CHAPTER 5 Trigonometric Ratios

What will you learn?

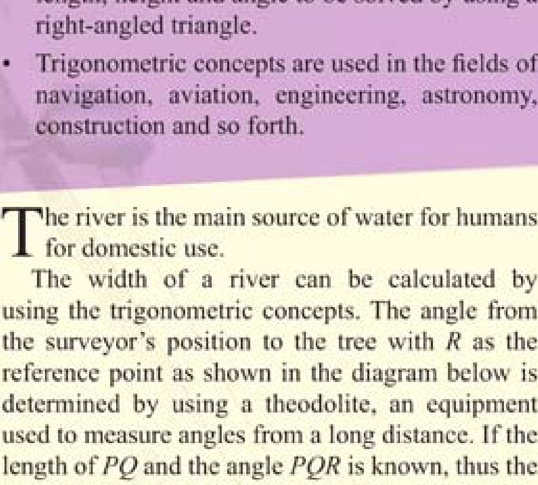
5.1 Sine, Cosine and Tangent of Acute Angles in Right-angled Triangles

Why do you learn this chapter?

- Trigonometric ratios allow problems related to length, height and angle to be solved by using a right-angled triangle.
- Trigonometric concepts are used in the fields of navigation, aviation, engineering, astronomy, construction and so forth.

The river is the main source of water for humans for domestic use.

The width of a river can be calculated by using the trigonometric concepts. The angle from the surveyor's position to the tree with R as the reference point as shown in the diagram below is determined by using a theodolite, an equipment used to measure angles from a long distance. If the length of PQ and the angle PQR is known, thus the width of the river, PR can be calculated easily using trigonometric methods.



Exploring Era

Al-Battani or Muhammad Ibn Jabir Ibn Sinan Abu Abdullah is the father of trigonometry. He was born in Battan, Damascus. He was an Arab prince and the ruler of Syria. Al-Battani was recognised as a well-known astronomer and Islamic mathematician. Al-Battani received early education from his father Jabir Ibn San'an who was also a famous scientist in his time. He successfully advanced trigonometry to a higher level and was the first to compile the table of cotangents.

http://bukutekskssm.my/Mathematics/F3/ExploringEraChapter5.pdf

WORD BANK

• degree	• *darjah*
• hypotenuse	• *hipotenus*
• cosine	• *kosinus*
• sine	• *sinus*
• tangent	• *tangen*
• Pythagoras theorem	• *teorem Pythagoras*

5.1 Sine, Cosine and Tangent of Acute Angles in Right-angled Triangles

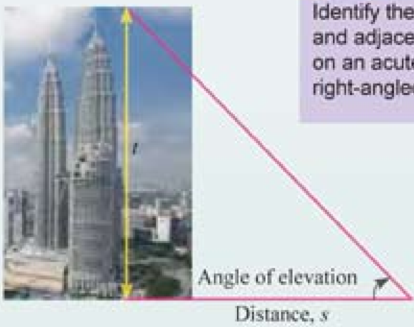
How do you identify the opposite side, adjacent side and hypotenuse?

LEARNING STANDARD

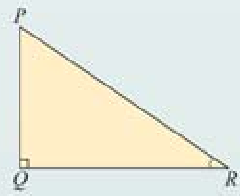
Identify the opposite side and adjacent side based on an acute angle in a right-angled triangle.

Do you know how the height of an object which is difficult to be measured such as buildings and mountains are determined?

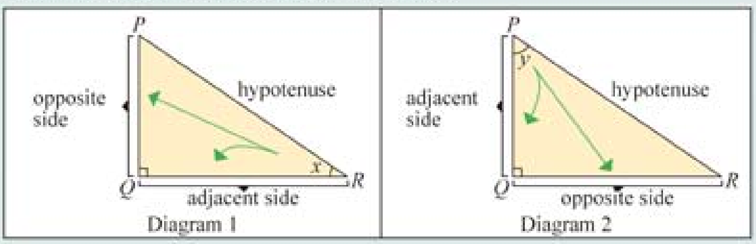
For example, in the diagram on the right, if the distance, s and the angle of elevation is known, then, the height, t of the building can be calculated by using the trigonometric concepts.



The diagram on the right shows a right-angled triangle PQR. As you have learnt in the chapter Pythagoras theorem in Form 1, the side PR is known as the **hypotenuse**, which is the longest side in the right-angled triangle PQR. Do the other two sides PQ and QR have special names like the longest side PR has?



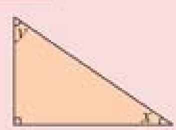
Examine Diagram 1 and Diagram 2 below.



TIPS

Acute angle
$0° < \theta < 90°$

TIPS



$x + y = 90°$
$\angle x$ and $\angle y$ are acute angles.

Based on $\angle PRQ$ in Diagram 1, QR is known as the **adjacent side** while PQ is known as **opposite side**.

Based on $\angle QPR$ in Diagram 2, PQ is the adjacent side while QR is the opposite side.

Take note that in both Diagram 1 and Diagram 2, the position of the hypotenuse PR is fixed, which is opposite the 90° angle.

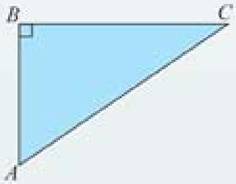
For a right-angled triangle:

(a) The hypotenuse is the longest side which is opposite the 90° angle.
(b) The adjacent side and the opposite side change based on the position of the referred acute angle.

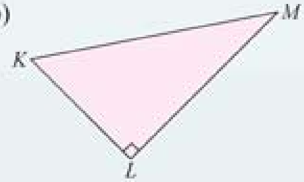
Example 1

Identify the opposite side, adjacent side and hypotenuse based on the given angle in the table below for all the following right-angled triangles.

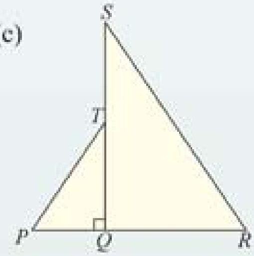
(a)



(b)



(c)

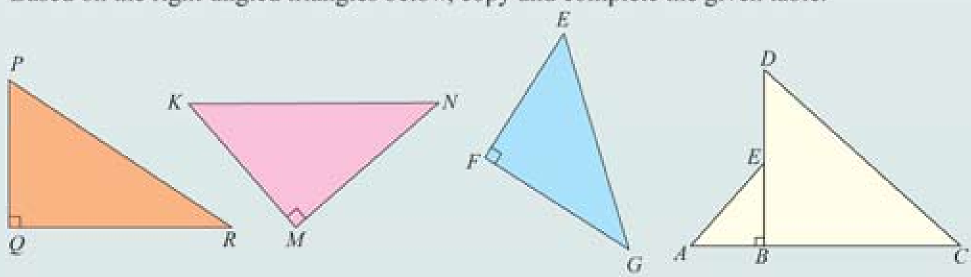


Solution:

Triangle	Angle	Hypotenuse	Opposite side	Adjacent side
ΔABC	$\angle BAC$	AC	BC	AB
	$\angle BCA$	AC	AB	BC
ΔKLM	$\angle LKM$	KM	LM	KL
	$\angle LMK$	KM	KL	LM
ΔPQT	$\angle TPQ$	PT	QT	PQ
ΔRQS	$\angle QRS$	RS	QS	QR

MIND TEST 5.1a

1. Based on the right-angled triangles below, copy and complete the given table.



Triangle	Angle	Hypotenuse	Opposite side	Adjacent side
ΔPQR	$\angle QPR$			
	$\angle PRQ$			
ΔKMN	$\angle MNK$			
	$\angle MKN$			
ΔEFG	$\angle FEG$			
	$\angle EGF$			
ΔABE	$\angle BAE$			
	$\angle AEB$			
ΔCBD	$\angle BCD$			
	$\angle BDC$			

What is the relationship between acute angles and the ratios of the sides of right-angled triangles?

LEARNING STANDARD

Make and verify the conjecture about the relationship between acute angles and the ratios of the sides of right-angled triangles, and hence define sine, cosine and tangent.

Brainstorming 1

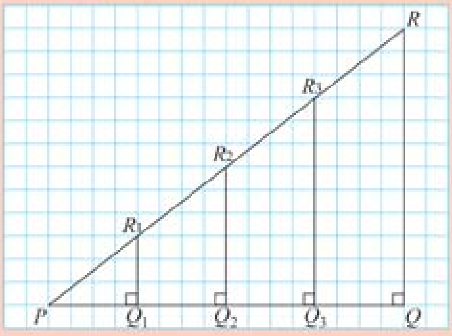
In groups

Aim: To identify the relationship between acute angles and the ratios of the sides of right-angled triangles.

Materials: Square grid paper, ruler and pencil.

Steps:

1. Draw a right-angled triangle PQR, where the length PQ is 16 units and the length QR is 12 units.
2. Draw a few straight lines parallel to RQ. Label them as R_1Q_1, R_2Q_2 and R_3Q_3 as shown in the diagram below.



TIPS

Use the Pythagoras theorem to determine the length of PR_1, PR_2, PR_3 and PR.

3. Complete the table below with the required measurements.

Acute angle	$\frac{\text{Opposite side}}{\text{Hypotenuse}}$	$\frac{\text{Adjacent side}}{\text{Hypotenuse}}$	$\frac{\text{Opposite side}}{\text{Adjacent side}}$
$\angle QPR$	$\frac{R_1Q_1}{PR_1} = \frac{3}{5}$	$\frac{PQ_1}{PR_1} = \frac{4}{5}$	$\frac{R_1Q_1}{PQ_1} = \frac{3}{4}$
	$\frac{R_2Q_2}{PR_2} =$	$\frac{PQ_2}{PR_2} =$	$\frac{R_2Q_2}{PQ_2} =$
	$\frac{R_3Q_3}{PR_3} =$	$\frac{PQ_3}{PR_3} =$	$\frac{R_3Q_3}{PQ_3} =$
	$\frac{RQ}{PR} =$	$\frac{PQ}{PR} =$	$\frac{RQ}{PQ} =$

Discussion:

1. What is the pattern of your answer to the ratio of the length of the opposite side to the hypotenuse, the ratio of the length of the adjacent side to the hypotenuse and the ratio of the length of the opposite side to the length of the adjacent side?
2. What happens if the size of the angle is changed? Justify your answer.

From Brainstorming 1, it is found that:

Given a fixed acute angle in right-angled triangles of different sizes:
(a) The ratio of the length of the opposite side to the hypotenuse is a constant.
(b) The ratio of the length of the adjacent side to the hypotenuse is a constant.
(c) The ratio of the length of the opposite side to the length of the adjacent side is a constant.

The relationships of the ratios obtained from Brainstorming 1 are trigonometric ratios known as **sine, cosine and tangent**, that is:

$$\text{sine} = \frac{\text{opposite side}}{\text{hypotenuse}}$$

$$\text{cosine} = \frac{\text{adjacent side}}{\text{hypotenuse}}$$

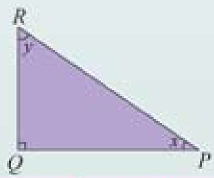
$$\text{tangent} = \frac{\text{opposite side}}{\text{adjacent side}}$$

REMINDER
- sin = sine
- cos = cosine
- tan = tangent

BULLETIN
The word **Trigonometry** originates from Greek words, that is,
Trigonon = triangle
Metron = to measure

Example 2

Complete the following table based on the diagram on the right.



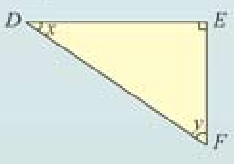
sin x	cos x	tan x	sin y	cos y	tan y

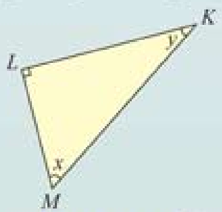
Solution:

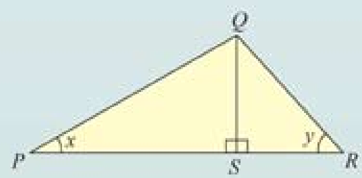
sin x	cos x	tan x	sin y	cos y	tan y
$\frac{QR}{PR}$	$\frac{PQ}{PR}$	$\frac{QR}{PQ}$	$\frac{PQ}{PR}$	$\frac{QR}{PR}$	$\frac{PQ}{QR}$

MIND TEST 5.1b

1. Complete the table based on the right-angled triangles below.







Triangles	sin x	cos x	tan x	sin y	cos y	tan y
ΔDEF						
ΔKLM						
ΔPQR						

What is the impact of changing the size of the angles on the values of sine, cosine and tangent?

LEARNING STANDARD

Make and verify the conjecture about the impact of changing the size of the angles on the values of sine, cosine and tangent.

Brainstorming 2

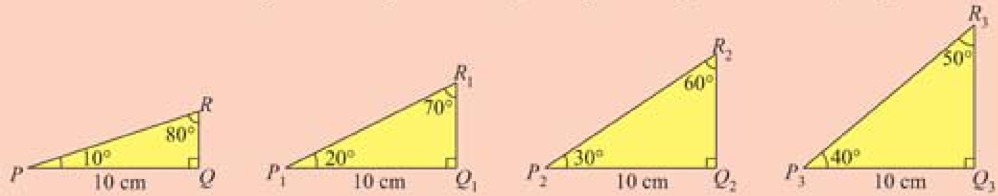
In pairs

Aim: To identify the impact of changing the size of the angles on the values of sine, cosine and tangent.

Materials: Square grid paper, ruler, protractor and pencil.

Steps:

1. Draw four right-angled triangles as shown below with the base length of 10 cm.
2. Make sure that the angles and lengths of all right-angled triangles are exactly as given.



3. Complete the table below.

sin 10°	sin 20°	sin 30°	sin 40°	sin 50°	sin 60°	sin 70°	sin 80°
$\frac{RQ}{PR}$ $= \frac{1.8}{10.2}$ $= 0.1765$							$\frac{PQ}{PR}$ $= \frac{10}{10.2}$ $= 0.9804$

cos 10°	cos 20°	cos 30°	cos 40°	cos 50°	cos 60°	cos 70°	cos 80°
$\frac{PQ}{PR}$ $= \frac{10}{10.2}$ $= 0.9804$							$\frac{RQ}{PR}$ $= \frac{1.8}{10.2}$ $= 0.1765$

tan 10°	tan 20°	tan 30°	tan 40°	tan 50°	tan 60°	tan 70°	tan 80°
$\frac{RQ}{PQ}$ $= \frac{1.8}{10}$ $= 0.1800$							$\frac{PQ}{RQ}$ $= \frac{10}{1.8}$ $= 5.5556$

Discussion:

1. Based on the values in the table for the trigonometric ratios you have completed, what conclusion can you make?

2. What is your conjecture on
   (a) the value of the sine ratio when the angle approaches 0° and 90°?
   (b) the value of the cosine ratio when the angle approaches 0° and 90°?
   (c) the value of the tangent ratio when the angle approaches 0° and 90°?

From Brainstorming 2, it is found that:

The larger the size of the acute angle
(a) the **larger the value of sine** and its value **approaches 1**.
(b) the **smaller the value of cosine** and its value **approaches zero**.
(c) the **larger the value of tangent**.

TIPS

$\sin 0° = 0$ $\quad\sin 90° = 1$
$\cos 0° = 1$ $\quad\cos 90° = 0$
$\tan 0° = 0$ $\quad\tan 90° = \infty$

Example 3

The diagram on the right shows two right-angled triangles. Determine whether all trigonometric ratios of angle x and angle y are equal. State the reason for your answer.

Solution:

$\sin x = \frac{3}{5}$ $\qquad \cos x = \frac{4}{5}$ $\qquad \tan x = \frac{3}{4}$

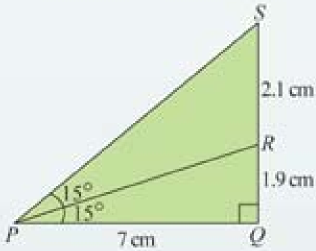
$\sin y = \frac{1.5}{2.5} = \frac{3}{5}$ $\qquad \cos y = \frac{2}{2.5} = \frac{4}{5}$ $\qquad \tan y = \frac{1.5}{2} = \frac{3}{4}$

The trigonometric ratios of angle x and angle y are equal because the length of corresponding sides of the two triangles are proportional.

MIND TEST 5.1c

1. The diagram on the right shows two right-angled triangles. Determine whether all trigonometric ratios of angle x and angle y are equal. State the reason for your answer.

2. The diagram on the right shows a right-angled triangle.



(a) Determine the trigonometric ratio for

i. sin 15° ii. cos 15° iii. tan 15°
iv. sin 30° v. cos 30° vi. tan 30°

(b) Is the increase in the value of the trigonometric ratio for angle 15° and angle 30° proportional to the increase in the angle?

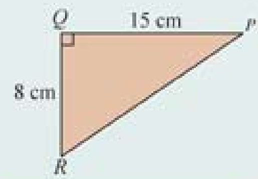
How do you determine the values of sine, cosine and tangent of acute angles?

LEARNING STANDARD

Determine the values of sine, cosine and tangent of acute angles.

Example 4

The diagram on the right shows a right-angled triangle PQR. Calculate the value of



(a) length of PR (b) $\sin \angle PRQ$ (c) $\cos \angle PRQ$ (d) $\tan \angle QPR$

Solution:

(a) length of PR

$PR = \sqrt{15^2 + 8^2}$
$= \sqrt{289}$
$= 17$ cm

(b) $\sin \angle PRQ$

$= \frac{15}{17}$

(c) $\cos \angle PRQ$

$= \frac{8}{17}$

(d) $\tan \angle QPR$

$= \frac{8}{15}$

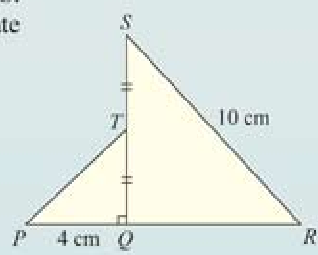
FLASHBACK

Pythagoras theorem

$c^2 = a^2 + b^2$
$a^2 = c^2 - b^2$
$b^2 = c^2 - a^2$

Example 5

The diagram on the right shows right-angled triangles PQT and RQS. PQR is a straight line. Given that the length of SQ is 6 cm, calculate the value of



(a) length of QR (b) length of PT (c) $\sin \angle QRS$
(d) $\cos \angle TPQ$ (e) $\tan \angle PTQ$ (f) $\tan \angle QSR$

Solution:

(a) length of QR

$QR = \sqrt{10^2 - 6^2}$
$= \sqrt{64}$
$= 8$ cm

(b) length of PT

$PT = \sqrt{4^2 + 3^2}$
$= \sqrt{25}$
$= 5$ cm

(c) $\sin \angle QRS$

$= \frac{6}{10}$
$= \frac{3}{5}$

(d) $\cos \angle TPQ$

$= \frac{4}{5}$

(e) $\tan \angle PTQ$

$= \frac{4}{3}$

(f) $\tan \angle QSR$

$= \frac{8}{6}$
$= \frac{4}{3}$

REMINDER

Ratio value should be given in the simplest term.

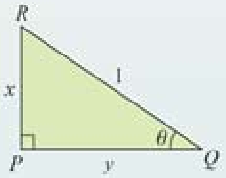
What is the relationship between sine, cosine and tangent?

For right-angled triangles, you have learnt that:

$$\text{sine} = \frac{\text{opposite side}}{\text{hypotenuse}}, \text{cosine} = \frac{\text{adjacent side}}{\text{hypotenuse}} \text{ and tangent} = \frac{\text{opposite side}}{\text{adjacent side}}$$

Do you know that the three trigonometric ratios above are related to one another? Tangent is the ratio of sine to cosine.

Study the diagram below.



It is known that,

(a) $\sin\theta = \frac{x}{1}$
$x = \sin\theta$

(b) $\cos\theta = \frac{y}{1}$
$y = \cos\theta$

(c) $\tan\theta = \frac{x}{y}$

Thus,

$$\tan\theta = \frac{\sin\theta}{\cos\theta}$$

DISCUSSION CORNER

If θ is $\angle QRP$, is the ratio of $\tan\theta$ still $\frac{\sin\theta}{\cos\theta}$? Discuss.

Example 6

If $\sin\theta = 0.6$ and $\cos\theta = 0.8$, calculate the value of $\tan\theta$.

Solution:

$$\tan\theta = \frac{\sin\theta}{\cos\theta}$$
$$= \frac{0.6}{0.8}$$
$$= \frac{3}{4}$$
$$= 0.75$$

QUIZ

If $\tan\theta = \frac{1}{2}$, state the possible values of $\sin\theta$ and $\cos\theta$.

SMART MIND

Given that $\sin\theta = x$, determine the possible values of $\cos\theta$ and $\tan\theta$.

Example 7

If $\sin\theta = \frac{3}{8}$ and $\tan\theta = \frac{3}{\sqrt{55}}$, calculate the value of $\cos\theta$.

Solution:

$$\tan\theta = \frac{\sin\theta}{\cos\theta}$$
$$\frac{3}{\sqrt{55}} = \frac{\frac{3}{8}}{\cos\theta}$$
$$\cos\theta = \frac{\frac{3}{8}}{\frac{3}{\sqrt{55}}}$$
$$\cos\theta = \frac{\sqrt{55}}{8}$$

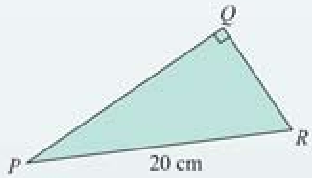
DISCUSSION CORNER

If $\tan\theta = 1$, what type of triangle is being represented?

Example 8

The diagram on the right shows a right-angled triangle PQR. Given that $PR = 20$ cm and $\sin \angle QPR = \frac{3}{5}$, calculate

(a) the length of QR
(b) $\cos \angle QPR$



Solution:

(a) $\sin \angle QPR = \frac{3}{5}$

$\frac{QR}{PR} = \frac{3}{5}$

$\frac{QR}{20} = \frac{3}{5}$

$QR = \frac{3(20)}{5}$

$QR = 12$ cm

(b) $PQ = \sqrt{20^2 - 12^2}$

$= \sqrt{256}$

$PQ = 16$ cm

Thus, $\cos \angle QPR = \frac{PQ}{PR}$

$= \frac{16}{20}$

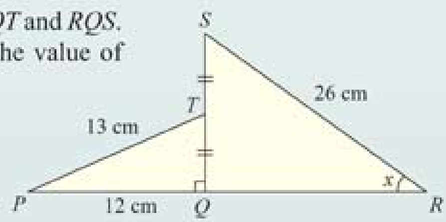
$= \frac{4}{5}$

SMART MIND

Given $\sin \theta = \frac{3}{5}$ and the length of hypotenuse is 20 cm, determine $\cos \theta$ and $\tan \theta$.

Example 9

The diagram on the right shows right-angled triangles PQT and RQS. Given that PQR and STQ are straight lines, calculate the value of $\cos x$.



Solution:

$\cos x = \frac{RQ}{SR}$ → Determine the value of RQ first

$TQ = \sqrt{13^2 - 12^2}$; $SQ = 2TQ$; $RQ = \sqrt{26^2 - 10^2}$

$= \sqrt{25}$; $= 2(5)$; $= \sqrt{576}$

$TQ = 5$ cm ; $SQ = 10$ cm ; $= 24$ cm

Thus, $\cos x = \frac{RQ}{SR}$

$= \frac{24}{26}$

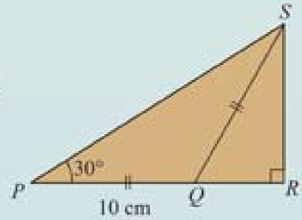
$= \frac{12}{13}$

DISCUSSION CORNER

Given that the hypotenuse of a right-angled triangle is $\sqrt{8}$ cm, determine $\tan \theta$ if $\cos \theta = \frac{1}{\sqrt{2}}$.

Example 10

The diagram on the right shows a right-angled triangle PRS. Given that PQR is a straight line and $\cos 60° = 0.5$, calculate the length of PS. State the answer correct to two decimal places.



Understanding the problem

Calculate the length of PS which is the hypotenuse of ΔPRS.

Planning a strategy

- $PS = \sqrt{PR^2 + SR^2}$
- SR and QR can be calculated if $\angle SQR$ or $\angle QSR$ is known.
- Identify the position of cos 60°.

Implementing the strategy

- $\angle QSP = \angle QPS = 30°$

  thus, $\angle PQS = 180 - 30° - 30° = 120°$
- $\angle SQR = 180° - \angle PQS$

  $\angle SQR = 180° - 120° = 60°$
- Given that cos 60° = 0.5

$\cos 60° = \frac{1}{2}$

$\frac{QR}{10} = \frac{1}{2}$

$QR = \frac{10\,(1)}{2}$

$= 5$ cm

$SR = \sqrt{10^2 - 5^2} = \sqrt{75}$ cm

Thus,

$PS = \sqrt{SR^2 + PR^2}$

$PS = \sqrt{(\sqrt{75})^2 + 15^2}$

$PS = 17.32$ cm

Making a conclusion

$PS = 17.32$ cm (2 d.p.)

MIND TEST 5.1d

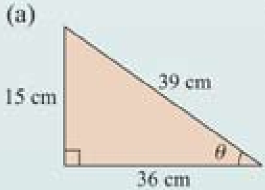
FLASHBACK

Pythagoras triples

A	B	C
3	4	5
5	12	13
6	8	10
8	15	17
7	24	25
9	40	41

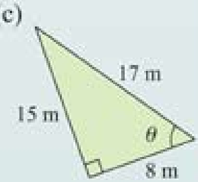
1. Calculate the values of $\sin \theta$, $\cos \theta$ and θ for each of the following right-angled triangles.

(a)

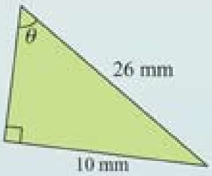


(b)

(c)

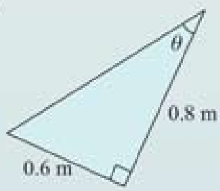


(d)



(e)

(f)



2. Calculate the value of x without drawing any right-angled triangles or using Pythagoras theorem or a calculator.

(a) $\sin \theta = \frac{1}{2}$, $\cos \theta = \frac{\sqrt{3}}{2}$, $\tan \theta = x$

(b) $\sin \theta = \frac{1}{\sqrt{2}}$, $\cos \theta = x$, $\tan \theta = 1$

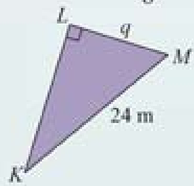
(c) $\sin \theta = x$, $\cos \theta = \frac{5}{8}$, $\tan \theta = \frac{\sqrt{39}}{5}$

(d) $\sin \theta = \frac{7}{9}$, $\cos \theta = x$, $\tan \theta = \frac{7}{4\sqrt{2}}$

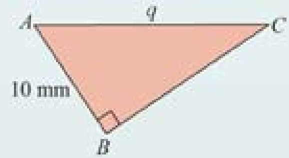
3. Determine the length of side q for each of the right-angled triangles below.

(a) $\sin \angle QRP = \frac{1}{3}$

(b) $\sin \angle LKM = \frac{7}{8}$

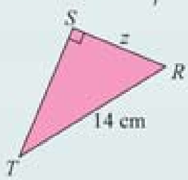


(c) $\sin \angle ACB = \frac{2}{5}$

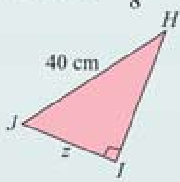


4. Determine the length of side z for each of the right-angled triangles below.

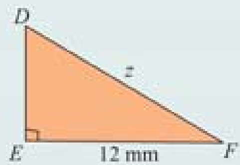
(a) $\cos \angle SRT = \frac{5}{7}$



(b) $\cos \angle HJI = \frac{3}{8}$



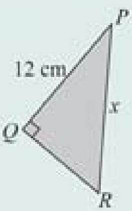
(c) $\cos \angle DFE = 0.4$



5. Calculate the value of x for each of the right-angled triangles below.

(a) $\tan \angle BAC = 0.9$

(b) $\tan \angle PRQ = \frac{3}{4}$

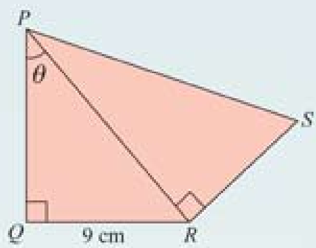


(c) $\tan \angle LKM = \frac{10}{3}$

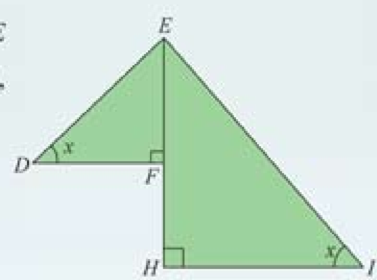
6. The diagram on the right shows right-angled triangles PQR and PRS. Given that $\tan \theta = \frac{3}{4}$ and $PS = \frac{5}{3} PR$, calculate, in cm, the length of

(a) PR

(b) RS



7. The diagram on the right shows right-angled triangles DFE and EHI. If $\tan x = \frac{5}{7}$, $DF = 21$ cm and $EF : EH = 1 : 2$, determine the length of EI in cm.



How do you determine the values of sine, cosine and tangent of 30°, 45° and 60° angles without using a calculator?

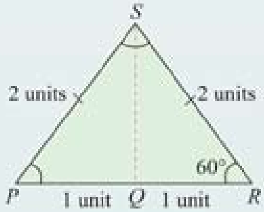


Diagram 1(a)

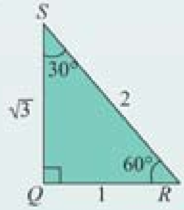


Diagram 1(b)

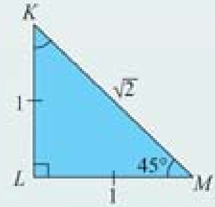


Diagram 2

LEARNING STANDARD

Determine the values of sine, cosine and tangent of 30°, 45° and 60° angles without using a calculator.

TIPS

$QS = \sqrt{2^2 - 1^2}$
$QS = \sqrt{3}$
$KM = \sqrt{1^2 + 1^2}$
$KM = \sqrt{2}$

Diagram 1(b) above is half of the equilateral triangle *PRS* where the length of *PQR* is 2 units. Diagram 2 shows an isosceles triangle *KLM*.

The table below shows the values of the trigonometric ratios of 30°, 45° and 60° angles that can be calculated without using a calculator, based on Diagram 1(b) and Diagram 2.

Ratio \ Angle	30°	60°	45°
$\sin \theta$	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$
$\cos \theta$	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$
$\tan \theta$	$\frac{1}{\sqrt{3}}$	$\sqrt{3}$	1

BULLETIN

Surd is an irrational number in the root form such as $\sqrt{2}$, $\sqrt{3}$ and $\sqrt{17}$. $\sqrt{3}$ is read as surd three.

Example 11

Calculate the following values without using a calculator.

(a) $\sin 45° + \cos 45°$
(b) $3 \cos 30° - 2 \sin 60°$
(c) $2 \tan 45° - 2 \cos 60°$
(d) $(2 \sin 60°)(4 \cos 30°) - 4 \tan 60°$
(e) $(3 \tan 30°)(4 \sin 60°) + 4 \sin 45°$

Solution:

(a) $\sin 45° + \cos 45°$

$= \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}$

$= \frac{2}{\sqrt{2}}$

$= \sqrt{2}$

> $\frac{2}{\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}}$
> $= \frac{2\sqrt{2}}{2}$
> $= \sqrt{2}$

(b) $3 \cos 30° - 2 \sin 60°$

$= 3\left(\frac{\sqrt{3}}{2}\right) - 2\left(\frac{\sqrt{3}}{2}\right)$

$= \frac{3\sqrt{3}}{2} - \frac{2\sqrt{3}}{2}$

$= \frac{\sqrt{3}}{2}$

(c) $2 \tan 45° - 2 \cos 60°$

$= 2(1) - 2\left(\frac{1}{2}\right)$

$= 2 - 1$

$= 1$

(d) $(2 \sin 60°)(4 \cos 30°) - 4 \tan 60°$

$= 2\left(\frac{\sqrt{3}}{2}\right)(4)\left(\frac{\sqrt{3}}{2}\right) - 4\sqrt{3}$

$= (\sqrt{3})(2)(\sqrt{3}) - 4\sqrt{3}$

$= 2(3) - 4\sqrt{3}$

$= 6 - 4\sqrt{3}$

(e) $(3 \tan 30°)(4 \sin 60°) + 4 \sin 45°$

$= 3\left(\frac{1}{\sqrt{3}}\right)(4)\left(\frac{\sqrt{3}}{2}\right) + 4\left(\frac{1}{\sqrt{2}}\right)$

$= \frac{6}{1} + \frac{4}{\sqrt{2}}$

$= 6 + 2\sqrt{2}$

> $= \frac{4}{\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}}$
> $= \frac{4\sqrt{2}}{2}$
> $= 2\sqrt{2}$

TIPS

$\sqrt{2} \times \sqrt{2} = \sqrt{2 \times 2}$
$= \sqrt{4}$
$= 2$

CHAPTER 5

MIND TEST 5.1e

1. Determine the following values without using a calculator.
   (a) $2 \cos 60° + \tan 45°$
   (b) $3 \cos 60° + 2 \tan 45°$
   (c) $2 \tan 45° + \cos 60°$
   (d) $3 \sin 30° - 2 \cos 60°$
   (e) $2 \sin 30° - 3 \cos 60°$
   (f) $4 \tan 45° - 2 \cos 60°$
   (g) $(2 \sin 60°)(3 \cos 60°) + 3 \tan 30°$
   (h) $(3 \tan 45°)(4 \sin 60°) - (2 \cos 30°)(3 \sin 30°)$
   (i) $4 \tan 45° + (2 \sin 45°)(6 \cos 45°)$
   (j) $(5 \tan 60°)(2 \sin 60°) - (3 \sin 45°)(4 \cos 45°)$

What is the unit of measure for angles?

Angles are measured in the unit of degrees (°). Angles can also be expressed in units of degrees (°), minutes (′) and seconds (″), that is,

$$1° = 60'$$
$$1' = 60''$$

Example 12

(a) Convert 30.2° to degrees and minutes.

(b) Convert the angle 43° 30′ to degrees.

Solution:

(a)
$$\begin{aligned} 30.2° &= 30° + 0.2° \\ &= 30° + (0.2 \times 60)' \\ &= 30° + 12' \\ &= 30°\ 12' \end{aligned}$$

(b)
$$\begin{aligned} 43°\ 30' &= 43° + 30' \\ &= 43° + \left(\frac{30}{60}\right)^{\circ} \\ &= 43° + 0.5° \\ &= 43.5° \end{aligned}$$

CHAPTER 5

MIND TEST 5.1f

1. Convert each of the following angles to degrees and minutes.
   (a) 37.80°
   (b) 74.6°
   (c) 58.1°
   (d) 60.2°
   (e) 41.5°
   (f) 16.9°
   (g) 5.4°
   (h) 72.3°
2. State each of the following angle in degrees.
   (a) 65° 54′
   (b) 47° 42′
   (c) 18° 12′
   (d) 69° 24′
   (e) 70° 6′
   (f) 36° 36′
   (g) 35° 30′
   (h) 20° 18′

How do you determine the values of sine, cosine and tangent?

LEARNING STANDARD

Perform calculations involving sine, cosine and tangent.

Do you know that a scientific calculator can be used to determine the trigonometric ratio of an angle?

Example 13

Use a scientific calculator to determine the following values correct to four decimal places.

(a) $\sin 45°\ 6'$
(b) $\cos 20.7°$
(c) $\tan 64°\ 12'$

Solution:

(a) $\sin 45° 6' = 0.7083$

SMART FINGER: sin 4 5 °′″ 6 °′″ = 0.7083398377

(b) $\cos 20.7° = 0.9354$

SMART FINGER: cos 2 0 . 7 = 0.9354440308

(c) $\tan 64° 12' = 2.0686$

SMART FINGER: tan 6 4 °′″ 12 °′″ = 2.068599355

BULLETIN

The button °′″ should be pressed only when the question is given in degrees and minutes.

MIND TEST 5.1g

1. Use a scientific calculator to determine the following values correct to four decimal places.

(a) $\sin 44°$ (b) $\cos 73.5°$ (c) $\tan 69.5°$ (d) $\sin 51° 24'$ (e) $\cos 30° 21'$ (f) $\tan 56° 24'$

How do you calculate the size of an angle by using trigonometric ratios sine, cosine and tangent?

If the value of the trigonometric ratio is given, you can use a scientific calculator to determine the size of the related angle.

Example 14

Use a scientific calculator to calculate the following x values.

(a) $\sin x = 0.8377$ (b) $\cos x = 0.7021$ (c) $\tan x = 2.4876$

Solution:

(a) $\sin x = 0.8377$

$x = \sin^{-1} 0.8377$

$x = 56.9°$ ← Answer in degrees.

$x = 56° 54'$ ← Answer in degrees and minutes.

REMINDER

If the unit of second is 30″ or more, the minute unit will be added by 1′.

SMART FINGER: shift sin 0 . 8 3 7 7 = 56.89803635 °′″ 56° 53′ 52.93″

52.93″ shows the value in seconds. Follow the steps below to round off the answer to the nearest minute.

$56° 53' \underbrace{52.93''}$

+1 ← > 30

$= 56° 54'$

CHAPTER 5

(b) $\cos x = 0.7021$

$x = \cos^{-1} 0.7021$

$x = 45.4°$

$x = 45° 24'$

SMART FINGER shift | cos | 0 | . | 7 | 0 | 2 | 1 | = | 45.40426895 | °′″ | 45° 24′15.37″

(c) $\tan x = 2.4876$

$x = \tan^{-1} 2.4876$

$x = 68.1°$

$x = 68° 6'$

SMART FINGER shift | tan | 2 | . | 4 | 8 | 7 | 6 | = | 68.10017426 | °′″ | 68° 6′0.63″

MIND TEST 5.1h

1. Using a scientific calculator, calculate the following x values.

(a) $\tan x = 0.2162$ (b) $\cos x = 0.5878$ (c) $\sin x = 0.4062$ (d) $\sin x = 0.9121$

(e) $\cos x = 0.9686$ (f) $\tan x = 3.8027$ (g) $\cos x = 0.5604$ (h) $\sin x = 0.1521$

(i) $\tan x = 0.7199$ (j) $\sin x = 0.9792$ (k) $\tan x = 1.0088$ (l) $\cos x = 0.099$

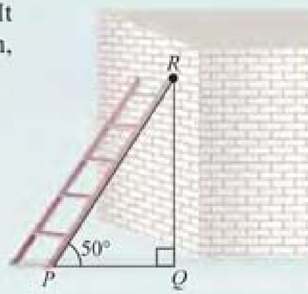
How do you solve problems involving sine, cosine and tangent?

LEARNING STANDARD

Solve problems involving sine, cosine and tangent.

Example 15

The diagram on the right shows a ladder leaning against a wall. It forms a right-angled triangle PQR. If the height of QR is 2.5 m, calculate the length of the ladder, PR in metres.
(State the answer correct to two decimal places).



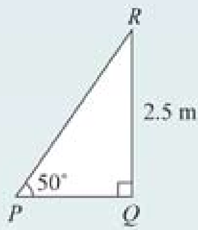
Solution:

$$\sin 50° = \frac{QR}{PR}$$

$$\sin 50° = \frac{2.5}{PR}$$

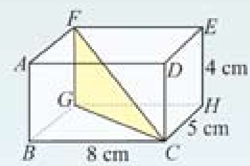
$$PR = \frac{2.5}{\sin 50°}$$

$$PR = 3.26 \text{ m (2 d.p.)}$$



Example 15

The diagram on the right shows a cuboid $ABCDEFGH$. It is given that BC = 8 cm, CH = 5 cm and the height of HE = 4 cm. If right-angled triangle FGC is formed in this cuboid, calculate the value of $\angle FCG$.



Understanding the problem

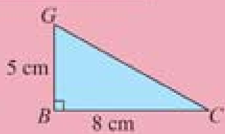
$\angle FCG$ can be calculated if any two sides CG, CF, FG are known.

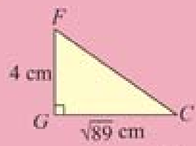
Planning a strategy

- $FG = EH$
- The length of CG is easier to calculate than the length of CF.
- $\tan \angle FCG = \dfrac{FG}{CG}$

Implementing the strategy

- $FG = EH$
  Thus,
  $FG = 4$ cm





$CG = \sqrt{BC^2 + BG^2}$
$= \sqrt{8^2 + 5^2}$
$CG = \sqrt{89}$

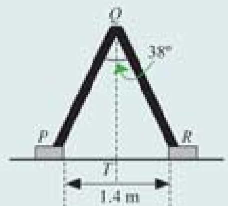
$\tan \angle FCG = \dfrac{4}{\sqrt{89}}$
$\angle FCG = \tan^{-1} \dfrac{4}{\sqrt{89}}$
$\angle FCG = 22.98°$

Making a conclusion

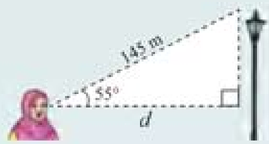
$\angle FCG = 22.98°$
or
$\angle FCG = 22° \, 59'$

MIND TEST 5.1i

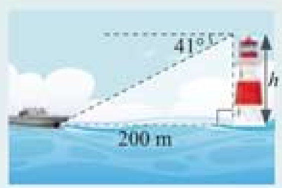
1. A foldable ladder which is placed on the floor forms an isosceles triangle PQR as shown in the diagram on the right. Given that T is the midpoint of PR, $\angle PQR = 38°$ and $PR = 1.4$ m, calculate the length of PQ, correct to two decimal places.



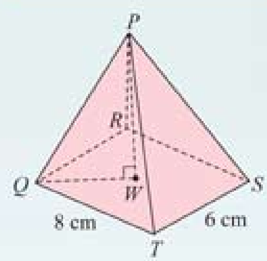
2. The diagram on the right shows Aisyah who is looking at a lamp post. Given that the angle of elevation at the tip of the lamp post from Aisyah's eyes is 55° and the distance between Aisyah's eyes and the tip of the lamp post is 145 metres, calculate the horizontal distance d in metres. State the answer correct to three significant figures.



3. The diagram on the right shows the position of a ship and a lighthouse. Given that the angle of depression of the ship from the lighthouse is 41° and the horizontal distance between the lighthouse and the ship is 200 m, calculate the height of the lighthouse, h in metres. State the answer correct to four significant figures.



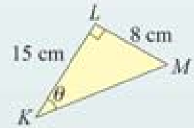
4. A right pyramid $PQRST$ has a rectangular base $QRST$. Given that W is the midpoint of QS and RT, the lengths of $QT = 8$ cm, $TS = 6$ cm and point P is vertically above point W, calculate
   (a) PT, in cm, if $PW = 12$ cm
   (b) the value of $\angle PTR$



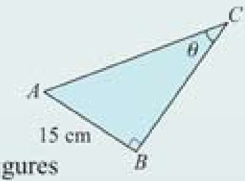
Dynamic Challenge

Test Yourself

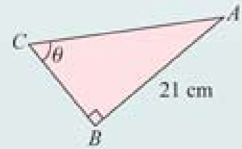
1. The diagram on the right shows a right-angled triangle *KLM*. Calculate
   (a) θ in degrees and minutes (b) $\sin (90° - \theta)$ (c) $\cos (90° - \theta)$



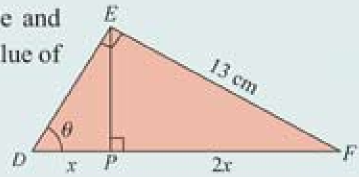
2. The diagram on the right shows a right-angled triangle *ABC*. Given that $\tan \theta = \frac{5}{12}$, calculate
   (a) the length of *AC* in cm
   (b) the value of $(90° - \theta)$
   (c) the value of θ in degrees and minutes correct to three significant figures



3. The diagram on the right shows a right-angled triangle *ABC*. Given that $AB = 21$ cm and $\sin \theta = \frac{7}{9}$, calculate
   (a) the length of *AC* in cm
   (b) the value of $\angle BAC$. State your answer to the nearest degree.



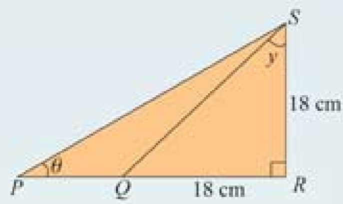
4. In the diagram on the right, *DEF* is a right-angled triangle and *DPF* is a straight line. Given that $PE = 5$ cm, calculate the value of
   (a) x in cm
   (b) θ in degrees and minutes



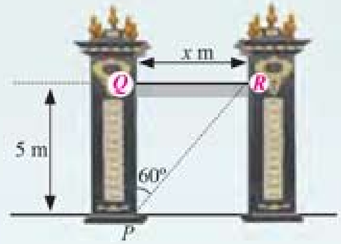
Skills Enhancement

1. Calculate the values of the following without using a calculator.
   (a) $8 \sin 60° - 3 \tan 60°$
   (b) $(\tan 30°)(2 \cos 30°) + 6 \sin 30°$
   (c) $(8 \cos 45°)(\sin 60°) + (8 \sin 45°)(\cos 30°)$

2. The diagram on the right shows a right-angled triangle *PRS*. *PQR* is a straight line. Given that $QR = RS = 18$ cm and $\tan \theta = \frac{3}{5}$, calculate
   (a) the length of *PQ*, in cm
   (b) the length of *PS*, in cm, correct to the nearest integer
   (c) the value of y

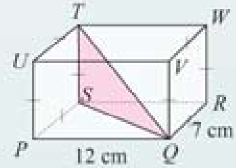


3. A gate has two vertical poles that are connected to a horizontal bridge with a distance of x metres. If the vertical height of the bridge from the ground surface is 5 m and the angle between the pole *PQ* and the inclined line *PR* is 60°, determine the value of x, in metres.

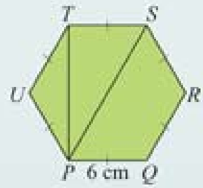


Self Mastery

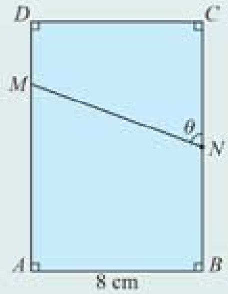
1. The diagram on the right shows a cuboid *PQRSTUVW*. *QRWV* and *PSTU* are squares. Given that $PQ = 12$ cm and $QR = 7$ cm, calculate
   (a) tan $\angle PQS$
   (b) the length of *TQ*, in cm, correct to four significant figures
   (c) the value of $\angle SQT$, in degrees and minutes



2. The diagram on the right shows a regular hexagon *PQRSTU* with sides 6 cm. Calculate
   (a) $\angle PTS$
   (b) $\angle TPS$
   (c) the length of *TP*, in cm, correct to three significant figures
   (d) the ratio of area of ΔPTU to area of ΔPTS



3. The diagram on the right shows a rectangle *ABCD*. It is given that $AB = 8$ cm, $BC = 2AB$ and *N* is the midpoint of *BC*.
   (a) If $MD = \frac{1}{4} AD$, calculate the length of *MN*, in cm. State your answer in surd form.
   (b) Calculate the value of θ, in degrees and minutes.
   (c) Shahril stated that the ratio of the area of trapezium *CDMN* to the area of trapezium *ABNM* is 1: 2. Is Shahril's statement true? State the reasons for your answer.

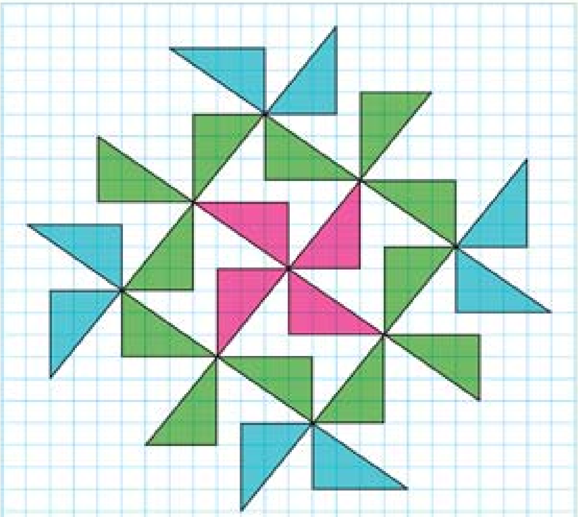


PROJECT

Materials: Grid paper 0.5 cm × 0.5 cm, pencil, ruler and colour pencil.

Steps:

1. Start by drawing a combination of right-angled triangles (pink).
2. Connect each vertex of the original combination with right-angled triangle (green).
3. Continue the pattern obtained in step 2 as many times as possible.
4. Colour and present your work in class.
5. Other groups are encouraged to use right-angled triangles of different sizes as the beginning pattern.



CONCEPT MAP

Trigonometric ratio

Ratio of sine

$\sin \theta = \dfrac{\text{opposite side}}{\text{hypotenuse}}$

Ratio of cosine

$\cos \theta = \dfrac{\text{adjacent side}}{\text{hypotenuse}}$

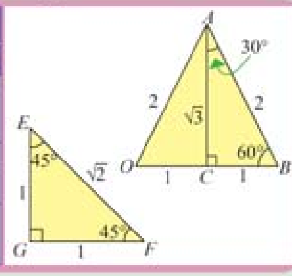
Ratio of tangent

$\tan \theta = \dfrac{\text{opposite side}}{\text{adjacent side}}$

When the **size** of an **acute angle θ increases,**
- the values of **$\tan \theta$ and $\sin \theta$ increase**
- the value of **$\cos \theta$ decreases**

Determine the angles from the values of sine, cosine and tangent

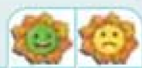
θ	Special angles		
	30°	45°	60°
$\tan \theta$	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$
$\sin \theta$	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$
$\cos \theta$	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$



θ°	0°	90°
$\sin \theta$	0	1
$\cos \theta$	1	0
$\tan \theta$	0	∞

SELF-REFLECT

At the end of this chapter, I can:



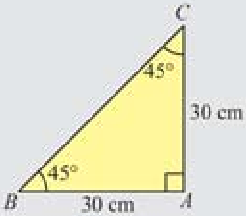
1.	Identify the opposite side and adjacent side based on an acute angle in a right-angled triangle.		
2.	Make and verify the conjecture about the relationship between acute angles and the ratios of the sides of right-angled triangles, and hence define sine, cosine and tangent.		
3.	Make and verify the conjecture about the impact of changing the size of the angles on the values of sine, cosine and tangent.		
4.	Determine the values of sine, cosine and tangent of acute angles.		
5.	Determine the values of sine, cosine and tangent of 30°, 45° and 60° angles without using a calculator.		
6.	Perform calculations involving sine, cosine and tangent.		
7.	Solve problems involving sine, cosine and tangent.		

EXPLORING MATHEMATICS

To measure the height of a pole, the following method can be used.

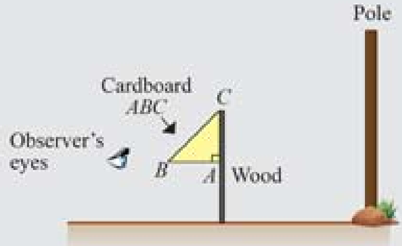
Step 1

Draw a right-angled triangle *ABC* on a cardboard as shown on the right with $AB = AC = 30$ cm.



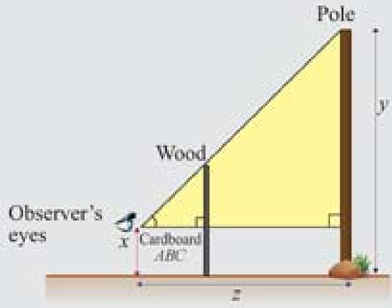
Step 2

$\angle ABC = \angle BCA = 45°$. Cut out the triangle and fix it to a wooden rod. Place the rod parallel to the pole to be measured.



Step 3

Align your view so that *BC* and the top of the pole are in a straight line. Use the wooden rod to make sure the position of triangle *ABC* is upright.



Step 4

If x represents the height of the eye from ground level, y represents the height of the pole and z represents the distance between the observer and the pole, then,

$$\tan 45° = \frac{y - x}{z}$$

$$y = z \tan 45° + x$$

The height of the pole can be easily determined without the need to measure the pole itself.