 5, 6\}$

$E = \{(1, 6), (2, 6), (3, 6), (3, 4), (3, 5), (4, 5), (5, 6)\}$

(b) $V = \{P, Q, R, S, T, U\}$

$E = \{(P, U), (P, T), (Q, U), (Q, T), (Q, R), (R, S), (R, T), (S, T)\}$

4. Draw a graph with multiple edges and loops based on the given information.

(a) $V = \{P, Q, R, S\}$

$E = \{(P, S), (P, S), (Q, R), (Q, R), (P, P), (S, S), (Q, Q), (R, R), (P, Q), (R, S)\}$

(b) $V = \{1, 2, 3, 4, 5\}$

$E = \{(1, 5), (1, 5), (3, 5), (3, 5), (1, 1), (3, 3), (2, 1), (2, 3), (2, 5), (4, 1), (4, 3), (4, 5)\}$

5. Draw a simple graph with the given degrees of vertices.

(a) 2, 3, 2, 3, 4 (b) 1, 2, 3, 3, 3, 2

6. Draw a graph with loops and multiple edges with the given degrees of vertices.

(a) 3, 3, 4 (b) 2, 2, 4, 6, 6

What is the difference between a directed graph and an undirected graph?

Learning Standard

Compare and contrast
- directed graphs and undirected graphs.
- weighted graphs and unweighted graphs

A **directed graph** is a graph in which a direction is assigned to the edge connecting two vertices. Directed graphs are usually used to represent the flow of a certain process. For example, road maps, airlines networks, electrical circuits, computer networks and organisation charts.

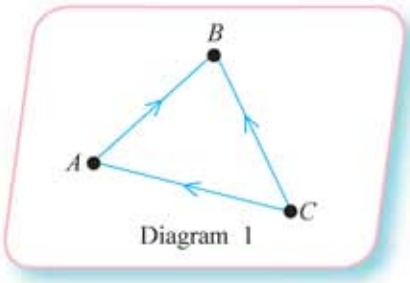


Diagram 1

INTERACTIVE ZONE

The blood circulation system in the human body is also categorised as a directed graph. Why?

Diagram 1 shows a simple directed graph. Based on the directions of the arrows, it can be seen that,

(a) for edge AB, A is the initial vertex and B is the terminal vertex.

(b) for edge CB, C is the initial vertex and B is the terminal vertex.

(c) all vertices are connected in one direction only.

INTERACTIVE ZONE

For electricity networks, power stations, transformer stations, substations and consumers are the vertices, and the cables and wires are the edges. Is an electricity network a directed graph or an undirected graph? Discuss.

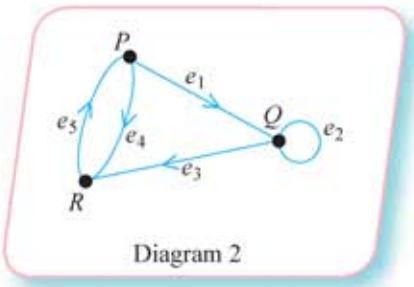


Diagram 2

Diagram 2 shows a directed graph with a loop and multiple edges. Based on the directions of the arrows, it can be seen that

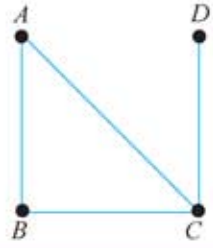
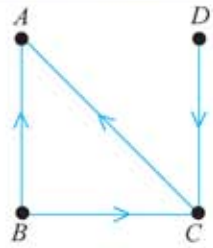
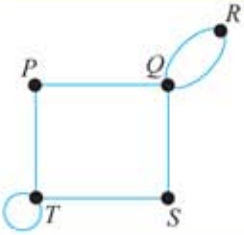
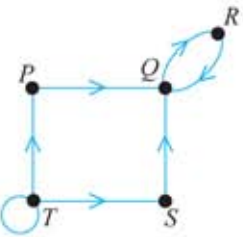
(a) $e_4 = (P, R)$; P is the initial vertex and R is the terminal vertex.

(b) $e_5 = (R, P)$; R is the initial vertex and P is the terminal vertex.

(c) $e_2 = (Q, Q)$; Q is the initial vertex and the terminal vertex because e_2 is a loop.

An **undirected graph** is a simple graph or a graph with loops and multiple edges drawn without any direction being assigned.

Differences between directed graphs and undirected graphs

Type of graph	Graph	Set V and Set E	Degrees
Simple graph	Undirected graph 	$V = \{A, B, C, D\}$ $E = \{(A, B), (A, C), (B, C), (C, D)\}$ The order of the vertices written is not important. Both pairs of vertices, (A, B) and (B, A) represent the edge AB.	$d(A) = 2$, $d(B) = 2$, $d(C) = 3$, $d(D) = 1$ $\Sigma d(V) = 8$
	Directed graph 	$V = \{A, B, C, D\}$ $E = \{(B, A), (C, A), (B, C), (D, C)\}$ The order of the vertices are written according to the direction of the edge. (B, A) and (A, B) represent different edges.	$d_{in}(A) = 2$ and $d_{out}(A) = 0$ Thus, $d(A) = 2 + 0$ $d(A) = 2$ $d_{in}(A)$ means the number of edges 'going into' vertex A. $d_{out}(A)$ means the number of edges 'coming out' from vertex A. $d_{in}(B) = 0$, $d_{out}(B) = 2$ $d_{in}(C) = 2$, $d_{out}(C) = 1$ $d_{in}(D) = 0$, $d_{out}(D) = 1$ $\Sigma d(V) = 8$
Graph with loops and multiple edges	Undirected graph 	$V = \{P, Q, R, S, T\}$ $E = \{(P, T), (P, Q), (Q, R), (Q, R), (Q, S), (S, T), (T, T)\}$	$d(P) = 2$, $d(Q) = 4$ $d(R) = 2$, $d(S) = 2$ $d(T) = 4$ $\Sigma d(V) = 14$
	Directed graph 	$V = \{P, Q, R, S, T\}$ $E = \{(P, Q), (Q, R), (R, Q), (S, Q), (T, S), (T, T), (T, P)\}$	$d_{in}(P) = 1$, $d_{out}(P) = 1$ $d_{in}(Q) = 3$, $d_{out}(Q) = 1$ $d_{in}(R) = 1$, $d_{out}(R) = 1$ $d_{in}(S) = 1$, $d_{out}(S) = 1$ $d_{in}(T) = 1$, $d_{out}(T) = 3$ Loop = 1 Loop = 1 $(T, P) = 1$ $(T, S) = 1$ $\Sigma d(V) = 14$

What are the differences between weighted graphs and unweighted graphs?

	Weighted graph	Unweighted graph
Type of graph	Directed graph and undirected graph	Directed graph and undirected graph
Edge	Associated with a value or a weight	Not associated with a value or a weight
Example	The edge represents: • distance between two cities. • travelling time. • the current in an electrical circuit. • cost.	The edge relates information like: • job hierarchy in an organisation chart. • flow map. • tree map. • bubble map.

Example 7

Draw a directed graph based on the given information.

(a) $V = \{P, Q, R, S, T, U\}$
$E = \{(P, Q), (P, R), (R, Q), (S, R), (S, Q), (S, T)\}$

(b) There is a loop at vertex Q and RS is a multiple edge such that
$d_{in}(P) = 1, d_{out}(P) = 1$
$d_{in}(Q) = 3, d_{out}(Q) = 2$
$d_{in}(R) = 0, d_{out}(R) = 3$
$d_{in}(S) = 3, d_{out}(S) = 1$

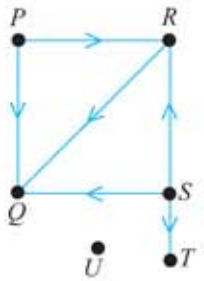
INFO ZONE

For Example 7(a), the vertex U appears in set V but not in set E. This means vertex U is not connected to any other vertices in the graph and it is known as an **isolated vertex**.

Solution:

(a)

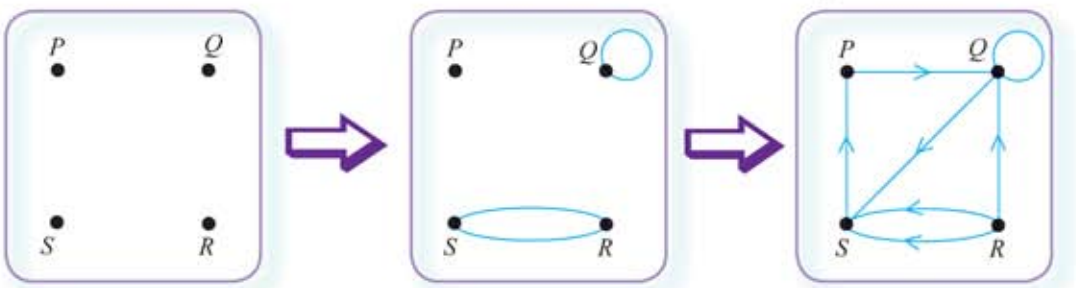
Vertex pair	Direction	
(P, Q)	P to Q	Two edges from vertex P
(P, R)	P to R	
(R, Q)	R to Q	One edge from vertex R
(S, R)	S to R	Three edges from vertex S
(S, Q)	S to Q	
(S, T)	S to T	
		Vertex T only connected to vertex S



(b) Total number of vertices = 4

RS – a multiple edge,
A loop at vertex Q.

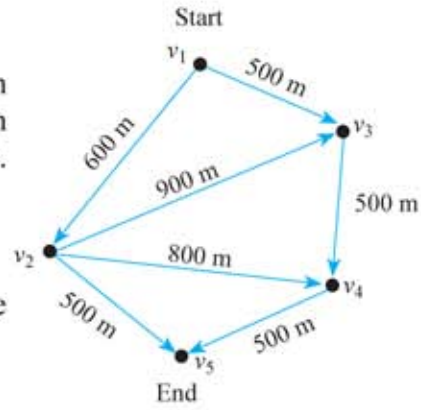
Complete the graph based on the number of edges going into and coming out from each vertex.



Example 8

The diagram on the right shows one-way paths that Izarul can choose for his running practice. Vertex v_1 is the starting position and vertex v_5 is the ending position before he goes home. Determine

(a) the shortest distance from v_1 to v_5.
(b) the longest distance from v_1 to v_5.
(c) the vertices that must be passed through if the distance of the one-way run is between 1.4 km and 2.1 km.



Solution:

(a) Shortest distance
$= v_1 \rightarrow v_2 \rightarrow v_5$
$= (600 + 500)$ m
$= 1\,100$ m
$= 1.1$ km

(b) Longest distance
$= v_1 \rightarrow v_2 \rightarrow v_3 \rightarrow v_4 \rightarrow v_5$
$= (600 + 900 + 500 + 500)$ m
$= 2\,500$ m
$= 2.5$ km

(c) v_1, v_3, v_4, v_5 and
v_1, v_2, v_4, v_5

Self Practice 5.1b

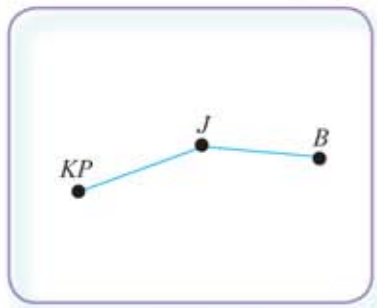
1. State two differences between directed graphs and undirected graphs.
2. What is the meaning of weight in a weighted graph?
3. Draw a directed graph based on the given information.
   (a) $V = \{P, Q, R, S, T, U, V\}$
   $E = \{(P, Q), (Q, R), (Q, S), (S, P), (S, R), (S, T), (U, T)\}$
   (b) (i) There is a loop at vertex Q and a loop at vertex S.
   (ii) QS is a multiple edge.
   (iii) $d_{in}(P) = 0$, $d_{out}(P) = 2$
   $d_{in}(Q) = 4$, $d_{out}(Q) = 2$
   $d_{in}(R) = 0$, $d_{out}(R) = 2$
   $d_{in}(S) = 4$, $d_{out}(S) = 2$
4. (a) Based on the information in Table 1 and Table 2, complete the weighted and undirected graph.

Name of place	Vertex
Kuala Pilah	KP
Bahau	B
Rompin	R
Batu Kikir	BK
Juasseh	J

Table 1

Vertex pair	Weight (km)
(J, B)	11.6
(KP, R)	40
(B, R)	20.7
(B, BK)	11.4
(BK, J)	6.6
(KP, J)	9.3

Table 2



Incomplete graph

(b) Mr Benny and Mr Muruges drive individually from Kuala Pilah to Rompin such that:
(i) Mr Benny uses the shortest route.
(ii) Mr Muruges takes the route which passes through Juasseh and Bahau.
Calculate the difference in distance, in km, for the journeys taken by Mr Benny and Mr Muruges from Kuala Pilah to Rompin.

How do you identify and draw subgraphs and trees?

Learning Standard

Identify and draw subgraphs and trees.

What do you understand about a subgraph?

A **subgraph** is part of a graph or the whole graph redrawn without changing the original positions of the vertices and edges. A graph H is said to be a subgraph of G if,

(a) the vertex set of graph H is a subset of the vertex set of graph G, that is $V(H) \subset V(G)$.
(b) the edge set of graph H is a subset of the edge set of graph G, that is $E(H) \subset E(G)$.
(c) the vertex pairs of the edges of graph H are the same as the edges of graph G.

In short,

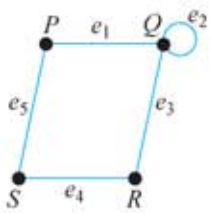
- a vertex in graph G is a subgraph of graph G.
- an edge in graph G along with the vertices it connects is a subgraph of graph G.
- each graph is a subgraph of itself.

INFO ZONE

The symbol $\subseteq$ also stands for subset.

Example 9

Determine whether Diagram 1, Diagram 2, Diagram 3 and Diagram 4 are the subgraphs of graph G.



Graph G

Diagram 1

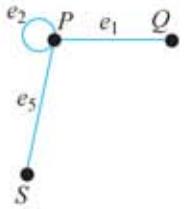


Diagram 2

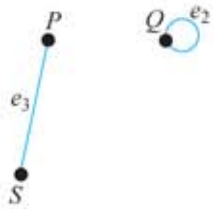


Diagram 3

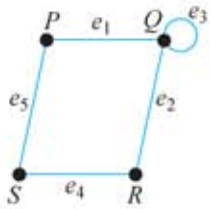


Diagram 4

Solution:

Diagram 1 – Yes, because the vertex pair for edge e_5 is the same.
$\{e_5\} \subset \{e_1, e_2, e_3, e_4, e_5\}$ and $\{P, S\} \subset \{P, Q, R, S\}$

Diagram 2 – No, because the position of loop e_2 is not on vertex Q.

Diagram 3 – No, because the edge connecting vertex P and vertex S is not e_3.

Diagram 4 – No, because the loop and the edge connecting vertex Q and vertex R are wrong.

What do you understand about a tree?

A **tree** of a graph is a subgraph of the graph with the following properties:

(a) A simple graph without loops and multiple edges.
(b) All the vertices are connected and each pair of vertices is connected by only one edge.
(c) Number of edges = Number of vertices – 1
Number of vertices = n
Number of edges = $n - 1$

INFO ZONE

The term tree was introduced by Arthur Cayley, an English mathematician, in the year 1857.

INFO ZONE

A family history chart is an example of a tree.

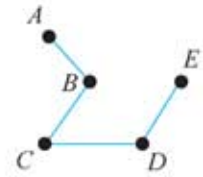


Diagram 1

Diagram 1 is a tree because
- all the vertices are connected.
- every pair of vertices is connected by an edge only.
- there are no loops or multiple edges.
- 5 vertices, 4 edges.

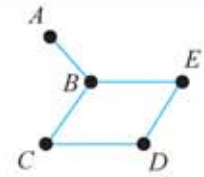
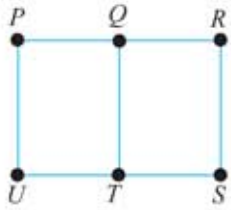


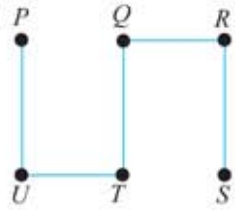
Diagram 2

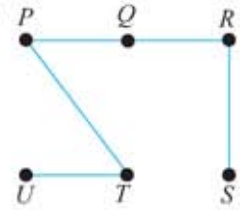
Diagram 2 is not a tree because
- vertex B and vertex E can be connected in two ways.
  (i) $B \rightarrow E$
  (ii) $B \rightarrow C \rightarrow D \rightarrow E$
- 5 vertices, 5 edges.

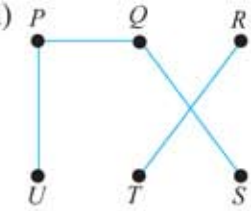
Example 10

Determine whether each of the following graphs is a tree. Justify your answer.

(a) 

(b) 

(c) 

(d) 

Solution:

(a) Not a tree. 6 vertices, 7 edges. Each pair of vertices can be connected in various ways.

(b) A tree. 6 vertices, 5 edges. Each pair of vertices is connected by one edge.

(c) A tree. 6 vertices, 5 edges. Each pair of vertices is connected by one edge.

(d) Not a tree. 6 vertices, 4 edges. Vertex R and vertex T are not connected to the other vertices.

Example 11

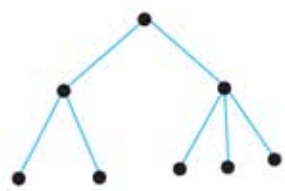
Draw a tree for the following information given.

(a) 6 vertices (b) 8 vertices (c) 4 edges (d) 9 edges

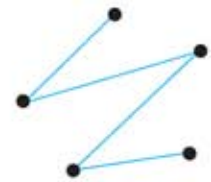
Solution:

(a) 6 vertices
5 edges

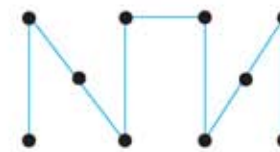
(b) 8 vertices
7 edges



(c) 4 edges
5 vertices

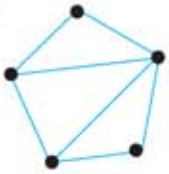


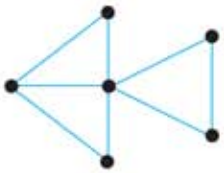
(d) 9 edges
10 vertices



Example 12

Draw two trees based on the graphs given below.

(a) 

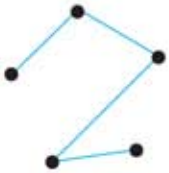
(b) 

INFO ZONE

Trees are used to determine the shortest path with the condition that the path passes through each vertex once only.

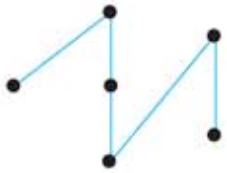
Solution:

(a) 5 vertices
7 edges (exceed by 3)

(i) 

(ii) 

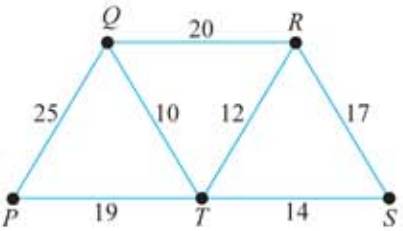
(b) 6 vertices
8 edges (exceed by 3)

(i) 

(ii) 

Example 13

The following diagram shows an undirected weighted graph. Draw a tree with a minimum total weight.

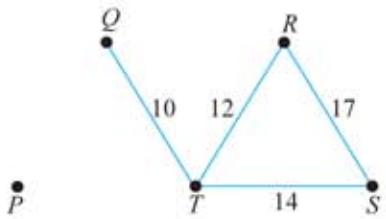


Solution:

Step 1

5 vertices, 7 edges

- 3 edges to be removed.
- Remove edges with the greatest weights (PQ, QR, PT)

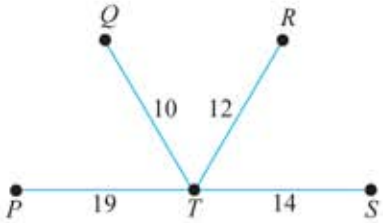


The graph above is not a tree because

- vertex P is not connected to the other vertices.
- three edges, RS, ST and RT, connect three vertices only.

Step 2

- Between the weights 19 and 25, keep weight 19 because its weight is smaller.
- Between the weights 12, 14 and 17, remove weight 17.

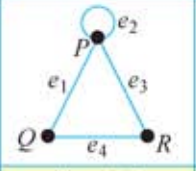
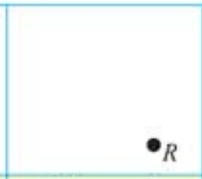
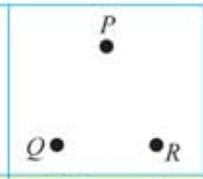
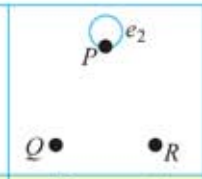
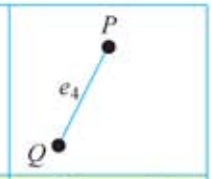


The graph obtained is a tree.

Minimum total weight of the tree
$= 10 + 12 + 14 + 19$
$= 55$

Self Practice 5.1c

1. Determine whether the given diagrams are the subgraphs of graph G.

					
Graph G	Diagram 1	Diagram 2	Diagram 3	Diagram 4	Diagram 5

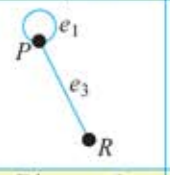
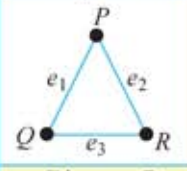
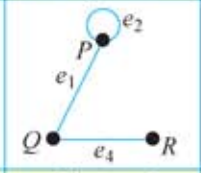
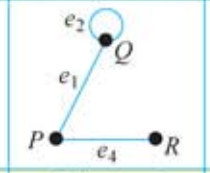
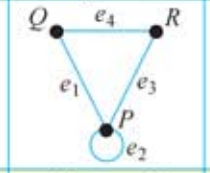
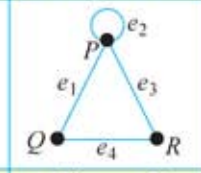
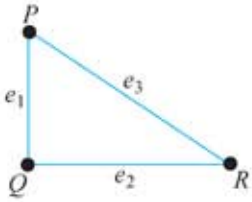
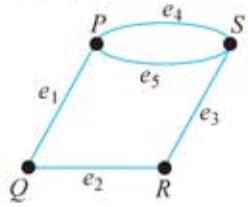
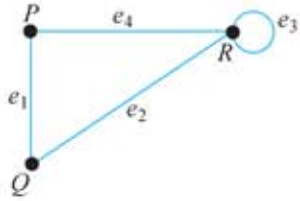

					

Diagram 6	Diagram 7	Diagram 8	Diagram 9	Diagram 10	Diagram 11

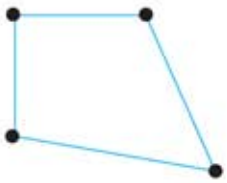
2. Draw five subgraphs for each given graph.

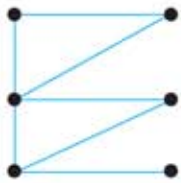
(a) 

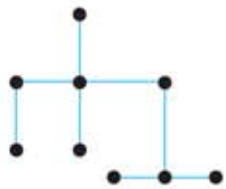
(b) 

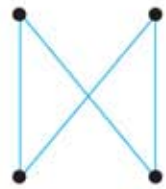
(c) 

3. Identify whether it is a tree or not a tree for the following diagrams.

(a) 

(b) 

(c) 

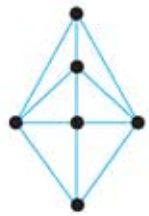
(d) 

4. Draw a tree based on the given information.

(a) 7 vertices (b) 9 vertices (c) 5 edges (d) 9 edges

5. Draw two trees based on the given graphs.

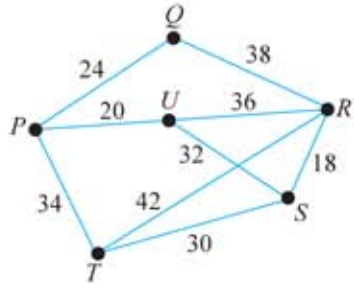
(a) 

(b) 

6. The diagram on the right shows an undirected weighted graph.

(a) Draw a tree with a minimum total weight.

(b) What is the minimum total weight?



How would you represent information in the form of networks?

Learning Standard

Represent information in the form of networks.

A network is a type of unique graph and can be used to represent overlapping and intersecting information. Networks are widely used in almost every area of our daily lives. A network that is drawn and displayed in graphic forms enables the interrelationships between various information or data structures to be understood easily.

Networks can be drawn as:
- directed weighted graph or directed unweighted graph
- undirected weighted graph or undirected unweighted graph

Transportation Networks

Transportation networks can be shown as weighted graphs and unweighted graphs. The weights can represent the distance, travelling time or cost of the journey. The well known navigation system in the weighted transportation networks is the GPS (Global Positioning System).

The diagram on the right shows the train transit network in Kuala Lumpur Sentral (KL Sentral). This undirected graph is an example of transportation networks with the vertices representing names of stations that are connected and the edges representing the types of trains.

Example 14

Mr Voon and his family plan to visit historical places in Melaka. The map shows three alternative routes with the distances and estimated times needed to travel from Tangkak to A Famosa, Melaka.

Assume P is a 46.3-kilometre route, Q is a 50.2-kilometre route and R is a 53.3-kilometre route. In your opinion, why does route P take the longest time compared to the other routes even though route P is the shortest route?

Solution:

Route P takes the longest time because the route passes through crowded town areas and there are more road users compared to the other two routes.

Social Networks

Social networks are becoming more popular among teenagers and adults. Social networks are used in areas like job opportunities, business opportunities, socialising, family relationships, education, social media and connecting with communities around the world.

Even though social networks are main platforms for various activities and are useful, you should be cautious and moderate in using social networks to avoid being distracted and being deceived easily.

Smart Mind

State a social network that you know of.

INTERACTIVE ZONE

Discuss the negative effects of using social networks.

Example 15

The table below shows the data of six pupils and the games that they like. Represent the information in the form of a network.

Name of pupil	Game
Edmund	Badminton, Chess
Azwan	Football, *Sepak takraw*
Rajan	Chess, Football
Aina	Chess, Netball
Maria	Badminton, Netball
Jenny	Netball, Volleyball

Smart Mind

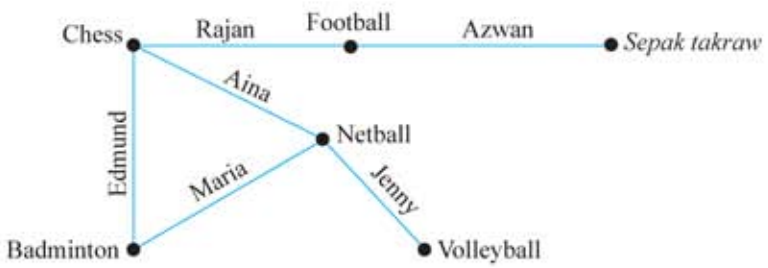
What are the meanings of LTE and 4G which are often used in wireless Internet networks?

INTERACTIVE ZONE

Solving criminal cases is also related to graphs. All evidence obtained are linked to one another. Discuss information that can be represented by the vertices and edges of graph.

Solution:

Let the vertices represent types of games and the edges represent the pupils' names.



TIPS

Choose information that is not repeated as the edges.

INTERACTIVE ZONE

Is the food chain a network? Discuss.

Self Practice 5.1d

Name of place	Vertex
Kuala Sepetang	*KS*
Matang	*M*
Kamunting	*K*
Simpang	*S*
Taiping	*T*
Changkat	*CJ*

Table 1

Vertex pair	Distance (km)
(*KS*, *M*)	8
(*K*, *M*)	8.4
(*K*, *T*)	5.2
(*S*, *T*)	5.9
(*S*, *M*)	3.5
(*S*, *KS*)	11
(*CJ*, *S*)	4.9
(*CJ*, *KS*)	15
(*CJ*, *T*)	10

Table 2

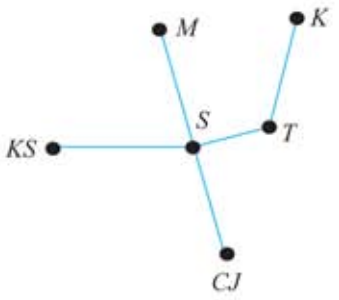


Diagram 1

1. Table 1 and Table 2 show names and distances of six places in Perak. Diagram 1 shows an incomplete undirected graph connecting the six places.

(a) Complete Diagram 1 by drawing an undirected weighted graph.

(b) Draw a tree with a minimum total weight which shows every place being visited once only.

(c) What is the minimum distance of the tree that you have drawn?

2. The table below shows four types of favourite food of several pupils.

Food	Name of pupil
Chicken rice	Mervin, Raj, Helen, Wong, Ain
Nasi lemak	Mervin, Nurul, Atiqah, Ain, Puspa
Fried rice	Helen, Julia, Nurul, Faruk, Puspa
Fried noodles	Faruk, Atiqah, Raj, Wong, Julia

(a) Based on the table, draw a graph with multiple edges.

(b) Between the types of food and the names of pupils, which group will you use to represent the vertices? Justify your answer.

(c) What is the relationship between the sum of degrees of your graph with the total number of food choices?

(d) Between table form and graph form, which form is clearer in showing the relationship between the types of food and the pupils?

3. (a) Draw an organisation chart of your class using your own creativity.

(b) State the type of graph you have used. Is the class organisation chart a network? Justify your answer.

How do you solve problems involving networks?

Learning Standard

Solve problems involving networks.

Example 16

The table below shows the choices of public transportation, estimated travelling time and estimated cost for a journey from Johor Bahru to Kota Bharu.

Type of transportation	Travelling time (24-hour system)	Estimated duration of journey	Estimated expenses
			Price of ticket per person/cost of fuel
Bus	2000 hours	9 hours	RM64 – RM75
Train	(1833 – 1147) hours	17 hours	With bed Child: RM32 – RM38 Adult: RM49 – RM55 Without bed Child: RM26 Adult: RM43
Cab	–	9 hours	RM120 (for one cab)

Based on the table, determine the type of transportation that should be chosen for the situation given below. Justify your answer.

(a) A journey involving an adult without time constraint.

(b) A journey involving an adult with time constraint.

(c) A journey involving two adults and two children.

Solution:

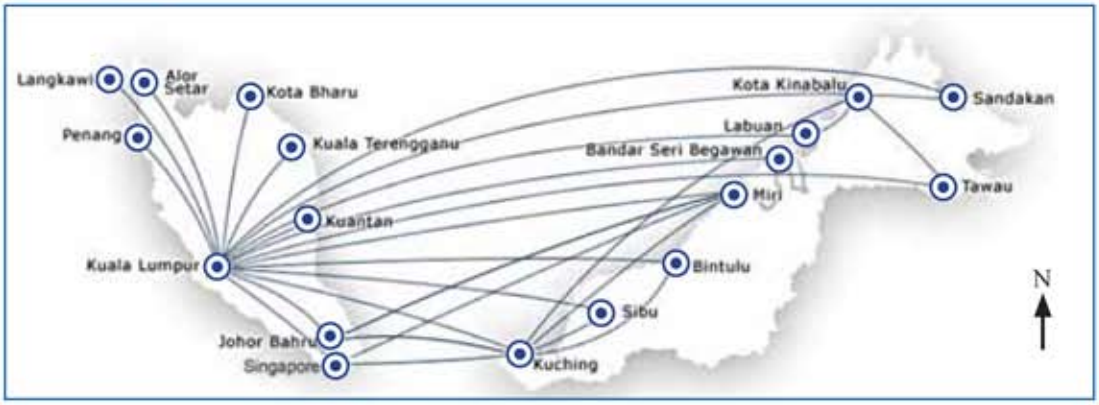
Type of transportation	Price of ticket per person (RM)	
	Minimum	Maximum
Bus	64	75
Train (With bed)	Child = 32 Adult = 49	Child = 38 Adult = 55
Train (Without bed)	Child = 26 Adult = 44	–

(a) Taking a train with bed is the best choice because the difference in price is only RM5 compared to taking a train without bed. The passengers can have a good rest throughout the whole journey. This choice is also cheaper compared to taking a cab.

(b) Taking a bus is the best choice because the duration of journey is shorter than that of a train and it is more economical than taking a cab. For safety purpose, it is not wise for an individual to take a cab for a long journey.

(c) Taking a cab is the most economical choice.

Example 17



The map above shows the domestic flight routes of a private airline in Malaysia.

(a) Mr Joshua works in Kuala Lumpur. He wants to visit his family in Kota Kinabalu. State the best route Mr Joshua can choose.

(b) What are the advantages and disadvantages for the choice of flight made?

Solution:

(a) The direct flight from Kuala Lumpur to Kota Kinabalu.

(b) The direct flight from Kuala Lumpur to Kota Kinabalu saves time and cost. The flight from Kuala Lumpur to Kota Kinabalu with transit takes a longer time and most probably the cost of the journey is also higher.

Self Practice 5.1e

1. Mr Maswi works in Kulai. He plans to visit his family in Miri on a certain weekend. On Friday, Mr Maswi's work ends at 12.30 noon. The table below shows the choices of domestic flight routes of a private airline in Malaysia on Friday and Saturday for that week.

Route	Friday		Saturday	
	Time	Price of ticket	Time	Price of ticket
Johor Bahru - Miri	No flight		(1705 – 1900) hrs	RM259.30
Johor Bahru - Kuching	(1630 – 1755) hrs	RM144.30	(0605 – 0730) hrs	RM174.30
	(1930 – 2055) hrs	RM124.30	(1205 – 1330) hrs	RM154.30
Kuching - Miri	(2010 – 2115) hrs	RM149.00	(0835 – 0940) hrs	RM84.00
	(2155 – 2300) hrs	RM149.00	(1145 – 1250) hrs	RM64.00

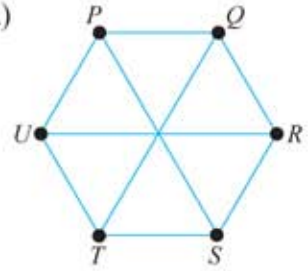
(a) Determine the most economical flight from Johor Bahru to Miri.

(b) Determine the best flight that Mr Maswi can choose if he needs to go back to Peninsular Malaysia on Sunday. Give your reason.

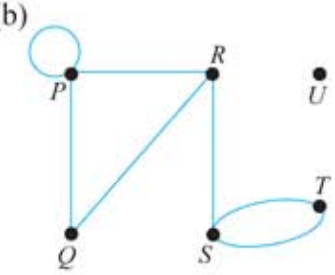
Comprehensive Practice

1. Three graphs are given below.

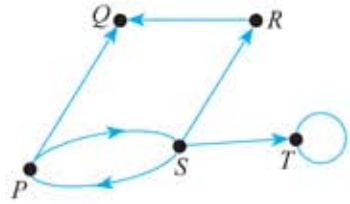
(a)



(b)



(c)



For each graph, determine

(i) set V. (ii) set E. (iii) sum of degrees.

2. Draw an undirected graph based on the given information.

(a) $V = \{P, Q, R, S, T, U\}$

$E = \{(P, Q), (Q, R), (R, S), (S, T)\}$

(b) $V = \{P, Q, R, S, T\}$

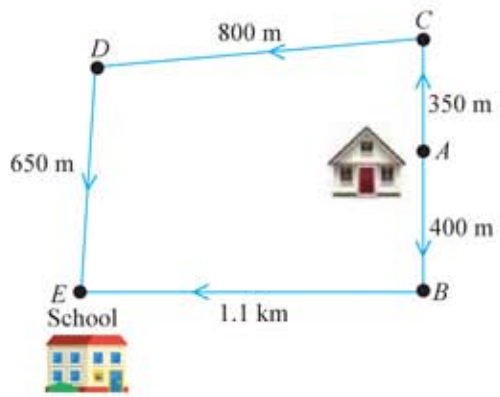
$E = \{(P, Q), (P, Q), (Q, R), (Q, T), (R, S), (S, S), (S, T)\}$

3. (a) Draw an undirected weighted graph to represent the federal roads connecting a few towns as shown in the table. You are advised to use an area map to determine the positions of the towns.

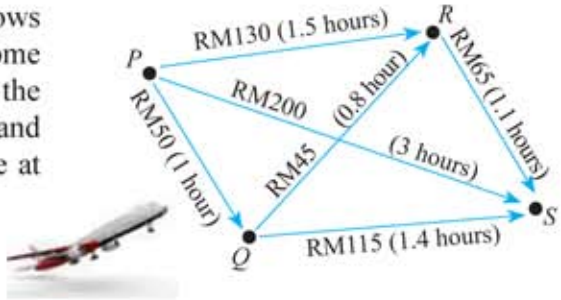
(b) Is the graph that you have drawn a tree? Justify your answer.

Name of town	Distance (km)
Temerloh - Mentakab	9.3
Temerloh - Bandar Bera	30.2
Bandar Bera - Teriang	4.9
Mentakab - Lanchang	21.1
Lanchang - Karak	21.9
Mentakab - Kuala Krau	34.6

4. The directed graph on the right shows the roads connecting Lani's house at A to the school at E. The edge BE is a municipal road and edges AB, AC, CD and DE are roads in the housing estates. Suggest the best route that Lani can choose to cycle to school. Justify your answer.



5. The directed weighted graph on the right shows the prices of tickets and the travel times for some choices of flights of a private airline. Vertex S is the destination of the flight from vertex P. Vertex Q and vertex R are the transit airports. The transit time at each of the airports is 45 minutes.



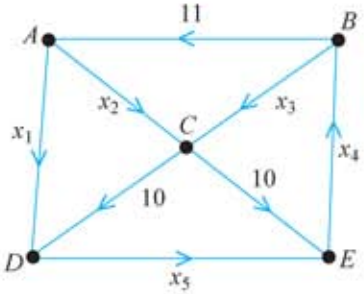
(a) State

(i) the most economical route.

(ii) the route that takes the shortest time.

(b) If you need to go to a destination at optimum cost, state the route that you will choose. Justify your answer.

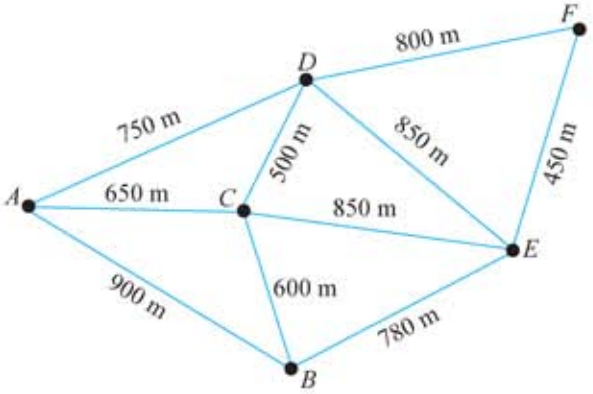
6. Write five linear equations based on the directed graph below. Given $x_5 = 15$, determine the values of x_1, x_2, x_3 and x_4.



TIPS

Use $\Sigma d_{in} = \Sigma d_{out}$ for each vertex.

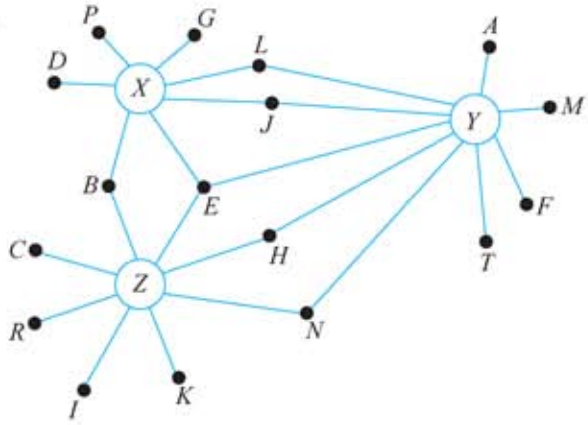
7. The following undirected graph shows six houses in a village. A salesperson needs to visit all the houses starting from house A and finishing at house F.



(a) Draw a directed graph to represent the shortest distance from A to F with the condition that all the paths are taken once only.

(b) Based on your graph, calculate the shortest distance in km.

8. The graph on the right shows the connections between the elements in set X, set Y and set Z, where the universal set, $\xi = X \cup Y \cup Z$.



(a) Represent the graph in a Venn diagram.

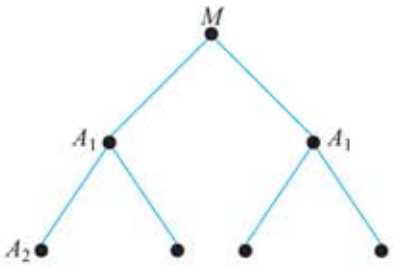
(b) Determine

(i) $(X \cup Y)'$

(ii) $(X \cap Z)' \cap Y'$

(iii) $(Y \cap Z) \cap (X \cap Y)$

9. Mr Ganesan is the manager of an insurance agency. He recruites two active insurance agents to sell the latest insurance scheme valued at RM100 per month. Besides selling insurance policies, each agent needs to recommend at least two new insurance agents. In the incomplete tree on the right, M represents the manager, A_1 represents the first level agents and A_2 represents the second level agents.



(a) If there are 30 agents in January, complete the given tree.

(b) The table on the right shows the percentages of basic commission received by an agent and the manager for an insurance policy sold.

Agent	25%
Manager	2%

(i) Calculate the total basic commission received by Mr Ganesan in January if the minimum number of insurance policies sold by an agent in January is 18.

(ii) What is the minimum number of policies that an agent needs to sell in order to receive a basic commission of at least RM1 000?

10. The diagram on the right shows the growth of a type of cell. It is given that on the first day, there are four cells. On the second day, each cell produces three cells. On the following day, each new cell produces another three new cells. The process of producing new cells repeats at the same rate.

(a) On which day will the total number of cells exceeds 50 for the first time?

(b) Calculate the total number of cells on the fifth day.

(c) Given the life span of a cell is 5 days, calculate the total number of cells on the eighth day.

PROJECT

1. A maze is an example of a network.
Diagram 1 shows an example of a maze whereas Diagram 2 is the corresponding network.

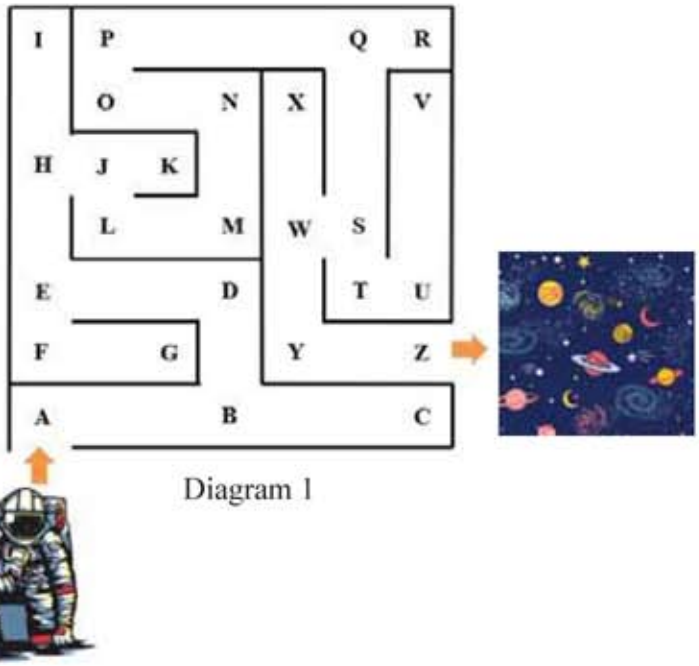


Diagram 1

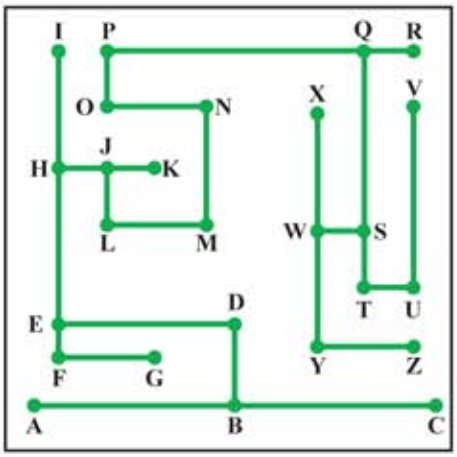
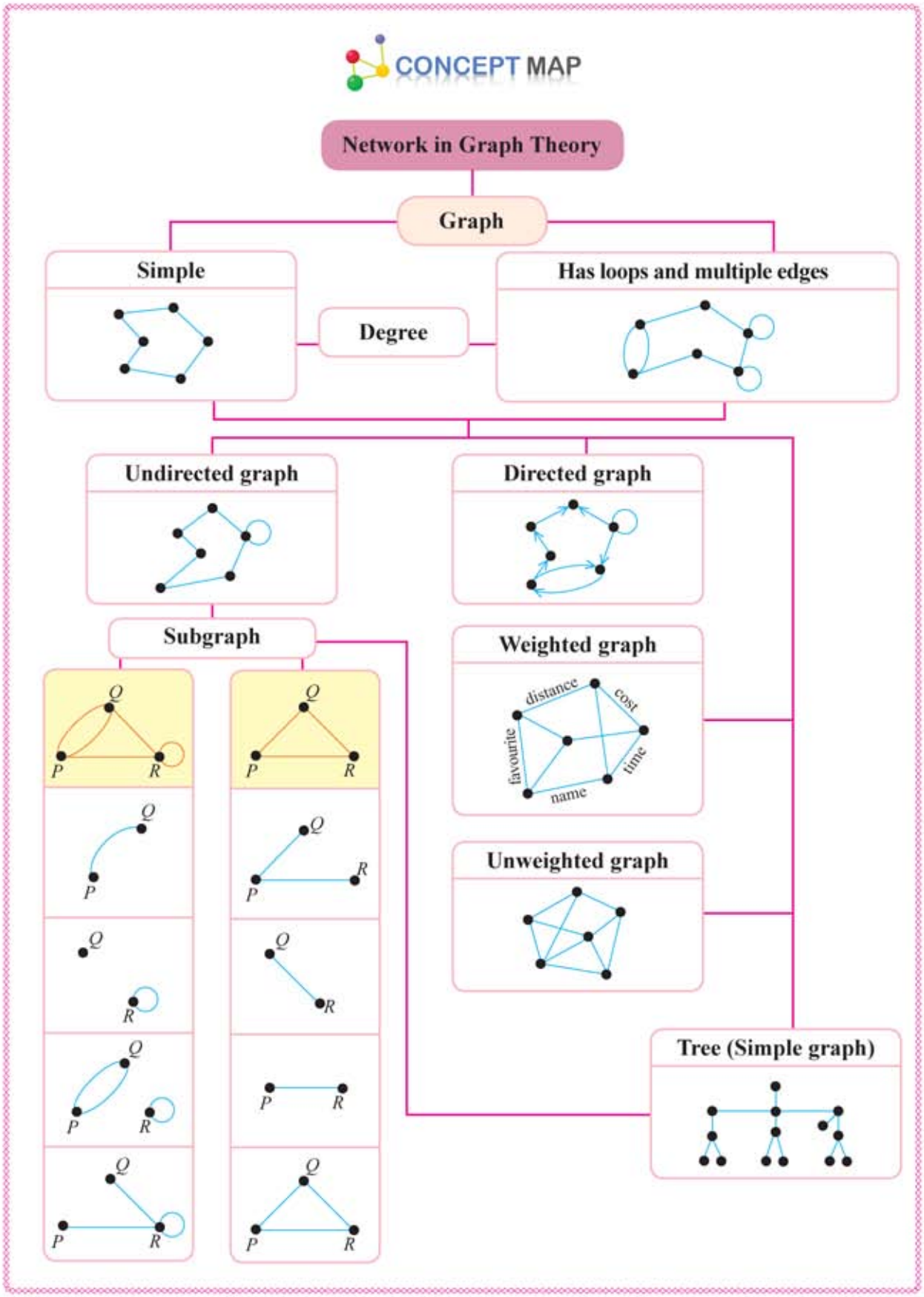


Diagram 2

2. Divide the class into groups.
3. Obtain examples of mazes with different levels of difficulty.
4. Label the vertices with suitable letters as shown in Diagram 1.
5. Draw the corresponding networks.
6. Exhibit your project work at the Mathematics Corner.


CONCEPT MAP
Network in Graph Theory
Graph
Simple
Degree
Has loops and multiple edges
Undirected graph
Directed graph
Subgraph
Weighted graph
distance
cost
favourite
time
name
Unweighted graph
Tree (Simple graph)
P
Q
R

Self Reflection

Complete the following statements with the answers provided.

subgraph | weighted graph | degree | simple graph | directed graph | edge | tree | loop

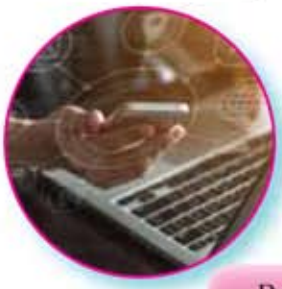
1. An _____ is a line which connects two vertices.
2. The _____ is the number of edges which connects two vertices.
3. A graph without a _____ and without multiple edges is known as a _____.
4. Edges of a _____ are associated with directions.
5. Edges of a _____ are associated with values or information.
6. A _____ is part of a graph or the whole graph redrawn.
7. Every pair of vertices in a _____ is connected by an edge only.

Mathematics Exploration

Gather information and prepare a multimedia presentation on one of the networks given below.

Social

Medicine



Banking

Language

Computer science

Include pictures, video or interesting materials to make your presentation more creative and informative.