CHAPTER 6

Linear Equations

What will you learn?

- Linear Equations in One Variable
- Linear Equations in Two Variables
- Simultaneous Linear Equations in Two Variables

Why study this chapter?

Various problems which involve daily situations can be solved by interpreting the information in a mathematical sentence expressed in the form of linear equations. Discuss with your teacher the situations or other fields that involve solution of linear equations.

21st century learning is one of the implementations in the Malaysia Education Blueprint (PPPM) 2013 - 2025. In the 21st century classroom, the students are arranged in groups to enable them to hold discussions and carry out various activities.

For example, a class consisting 30 students is needed to be arranged in groups to achieve the goals.

It is given that the number of boys in the class is 6 more than the number of girls. How do you determine the number of groups that can be formed such that each group has 2 girls?

Walking through Time

Diophantus of Alexandria

Diophantus of Alexandria was a Greek mathematician who contributed significantly to the knowledge of solving algebraic equations. He was known as the Father of Algebra.

For more information:

http://goo.gl/9AoB9

Word Link

• trial and improvement	• *cuba jaya*
• equality	• *kesamaan*
• numerical value	• *nilai berangka*
• working backwards / backtracking	• *pematahbalikan*
• variable	• *pemboleh ubah*
• substitution	• *penggantian*
• elimination	• *penghapusan*
• solution	• *penyelesaian*
• linear equation	• *persamaan linear*
• simultaneous linear equations	• *persamaan linear serentak*
• root of an equation	• *punca persamaan*

Open the folder downloaded from page vii for the audio of Word Link.

6.1 Linear Equations in One Variable

LEARNING STANDARDS

Identify linear equations in one variable and describe the characteristics of the equations.

What are linear equations in one variable?

Diagram (a)

Diagram (b)

Note that the amount of money in Diagram (a) is equal to the amount of money in Diagram (b). This situation can be written as:

$$\text{Amount of money in Diagram (a)} = \text{Amount of money in Diagram (b)}$$

The symbol '=' is used to show the relationship between two quantities that have the same value. The mathematical sentence that involves equality is known as **equation**.

For example, $x + 2 = 5$ and $y - 7 = 11$

Did You Know

The symbol '$\neq$' is used for relationship that consists of different values. For example, $2 \neq 5$

CHAPTER 6

Exploration Activity 1

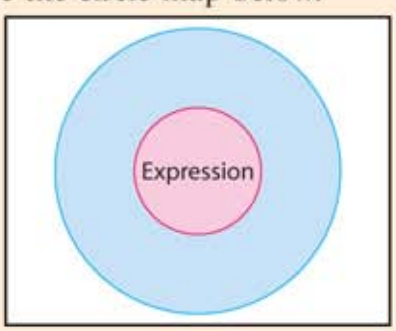
Individual

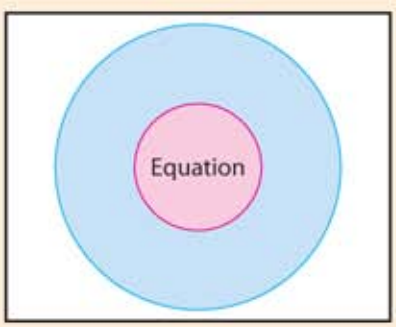
Aim: To identify algebraic expressions and equations.

1. Observe the mathematical sentences written on the cards in the diagram below.

$7h + 2$	$x + 5 = 8$	$\frac{y}{2} - 10 = 1$	$k^2 + 1 = 6$	$6p + 4q = 9$
$x - \frac{2y}{3} = 7$	$3n + 1 = 6n$	$5a^3 - 4$	$3h - 1 = 5k$	$x + 2x$

2. Classify the mathematical sentences into expressions and equations. Copy and complete the circle map below.





3. Compare your answers with those of your friends.

Look at the following equations obtained from the results of Exploration Activity 1.

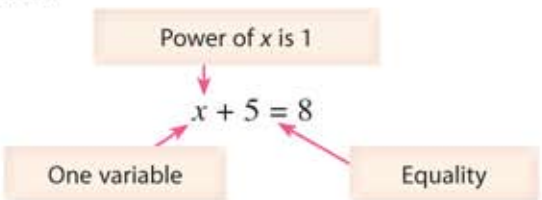
$x + 5 = 8$	$\frac{y}{2} - 10 = 1$	$6p + 4q = 9$
$x - \frac{2y}{3} = 7$	$3n + 1 = 6n$	$3h - 1 = 5k$

These equations are known as **linear equations** since the power of the variables is one.

Among the linear equations, it is found that the following equations have only one variable and the power of the variable is one.

$x + 5 = 8$	$\frac{y}{2} - 10 = 1$	$3n + 1 = 6n$

These equations are known as **linear equations in one variable**.



Communication Corner

Discuss why $k^2 + 1 = 6$ is a non-linear equation.

Did You Know

Expressions such as xy, $\frac{1}{x}$ and $\frac{x}{y}$ are non-linear expressions.

SMART TIPS

The characteristics of a linear equation in one variable:
- Has only one variable
- The power of the variable is one

Example 1

Explain whether each of the following equations is a linear equation in one variable:

(a) $3x + 2 = 5$ (b) $p - 4q = 6$

(c) $2(k - 7) = \frac{k}{3}$ (d) $y^2 + 3y = 1$

Solution

(a) Yes, because the equation has one variable, x and the power of x is 1.

(b) No, because the equation has two variables, p and q.

(c) Yes, because the equation has one variable, k and the power of k is 1.

(d) No, because the highest power of the variable y is 2.

Let's Discuss

Discuss whether each of the following equations is a linear equation in one variable.

(a) $x = 0$

(b) $\frac{1}{x} + 2 = 6$

(c) $x(x - 1) = 9$

Self Practice 6.1a

1. Explain whether each of the following equations is a linear equation in one variable:

(a) $m + 5 = 12$ (b) $3(p - 2) = -7$ (c) $9x + 8y = 10$ (d) $k^2 - 5k = 4$

How do you form linear equations based on a given situation and vice-versa?

LEARNING STANDARDS

Form linear equations in one variable based on a statement or a situation, and vice-versa.

Example 2

Form a linear equation in one variable for each of the following:

(a) Subtract 8 from a number, its remainder is 2.

(b) Peck Chin bought five pens which cost y sen each and an exercise book which costs RM3. The total amount that she paid was RM7.

Solution

(a) Let the number be x.

$x - 8 = 2$

(b) The price of five pens $= 5 \times y$

$= 5y$

$5y + 300 = 700$

RM3 = 300 sen; RM7 = 700 sen

SMART TIPS

- Each term in a linear equation must have the same unit.
- It is unnecessary to write the unit while forming linear equations.

Example 3

Write a statement or situation for each of the following equations.

(a) $2x + 5 = 19$, where x is a number.

(b) $y - 2 = 8$, where y is the age of Rajes now.

Solution

(a) Twice a number added to 5, the result is 19.

(b) Rajes was 8 years old 2 years ago.

Self Practice 6.1b

1. Form a linear equation in one variable for each of the following:

(a) The quotient of a number and 6 is 12.

(b) The price of chicken is RMy per kilogram. Rozita bought 5 kg of chicken and made a total payment of RM40.

(c) The perimeter of a rectangle with length $2x$ cm and width 5 m is 14 m.

2. Write a statement or situation for each of the following equations:

(a) $p - 1 = 6$, where p is a number.

(b) $x + 10 = 78$, where x is Edri's score in a test.

(c) $4m = 50$, where m is the mass, in kg, of a pack of rice.

How do you solve linear equations in one variable?

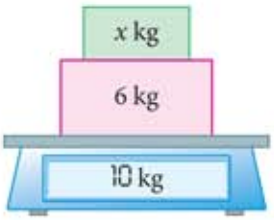
LEARNING STANDARDS

Solve linear equations in one variable.

Two blocks with mass of 6 kg and x kg respectively are placed on the scales as shown in the diagram.

The reading shown by the scales is equal to the total mass of both the blocks. Therefore, the linear equation formed is $x + 6 = 10$.

If we replace the x kg block with a block of a mass of 4 kg, the reading on the scales is equal to 10 kg. This shows that 4 is a numerical value that satisfies the equation $x + 6 = 10$. This numerical value is known as the solution of the equation $x + 6 = 10$.



The **solution of linear equation** is the numerical value that satisfies the equation.

The linear equation in one variable can be solved by using three methods as follows:
(a) Trial and improvement method
(b) Application of equality concept
(c) Backtracking method

Did You Know?

Solution of linear equation is also known as the **root** of the equation.

Example 4

Solve the linear equation $2x + 1 = 7$ by using
(a) trial and improvement method
(b) application of equality concept
(c) backtracking method

Solution

(a) **Trial and improvement method**

$2x + 1 = 7$

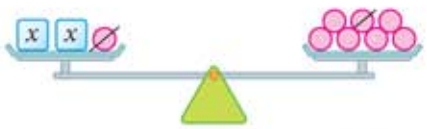
Value of x	Left hand side $= 2x + 1$	Review	Reflection
1	$2(1) + 1 = 3$	$3 \neq 7$, the value 3 is less than 7.	Substitute x with a value greater than 1.
2	$2(2) + 1 = 5$	$5 \neq 7$, the value 5 is close to 7.	Substitute x with a value greater than 2.
3	$2(3) + 1 = 7$	Left hand side = 7	$x = 3$ is the solution.

Thus, $x = 3$

(b) **Application of equality concept**

$2x + 1 = 7$

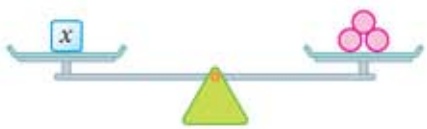
$2x + 1 - 1 = 7 - 1$



$2x = 6$

$\frac{2x}{2} = \frac{6}{2}$

Thus, $x = 3$



SMART TIPS

In a linear equation, the value on the left hand side is always equal to the value on the right hand side. Therefore, the mathematical operations performed on both sides of the linear equation must be the same in order to obey the equality concept.

(c) **Backtracking method**

$2x + 1 = 7$

Let the initial value = x and the final value = 7.

List the mathematical operations starting from the value of x to become 7.

$x \longrightarrow \times 2 \longrightarrow + 1 \longrightarrow = 7$

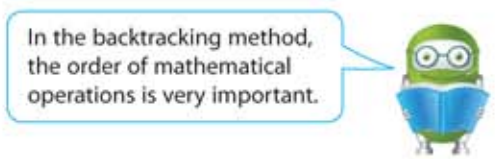
By using backtracking method, we have to consider the reverse operation so that 7 can become the value of x.

$x \longleftarrow \div 2 \longleftarrow - 1 \longleftarrow = 7$

So the value of x can be determined by writing

$7 - 1 = 6 \longrightarrow 6 \div 2 = 3$

Thus, $x = 3$



SMART TIPS

The backtracking method uses working backwards strategy and usually is used to solve problems that have known final value but the initial value is unknown.

Let's Discuss

Is backtracking method appropriate for solving all the linear equations? Discuss.

Example 5

Solve the following equations:

(a) $\frac{2x + 13}{5} = 7$ (b) $5(x - 4) = x + 16$

Solution

(a) $\frac{2x + 13}{5} = 7$

$\frac{2x + 13}{5} \times 5 = 7 \times 5$ ← Multiply both sides by 5.

$2x + 13 = 35$

$2x + 13 - 13 = 35 - 13$ ← Subtract 13 from both sides of the equation.

$2x = 22$

$\frac{2x}{2} = \frac{22}{2}$ ← Divide both sides by 2.

$x = 11$

(b) $5(x - 4) = x + 16$

$5x - 20 = x + 16$ ← Expand the equation.

$5x - 20 + 20 = x + 16 + 20$ ← Add 20 to both sides.

$5x = x + 36$

$5x - x = x + 36 - x$ ← Subtract x from both sides of the equation.

$4x = 36$

$\frac{4x}{4} = \frac{36}{4}$ ← Divide both sides by 4.

$x = 9$

Think Smart

Can you solve the equation $3(2x + 3) = 6x + 9$? Explain.

SMART TIPS

Check your answer by substituting the value of the variable obtained into the equation and then test whether the value on the left hand side is equal to the value on the right hand side.

Visit http://goo.gl/nB5Sq for the game about solving linear equations in one variable.

Self Practice 6.1c

1. Solve the following linear equations:

(a) $x - 10 = 3$ (b) $4x - 1 = 7$ (c) $\frac{x}{6} + 3 = 5$

(d) $\frac{x + 8}{2} = 9$ (e) $3(x + 2) = 5x$ (f) $\frac{2}{3}x - 4 = x + 1$

How do you solve problems?

LEARNING STANDARDS

Solve problems involving linear equations in one variable.

MATHEMATICS APPLICATION TIME

After 10 years, Jalil's age will be thrice his age this year. What is Jalil's age?

Solution

Understanding the problem

After 10 years, Jalil's age is thrice his age this year.

Devising a plan

Let Jalil's current age = x years old

After 10 years, Jalil's age = $3 \times x$

$= 3x$

Linear equation in one variable that can be formed is $x + 10 = 3x$.

Implementing the strategy

$$x + 10 = 3x$$

$$x + 10 - x = 3x - x$$

$$10 = 2x$$

$$\frac{10}{2} = \frac{2x}{2}$$

$$5 = x$$

Jalil is 5 years old.

Doing reflection

When $x = 5$, $5 + 10 = 3(5)$

$= 15$

SMART TIPS

The steps to solve problems involving linear equations in one variable:

- Identify the variable in the problem and represent the variable with a letter.
- ↓
- Form a linear equation based on the information given.
- ↓
- Solve the equation to find the value of the variable.

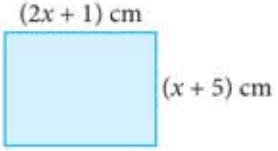
Think Smart

The sum of three consecutive odd numbers is 63. What are the numbers?

Self Practice 6.1d

1. In a mathematics test, Azmah scored 17 marks more than Yazid while Suzana's score is twice of Yazid's score. If their total score is 161, what is Azmah's score?

2. The diagram shows a rectangle. If the perimeter of the rectangle is 66 cm, what is the area of the rectangle?



Mastery Q 6.1

Open the folder downloaded from page vii for extra questions of Mastery Q 6.1.

1. Write the linear equation in one variable for each of the given situations:

(a) The difference between x and 8 is 15, where x is greater than 8.

(b) The price of an exercise book is thrice the price of a pen. Yahya buys 2 exercise books and 8 pens and makes a total payment of RM42.

(c) Harjit's age is p years old. He was born when his mother was 34 years old. Now, his mother's age is thrice his age.

2. Solve the following equations using the trial and improvement method:

(a) $x - 6 = 10$ (b) $2x + 3 = 11$ (c) $14 - 3x = 8$

3. Solve the following equations by applying the equality concept:

(a) $5x + 1 = 3x$ (b) $2y - 4 = 5y + 8$ (c) $\frac{2}{3}x - 9 = 7$

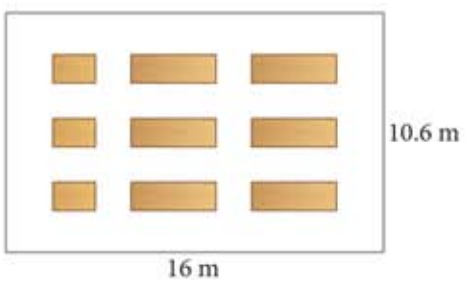
4. Solve the following equations using the backtracking method:

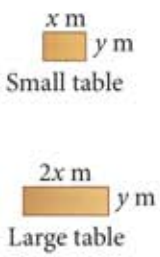
(a) $3x + 2 = 12$ (b) $2(x + 4) = 22$ (c) $\frac{7y - 6}{9} = 4$

5. There are 35 students participating in a community activity to clean up their school. The number of girls who participated is 5 less than the number of boys. How many boys participated in the activity?

6. The time spent by Nadia to answer the geography quiz is 30 minutes more than the time spent to answer the history quiz. The time spent to answer the history quiz is half the time spent to answer the geography quiz. What is the total time that Nadia took to answer both quizzes?

7.





The diagram shows the plan for the arrangement of tables in a room. The distance between the tables is 1.5 m and the distance between the tables and the wall is 2 m. Determine the surface areas of a small table and a large table.

6.2 Linear Equations in Two Variables

What are linear equations in two variables?

LEARNING STANDARDS

Identify linear equations in two variables and describe the characteristics of the equations.

Look at the following linear equations obtained from the results in Exploration Activity 1.

$6p + 4q = 9$ $\quad x - \frac{2y}{3} = 7$ $\quad 3h - 1 = 5k$

These linear equations have two variables and the power of each variable is 1. These equations are known as **linear equations in two variables**.

Linear equation in two variables is a linear equation that has two variables and the power of each variable is one.

Linear equation in two variables can be written in the general form of

$$ax + by = c \quad \text{where } a \text{ and } b \text{ are non-zero.}$$

Which of the following linear equations in two variables is written in the general form?

$6p + 4q = 9$ $\quad x - \frac{2y}{3} = 7$ $\quad 3h - 1 = 5k$

Example 6

Explain whether the following equations are linear equations in two variables:

(a) $3p - q = 6$ (b) $7x^2 + 5y = 9$

(c) $2(k + 8) = 5k$ (d) $\frac{y}{6} + 4 = -2x$

Let's Discuss

Discuss why $xy + 2y = 3$ and $\frac{1}{x} + \frac{1}{y} = 5$ are not linear equations in two variables.

Solution

(a) Yes, because the equation has two variables, p and q, and the power of each variable is 1.

(b) No, because the power of variable x is 2.

(c) No, because the equation has only one variable.

(d) Yes, because the equation has two variables, x and y, and the power of each variable is 1.

Self Practice 6.2a

1. Explain whether the following equations are linear equations in two variables:

(a) $h - 7k = 6$ (b) $3m + 5 = 9m - 1$ (c) $\frac{2x}{3} - 8 = 4y$ (d) $p(p + 3) = 2q$

How do you form linear equations in two variables?

Example 7

LEARNING STANDARDS

Form linear equations in two variables based on a statement or a situation, and vice-versa.

Form a linear equation in two variables for each of the following:

(a) The difference between two numbers is 18.

(b) The amount of pocket money saved by Ahmad is RMx and the amount of pocket money saved by Norita is RMy. Their total savings is RM600.

(c) A bus and a van can accommodate m and n passengers respectively. The total number of passengers for two buses and five vans is 100.

Solution

(a) Let the two numbers be p and q respectively.
$p - q = 18$

(b) Total savings = 600
$x + y = 600$

(c) Total number of passengers = 100
$2m + 5n = 100$

Example 8

Write the situation for each of the following linear equations in two variables.

(a) $p - q = 6$, where p and q represent the number of boys and the number of girls respectively who participated in an expedition held during the last school holidays.

(b) $4x + 5y = 35$, where x and y represent the price, in RM, of a plate of *nasi lemak* and a plate of fried noodles respectively.

Solution

(a) In an expedition held during the last school holidays, the number of boys is 6 more than the number of girls.

(b) The total price for 4 plates of *nasi lemak* and 5 plates of fried noodles is RM35.

Self Practice 6.2b

1. Form a linear equation in two variables for each of the following:

(a) In a long jump event, the total points obtained by Satria team and Perdana team were 258.

(b) The difference between the largest angle and the smallest angle in a triangle is 15°.

(c) The total price of tickets for a water theme park for eight adults and five children is RM265.

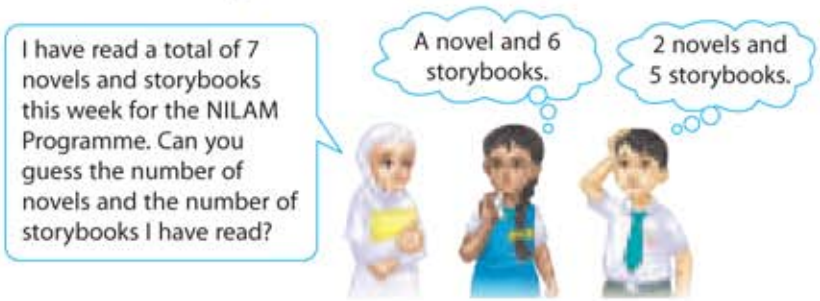
(d) The total marks for a test paper which consists of Section A and Section B is 40. Each correct answer in Section A and Section B contributes 1 mark and 2 marks respectively.

2. Write the situation for each of the following linear equations:
   (a) $x + y = 465$, where x and y represent the number of aluminium cans and the number of glass bottles collected respectively in a recycling campaign.
   (b) $p - q = 3$, where p and q represent the length and width respectively, in cm, of a rectangle.

How do you determine the possible solutions of linear equations in two variables?

LEARNING STANDARDS

Determine and explain possible solutions of linear equations in two variables.



The equation that can be formed from the situation above is $x + y = 7$ and it has a few pairs of different values for x and y. All the pairs of values of x and y are possible solutions for the equation.

A **linear equation in two variables** has **many possible pairs of values of solutions**.

Example 9

Write three possible pairs of solutions for $2x + y = 6$.

Solution

$2x + y = 6$

When $x = 0$,	When $x = 1$,	When $x = 2$,
$2(0) + y = 6$	$2(1) + y = 6$	$2(2) + y = 6$
$y = 6$	$y = 6 - 2$	$y = 6 - 4$
	$y = 4$	$y = 2$

Thus, three possible pairs of solutions are $x = 0, y = 6$; $x = 1, y = 4$ and $x = 2, y = 2$.

SMART TIPS

Each pair of solutions can be writen in the ordered pair (x, y). For example, (0, 6), (1, 4) and (2, 2).

Smart

A scientific calculator can be used to determine the values of solutions for linear equations in two variables.

1. Write equation in the form $y = 6 - 2x$.
   Press
   [ALPHA] [Y] [ALPHA] [=] [6] [−] [2] [ALPHA] [X] [CALC]
   Screen displayed
   X?
2. Enter the value x. For example 0.
   Press [0] [=]
   Screen displayed
   6 − 2X
   6
   Thus, $y = 6$.
3. Press [CALC] and enter the different values of x for the other pairs of solutions.

Example 10

Osman has bought a total of five jerseys for Harimau Team and Kancil Team. What are the possible numbers of jerseys that can be bought for each team?

Solution

Let the number of jerseys for Harimau Team = x
and the number of jerseys for Kancil Team = y

Thus, $x + y = 5$

Number of jerseys for Harimau Team	Number of jerseys for Kancil Team
$x = 1$	$y = 4$
$x = 2$	$y = 3$
$x = 3$	$y = 2$
$x = 4$	$y = 1$

SMART TIPS

The steps to determine the possible solutions of linear equations in two variables:

Choose a value for one of the variables.

↓

Substitute the value into the linear equation.

↓

Solve the equation to find the value of the other variable.

Let's Discuss

Besides the pairs of solutions listed above, is there any other pair of solutions for the number of jerseys that Osman bought? Discuss.

Self Practice 6.2c

1. Write three possible pairs of solutions for the following equations:

   (a) $x - y = 7$ (b) $y = 1 - 2x$ (c) $3x - \frac{y}{2} = 6$

2. On sports day, Sani won 4 medals all of which were gold and bronze medals. How many gold and bronze medals did Sani possibly win?

How do you represent graphically linear equations in two variables?

LEARNING STANDARDS

Represent graphically linear equations in two variables.

Exploration Activity 2

Aim: To represent graphically linear equations in two variables.

Instruction:
- Explore by yourself before the lesson begins and discuss in groups of four during the lesson.
- Open the folder downloaded from page vii.

1. Open the file *linear equation graphically.pdf* and print the file.
2. Complete the table for each given equation.

(a) $x - y = -2$

x	0	1	2	3	4	5
y	2					
(x, y)	(0, 2)					

(b) $x - 2y = 8$

x	0	2	4	6	8	10
y						
(x, y)						

(c) $x + y = -5$

x						
y	0	−1	−2	−3	−4	−5
(x, y)						

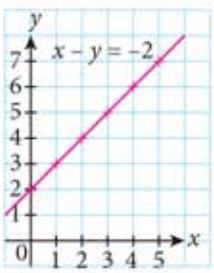
(d) $2x + y = 2$

x						
y	10	8	6	4	2	0
(x, y)						

3. Based on the ordered pairs obtained from the tables above, plot all the points on the printed grid paper.
4. Draw a graph by connecting all the points.
5. Study the shape of your graphs. What is your observation?
6. Compare the shape of the graphs obtained from the given linear equations.
7. Discuss your findings.

From the results of Exploration Activity 2, it is found that straight line graphs are obtained when all the ordered pairs of the linear equations in two variables are plotted and connected.

All the points on the straight line are the solutions of the linear equation.



Self Practice 6.2d

1. Which of the following diagrams represent graphically a linear equation in two variables?

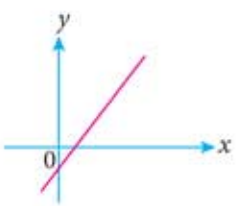


Diagram (a)

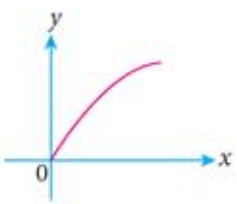


Diagram (b)

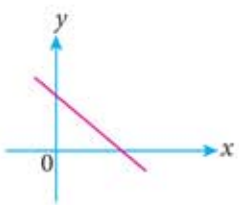


Diagram (c)

Mastery Q 6.2

Open the folder downloaded from page vii for extra questions of Mastery Q 6.2.

1. Form a linear equation in two variables for each of the following:
   (a) The total price of x headscarves at RM30 each is RM8 more than the total price of y shawls at RM20 each.
   (b) The total age of a father and his twin children is 130.
   (c) In a school, the number of female teachers is twice the number of male teachers.

2. Write two possible pairs of solutions for each of the following equations:
   (a) $x + y = 7$ (b) $y - 2 = 5x$

3. Draw graphs to represent each of the following linear equations based on the given values of x:
   (a) $x - y = 2$; $x = 0, 1, 2, 3, 4, 5$ (b) $2x + y = 4$; $x = -2, -1, 0, 1, 2$
   (c) $y - \frac{x}{2} = 3$; $x = -6, -4, -2, 0, 2$

4. A shirt costs RM20 and a pair of pants costs RM10. Find the possible number of shirts and pants that Sheimah can buy with a total payment of RM80. What is the maximum number of shirts that Sheimah can buy?

5. Pei San saves only 10 sen and 20 sen coins in her coin box. Her total savings is RM5. Draw a graph to represent the situation.

6.3 Simultaneous Linear Equations in Two Variables

How do you form simultaneous linear equations and represent them graphically?

LEARNING STANDARDS

Form simultaneous linear equations based on daily situations. Hence, represent graphically the simultaneous linear equations in two variables and explain the meaning of simultaneous linear equations.

Faizah rears a total of 7 chickens and ducks in a coop. The cost of rearing a chicken is RM2 per week whereas the cost of rearing a duck is RM1 per week. The total cost for rearing the chickens and ducks is RM12 per week. How many chickens and ducks can Faizah rear?

Based on the situation above, let x and y be the number of chickens and ducks being reared respectively,

thus $x + y = 7$ ← Total number of chickens and ducks is 7.

and $2x + y = 12$ ← Total rearing cost of chickens and ducks is RM12 per week.

Both the equations formed are linear equations in two variables.

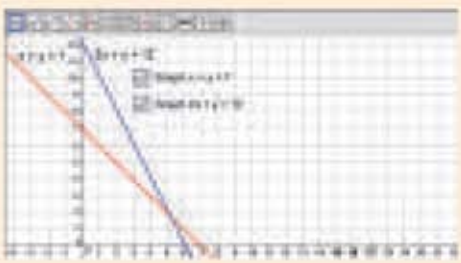
To determine the number of chickens and the number of ducks, we need to find the values of x and y that satisfy both the linear equations.

Exploration Activity 3a

Aim: To explore simultaneous linear equations.

Instruction:
- Explore by yourself before the lesson begins and discuss in groups of four during the lesson.
- Open the folder downloaded from page vii.

1. Open the file *intersecting.ggb* using *GeoGebra*.
2. Click on both the boxes to display the straight line graph for the equations $x + y = 7$ and $2x + y = 12$.
3. Observe the two straight lines displayed and state the point of intersection (x, y) of both the straight lines.



4. Determine whether the point of intersection (x, y) is the solution for both the linear equations.

From the results of Exploration Activity 3a, it is found that the point of intersection of the two straight lines is the common pair of solutions for both the linear equations.

Linear equations $x + y = 7$ and $2x + y = 12$ are **simultaneous linear equations** in two variables because both the linear equations have two similar variables.

The solution of simultaneous linear equations that has one point of intersection is known as a **unique solution**.

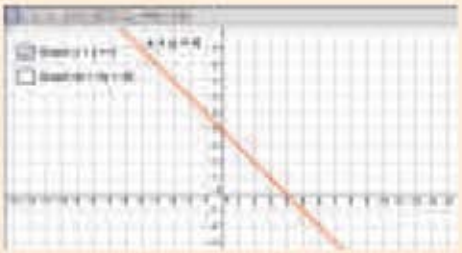
Exploration Activity 3b

Aim: To explore simultaneous linear equations.

Instruction:
- Explore by yourself before the lesson begins and discuss in groups of four during the lesson.
- Open the folder downloaded from page vii.

1. Open the file *parallel.ggb* using *GeoGebra*.
2. Click on both the boxes to display the straight line graph for the equations $2x - 3y = -6$ and $4x - 6y = 18$.
3. Observe the two straight lines displayed.

4. Open the file *overlapping.ggb* using *GeoGebra*.
5. Click on both the boxes to display the straight line graph for the equations $x + y = 4$ and $5x + 5y = 20$.
6. Observe the two straight lines displayed.



7. Copy and record all your results in the following table.

Linear equations displayed	Condition of both straight lines	Point of intersection
$2x - 3y = -6$ $4x - 6y = 18$		
$x + y = 4$ $5x + 5y = 20$		

8. Discuss with your friends your findings.

Exploration Activities 3a and 3b show that there are three cases involving solution of simultaneous linear equations as shown in the table below.

Condition of both straight lines	Type of solution
Intersecting	Unique solution
Parallel	No solution
Overlapping	Infinite solutions

Self Practice 6.3a

1. Form simultaneous linear equations for each given daily situation. Hence, represent graphically the simultaneous linear equations in two variables.
   (a) Puan Siti intends to reward her students with 6 Malay dictionaries and English dictionaries. The prices of a Malay dictionary and an English dictionary are RM20 and RM40 respectively. Puan Siti buys x Malay dictionaries and y English dictionaries at a total cost of RM160.
   (b) A total of 12 students comprising boys and girls are divided equally into two groups. The number of boys in each group is 2 more than the number of girls.

How do you solve simultaneous linear equations in two variables?

LEARNING STANDARDS

Solve simultaneous linear equations in two variables using various methods.

The simultaneous linear equations in two variables can be solved by using

(i) graphical method
(ii) substitution method
(iii) elimination method

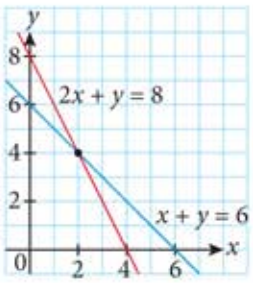
Example 11

Solve the following simultaneous linear equations by using graphical method.

$$x + y = 6 \quad \text{and} \quad 2x + y = 8$$

Solution

From the graph drawn, the point of intersection is (2, 4). Thus, the solution is $x = 2$ and $y = 4$.



SMART TIPS

The steps for solving simultaneous linear equations in two variables using substitution method:

- Express one of the variables in terms of the other variable.
- Substitute the expression into the other linear equation.
- Solve the linear equation in one variable.
- Substitute the value obtained into the expressed equation to find the value of the other variable.

Example 12

Solve the following simultaneous linear equations by using substitution method.

$$x - 3y = 7 \quad \text{and} \quad 5x + 2y = 1$$

Solution

$$x - 3y = 7 \quad \ldots\ldots\ldots\ldots \text{①}$$
$$5x + 2y = 1 \quad \ldots\ldots\ldots\ldots \text{②}$$

From ①, $x = 7 + 3y \quad \ldots\ldots \text{③}$ ← Express x in terms of y.

Substitute ③ into ②.

$$5(7 + 3y) + 2y = 1$$
$$35 + 15y + 2y = 1$$
$$35 + 17y = 1$$
$$17y = 1 - 35$$
$$17y = -34$$
$$y = \frac{-34}{17}$$
$$y = -2$$

Substitute $y = -2$ into ③.

$$x = 7 + 3(-2)$$
$$= 1$$

Thus, $x = 1$ and $y = -2$.

Example 13

Solve the following simultaneous linear equations by using elimination method.

(a) $x + 2y = 9$ and $3x - 2y = 15$

(b) $2x + 5y = 14$ and $3x + 4y = 7$

Solution

(a) $x + 2y = 9$ ..... ①

$3x - 2y = 15$ ..... ②

Identify the variable with the same coefficient.

① + ②: $4x + 0 = 24$

Eliminate the variable y by adding ① and ②.

$4x = 24$

$x = 6$

Substitute $x = 6$ into ①.

$6 + 2y = 9$

$2y = 9 - 6$

$2y = 3$

$y = \frac{3}{2}$

Thus, $x = 6$ and $y = \frac{3}{2}$.

(b) $2x + 5y = 14$ ..... ①

$3x + 4y = 7$ ..... ②

① × 3: $6x + 15y = 42$ ..... ③

② × 2: $6x + 8y = 14$ ..... ④

- Multiply ① and ② to equate the coefficient of x.
- The LCM for 2 and 3 is 6.

③ − ④: $0 + 7y = 28$

Eliminate the variable x by subtracting ④ from ③.

$7y = 28$

$y = 4$

Substitute $y = 4$ into ①.

$2x + 5(4) = 14$

$2x + 20 = 14$

$2x = 14 - 20$

$2x = -6$

$x = -3$

Thus, $x = -3$ and $y = 4$.

SMART TIPS

The steps for solving simultaneous linear equations in two variables using elimination method:

- Multiply one or both equations with a number so that the coefficient of one of the variables is equal.
- Add or subtract both the equations to eliminate one of the variables.
- Solve the linear equation in one variable.
- Substitute the value obtained into the original equation to find the value of the other variable.

Scan the QR Code or visit https://youtu.be/L0DJskZw3y0 to learn about the solution of Example 13(b) by using scientific calculator.

Self Practice 6.3b

1. Solve the following simultaneous linear equations:

(a) $y = 3x + 1$
$x + 2y = 16$

(b) $x + y = 5$
$2x - y = 22$

(c) $4x + 3y = 8$
$x - 3y = 2$

(d) $8x + 3y = -4$
$5x + 2y = 6$

How do you solve problems?

MATHEMATICS APPLICATION TIME

LEARNING STANDARDS

Solve problems involving simultaneous linear equations in two variables.

Yesterday I brought my wife and three children who are under 12 years old to the Zoo Negara, and the total cost of tickets was RM97.

Last week, I brought my wife and a 5-year-old child together with two of my friends to the Zoo Negara. The total cost of tickets was RM139.

Based on the conversation above, how much is the ticket for an adult and a child?

Solution

Understanding the problem

- Total cost of tickets for 2 adults and 3 children is RM97.
- Total cost of tickets for 4 adults and one child is RM139.

Devising a plan

Let the cost of the ticket for an adult = RMx
and the cost of the ticket for a child = RMy
Form two simultaneous linear equations and solve them.

Implementing the strategy

$2x + 3y = 97$ ..... ①
$4x + y = 139$ ..... ②
① × 2: $4x + 6y = 194$ ..... ③
$4x + y = 139$ ..... ②

③ − ②: $0 + 5y = 55$
$5y = 55$
$y = 11$

Substitute $y = 11$ into the equation ①.

$2x + 3(11) = 97$
$2x + 33 = 97$
$2x = 64$
$x = 32$

Thus, the cost of the ticket for an adult is RM32 and the cost of the ticket for a child is RM11.

Doing reflection

Total cost of tickets for 2 adults and 3 children = 2(32) + 3(11)
= RM97

Total cost of tickets for 4 adults and 1 child = 4(32) + 11
= RM139

Self Practice 6.3c

1. During the *Hari Keusahawanan* in a school, 800 booklets of coupons were sold. The price of each booklet of coupons was RM30 and RM50 respectively. The total amount collected was RM30 000. How many booklets of RM30 and RM50 coupons were sold?

2. The length of a rectangular swimming pool is p m and its width is q m. It is given that the length of the swimming pool is twice its width. If the perimeter of the swimming pool is 150 m, find the values of p and q.

Mastery Q 6.3

Open the folder downloaded from page vii for extra questions of Mastery Q 6.3.

1. Form simultaneous linear equations based on the following statement.

   The difference between two numbers is 5. When the larger number is multiplied by 2, the sum of both the numbers is 7.

   Hence, represent graphically the simultaneous linear equations in two variables and state the type of the solution.

2. Solve the following simultaneous linear equations:

   (a) $x + 4y = 14$
   $3x + 2y = 12$

   (b) $3m - 2n = 19$
   $5m + 7n = 11$

   (c) $\frac{1}{3}p + q = 4$
   $\frac{p - q}{4} = 2$

   (d) $\frac{f}{2} + \frac{g}{5} = 3$
   $2g - f = 10$

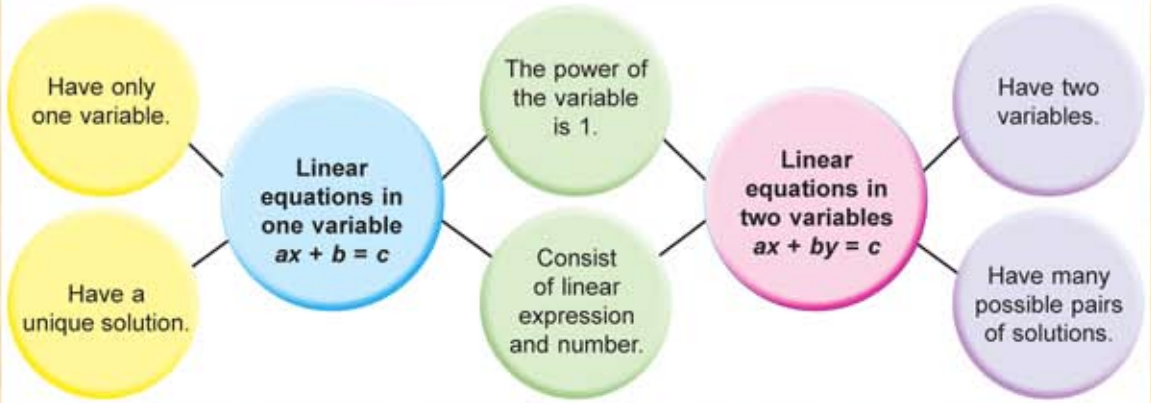
3. A wire with a length of 100 cm is cut into three parts. The length of wire for the first and second parts are the same. The length of the third part of the wire exceeds the sum of the first two parts by 4 cm. Calculate the length of each part.

4. Lai Yee and Khadijah have 60 stamps. When Lai Yee gives 5 stamps to Khadijah, the number of Lai Yee's stamps is twice the number of Khadijah's stamps. How many stamps does each of them have at first?

5. After six years, the age of Devaki's father is thrice the age of Devaki. If the age of Devaki's father was seven times the age of Devaki two years ago, what are their ages this year?

6. Sarah has more money than Hui Chin. If Sarah gives RM10 to Hui Chin, both of them will have the same amount of money. If Hui Chin gives RM5 to Sarah, Sarah will have four times the amount of money that Hui Chin has. How much does each of them have?

SUMMARY



CHAPTER 6

At the end of this chapter, I can...	Very good	Work harder
identify linear equations in one variable and describe the characteristics of the equations.		
form linear equations in one variable based on a statement or a situation, and vice-versa.		
solve linear equations in one variable.		
solve problems involving linear equations in one variable.		
identify linear equations in two variables and describe the characteristics of the equations.		
form linear equations in two variables based on a statement or a situation, and vice-versa.		
determine and explain possible solutions of linear equations in two variables.		
represent graphically linear equations in two variables.		
form simultaneous linear equations based on daily situations. Hence, represent graphically the simultaneous linear equations in two variables and explain the meaning of simultaneous linear equations.		
solve simultaneous linear equations in two variables using various methods.		
solve problems involving simultaneous linear equations in two variables.		

Let's PRACTISE

Test Yourself

1. Meena has a packet of sweets. She gives half of the sweets to her brother. After eating 3 sweets, there are 5 sweets left in the packet. How many sweets were there in the packet initially?

2. In a cross-country run organized by a school, the students who completed the run within one hour would obtain 2 points for their team. A total of 280 students managed to obtain the points. The number of boys who obtained the points was 60 more than the number of girls. How many points did the girls obtain during the cross-country run?

Self Mastery

3. The total savings of Ella and Zahida was RM2 000. Ella and Zahida each donated $\frac{1}{4}$ and $\frac{1}{5}$ of their savings to an old folks home. The total amount donated by them was RM440. What are the balances of Ella's and Zahida's savings now?

4. An online dealer sells two types of clothes, *baju kurung* and *baju kebaya*. The profit obtained from a *baju kurung* is RM10 less than the profit obtained from a *baju kebaya*. He earns a profit of RM275 from the sale of 5 *baju kurung* and 8 *baju kebaya* during the first week. In the second week, he sells 9 *baju kurung* and 7 *baju kebaya*. What is his profit in the second week?

5. The perimeter of a rectangle is 56 cm. When its length is reduced by 2 cm and its width is increased by 4 cm, a square is formed. What is the area of the rectangle?

6. The incomplete receipt shows the expenses of Liza and Kei Ling at Sedap Restaurant. The price of a cup of coffee is RM1 more than the price of a piece of curry puff. What are the prices of a cup of coffee and a piece of curry puff?

Sedap Restaurant	
2 cups of coffee	RM
4 curry puffs	RM
Total	RM6.80

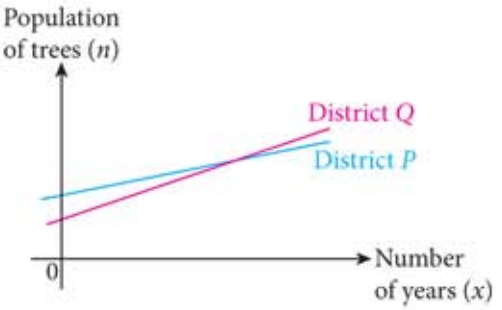
7. Encik Rizal and his wife brought their children below 12 years old along with them for a holiday in Pulau Redang. They spent a total amount of RM1 150 for the package tour. The fees for an adult and a child are RM350 and RM150 respectively. How many children did he bring with him?

Challenge Yourself

8. Asnita : My age is x years more than your age.
Reslynna : I was 13 years old x years ago.
Asnita : My age will be 31 years old after x years.

Based on their conversation, what are the ages of Asnita and Reslynna?

9. The straight line graph represents the population of trees planted in the districts P and Q since the year 2010. The linear equations represented by the straight line graphs of districts P and Q are $n - 3x = 16$ and $n - 5x = 10$ respectively, where n is the population of trees planted and x is the number of years after 2010. In which year would the population of trees planted in both the districts be the same? State the population of the trees in that year.



10. Cikgu Latif recorded the time for a 100 m sprint by three students Amir, Ben and Ravi. The recorded time of Ben and Ravi is the same. The average time for the three students is 13.3 s. However, Cikgu Latif noticed that Amir's time was recorded wrongly as 15.3 s instead of 13.5 s. Calculate the actual average time of the three students.

11. A wholesaler supplies two kinds of fruits, pineapples and watermelons, to stalls A and B. The mass of the fruits supplied are as shown in the table.

Stall	Mass (kg)	
	Pineapple	Watermelon
A	15	40
B	25	60

The total payments received by the wholesaler from stalls A and B are RM90 and RM140 respectively. Determine the price per kilogram for each kind of fruit.

12. Application Commerce

A company produces two types of pen drive, P and Q. In the year 2015, the profit earned from both the pen drives was RM350 000. In the year 2016, the profit earned from type P pen drive increased by 25% while the profit earned from type Q pen drive decreased by 10%. If the total profit for the year 2016 was RM395 500, find the profit earned from each type of the pen drive in the year 2016.

ASSIGNMENT

Linear equations are often used to solve various daily problems. Besides that, linear equations are also used in other fields such as management, finance, computer knowledge, science, engineering, construction and health.

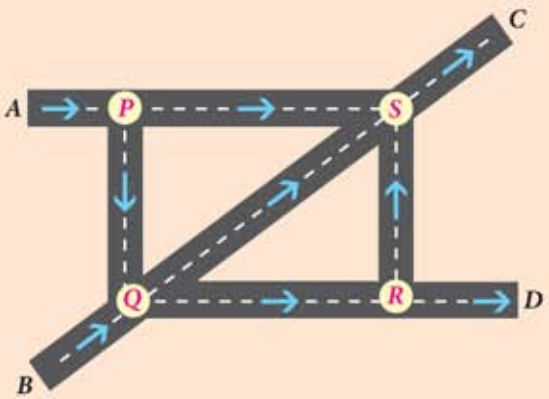
Carry out a study and write a report about the importance of linear equations in the stated fields.

Scan the QR Code or visit http://goo.gl/nDPH2m to explore the usage of linear equations in various fields.

Exploring MATHEMATICS

Linear equations can be used to study and solve the traffic problems.

The diagram shows a road map with four junctions, P, Q, R and S, where each road is a one-way road.



The flow rate of vehicles during the day is given in the table.

Traffic	Average flow rate of vehicles (Number of vehicles per hour)
Vehicles from A	110
Vehicles from B	75
Vehicles that move along PQ	35
Vehicles that move along QS	80
Vehicles that move along RS	20

By using the equality concept, form some linear equations from the given information. Hence, determine the average flow rate of vehicles that travel to C and D.