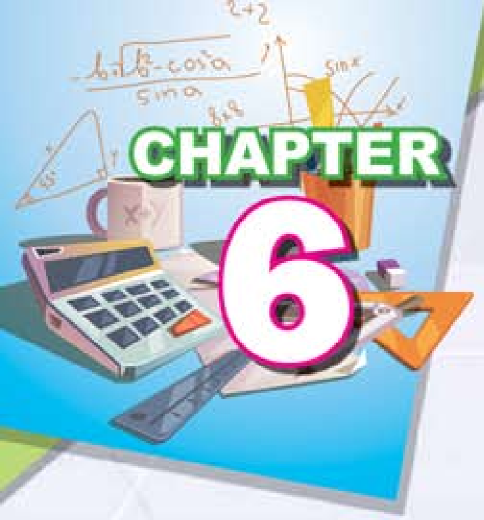


CHAPTER 6

Linear Inequalities in Two Variables

You will learn

- Linear Inequalities in Two Variables
- Systems of Linear Inequalities in Two Variables

Business and entrepreneurship are the main pulses of a country's economy to provide career opportunities and to improve standard of living of people. A businessman or an entrepreneur needs to be proficient in communication, leadership, accounting, marketing and most importantly in planning. Careful planning enables a businessman or an entrepreneur to obtain a high return, which is a maximum profit at minimal cost.

Do you know how an entrepreneur plans to make a high profit by minimising expenditures?

Why Study This Chapter?

Knowledge of linear inequalities is fundamental to the field of linear programming. This knowledge is very important in business, corporate world and other areas that require optimum results, which is a maximum profit at minimal cost.

Walking Through Time

Thomas Harriot
(1560 – 1621)

Thomas Harriot was an English astronomer, mathematician and translator. He contributed to the advancement of the scientific field. Thomas Harriot was the first person who introduced the inequality symbols greater than, >, and less than, <.

http://bt.sasbadi.com/m4155

WORD BANK

• linear inequality	• *ketaksamaan linear*
• linear	• *linear*
• variable	• *pemboleh ubah*
• region	• *rantau*
• linear inequality system	• *sistem ketaksamaan linear*

6.1 Linear Inequalities in Two Variables

How do you represent a situation in the form of a linear inequality?

Inequalities are used to describe the relationship between two quantities that are not equal.

> **Learning Standard**
>
> Represent situations in the form of linear inequalities.

Diagram 1

Diagram 1 shows a common warning sign on the road or highway. The warning sign is placed at an entrance of a tunnel. What is the maximum height of a vehicle that can pass through the tunnel?

> **MY MEMORY**
>
> $>$ greater than
> $<$ less than
> $\geqslant$ greater than or equal to
> $\leqslant$ less than or equal to

Diagram 2 shows the maximum mass of a 3-tonne lorry when it is loaded and not loaded. What is the relationship between the mass of an unloaded lorry, the mass of a loaded lorry and the mass of the load?

Diagram 2

For the situation in Diagram 1, let the overall height of a vehicle is represented by a variable h, thus $h < 4.75$ m. In this situation, only one quantity is involved, that is the height in metres.

The situation in Diagram 2 involves the mass in kg but in two different conditions. Let,

y = gross vehicle weight (BDM, *berat dengan muatan*)
x = kerb weight (BTM, *berat tanpa muatan*)

Value of load = BDM – BTM
= (7 500 – 3 410) kg
= 4 090 kg

Thus, the situation in Diagram 2 can be stated as

$$y \leqslant x + 4\ 090$$

That is, x and y are two variables representing two quantities with the same unit.

> **INFO ZONE**
>
> A variable is a factor such as an element, a feature or an integer that must be taken into account in a calculation.

> **MY MEMORY**
>
> The highest power of a variable in a linear equation is 1.

Example 1

Taufik wants to buy some revision books and exercise books at a book exhibition. He finds that the price of a reference book is RM14 and the price of an exercise book is RM9. The maximum amount of money that Taufik can spend is RM100. Represent the above situation in an appropriate form of linear inequality.

> **MY MEMORY**
>
> Linear equation in two variables:
> $ax + by = c$

Solution:

Let, x = reference book and y = exercise book.

Hence, $$14x + 9y \leqslant 100$$

The price of a reference book is RM14. ($14x$)

The price of an exercise book is RM9. ($9y$)

The total price of the reference books and the exercise books is less than or equal to RM100. ($\leqslant 100$)

The linear inequality $14x + 9y \leqslant 100$ can also be written as;

$$14x \leqslant 100 - 9y \text{ or } 9y \leqslant 100 - 14x$$

Example 2

Puan Hidayah wants to buy mathematics posters to hang in the Mathematics Room. The cost of a small poster is RM12.50 and for a large poster is RM18.50. Represent the purchase of both posters in an appropriate form of linear inequality if the allocation from the school is RM150.00.

Solution:

Let,

k = small poster and b = large poster.

Hence, $12.50k + 18.50b \leqslant 150$

$$\frac{25}{2}k + \frac{37}{2}b \leqslant 150$$

Multipy both sides of the inequality by 2 to cancel the fractional denominator:

$$2 \times \frac{25}{2}k + 2 \times \frac{37}{2}b \leqslant 2 \times 150$$

$$25k + 37b \leqslant 300$$

Self Practice 6.1a

1. Represent the given situations in the form of linear inequalities using suitable variables.
   (a) Madam Linda Lim wants to prepare fish and prawn dishes in conjunction with the Chinese New Year celebration. The cost of a kilogram of mackerel is RM25 and a kilogram of prawns is RM45. Madam Linda Lim wants to spend RM250 to buy fish and prawns.
   (b) Encik Halim wants to rear chickens and ducks on a small scale to generate some extra income for managing his growing family expenses. A chick costs RM2 and a duckling costs RM1.50. Encik Halim has a capital of RM500 to purchase the chicks and ducklings.
   (c) Madam Letchumy wants to contribute RM50 for two types of *kuih* in conjunction with Canteen Day at her son's school. The cost of a curry puff is 30 sen and a *kuih kasturi* is 40 sen.
   (d) Puan Yati sells *nasi lemak*. The selling prices of a packet of *nasi lemak* with egg is RM1.50 and a packet of *nasi lemak* with chicken is RM3.50. The total daily sales of *nasi lemak* should exceed RM120 in order to make a minimum profit.

How do you verify the conjecture about points in the region that satisfy a linear inequality?

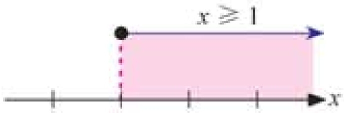


Diagram 1

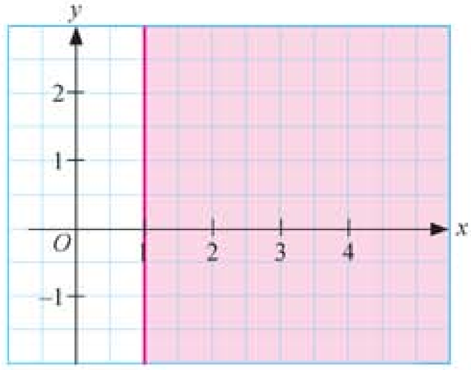


Diagram 2

Learning Standard

Make and verify the conjecture about the points in the region and the solution of certain linear inequalities.

In Form 1, you learned how to represent an inequality in one variable using a number line as in Diagram 1. Do you know that an inequality can also be represented on a Cartesian plane by shading the region that satisfies the inequality as in Diagram 2? All the x-coordinates in the shaded region satisfy the inequality $x \geqslant 1$.

TIPS

Inequality sign	Type of line
$>$, $<$	Dashed line
$\geqslant$, $\leqslant$	Solid line

Diagram 3 shows the types of regions generated on a Cartesian plane when a straight line is drawn.

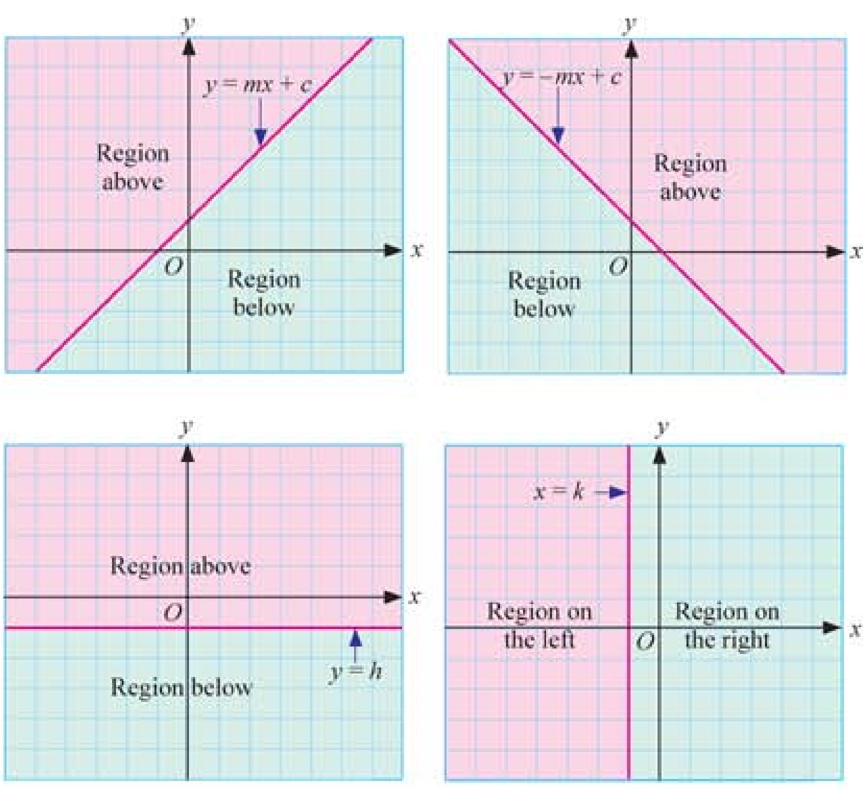


Diagram 3

MY MEMORY

General form of a straight line:
$y = mx + c$
m = gradient
c = y-intercept

Smart Mind

The gradient of the straight line $y = h$ is zero. Why?

INTERACTIVE ZONE

Why is the gradient of the straight line $x = k$ undefined? Discuss.

Mind Stimulation 1

Aim: To verify the conjecture about the points in the region that satisfy a linear inequality.

Materials: Activity sheets, graph papers

Scan the QR Code to carry out this activity. *http://bt.sasbadi.com/m4159*

Steps:

1. Divide the class into groups.
2. Each group is given a piece of graph paper and an activity sheet.
3. Pupils are asked to draw a straight line that represents the given linear equation for $-5 \leqslant x \leqslant 5$ on the graph paper and to plot the points obtained in the table on the activity sheet. (Example 1)

Point	y-coordinate	Value of $x + 2$	Position of point (From graph)			Point that satisfies		
			On the straight line	Region above	Region below	$y = x + 2$	$y > x + 2$	$y < x + 2$
(–5, 4)	4	$-5 + 2 = -3$		✓			$(4 > -3)$	
(1, 3)	3	$1 + 2 = 3$	✓			$(3 = 3)$		
(0, –2)	–2	$0 + 2 = 2$			✓			$(-2 < 2)$
(4, 7)								
(–3, 0)								
(3, 5)								

Example 1

4. Three Stray, One Stay activity can be carried out so that all pupils have the opportunity to explain the results.

Discussion:
What is the relationship between the position of a point on the straight line, in the region above or in the region below with the given linear equation or linear inequality?

From the activity in Mind Stimulation 1, it is found that:

- All the points that lie on the straight line satisfy the equation $y = mx + c$.
- All the points that lie in the region above satisfy the inequality $y > mx + c$.
- All the points that lie in the region below satisfy the inequality $y < mx + c$.

Example 3

Draw the straight line $y = -2x + 6$ for $-1 \leq x \leq 5$. Plot the points (1, −2), (4, −2), (0, 1), (1, 4), (4, 3) and (2, 6). Determine whether the points plotted satisfy $y = -2x + 6$, $y > -2x + 6$ or $y < -2x + 6$.

CHAPTER 6

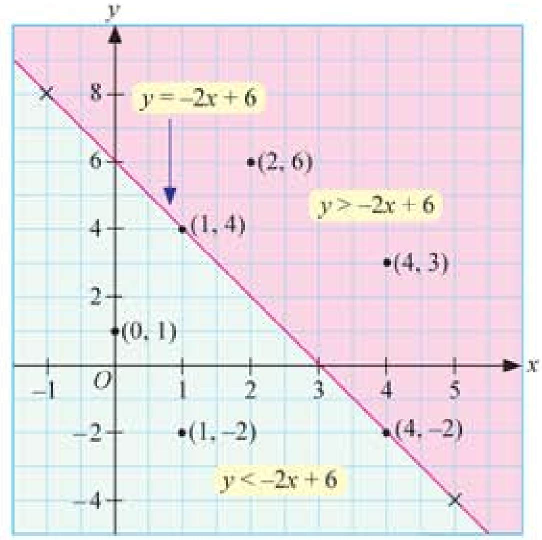
Solution:

$y = -2x + 6$

x	-1	5
y	8	-4

When $x = -1$,
$y = -2(-1) + 6$
$y = 8$

When $x = 5$,
$y = -2(5) + 6$
$y = -4$



- Points (1, 4) and (4, −2) satisfy the equation $y = -2x + 6$.
- Points (2, 6) and (4, 3) satisfy the inequality $y > -2x + 6$.
- Points (0, 1) and (1, −2) satisfy the inequality $y < -2x + 6$.

Example 4

Given the linear equation $y = -3x + 6$, without drawing the graph of the straight line, determine whether the given points satisfy $y = -3x + 6$, $y > -3x + 6$ or $y < -3x + 6$.

(a) (2, 5) (b) (1, 2) (c) (–1, 9) (d) (0, 8)

Solution:

(a) (2, 5)

y	$-3x + 6$
5	$-3(2) + 6$ $= 0$
$5 > 0$	
thus, point (2, 5) satisfies $y > -3x + 6$	

(b) (1, 2)

y	$-3x + 6$
2	$-3(1) + 6$ $= 3$
$2 < 3$	
thus, point (1, 2) satisfies $y < -3x + 6$	

(c) (–1, 9)

y	$-3x + 6$
9	$-3(-1) + 6$ $= 9$
$9 = 9$	
thus, point (−1, 9) satisfies $y = -3x + 6$	

(d) (0, 8)

y	$-3x + 6$
8	$-3(0) + 6$ $= 6$
$8 > 6$	
thus, point (0, 8) satisfies $y > -3x + 6$	

Self Practice 6.1b

1. Draw the straight line $y = \frac{2}{3}x - 2$ for $0 \leqslant x \leqslant 3$. Plot the points (3, 1), (2, –2), (1.5, –1), (3, –2) and (1, –1). Determine whether the points plotted satisfy $y = \frac{2}{3}x - 2$, $y > \frac{2}{3}x - 2$ or $y < \frac{2}{3}x - 2$.

2. Draw the straight line $y = -\frac{1}{2}x + 2$ for $-4 \leqslant x \leqslant 6$. Plot the points (–3, 5), (–3, 1), (1, –2), (2, 1) and (4, 5). Determine whether the points plotted satisfy $y = -\frac{1}{2}x + 2$, $y > -\frac{1}{2}x + 2$ or $y < -\frac{1}{2}x + 2$.

3. Given the linear equation $y = 4x - 5$, without drawing the graph of the straight line, determine whether the given points satisfy $y = 4x - 5$, $y > 4x - 5$ or $y < 4x - 5$.
   (a) (2, 4) (b) (3, 7) (c) (0, –6) (d) (–2, 0) (e) (4, 5)

4. Given the linear equation $y = -3x + 4$, without drawing the graph of the straight line, determine whether the given points satisfy $y = -3x + 4$, $y > -3x + 4$ or $y < -3x + 4$.
   (a) (–2, 3) (b) (1, 1) (c) (–1, 8) (d) (0, 1) (e) (–0.5, 7)

How do you determine and shade the region that satisfies a linear inequality?

Learning Standard

Determine and shade the region that satisfies a linear inequality.

You have learned that if a straight line representing a linear equation $y = mx + c$ is drawn on a Cartesian plane, all the points on the Cartesian plane can be categorised into three groups, which are;

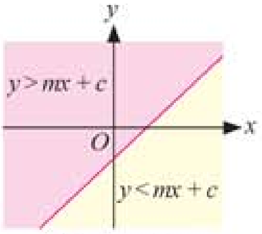


Diagram 1

- Points that lie on the straight line satisfy the equation $y = mx + c$.
- Points located in the region above the straight line satisfy the inequality $y > mx + c$.
- Points located in the region below the straight line satisfy the inequality $y < mx + c$.

For the straight lines $y = h$ and $x = k$ that are drawn on a Cartesian plane where h and k are constants, all the points on the Cartesian plane can also be categorised as follows:

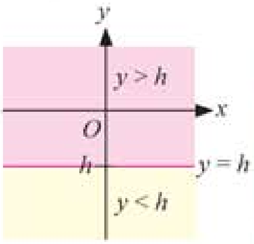
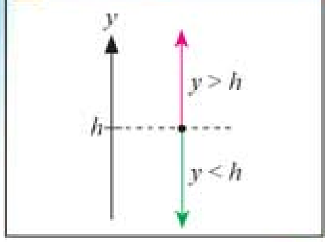


Diagram 2

- Points that lie on the straight line satisfy the equation $y = h$.
- Points located in the region above the straight line satisfy the inequality $y > h$.
- Points located in the region below the straight line satisfy the inequality $y < h$.

MY MEMORY



CHAPTER 6

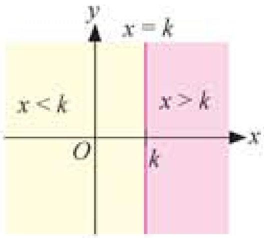
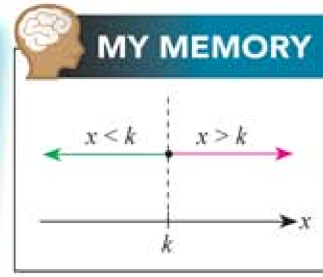


Diagram 3

- Points that lie on the straight line satisfy the equation $x = k$.
- Points located in the right region of the straight line satisfy the inequality $x > k$.
- Points located in the left region of the straight line satisfy the inequality $x < k$.



What is the relationship between a point on a Cartesian plane with the inequality $y > mx + c$, $y < mx + c$, $y \geqslant mx + c$ or $y \leqslant mx + c$?

$>, <$

- Points that lie in the region above or below a straight line $y = mx + c$.
- The straight line is drawn using a dashed line.

$\geqslant, \leqslant$

- Points that lie on the straight line $y = mx + c$ including the region above or below.
- The straight line is drawn using a solid line.

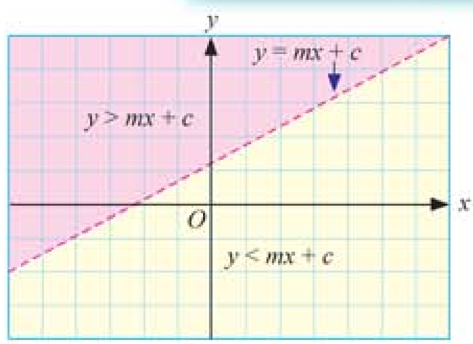


Diagram 4

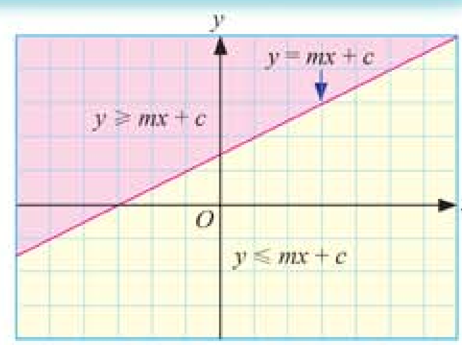


Diagram 5

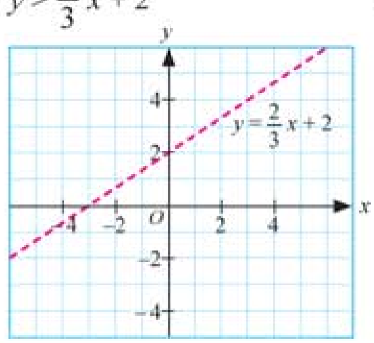
The regions do not include points that lie on the straight line $y = mx + c$.

The regions include points that lie on the straight line $y = mx + c$.

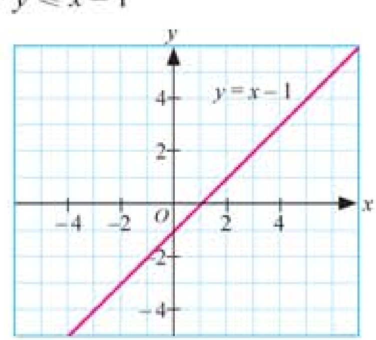
Example 5

Shade the region that represents each of the following inequalities.

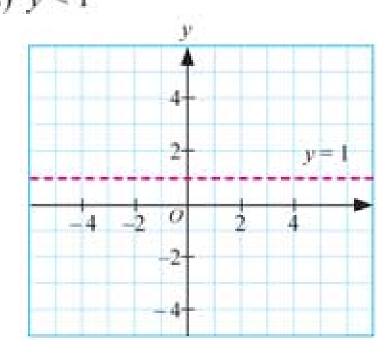
(a) $y > \frac{2}{3}x + 2$



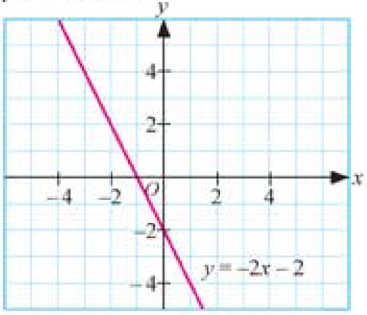
(b) $y \leqslant x - 1$



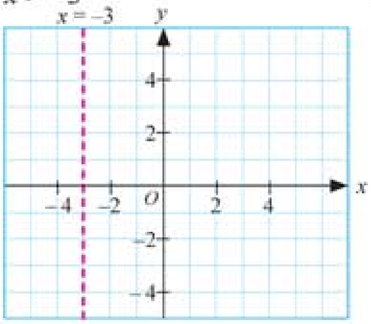
(c) $y < 1$



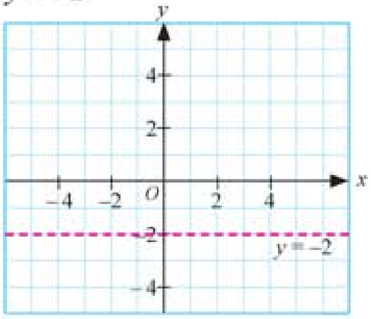
(d) $y \geqslant -2x - 2$



(e) $x > -3$

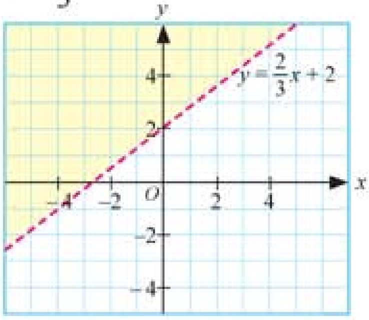


(f) $y > -2$

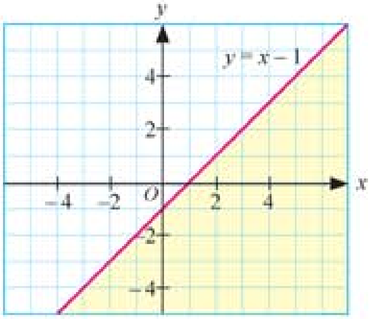


Solution:

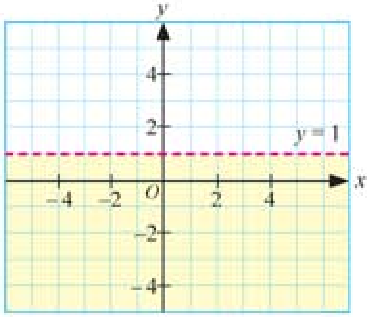
(a) $y > \frac{2}{3}x + 2$



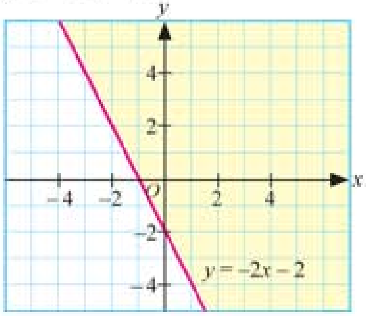
(b) $y \leqslant x - 1$



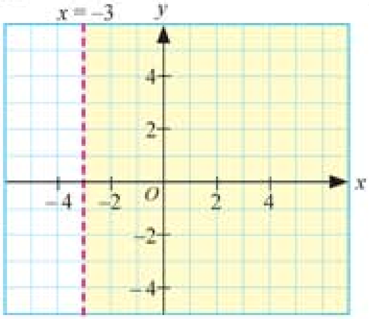
(c) $y < 1$



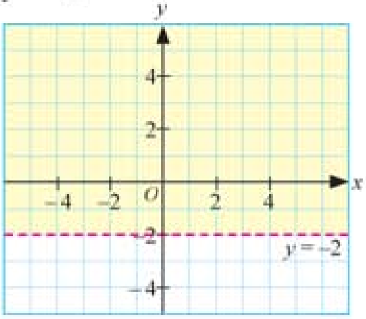
(d) $y \geqslant -2x - 2$



(e) $x > -3$



(f) $y > -2$



Example 6

Draw the graphs and shade the region that represents each of the following inequalities.

(a) $y \leqslant 2x + 3$ (b) $y > x + 5$ (c) $y \leqslant 2x$
(d) $x - y > 4$ (e) $y \geqslant 0$ (f) $x < 4$

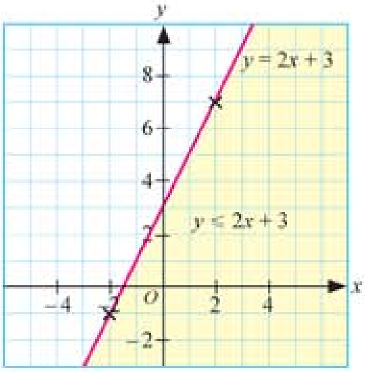
TIPS

Convert the given linear inequality to the form of a linear equation to draw the straight line.

Solution:

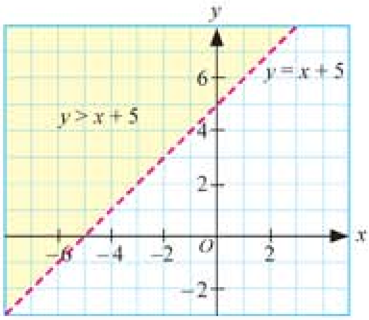
(a) $y = 2x + 3$

x	–2	2
y	–1	7



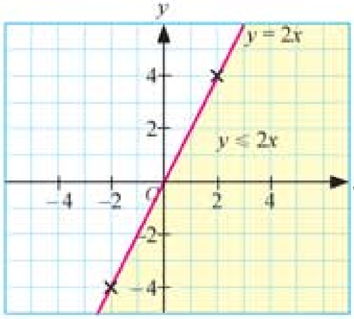
(b) $y = x + 5$

x	0	–5
y	5	0



(c) $y = 2x$

x	-2	2
y	-4	4



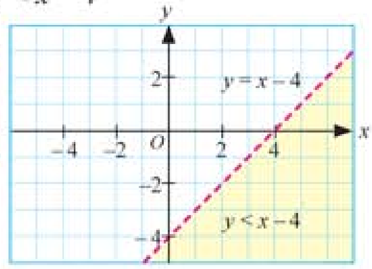
(d) $x - y = 4$

Change the coefficient of y to a positive value so that it is easier to choose the region.

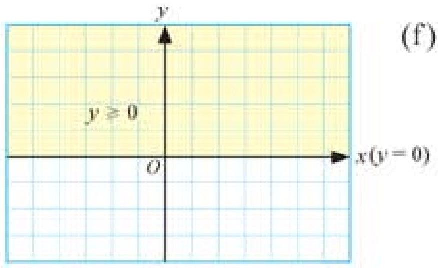
$$x - y > 4$$
$$(\times -1): \; -x + y < -4$$
$$y < x - 4$$

$y = x - 4$

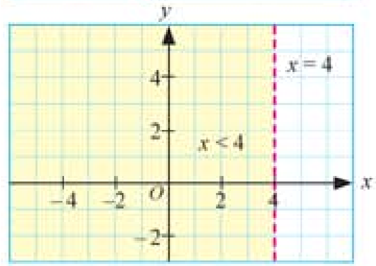
x	0	4
y	-4	0



(e) $y = 0$ (x-axis)



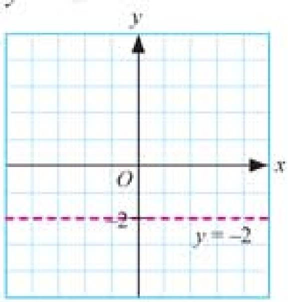
(f) $x = 4$



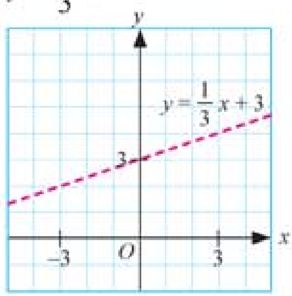
Self Practice 6.1c

1. Shade the region that represents each of the following inequalities.

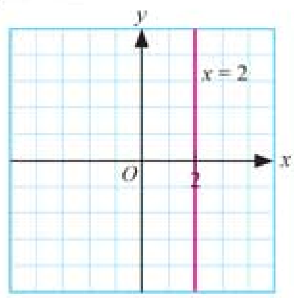
(a) $y < -2$



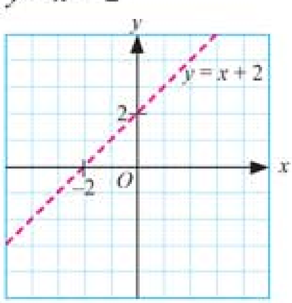
(b) $y < \frac{1}{3}x + 3$



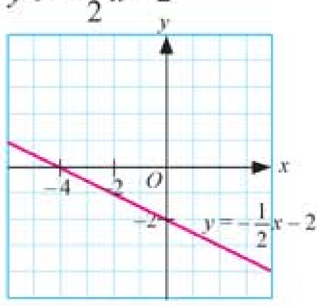
(c) $x \leqslant 2$



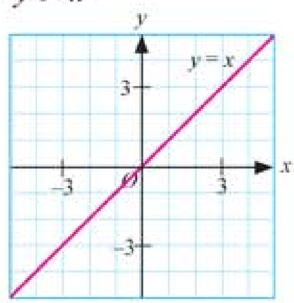
(d) $y > x + 2$



(e) $y \geqslant -\frac{1}{2}x - 2$



(f) $y \geqslant x$



2. Draw the graphs and shade the region that represents each of the following inequalities.

(a) $x \leqslant 0$ (b) $y > \frac{1}{2}x$ (c) $x + y \geqslant -3$ (d) $2y < x + 4$

(e) $y \leqslant -x + 2$ (f) $2y + x \geqslant 2$ (g) $x - y \geqslant 2$ (h) $x - y < -3$

6.2 Systems of Linear Inequalities in Two Variables

What is the meaning of system of linear inequalities?

Learning Standard

Represent situations in the form of system of linear inequalities.

A travel agency needs to transport 150 tourists and luggage with a total mass of 2 000 kg. The agency provides buses and vans to transport the tourists. A bus can carry 32 passengers and luggage with 350 kg and a van can carry eight passengers and luggage with 100 kg. The maximum number of buses provided is four. What is the maximum number of buses and vans required at a minimal cost?

The above problem can be solved by constructing several related linear inequalities and determining the region that satisfies all the linear inequalities. A combination of two or more linear inequalities is known as a **system of linear inequalities**.

How do you determine the appropriate inequality for a certain situation?

Example of situation	Linear inequality
y is greater than x	$y > x$
y is less than x	$y < x$
y is not less than x	$y \geqslant x$
y is not more than x	$y \leqslant x$
y is at least k times x	$y \geqslant kx$
y is at most k times x	$y \leqslant kx$
Maximum of y is k	$y \leqslant k$
Minimum of y is k	$y \geqslant k$
Sum of x and y is greater than k	$x + y > k$
Difference between y and x is less than k	$y - x < k$
y is more than x by at least k	$y - x \geqslant k$

Example 7

SMK Seri Permata is hosting a Leadership Camp during the mid-year holidays. y Form 4 pupils and x Form 5 pupils participate in the camp. The pupils are selected according to the following conditions:

(a) The total number of participants for the camp is at most 40.

(b) The number of Form 5 pupils is not less than the number of Form 4 pupils.

(c) At least 15 Form 4 pupils participate in the camp.

Write all the linear inequalities that satisfy the above conditions.

> **Indicator**
>
> $x \geqslant 0$ and $y \geqslant 0$ because x and y represent quantities.

Solution:

(a) $x + y \leqslant 40$ (b) $x \geqslant y$ (c) $y \geqslant 15$

Example 8

A computer shop sells refill ink of brands X and Y for printers. The seller needs to place an order of x number of refill ink of brand X and y number of refill ink of brand Y which cost RM14 and RM18 respectively from a supplier. The conditions for the order are as follows:

(a) The total number of ink ordered does not exceed 25 units.

(b) The number of ink of brand Y is at least twice that of brand X.

(c) The number of ink of brand Y is not more than 8 units.

(d) The total purchase does not exceed RM400.

Write all the linear inequalities that satisfy the above conditions.

Solution:

(a) $x + y \leqslant 25$ (b) $y \geqslant 2x$ (c) $y \leqslant 8$ (d) $14x + 18y \leqslant 400$

Self Practice 6.2a

1. Mr Wong bought two types of shirts from a supplier to sell in his shop. Each shirt of brand X costs RM8 and each shirt of brand Y costs RM12. The conditions for the purchase of the shirts are as follows:

(a) The total number of shirts bought is not more than 50.

(b) The number of shirts of brand X is at least twice the number of shirts of brand Y.

(c) The total purchase does not exceed RM850.

Write all the linear inequalities that satisfy the above conditions.

2. A factory produces two types of sports shoes. The sports shoes of type X are for children and type Y are for adults. The conditions for the production of both types of sports shoes in a week are as follows:

(a) The maximum production of shoes is 500 pairs.

(b) The number of sports shoes for children is at most three times the number of sports shoes for adults.

(c) The minimum production of sports shoes for adults is 200 pairs.

Write all the linear inequalities that satisfy the above conditions.

3. Encik Musa has a piece of land for growing vegetables. He wants to grow two types of chilli plants, green chilli and *cili padi*. The conditions for growing the two types of chilli plants are as follows:

(a) The total number of chilli plants that can be grown is at most 250.

(b) The number of green chilli plants is at least three times the number of *cili padi* plants.

(c) The minimum number of green chilli plants is 100.

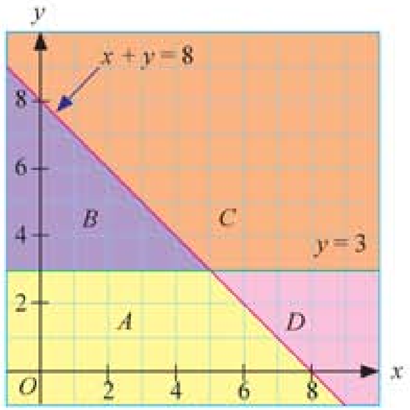
Write all the linear inequalities that satisfy the above conditions.

How do you determine the conjecture about the points in the region that satisfy a system of linear inequalities?

Learning Standard

Make and verify the conjecture about the points in the region and solution of linear inequality system.

The points that satisfy all the linear inequalities in a system of linear inequalities can be determined by identifying the appropriate region.



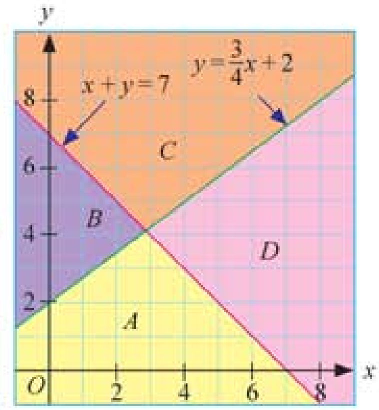
The above diagram shows the regions in a system of linear inequalities. Only one region from regions A, B, C and D satisfies both the linear inequalities $x + y \leqslant 8$ and $y \geqslant 3$. You can determine the region by substituting at least one point in the region into the system of linear inequalities.

Region	Point	Inequality $x + y \leqslant 8$	True/False	Inequality $y \geqslant 3$	True/False
A	(2, 1)	$2 + 1 \leqslant 8$	True	$1 \geqslant 3$	False
	(4, 2)	$4 + 2 \leqslant 8$	True	$2 \geqslant 3$	False
B	(2, 5)	$2 + 5 \leqslant 8$	True	$5 \geqslant 3$	True
	(3, 4)	$3 + 4 \leqslant 8$	True	$4 \geqslant 3$	True
C	(2, 7)	$2 + 7 \leqslant 8$	False	$7 \geqslant 3$	True
	(7, 4)	$7 + 4 \leqslant 8$	False	$4 \geqslant 3$	True
D	(7, 2)	$7 + 2 \leqslant 8$	False	$2 \geqslant 3$	False
	(8, 1)	$8 + 1 \leqslant 8$	False	$1 \geqslant 3$	False

From the table, it is found that only points from region B satisfy both the inequalities that are tested. Thus, region B satisfies the inequalities $x + y \leqslant 8$ and $y \geqslant 3$.

Example 9

Based on the diagram, determine the region that satisfies the inequalities $y \leqslant \frac{3}{4}x + 2$ and $x + y \geqslant 7$.



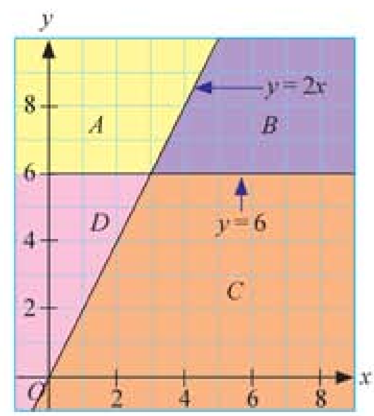
Solution:

Region	Point	$y \leqslant \frac{3}{4}x + 2$	True/ False	$x + y \geqslant 7$	True/ False
A	(4, 2)	$2 \leqslant \frac{3}{4}(4) + 2$	True	$4 + 2 \geqslant 7$	False
B	(1, 5)	$5 \leqslant \frac{3}{4}(1) + 2$	False	$1 + 5 \geqslant 7$	False
C	(4, 7)	$7 \leqslant \frac{3}{4}(4) + 2$	False	$4 + 7 \geqslant 7$	True
D	(8, 6)	$6 \leqslant \frac{3}{4}(8) + 2$	True	$8 + 6 \geqslant 7$	True

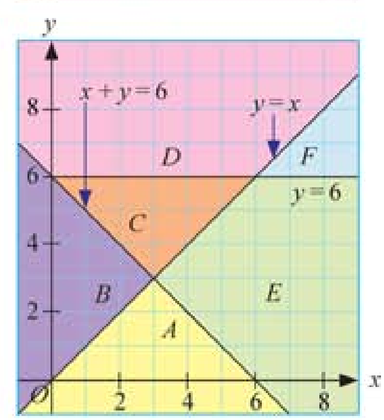
Region D satisfies both the inequalities $y \leqslant \frac{3}{4}x + 2$ and $x + y \geqslant 7$.

Self Practice 6.2b

1. Based on the diagram, determine the region that satisfies each of the following systems of linear inequalities.
   (a) $y \geqslant 2x$ and $y \leqslant 6$.
   (b) $y \geqslant 2x$ and $y \geqslant 6$.
   (c) $y \leqslant 2x$ and $y \leqslant 6$.
   (d) $y \leqslant 2x$ and $y \geqslant 6$.



2. Based on the diagram, determine the region that satisfies each of the following systems of linear inequalities.
   (a) $y \leqslant 6, y \leqslant x$ and $x + y \geqslant 6$.
   (b) $y \leqslant 6, y \geqslant x$ and $x + y \geqslant 6$.
   (c) $y \leqslant 6, y \leqslant x$ and $x + y \leqslant 6$.
   (d) $y \geqslant 6, y \geqslant x$ and $x + y \geqslant 6$.



How do you determine and shade the region that satisfies a system of linear inequalities?

Learning Standard

Determine and shade the region that satisfies a linear inequality system.

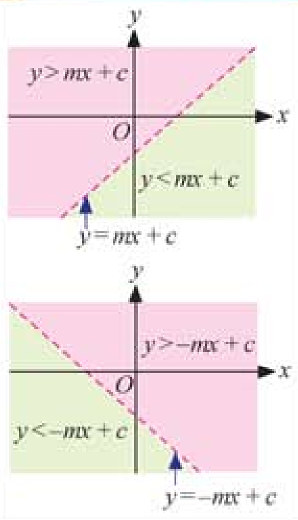
The region that satisfies a system of linear inequalities can be determined by the following steps:

(a) Mark the region involved for each linear inequality with different and easily spotted markings.

(b) Identify the **common region** for all the markings involved.

(c) Shade the common region completely. Make sure that the shading is not outside the common region.

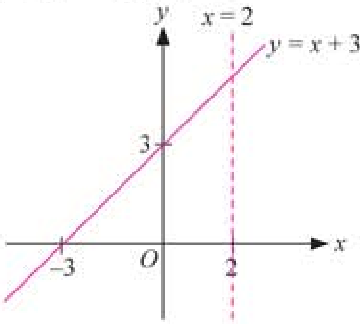
MY MEMORY



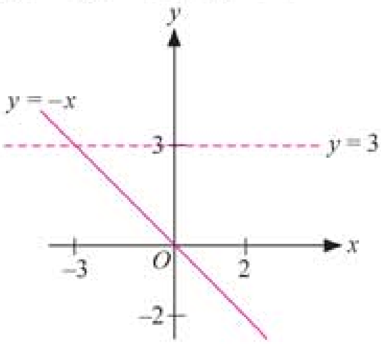
Example 10

Shade the region that satisfies each of the following systems of linear inequalities.

(a) $y \leqslant x + 3$ and $x < 2$



(b) $y \geqslant -x$, $y < 3$ and $y \geqslant 0$

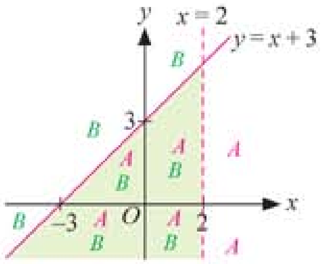


Solution:

(a) (i) Mark the region that satisfies $y \leqslant x + 3$ with the letter A.

(ii) Mark the region that satisfies $x < 2$ with the letter B.

(iii) Shade the common region marked by both the letters A and B.



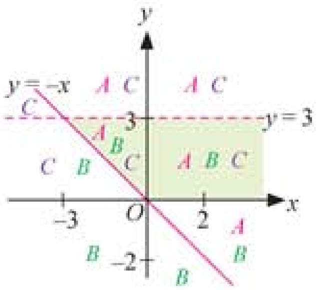
(b) (i) A represents the region $y \geqslant -x$.

B represents the region $y < 3$.

C represents the region $y \geqslant 0$.

$y \geqslant 0$ is the region above straight line $y = 0$, that is the x-axis.

(ii) Shade the common region marked with the three letters A, B and C.



Example 11

Draw and shade the region that satisfies the system of linear inequalities $2y \geqslant x$, $x + y < 4$ and $x \geqslant 0$.

Solution:

(a) Convert the linear inequalities to linear equations and draw the straight lines that represent the equations.

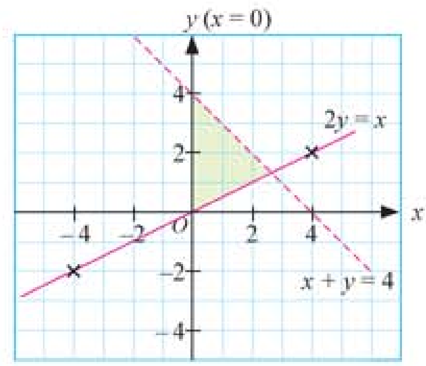
$2y = x$

x	-4	4
y	-2	2

$x + y = 4$

x	0	4
y	4	0

$x = 0$ (y-axis)



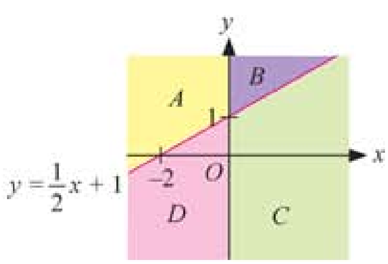
(b) Draw the straight lines of the equations involved.

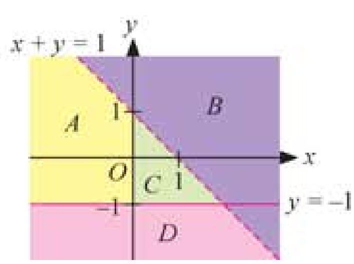
(c) Identify the common region and then shade the region.

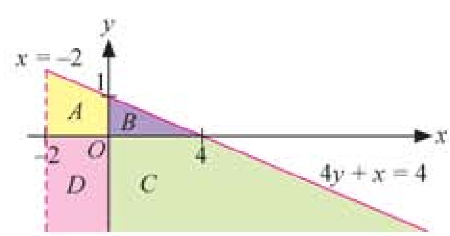
Self Practice 6.2c

1. Identify the region that satisfies each of the following systems of linear inequalities.

(a) $y \leqslant \frac{1}{2}x + 1$ and $x \geqslant 0$ (b) $x + y < 1$, $y \geqslant -1$ and $x \geqslant 0$ (c) $4y + x \leqslant 4$, $x > -2$, $x \leqslant 0$ and $y \geqslant 0$

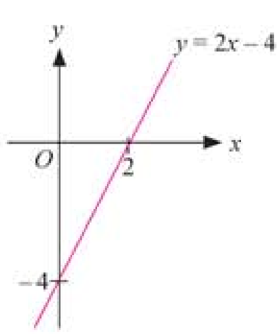


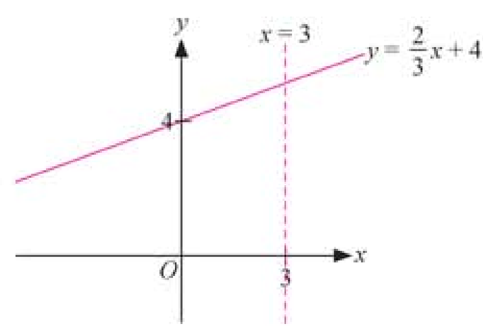




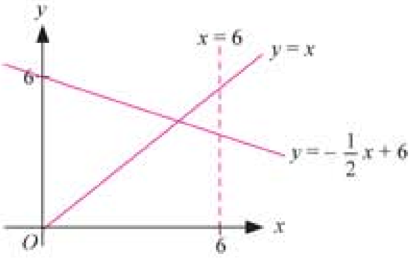
2. Shade the region that satisfies each of the following systems of linear inequalities.

(a) $y \geqslant 2x - 4$, $x \geqslant 0$ and $y \leqslant 0$ (b) $y \leqslant \frac{2}{3}x + 4$, $x < 3$, $x \geqslant 0$ and $y \geqslant 0$

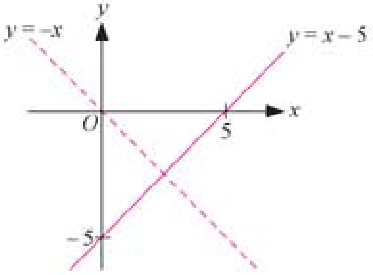




(c) $y \leqslant -\frac{1}{2}x + 6$, $y \leqslant x$, $x < 6$ and $y \geqslant 0$



(d) $y < -x$, $y \leqslant x - 5$ and $y > -5$



3. Sketch and shade the region that satisfies each of the following systems of linear inequalities.

(a) $y < -2x + 6$, $x \geqslant 0$ and $y \geqslant 0$

(b) $y \geqslant -\frac{1}{2}x + 2$, $y \leqslant x + 2$ and $x < 4$

(c) $y \leqslant -x + 8$, $y \geqslant -2x + 8$ and $x < 4$

(d) $y - x \leqslant 6$, $y \geqslant x$, $y \leqslant -x + 6$ and $y \geqslant 0$

How do you solve problems involving system of linear inequalities in two variables?

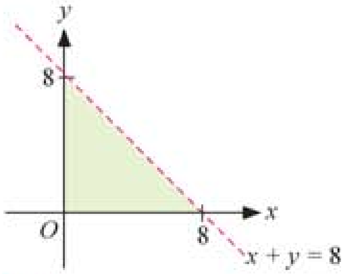
Learning Standard

Solve problems involving systems of linear inequalities in two variables.

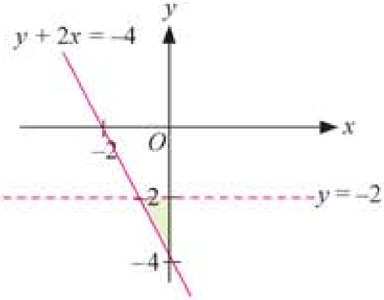
Example 12

State three linear inequalities that define the shaded region in each of the following diagrams.

(a)



(b)



TIPS

Linear equations for
- x-axis ⇨ $y = 0$
- y-axis ⇨ $x = 0$

Solution:

(a) The three straight lines involved are $x + y = 8$, the x-axis and the y-axis.

(i) The shaded region is below the straight line $x + y = 8$ and is drawn with a dashed line, thus $x + y < 8$.

(ii) The shaded region is above the x-axis, thus $y \geqslant 0$.

(iii) The shaded region is to the right of the y-axis, thus $x \geqslant 0$.

The three linear inequalities that satisfy the shaded region are $x + y < 8$, $y \geqslant 0$ and $x \geqslant 0$.

(b) The three straight lines involved are $y + 2x = -4$, $y = -2$ and the y-axis.

(i) The shaded region is above the straight line $y + 2x = -4$ and is drawn with a solid line, thus $y + 2x \geqslant -4$.

(ii) The shaded region is below the straight line $y = -2$ and is drawn with a dashed line, thus $y < -2$.

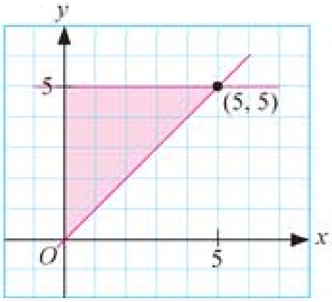
(iii) The shaded region is to the left of the y-axis, thus $x \leqslant 0$.

The three linear inequalities that satisfy the shaded region are $y + 2x \geqslant -4$, $y < -2$ and $x \leqslant 0$.

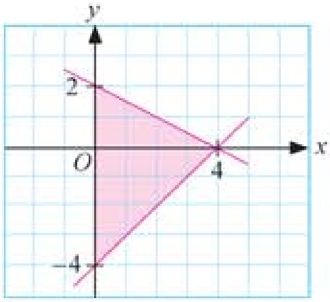
Example 13

Write three linear inequalities that satisfy the shaded region in each diagram below.

(a)



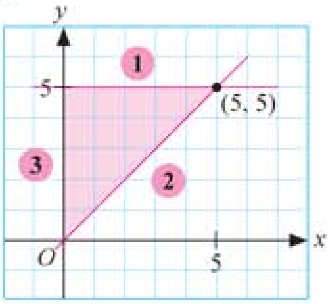
(b)



> **MY MEMORY**
>
> Gradient
>
> $m = \frac{y_2 - y_1}{x_2 - x_1}$

Solution:

(a)



(i) Equation 1

The straight line is parallel to the x-axis.

Thus, $y = 5$

(ii) Equation 2

Gradient, $m = \frac{5 - 0}{5 - 0} = 1$

y-intercept, $c = 0$

Thus, $y = x$

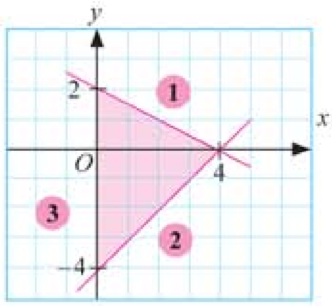
(iii) Equation 3

The straight line is the y-axis.

Thus, $x = 0$

The three linear inequalities that satisfy the shaded region are $y \leqslant 5$, $y \geqslant x$ and $x \geqslant 0$.

(b)



(i) Equation 1

Gradient, $m = \frac{2 - 0}{0 - 4} = -\frac{1}{2}$

y-intercept, $c = 2$

Thus, $y = -\frac{1}{2}x + 2$

(ii) Equation 2

Gradient, $m = \frac{-4 - 0}{0 - 4} = 1$

y-intercept, $c = -4$

Thus, $y = x - 4$

(iii) Equation 3

The straight line is the y-axis.

Thus, $x = 0$

The three linear inequalities that satisfy the shaded region are $y \leqslant -\frac{1}{2}x + 2$, $y \geqslant x - 4$ and $x \geqslant 0$.

Example 14

Madam Carol needs to select at most 20 pupils for a choir competition. The number of girls is at least twice the number of boys.

(a) Write two linear inequalities other than $x \geqslant 0$ and $y \geqslant 0$, which represent the conditions of the selection of the competitors.

(b) Draw and shade the region that satisfies the above system of linear inequalities.

(c) From the graph,

(i) determine the minimum and maximum number of girls when the number of boys is five.

(ii) determine whether the conditions of the selection are adhered to if eight boys want to participate in the choir competition. Justify your answer.

Solution:

(a) Let x = number of girls and y = number of boys

$x + y \leqslant 20$

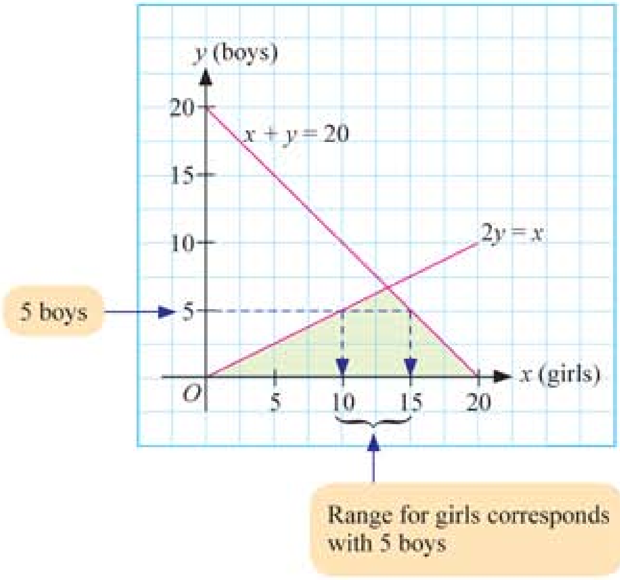
$x \geqslant 2y$ or $2y \leqslant x$ ← Make y the subject of the formula so that the region can be marked correctly.

(b) $x + y = 20$

x	0	20
y	20	0

$2y = x$

x	0	20
y	0	10



(c) (i) $y = 5$ boys
$x = 10$ girls (minimum)
$x = 15$ girls (maximum)

(ii) No, because the value of $y = 8$ is outside the shaded region.

Indicator

The graph of an inequality is drawn in the first quadrant only because the situation involves the variables x and y which represent quantities.

INFO ZONE

A system of linear inequalities is the basis used in the field of business to obtain the maximum profit using the minimum cost that involves limited resources such as manpower, raw materials and finance.

Smart Mind

Construct appropriate linear inequalities for the following conditions.

(a) y is at most three times x.

(b) The sum of x and y is less than 100.

(c) x exceeds y by at least 20.

(d) The difference between y and x is less than 50.

CHAPTER 6

Example 15

The maximum number of passengers on a train in a theme park is 30. As a security measure, the required number of adults is always greater than or equal to the number of children.

(a) Write two linear inequalities, other than $x \geqslant 0$ and $y \geqslant 0$, which represent the given situation.

(b) Draw and shade the region that satisfies the above system of linear inequalities.

(c) From the graph, determine the maximum number of children that is allowed to board the train.

(d) If there are 18 children, can all the children board the train at the same time? Justify your answer.

Solution:

Understanding the problem

(a) Determine two linear inequalities according to the conditions in the situation.

(b) Draw and shade the region for the system of linear inequalities.

(c) Determine the related value from the graph.

Planning a strategy

(a) Let x = number of children and y = number of adults

(b) A graph of linear inequalities is drawn in the first quadrant.

(c) The value is determined from the common region.

Implementing the strategy

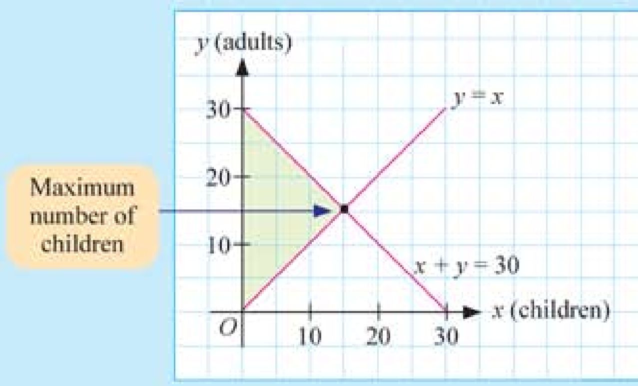
(a) $x + y \leqslant 30$
$y \geqslant x$

(b) $x + y = 30$

x	0	30
y	30	0

$y = x$

x	0	30
y	0	30



(c) The maximum number of children is 15.

(d) No, because the maximum number of children is only 15.
or
No, because the value of $x = 18$ is outside the shaded region.

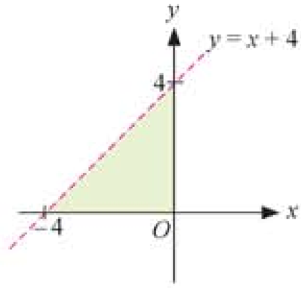
Conclusion

All the points in the common region satisfy the system of linear inequalities constructed based on the given conditions in the situation.

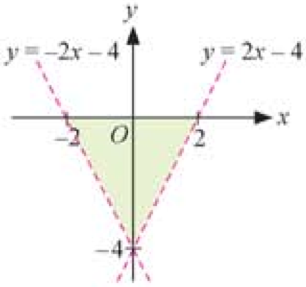
Self Practice 6.2d

1. State three linear inequalities that define the shaded region in each of the following diagrams.

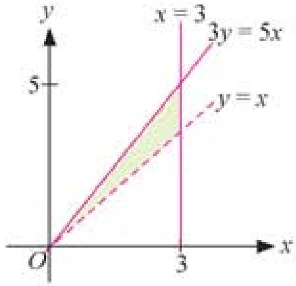
(a)



(b)

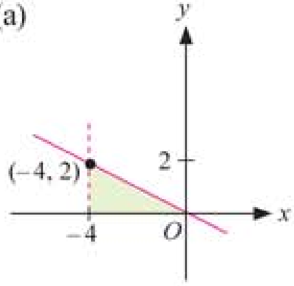


(c)

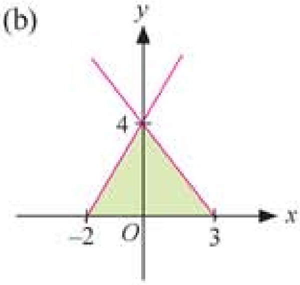


2. Write the inequalities that satisfy the shaded region in each of the following diagrams.

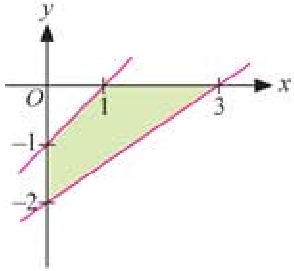
(a)



(b)



(c)



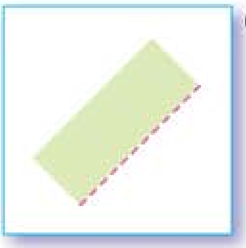
3. Mr Timothy wants to buy x doughnuts and y curry puffs to be donated to the school in conjunction with Canteen Day. The total number of both types of pastries is at most 150 and the number of doughnuts is at least twice the number of curry puffs.
   (a) Write two linear inequalities, other than $x \geqslant 0$ and $y \geqslant 0$, which represent the conditions of the purchase of the pastries by Mr Timothy.
   (b) Draw and shade the region that satisfies the above system of linear inequalities.
   (c) From the graph, determine
      (i) the maximum number of curry puffs purchased.
      (ii) the minimum and maximum number of doughnuts that Mr Timothy can buy if he buys 25 curry puffs.

4. Mrs Kiran Kaur needs to buy curtain fabrics for her new house. She buys x metres of floral fabrics and y metres of abstract fabrics. The total length of both types of fabrics is not exceeding 120 metres. The length of the abstract fabrics is at least one third of the length of the floral fabrics.
   (a) Write two linear inequalities, other than $x \geqslant 0$ and $y \geqslant 0$, which represent the conditions of the purchase of curtain fabrics by Mrs Kiran Kaur.
   (b) Draw and shade the region that satisfies the above system of linear inequalities.
   (c) From the graph, determine the maximum length, in metres, of the floral fabrics purchased.
   (d) Mrs Kiran Kaur buys 60 metres of abstract fabrics and 80 metres of floral fabrics. Does the above purchase satisfy the system of linear inequalities that you have constructed? Justify your answer.

Comprehensive Practice

1. Write the linear inequalities that match the given sketch of the regions.

(a)



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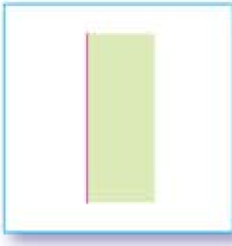

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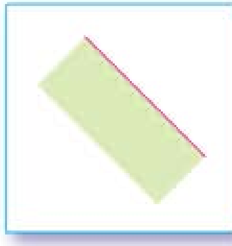

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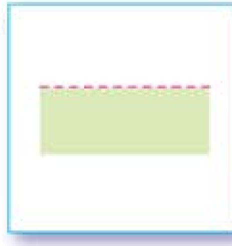

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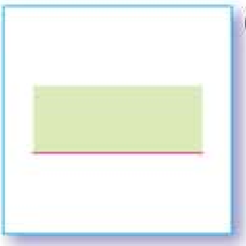

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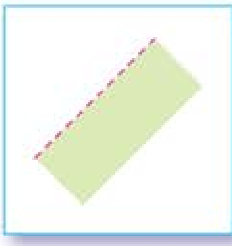
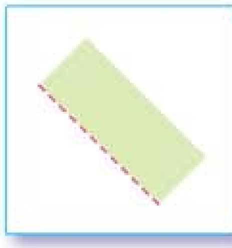
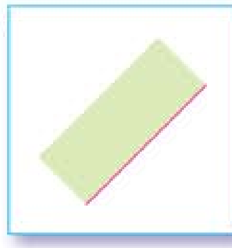


(g)



(h)



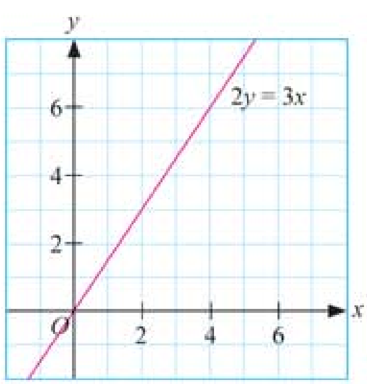
$y > -x - 3$	$y < 4$	$y \leqslant 4 - x$	$x \geqslant 0$	$y \geqslant 0$
$\frac{1}{2}y - x \geqslant 4$	$y < 2x - 5$	$3y + x > 4$	$x \leqslant 2 - y$	$y < -1$
$2y > x + 5$	$-y > 8 - 2x$	$y + x \leqslant 2$	$2y < x$	$2y \geqslant x$
$y \leqslant -\frac{1}{2}x$	$x \geqslant -5$	$-y \leqslant 4 - x$	$y \geqslant 10$	$y - x > 8$

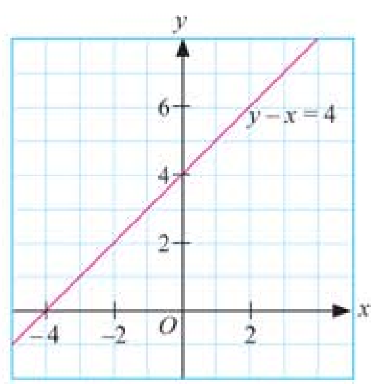
2. Shade the common region for each of the following systems of linear inequalities.

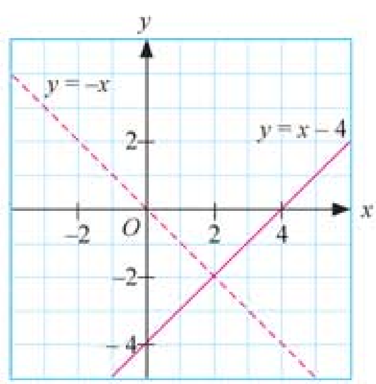
(a) $2y \leqslant 3x$, $x < 4$ and $y \geqslant 0$

(b) $y - x \geqslant 4$, $y < 4$ and $x > -4$

(c) $y > -x$, $y \geqslant x - 4$, $y \leqslant 2$ and $y \geqslant 0$

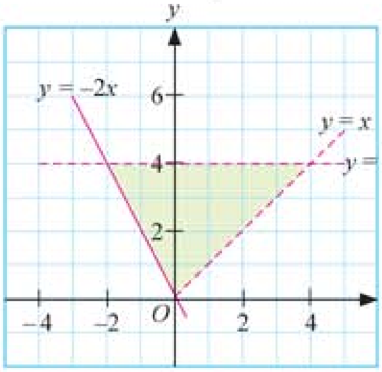




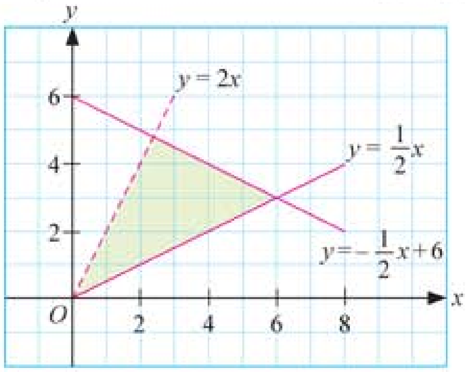


3. Write three linear inequalities that satisfy the shaded region in each of the following diagrams.

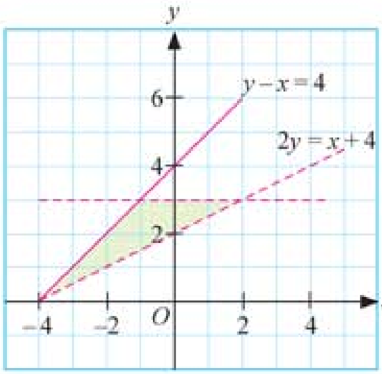
(a)



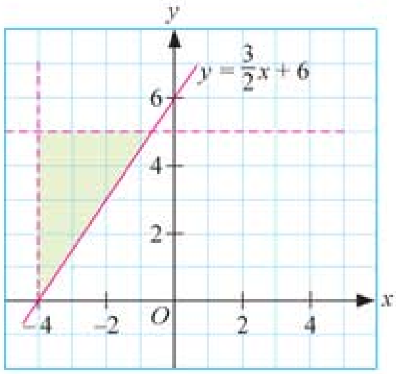
(b)



(c)



(d)



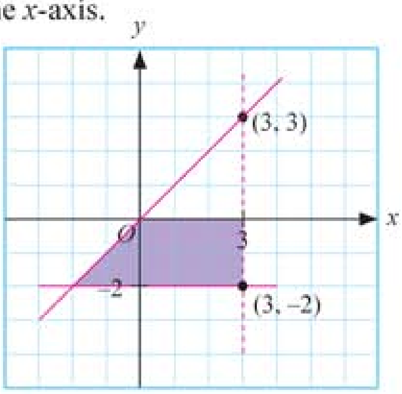
4. Draw and shade the common region for each of the following systems of linear inequalities.

(a) $y + x \geqslant 10,\ y \geqslant x$ and $y < 10$

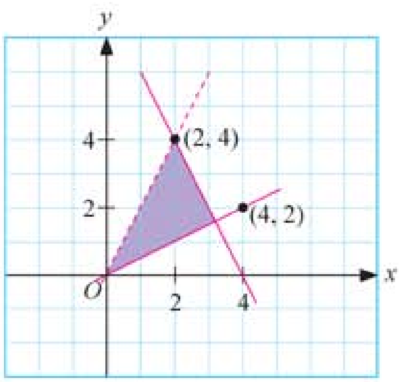
(b) $y \leqslant x + 6, y \geqslant \frac{2}{3}x + 4, y > -x$ and $x \leqslant 0$

5. Write the linear inequalities that satisfy the image of the shaded region under a reflection in the x-axis.

(a)

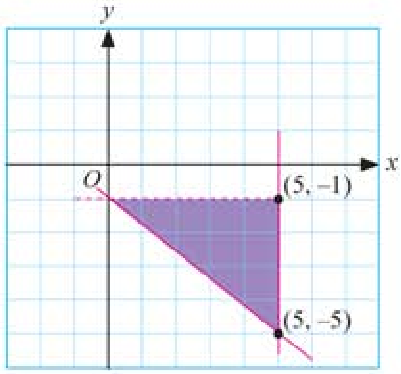


(b)

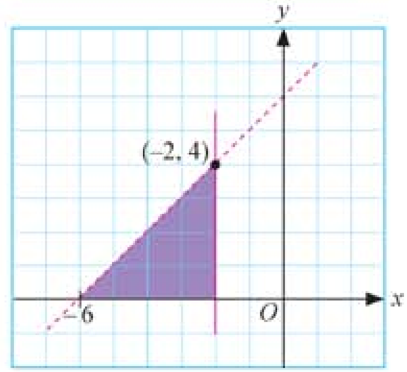


6. Write the linear inequalities that satisfy the image of the shaded region under a reflection in the y-axis.

(a)



(b)



7. Puan Jasmin is a seamstress. She sews two types of *baju kurung*; *baju kurung pesak* and *baju kurung moden*. Assuming Puan Jasmin sews x pairs of *baju kurung pesak* and y pairs of *baju kurung moden* in a certain month. The information below is related to both types of *baju kurung* sewn by Puan Jasmin.

- The total number of *baju kurung* sewn is at most 40 pairs.
- The maximum number of *baju kurung pesak* is 25 pairs.
- The minimum number of *baju kurung moden* is 10 pairs.

(a) Based on the above information, write three linear inequalities, other than $x \geqslant 0$ and $y \geqslant 0$, which represent the above situation.

(b) Draw and shade the common region that satisfies the linear inequalities constructed.

(c) From the graph, determine the minimum and maximum number of *baju kurung moden* that may be sewn if the number of *baju kurung pesak* is 10 pairs.

(d) The cost of sewing a pair of *baju kurung pesak* is RM50 and a pair of *baju kurung moden* is RM75. Based on the common region, calculate the maximum income that can be earned by Puan Jasmin if she successfully sewed 15 pairs of *baju kurung pesak*.

8. Encik Aiman sells x metres of batik cloth that he purchased from supplier X and y metres of batik cloth from supplier Y. The total length of the batik cloth purchased is at most 1 000 metres. The batik cloth supplied by supplier Y is at least half of that supplied by supplier X.

(a) Based on the above purchase information, write two related linear inequalities other than $x \geqslant 0$ and $y \geqslant 0$.

(b) Draw and shade the common region that satisfies the linear inequalities constructed.

(c) From the graph, determine the minimum and maximum lengths in metres of batik cloth that is supplied by supplier Y if Encik Aiman purchased 500 metres of batik cloth from supplier X.

(d) Supplier X faces a shortage of stock. Encik Aiman is forced to purchase at least $\frac{3}{4}$ of the batik cloth from supplier Y.

(i) Write a linear inequality representing the above situation.

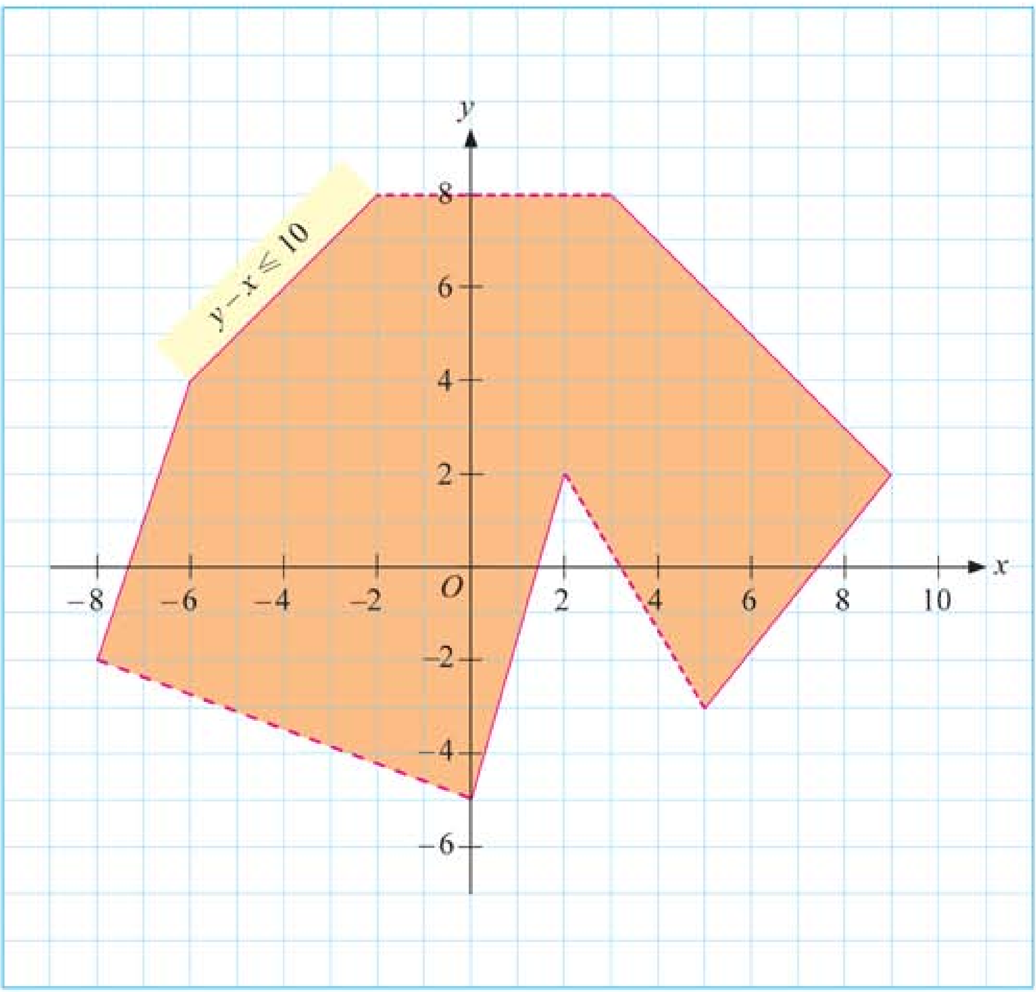
(ii) Draw a straight line representing the inequality in (d)(i).

PROJECT

1. Divide the class into groups.
2. Each group is required to draw an octagon on a Cartesian plane using a grid paper (Example 1) and write the linear inequalities that represent the shaded region on a separate sheet of paper.
3. All the materials supplied are put into envelopes and distributed to the other groups.
4. Each group is required to match the correct linear inequalities by the sides of the polygon (Example 1) in a certain period of time.
5. The leader or a representative from the original group (provider) is required to check the matching of the linear inequalities and give the score.

TIPS

Write the linear inequalities in the general form.



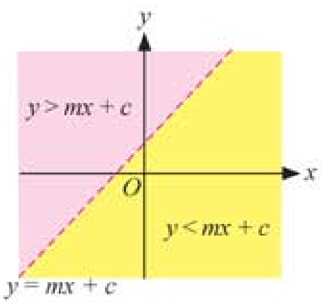
Example 1

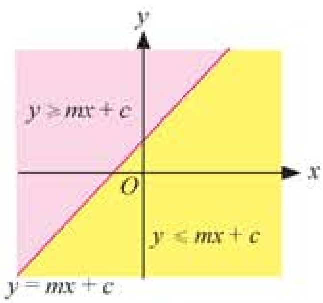
CONCEPT MAP

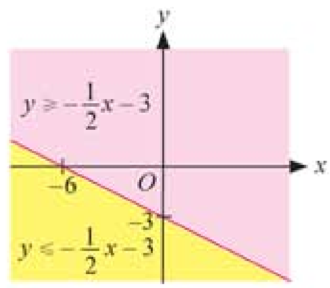
Linear Inequalities in Two Variables

Dashed line ⇨ points on the straight line $y = mx + c$ are not included in the region.

Solid line ⇨ points on the straight line $y = mx + c$ are included in the region.

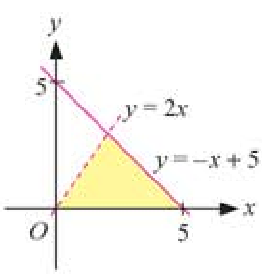




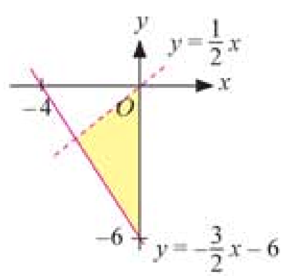


Systems of Linear Inequalities in Two Variables

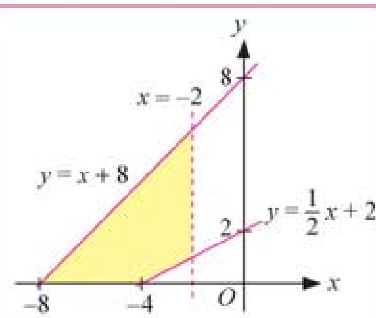
Common region ⇨ region that satisfies all the linear inequalities involved in a system of linear inequalities.



1. $y \geqslant 0$
2. $y \leqslant -x + 5$
3. $y < 2x$



1. $x \leqslant 0$
2. $y < \frac{1}{2}x$
3. $y \geqslant -\frac{3}{2}x - 6$



1. $y \geqslant 0$
2. $y \geqslant \frac{1}{2}x + 2$
3. $y \leqslant x + 8$
4. $x < -2$

Self Reflection

1. The [ ] lines are used to draw the linear inequalities in the form $y > mx + c$ and $y < mx + c$.

2. The [ ] lines are used to draw the [ ] inequalities in the form $y \geqslant mx + c$ and $y \leqslant mx + c$.

3. A region that satisfies a [ ] of linear inequalities is known as the [ ] region.

Mathematics Exploration

1. Download a dynamic geometry software.
2. Type a linear inequality. You can click on the keyboard at the bottom of the display.
3. Press Enter each time you want to add another linear inequality to form a common region.
4. You can also see the common region for a combination of linear inequalities and non-linear inequalities.