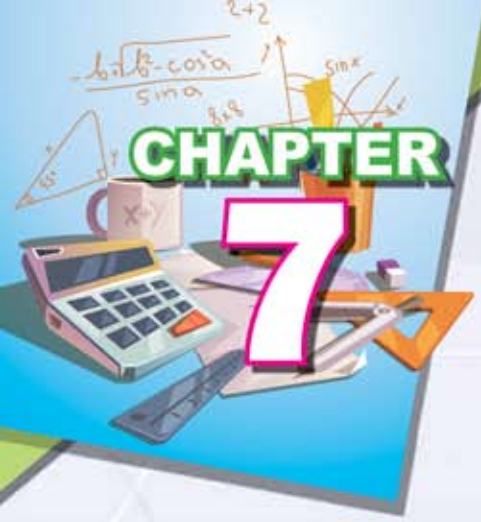


CHAPTER 7

Graphs of Motion

You will learn

- Distance-Time Graphs
- Speed-Time Graphs

Mohamad Ridzuan Puzi, our national Paralympic athlete, created history by setting a world record in the 100 m T36 men's event and the Asian Games record in the long jump event by winning three overall gold medals. The best record in the blue ribbon event, that is 11.87 s, is a new world record. Ridzuan also broke the long jump Asian Games record with his best personal record of 5.59 m and won a gold medal in the 400-metre event. Mohamad Ridzuan Puzi became the first national athlete to be crowned the 2018 Best Asian Para Athlete (Male) in a special ceremony held in the United Arab Emirates (UAE) on 5 February 2019.

What is the technique used by runners to win a certain race?

Why Study This Chapter?

Knowledge about motion is important in the automotive field, sports science, physics, engineering and astronomy.

WORD BANK

• distance-time graph	• *graf jarak-masa*
• speed-time graph	• *graf laju-masa*
• distance	• *jarak*
• speed	• *laju*
• uniform speed	• *laju seragam*
• deceleration	• *nyahpecutan*
• acceleration	• *pecutan*

Walking Through Time

Nicholas Oresme
(1323 – 1382)

Nicholas Oresme was an important mathematician and scientist from France in the 14th century. He used the rectangular coordinate system and is said to be the first person who produced speed-time graphs.

http://bt.sasbadi.com/m4183

7.1 Distance-Time Graphs

What do you understand about distance-time graphs?

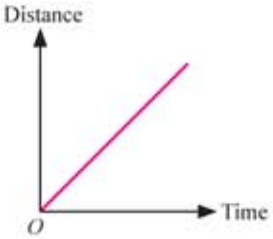
Learning Standard

Draw distance-time graphs.

Have you ever used public transport to go to a certain destination? The ticket for the journey, especially flight ticket, has a display of departure time and the estimated time of arrival at your destination. For example, the estimated duration of a domestic flight from Kuala Lumpur International Airport 2 (KLIA2) to Miri is 2 hours 20 minutes. Do you know how the duration is estimated?

Do you still remember the relationship between speed, distance and time that you have studied in Form 2?

Speed is a rate which involves distance and time. The relationship between speed and time can be represented by a distance-time graph. A distance-time graph enables the motion of an object to be represented in the graphic form which is easy to be understood.



In a distance-time graph:

- the vertical axis represents distance
- the horizontal axis represents time
- the gradient of the graph represents the **rate of change in distance with respect to time**, that is **speed**.

How do you draw a distance-time graph?

A distance-time graph can be drawn if the following information regarding the motion is obtained.

(a) Distance-time table

(b) Equation that represents the relationship between distance and time

Draw a distance-time graph based on a distance-time table

Example 1

Azree wants to be a track cyclist and hopes to make Malaysia well-known in the world like Azizulhasni Awang, our national track cycling champion. The table below shows the distance and the time taken by Azreen during the training.

Time (minutes)	0	30	60	90	120
Distance (km)	0	10	20	30	40

Draw a distance-time graph based on the above table.

Malaysiaku

Azizulhasni Awang is also nicknamed *The Pocket Rocketman*. Why?

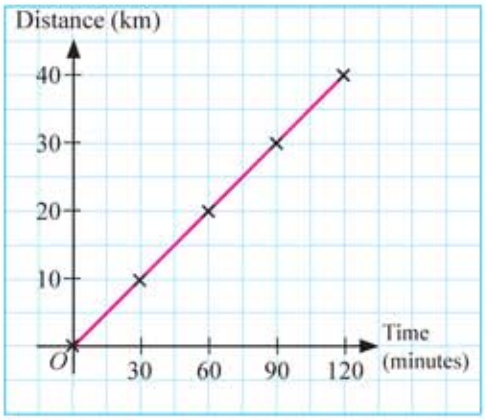
Solution:

Steps

(a) Choose suitable scales to represent the given distance and time.

(b) Plot a point that represents each pair of values of distance and time on a grid paper or graph paper.

(c) Join the points plotted using a ruler to obtain the distance-time graph as shown.



Draw distance-time graphs based on equations that represent the relationship between distance and time

Example 2

Mr Selva drove his car for a distance of 240 km from Kuala Lumpur to Kuantan in 3 hours to visit his mother. The distance, s km, that Mr Selva travelled in t hours is given by the equation $s = 80t$. Draw a distance-time graph to represent Mr Selva's journey from Kuala Lumpur to Kuantan.

Solution:

Steps

(a) Construct a distance-time table as shown below using the equation $s = 80t$.

Time, t (hours)	0	1	2	3
Distance, s (km)	0	80	160	240

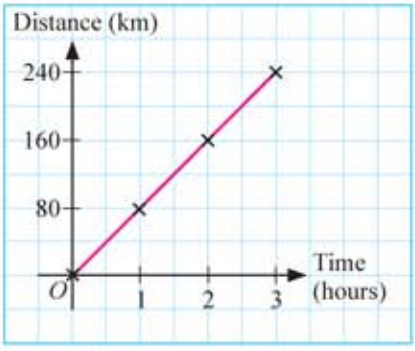
$s = 80t$ when $t = 0$, $s = 80(0)$, $s = 0$

$s = 80t$ when $t = 3$, $s = 80(3)$, $s = 240$

TIPS

A straight line can be drawn by joining at least two of the plotted points.

(b) Draw the distance-time graph as shown by joining the plotted points.



Example 3

Jaswinder Singh takes part in an 8-kilometre cross country event organised by the school during National Sports Day. The relationship between distance from the finishing line and the time of the run is $s = 8 - \frac{1}{5}t$, where s is the distance in km and t is the time in minutes. Draw a distance-time graph to represent Jaswinder Singh's run for the duration $0 \leqslant t \leqslant 40$.

Solution:

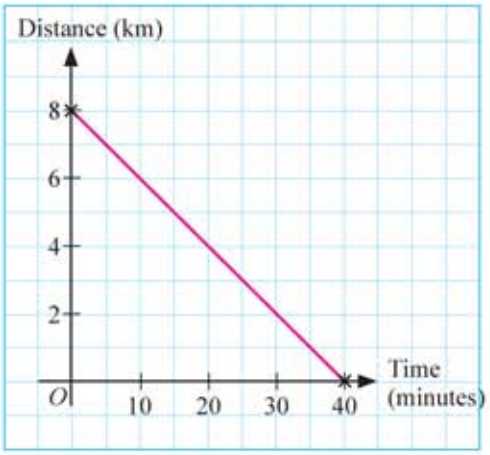
Given $s = 8 - \frac{1}{5}t$

Time, t (minutes)	0	40
Distance, s (km)	8	0

2 points that satisfy the equation are enough to draw the straight line.

$s = 8 - \frac{1}{5}t$
when $t = 0$,
$s = 8 - \frac{1}{5}(0)$
$s = 8$

$s = 8 - \frac{1}{5}t$
when $t = 40$,
$s = 8 - \frac{1}{5}(40)$
$s = 0$



Self Practice 7.1a

1. The table below shows the time taken by Haji Ali to walk from his house to the mosque for prayers. He needs 20 minutes to walk to the mosque which is located 300 m from his house. Draw a distance-time graph based on the given table.

Time (minutes)	0	5	10	15	20
Distance (metres)	0	75	150	225	300

2. Encik Nyambek drives his car to his office which is located 45 km from his house in Bekenu. The table below shows the time taken by Encik Nyambek to reach his office in Miri from Bekenu. Draw a distance-time graph based on the given table.

Time (hours)	0	0.2	0.4	0.6	0.8	1.0
Distance (km)	0	9	18	27	36	45

3. The motion of a particle for a certain period is represented by $s = 8t + 5$ where s is the distance in cm and t is the time in seconds. Draw a distance-time graph to represent the motion of the particle for a period of 5 seconds.

4. Leong cycles to Zainal's house which is located 1.6 km from his house. Leong's journey from Zainal's house is given by the equation $s = 1.6 - 0.2t$ where s is the distance in km and t is the time in minutes. Draw a distance-time graph to represent Leong's journey for the period $0 \leqslant t \leqslant 8$.

How do you interpret distance-time graphs?

Study the two distance-time graphs below.

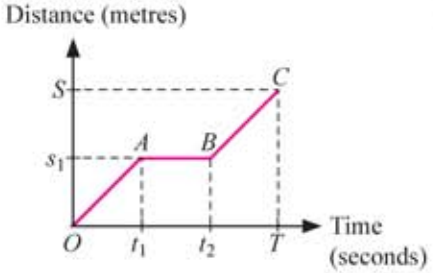


Diagram 1

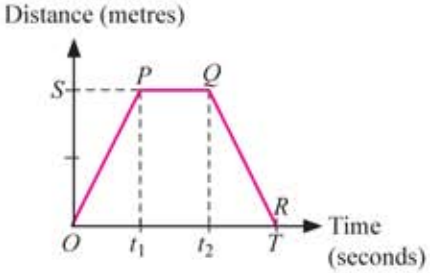


Diagram 2

Diagram 1 shows a motion from O to C for a distance of S metres in a period of T seconds.	Diagram 2 shows a motion from O to R passing through P and Q.
OA ⇨ **positive gradient** of graph ⇨ **gradient** of OA represents **speed** of motion ⇨ motion at **uniform speed** ⇨ motion for a distance of s_1 metres in a period of t_1 seconds	OP ⇨ **positive gradient** of graph ⇨ motion for a distance of S metres in a period of t_1 seconds
AB ⇨ **zero gradient** ⇨ no change in distance means motion stops (**stationary**) ⇨ stationary for the period $(t_2 - t_1)$ seconds	PQ ⇨ **zero gradient** ⇨ no change in distance (**stationary**) ⇨ stationary for the period $(t_2 - t_1)$ seconds
BC ⇨ positive gradient ⇨ motion continues to C	QR ⇨ **negative gradient** ⇨ negative speed shows object moves back to original place or moves in the opposite direction
OC ⇨ motion of S metres in a period of T seconds	OR ⇨ motion of $2S$ metres (to and fro) in a period of T seconds

Learning Standard

Interpret distance-time graphs and describe the motion based on the graphs.

MY MEMORY

$$\text{Speed} = \frac{\text{Distance}}{\text{Time}}$$

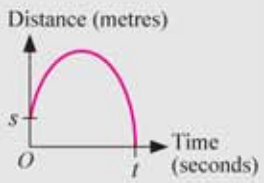
$$\text{Gradient} = \frac{\text{Vertical distance}}{\text{Horizontal distance}}$$

TIPS

Positive gradient and negative gradient of a distance-time graph indicate directions of motion.

Smart Mind

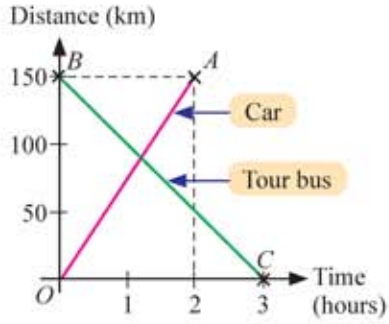
The distance-time graph below shows the motion of an object.



What type of equation will give us the distance-time graph as shown above? Discuss and state one example of the motion.

Example 4

The distance-time graph shows the motion of a car and a tour bus. Graph OA represents the motion of the car from Puchong to Melaka. Graph BC represents the motion of the tour bus from Melaka to Puchong. Determine the difference in speed, in km h^{-1}, of the two vehicles.



Solution:

Speed of car

$$= \frac{\text{difference in distance travelled}}{\text{corresponding difference in time}}$$

$$= \frac{(150 - 0)\text{ km}}{(2 - 0)\text{ hours}}$$

$= 75$ km per hour

$= 75$ km h^{-1}

Speed of tour bus $= \dfrac{\text{difference in distance travelled}}{\text{corresponding difference in time}}$

$$= \frac{(0 - 150)\text{ km}}{(3 - 0)\text{ hours}}$$

$= -50$ km per hour

$= 50$ km h^{-1}

The negative sign means the tour bus and the car move in the opposite direction.

Hence, difference in speed $= (75 - 50)$ km h^{-1}

$= 25$ km h^{-1}

TIPS

km h^{-1} can also be written as km/h.

Example 5

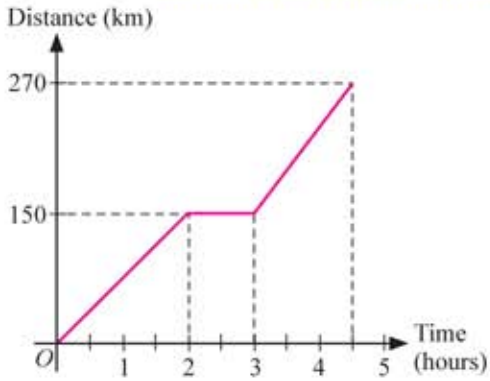
The distance-time graph shows the motion of a car for a period of 4.5 hours.

(a) Determine

(i) the duration when the car is stationary.

(ii) the speed, in km h^{-1}, of the car in the first hour.

(b) Describe the motion of the car for the last 90 minutes.



Solution:

(a) (i) Stationary period = period the car stops

$= (3 - 2)$ hour

$= 1$ hour

(ii) Speed of car in the first hour = speed of car in the first 2 hours

$$= \frac{(150 - 0)\text{ km}}{(2 - 0)\text{ hours}}$$

$= 75$ km h^{-1}

From the graph, the distance travelled in the first hour is not known. Therefore, the speed is determined by the distance travelled in the first 2 hours. (The gradients on a straight line are the same).

(b) Speed of car $= \dfrac{(270 - 150)\text{ km}}{(4.5 - 3)\text{ h}}$

last 90 minutes

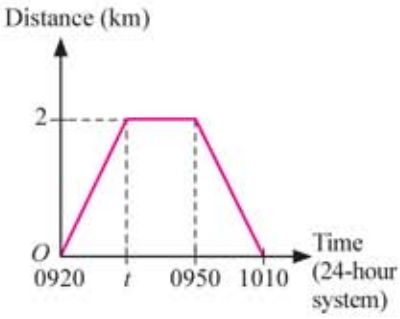
$= 80$ km h^{-1}

The car travels for 120 km with a speed of 80 km h^{-1} in the last 90 minutes.

CHAPTER 7

Example 6

Sahana cycles to the post office to send Hari Raya Aidilfitri greeting cards to her close friends. The distance-time graph shows Sahana's journey from her house to the post office and back.



(a) Determine

(i) the total distance, in km, for Sahana's whole journey.

(ii) the value of t, if Sahana cycles with a speed of 8 km h^{-1} to the post office.

(b) Describe Sahana's journey from the post office back to her house.

Solution:

(a) (i) Total distance travelled = 2 km + 2 km
= 4 km

(ii) $\text{Time} = \dfrac{\text{Distance}}{\text{Speed}}$

$= \dfrac{2 \text{ km}}{8 \text{ km h}^{-1}}$

$= 0.25$ hour

$= 15$ minutes

Thus, $t = 0920 + 0015$
$= 0935$

(b) $\text{Rate of change in distance} = \dfrac{(0 - 2) \text{ km}}{\left(\dfrac{1010 - 0950}{60}\right) \text{ hour}}$

1 minute = $\frac{1}{60}$ hour

$= -6 \text{ km h}^{-1}$

$= 6 \text{ km h}^{-1}$

Sahana cycles for a distance of 2 km in 20 minutes with a speed of 6 km h^{-1}.

TIPS

$\text{Speed} = \dfrac{\text{Distance}}{\text{Time}}$

$\text{Time} = \dfrac{\text{Distance}}{\text{Speed}}$

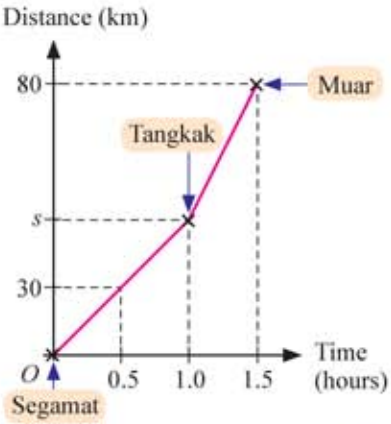
Motion with different speeds

The speed of a motion usually changes in a journey. In such situation, **average speed** is used.

$$\text{Average speed} = \frac{\text{Total distance}}{\text{Total time}}$$

Example 7

Zabedah wants to visit her friend in Muar. The distance-time graph shows her journey by car from Segamat to Muar passing through Tangkak.



(a) Calculate the average speed, in km h^{-1}, of Zabedah's journey from Segamat to Muar.

(b) If the rate of change in distance of the car from Segamat to Tangkak is 50 km h^{-1}, calculate the distance, in km, between Tangkak and Muar.

(c) Describe the motion of the car from Segamat to Muar.

Solution:

(a) Average speed $= \dfrac{\text{Total distance}}{\text{Total time}}$

$= \dfrac{80 \text{ km}}{1.5 \text{ h}}$

$= 53\dfrac{1}{3} \text{ km h}^{-1}$

TIPS

$\text{Speed} = \dfrac{\text{Distance}}{\text{Time}}$

Distance = Speed × Time

(b) Total distance = Average speed × Total time

$= 50 \text{ km h}^{-1} \times 1 \text{ h}$

$= 50 \text{ km}$ ← Distance between Segamat and Tangkak

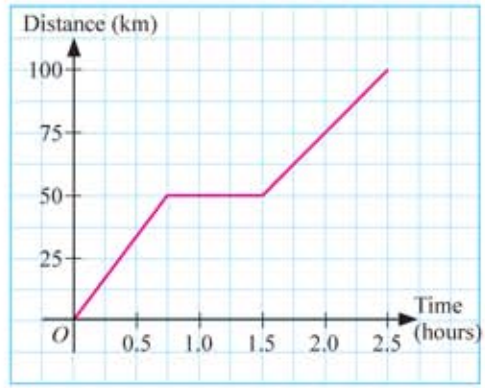
Distance between Tangkak and Muar = (80 – 50) km

= 30 km

(c) The car moves for a distance of 80 km in 1.5 hours with an average speed of $53\dfrac{1}{3} \text{ km h}^{-1}$.

Self Practice 7.1b

1. The distance-time graph shows the journey of Encik Rejab and his family by car from Kota Kinabalu to Keningau to celebrate *Pesta Kaamatan*.



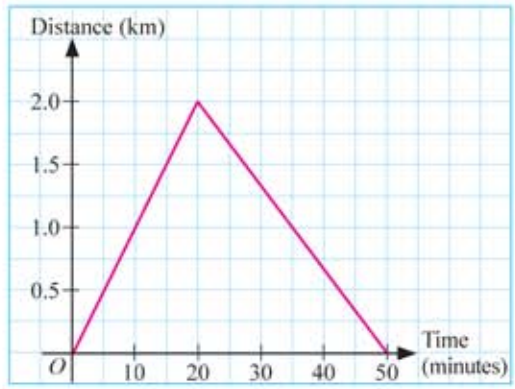
(a) Calculate the speed, in km h^{-1}, of the car for the last 1 hour.

(b) Describe the motion of Encik Rejab's car for the period of 45 minutes after travelling the first 50 km of the journey.

(c) (i) Calculate the average speed, in km h^{-1}, for the journey from Kota Kinabalu to Keningau.

(ii) Hence, describe the motion of the car for the whole journey.

2. Encik Rashid exercises every day to stay healthy. The distance-time graph shows the distance and time of Encik Rashid's run from his house to the playground and then back to his house.

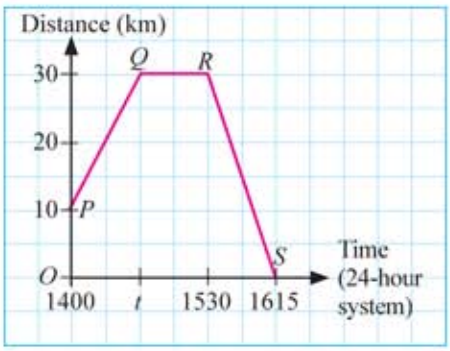


(a) Calculate the difference between the speed of Encik Rashid's run from his house to the playground and the speed of his run from the playground back to his house in km h^{-1}.

(b) Calculate the average speed, in km h^{-1}, for Encik Rashid's whole run.

(c) Describe Encik Rashid's motion for the period of 50 minutes.

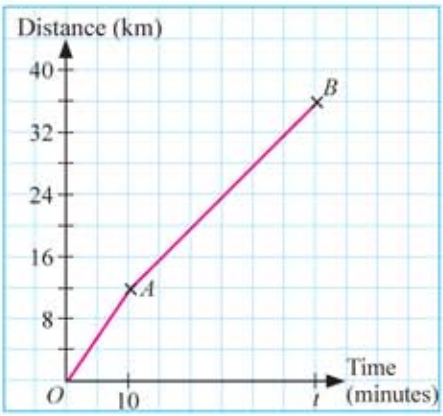
3. The distance-time graph shows Puan Rozita's journey by car for a period of $2\frac{1}{4}$ hours. PQ represents Puan Rozita's journey from her workplace to a market and RS represents the return journey to her house.



(a) Calculate the value of t, if the speed of the car for Puan Rozita's journey from her workplace to the market is 50 km h^{-1}.

(b) Describe the motion of the car represented by
(i) the straight line QR.
(ii) the straight line RS.

4. Encik Yusri works in a law firm. Every day Encik Yusri sends his child to school on his way to workplace by car. OA represents the journey from his house to school and AB represents the journey from the school to his workplace.



(a) Calculate the value of t, if the rate of change in distance of the car from the school to his workplace is 48 km h^{-1}.

(b) Describe the motion of the car for the whole journey from his house to his workplace.

How do you solve problems involving distance-time graphs?

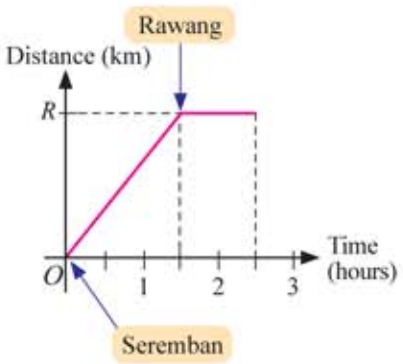
Example 8

Learning Standard
Solve problems involving distance-time graphs.

The incomplete distance-time graph shows Mr Tan's journey from Seremban to Lumut. Mr Tan stops at Rawang for lunch and a short break before he continues his journey to Lumut.

(a) If the average speed of Mr Tan's car from Seremban to Rawang is $66\frac{2}{3}$ km h^{-1}, calculate the distance, in km, between Seremban and Rawang.

(b) The distance between Seremban and Lumut is 300 km and Mr Tan drives at an average speed of 80 km h^{-1} to reach Lumut from Rawang. Complete the given distance-time graph to represent Mr Tan's whole journey.



Solution:

Understanding problem

(a) Calculate the distance between Seremban and Rawang in km.

(b) Complete the distance-time graph from Rawang to Lumut.

Planning strategy

(a) Speed $= \frac{\text{Distance}}{\text{Time}}$

Distance = Speed × Time

(b) • Determine the distance between Rawang and Lumut.

• Time $= \frac{\text{Distance}}{\text{Time}}$

• Complete the distance-time graph.

Carrying out strategy

(a) Distance = Speed × Time

$= 66\frac{2}{3}$ km h^{-1} × 1.5 hours

= 100 km

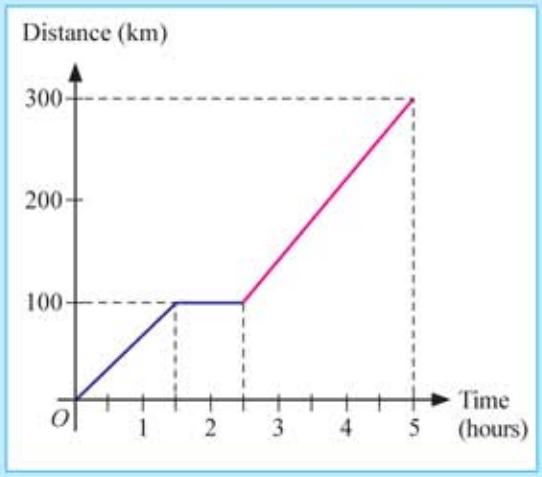
(b) Distance between Rawang and Lumut

= 300 km − 100 km = 200 km

Time $= \frac{\text{Distance}}{\text{Speed}}$

$= \frac{200 \text{ km}}{80 \text{ km h}^{-1}}$

= 2.5 hours



Conclusion

(a) The distance between Seremban and Rawang is 100 km.

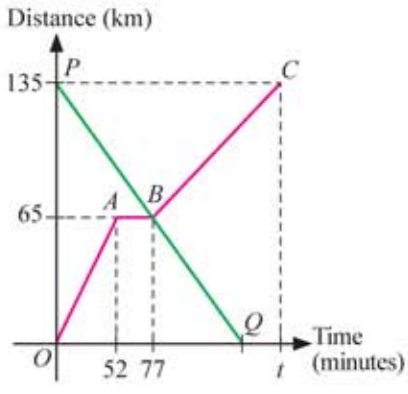
(b) The distance between Rawang and Lumut is 200 km and the time taken is 2.5 hours.

Example 9

The distance-time graph shows the journey of two cars between Kuala Lipis and Cameron Highlands. Graph *PBQ* represents Encik Manaf's journey together with his family from Cameron Highlands to Kuala Lipis to attend his cousin's wedding. Graph *OABC* represents the journey of Encik Raven's family from Kuala Lipis to Cameron Highlands for a holiday.

(a) The rates of change in distance for *OA* and *BC* are the same. Calculate the value of *t*.

(b) The average speed of Encik Manaf's journey is 72 km h^{-1}. Calculate the difference in time, in minutes, for the two journeys to reach their respective destinations.



Solution:

Understanding problem

(a) Calculate t, that is time in minutes.

(b) Difference in time of the two cars' journey to reach their respective destinations.

Planning strategy

(a) Gradient of OA = Gradient of BC

(b)
- Time taken by Encik Raven = t.
- Determine the time taken by Encik Manaf.
- Time = $\frac{\text{Distance}}{\text{Speed}}$

Carrying out strategy

(a) $\frac{(65-0)\text{ km}}{(52-0)\text{ min}} = \frac{(135-65)\text{ km}}{(t-77)\text{ min}}$

$$\frac{65}{52} = \frac{70}{t-77}$$

$$t - 77 = \frac{70(52)}{65}$$

$$t = 56 + 77$$

$$t = 133$$

(b)
- Total time for Encik Raven's journey, $t = 133$ minutes.
- Total time for Encik Manaf's journey in minutes.

$$\text{Time} = \frac{135\text{ km}}{72\text{ km h}^{-1}}$$

$$= 1.875\text{ hours} \times 60$$

$$= 112.5\text{ minutes}$$

- Difference in time = 133 – 112.5 = 20.5 minutes

Conclusion

(a) $t = 133$

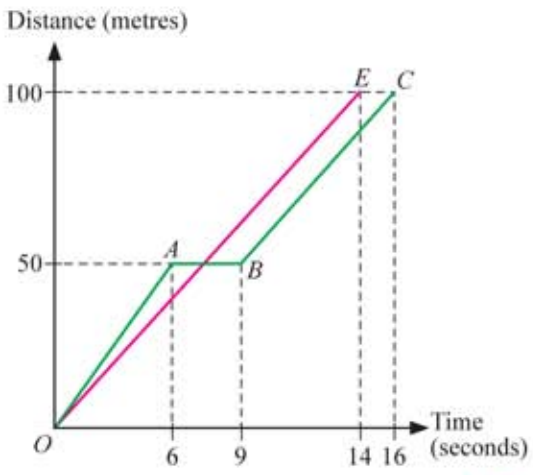
(b) The difference in time for the two journeys to reach their respective destinations is 20.5 minutes.

Self Practice 7.1c

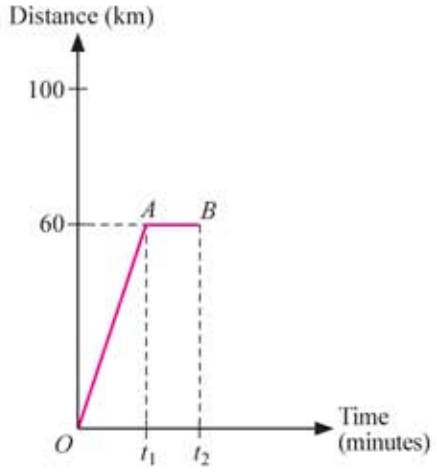
1. The distance-time graph shows the time taken by the two best participants in the 100-metre event during the Sports Championship of SMK Sinar Harapan. Graph OE represents Rizal's run and graph $OABC$ represents Jeffery's run. AB represents the time elapsed before Jeffery continued his run after a fall.

(a) Calculate the time lost, in seconds, by Jeffery in the competition.

(b) Did Jeffry have the chance to become the champion in the 100-metre event if he did not fall down and manage to maintain his speed for the whole run? Justify your answer.



2. The incomplete distance-time graph shows Encik Jumali's journey for a distance of 100 km.



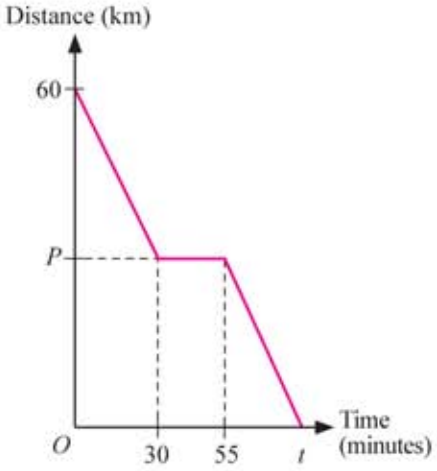
(a) It is given that the rate of change in distance for the first 60 km is 72 km h^{-1}. Calculate the value of t_1.

(b) If Encik Jumali's car is stationary for 20 minutes, calculate the value of t_2.

(c) His journey continues from B to its destination with an average speed of 75 km h^{-1}. Complete the distance-time graph for the whole journey of Encik Jumali.

(d) If the journey from O starts at 9:30 in the morning, calculate the time Encik Jumali arrives at his destination.

3. Encik Jamal goes to Padang Besar with his family. During the return journey to Jitra, they stop at Bukit Kayu Hitam for a tea break. The distance-time graph shows the return journey from Padang Besar to Jitra.



(a) Calculate the duration in which Encik Jamal's car is stationary.

(b) It is given that the average speed for the journey from Padang Besar to Bukit Kayu Hitam is 66 km h^{-1}.

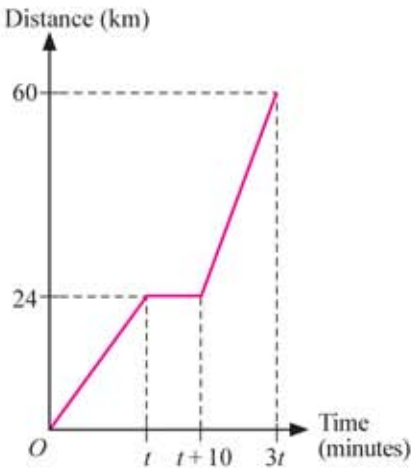
(i) Determine the value of P.

(ii) Calculate the distance between Padang Besar and Bukit Kayu Hitam.

(c) If Encik Jamal drives at an average speed of 64.8 km h^{-1} for the return journey from Bukit Kayu Hitam to his house in Jitra, calculate the value of t.

(d) Calculate the average speed, in km h^{-1}, for the whole journey.

4. The distance-time graph shows Mr Moorthy's journey by car for a distance of 60 km in $3t$ minutes. It is given that the rate of change in distance before and after the stationary period are the same.



(a) Calculate the value of t.

(b) Calculate the average speed, in km h^{-1}, for the whole journey of Mr Moorthy.

(c) Describe the motion of the car after the stationary period.

7.2 Speed-Time Graphs

What do you understand about speed-time graphs?

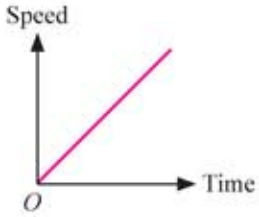
Learning Standard

Draw speed-time graphs.

Have you observed the motion of the needle in a speedometer of a car when your parents drive the car? The needle shows that the value of the speed changes when the oil pedal or brake pedal is pressed. Study the diagrams of the speedometer below.

Diagram 1 Diagram 2 Diagram 3 Diagram 4

The needle in Diagram 1 shows a value of 0, which means the vehicle is stationary. The increase in value as shown in Diagram 2 and Diagram 3 means the speed of the vehicle is increasing. The speed in Diagram 4 is decreasing compared to the speed shown in Diagram 3 for a certain period. The rate of change of speed of a motion can be shown by drawing a speed-time graph.



In a speed-time graph:

- the vertical axis represents the speed of a motion.
- the horizontal axis represents the time taken.
- the gradient of graph represents the **rate of change of speed with respect to time**, that is **acceleration**.

How do you draw a speed-time graph?

A speed-time graph can be drawn if the following information about the motion is obtained.

(a) Speed-time table
(b) Equation that represents the relationship between speed and time

How do you draw a speed-time graph based on a speed-time table?

Example 10

The table below shows the change of speed of Encik Azizul's car for a period of 5 seconds.

Time (seconds)	0	1	2	3	4	5
Speed ($m\ s^{-1}$)	0	10	20	30	40	50

Draw a speed-time graph based on the given table.

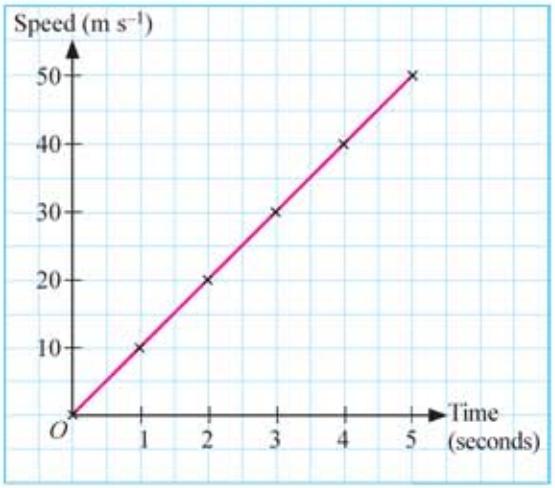
Solution:

Steps

(a) Choose appropriate scales to represent the given speed and time.

(b) Plot a point to represent each pair of values of speed and time on a grid paper or graph paper.

(c) Join the plotted points using a ruler to obtain the speed-time graph as shown.



Example 11

The rate of change of speed of an aeroplane that is landing is given by the equation $v = 800 - 1\,600t$ where v is the speed in km h^{-1} and t is time in hours. Draw a speed-time graph to represent the landing of the aeroplane for the period $0 \leqslant t \leqslant 0.5$.

Solution:

Steps

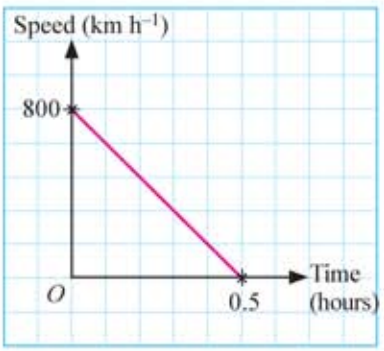
(a) Construct a speed-time table as shown below using the equation $v = 800 - 1\,600t$.

Time, t (hours)	0	0.5
Speed, v (km h^{-1})	800	0

$v = 800 - 1\,600t$ when $t = 0$, $v = 800 - 1600(0)$, $v = 800$

$v = 800 - 1\,600t$ when $t = 0.5$, $v = 800 - 1\,600(0.5)$, $v = 0$

(b) Draw the speed-time graph by plotting the points based on the table constructed.



Self Practice 7.2a

1. Draw a speed-time graph based on the given table.

(a)

Time (seconds)	0	1	2	3	4	5
Speed (m s^{-1})	3	4	5	6	7	8

(b)

Time (minutes)	0	1	2	3	4
Speed (km min^{-1})	30	25	20	15	10

2. Draw a speed-time graph by constructing a speed-time table for each of the following equations. It is given that v is the speed in m s^{-1} and t is the time in seconds.

(a) $v = 60 - 2t$; $0 \leqslant t \leqslant 30$.

(b) $v = 3t$; $0 \leqslant t \leqslant 5$.

What is the relationship between the area under a speed-time graph and the distance travelled?

Learning Standard

Make a relationship between the area under speed-time graph and the distance travelled, and hence determine the distance.

The diagrams below show two graphs:

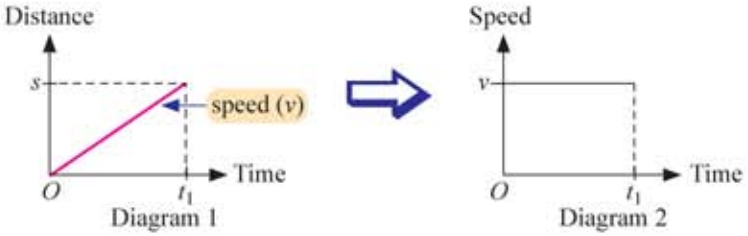


Diagram 1 Diagram 2

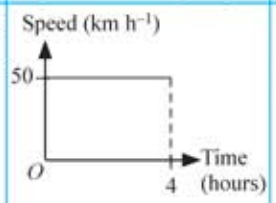
From the gradient of the distance-time graph in Diagram 1, we can determine the speed of the motion. This information from the distance-time graph can be used to draw the speed-time graph as shown in Diagram 2. Do you know that the distance, s, travelled by a motion can be determined from the speed-time graph?

Mind Stimulation 1

Aim: To determine the relationship between the area under speed-time graph and the distance travelled.

Steps:

1. Divide the class into groups.
2. Read and understand each of the given statements. Calculate the average speed in km h^{-1}.
3. Sketch the speed-time graphs based on the given statements.
4. Calculate the area under the speed-time graph and the distance travelled like example (a).

Statement	Speed-time graph	Area under the graph	Distance travelled
(a) Encik Faizal drives a distance of 200 km in 4 hours. Speed $= \frac{200\text{ km}}{4\text{ hours}} = 50\text{ km h}^{-1}$		Area $= 4\text{ h} \times 50\text{ km h}^{-1}$ $= 200\text{ km}$	Distance travelled = speed × time $= 50\text{ km h}^{-1} \times 4\text{ h}$ $= 200\text{ km}$
(b) A tour bus moves 150 km in 2 hours.			
(c) Mrs Malini cycles 8 km in 40 minutes.			
(d) Mr Gomez runs 4 km in 30 minutes.			

Discussion:

1. What is the relationship between the area under speed-time graph and the distance travelled?
2. Present your group's findings through Gallery Walk activity.
3. Are your group's findings the same as the other groups' findings?

From the activity in Mind Stimulation 1, it is found that:

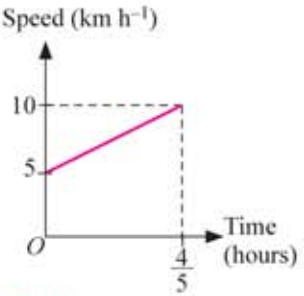
> The area under a speed-time graph is the same as the distance travelled for the same time interval.

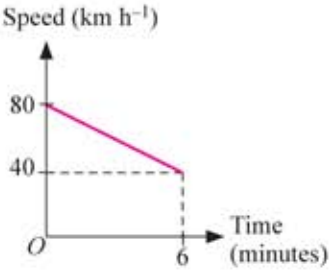
In general,

> For a speed-time graph:
> Area under the graph = Distance travelled

Example 12

Calculate the distance travelled in each motion based on the following speed-time graphs.

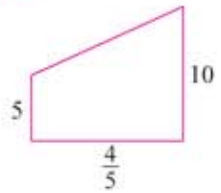
(a) 

(b) 

Indicator

Make sure the unit for time used in the speed is the same as the unit for time.

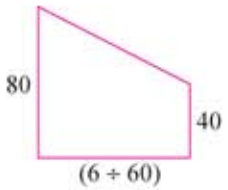
Solution:

(a) 

Distance = area of trapezium

$= \frac{1}{2} \times \left(\frac{4}{5}\right) \text{h} \times (5 + 10) \text{ km h}^{-1}$

$= 6 \text{ km}$

(b) 

Distance = area of trapezium

$= \frac{1}{2} \times \left(\frac{6}{60}\right) \text{h} \times (40 + 80) \text{ km h}^{-1}$

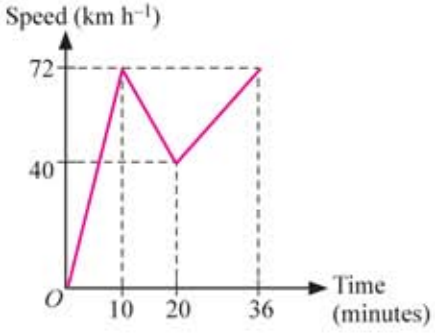
$= 6 \text{ km}$

Example 13

The speed-time graph shows the speed of Mrs Liew's car for a period of 36 minutes. Calculate

(a) the total distance, in km, travelled by Mrs Liew's car for the period of 36 minutes.

(b) the average speed, in km h^{-1}, of Mrs Liew's car for the period of 36 minutes.



Solution:

(a) Total distance
= area under the graph
$= \left[\frac{1}{2} \times \left(\frac{10}{60}\right) \text{ h} \times 72 \text{ km h}^{-1}\right] + \left[\frac{1}{2} \times \left(\frac{10}{60}\right) \text{ h} \times (40 + 72) \text{ km h}^{-1}\right] + \left[\frac{1}{2} \times \left(\frac{16}{60}\right) \text{ h} \times (40 + 72) \text{ km h}^{-1}\right]$
$= \left(6 + \frac{28}{3} + \frac{224}{15}\right) \text{ km}$
$= 30.27 \text{ km}$

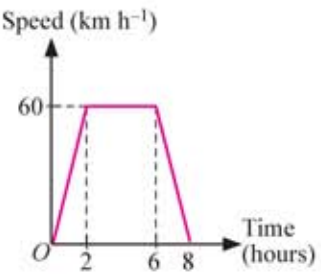
(b) Average speed $= \frac{\text{Total distance}}{\text{Total time}}$
$= \frac{30.27 \text{ km}}{(36 \div 60) \text{ h}}$
$= 50.45 \text{ km h}^{-1}$

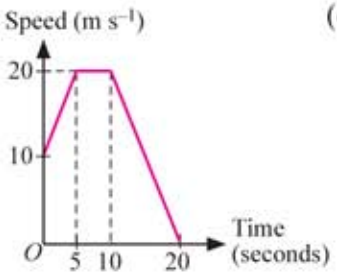
MY MEMORY

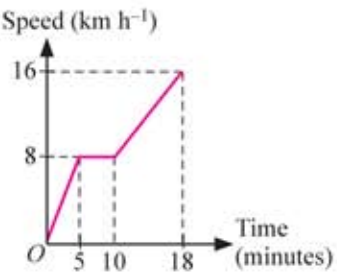
60 minutes = 1 hour
1 minute = $\frac{1}{60}$ hour

Self Practice 7.2b

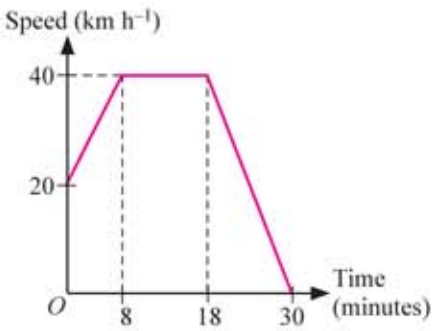
1. Calculate the distance travelled, in km, in each motion based on the given speed-time graph.

(a)


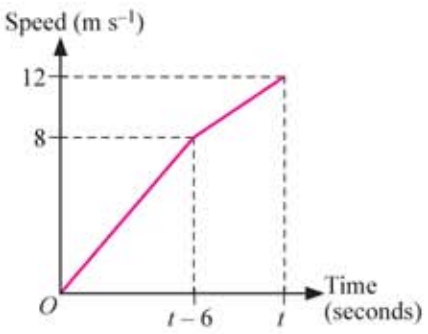
(b)


(c)


2. The speed-time graph shows the speed of Encik Mustaffa's motorcycle for a period of 30 minutes when he fetches his child after extra class. Calculate
(a) the total distance, in km, for a period of 30 minutes.
(b) the average speed, in km h^{-1}, for a period of 30 minutes.

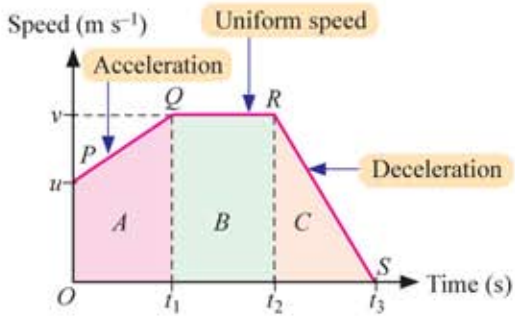


3. Sarves competes in the 100-metre event during the sports event in his school. The speed-time graph shows the speed of Sarves up to the finishing line. Calculate
(a) the value of t.
(b) the average speed of Sarves in km h^{-1}.



How do you interpret speed-time graphs?

Study the speed-time graph below.



The interpretation of the speed-time graph:

PQ ⇨ The **speed of object increases** from u m s^{-1} to v m s^{-1}.

⇨ The gradient of graph is positive, hence the rate of change of speed is positive.

⇨ **Acceleration** = $\dfrac{\text{Change of speed}}{\text{Change in time}}$

⇨ The **area** of trapezium A, that is the area under graph PQ represents the **distance** travelled in the period of t_1 seconds.

QR ⇨ There is **no change of speed** (zero gradient).

⇨ The object moves at a **uniform speed**.

⇨ The area of rectangle B, that is the area under graph QR represents the distance travelled in the period of $(t_2 - t_1)$ seconds.

RS ⇨ **Speed of the object decreases**.

⇨ The gradient of graph is negative, hence the rate of change of speed is negative.

⇨ **Deceleration** = $\dfrac{\text{Change of speed}}{\text{Change in time}}$

⇨ There is no change in the direction, that is the motion of the object remains in the same direction.

⇨ The area of triangle C, that is the area under graph RS represents the distance travelled in the period of $(t_3 - t_2)$ seconds.

Learning Standard

Interpret speed-time graphs and describe the movement based on the graphs.

INFO ZONE

Distance
- The length of the space between two points

Displacement
- **Vector** distance from a fixed point measured in **a certain direction**

INFO ZONE

Speed
- Rate of change in distance with respect to time.
- Speed = $\dfrac{\text{Distance}}{\text{Time}}$

Velocity
- Rate of change in displacement with respect to time.
- Velocity = $\dfrac{\text{Displacement}}{\text{Time}}$

MY MEMORY

Speed = $\dfrac{\text{Distance}}{\text{Time}}$

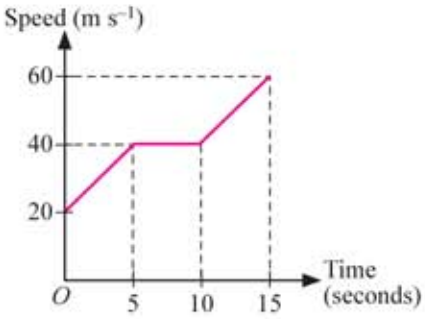
Acceleration = $\dfrac{\text{Speed}}{\text{Time}}$

Indicator

The direction of motion of an object remains the same during acceleration or deceleration.

Example 14

The speed-time graph shows the motion of Puan Salina's car for a period of 15 seconds.



(a) Calculate the rate of change of speed, in m s^{-2}, for the first 5 seconds.

(b) Describe the motion of the car for the second 5 seconds.

(c) Calculate the total distance, in metres, travelled in the period of 15 seconds.

Solution:

(a) Rate of change of speed $= \dfrac{\text{Change of speed}}{\text{Change in time}}$

$= \dfrac{(40 - 20)\text{ m s}^{-1}}{(5 - 0)\text{ s}}$

$= 4\text{ m s}^{-2}$

(b) The car moves at a uniform speed of 40 m s^{-1} for the period of 5 seconds.

(c) Total distance travelled = area under the graph

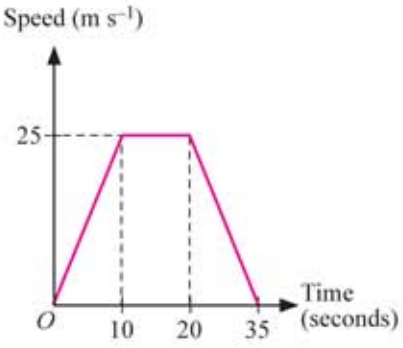
$= \left[\frac{1}{2} \times 5 \times (20 + 40)\right] + [(10 - 5) \times 40] + \left[\frac{1}{2} \times (15 - 10) \times (40 + 60)\right]$

$= (150 + 200 + 250)\text{ m}$

$= 600\text{ m}$

Example 15

Mr Daniel Wong drives his car to a convenience store to buy newspaper. The speed-time graph shows the motion of the car from his house to the road junction before reaching his destination.



(a) Describe the motion of Mr Daniel Wong's car for the first 10 seconds.

(b) What happens to the motion of Mr Daniel Wong's car from the 10th second till the 20th second?

(c) Calculate the rate of change of speed, in m s^{-2}, for the last 5 seconds.

(d) Calculate the distance, in metres, travelled during deceleration and describe the motion of the car for the period.

Solution:

(a) Rate of change of speed for the first 10 seconds

$= \dfrac{\text{Change of speed}}{\text{Change in time}}$

$= \dfrac{(25 - 0)\text{ m s}^{-1}}{10 - 0\text{ s}}$

$= 2.5\text{ m s}^{-2}$

The car accelerates at a rate of 2.5 m s^{-2} for the first 10 seconds.

(b) Mr Daniel Wong's car moves at a uniform speed of 25 m s^{-1} from the 10th second till the 20th second.

(c) Rate of change of speed $= \dfrac{(0 - 25)\text{ m s}^{-1}}{(35 - 20)\text{ s}}$

$= -\dfrac{5}{3}\text{ m s}^{-2}$

The rates of change in speed for the last 5 seconds and for the last 15 seconds are the same.

(d) Distance travelled during deceleration = distance travelled in the last 15 seconds

$= \dfrac{1}{2} \times (35 - 20) \times 25$

$= \left[\dfrac{1}{2} \times 15 \times 25\right]\text{ m}$

$= 187.5\text{ m}$

The answer can be written as
- acceleration $= -\dfrac{5}{3}\text{ m s}^{-2}$ or
- deceleration $= \dfrac{5}{3}\text{ m s}^{-2}$

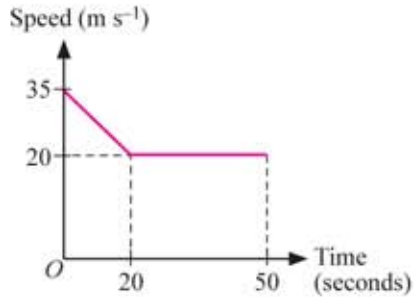
The car travels for 187.5 m in 15 seconds with a deceleration of $\dfrac{5}{3}\text{ m s}^{-2}$.

Self Practice 7.2c

1. The speed-time graph shows the motion of a motorcycle for a period of 50 seconds. Describe the motion of the motorcycle

(a) for the first 20 seconds.

(b) when it moves at a uniform speed.

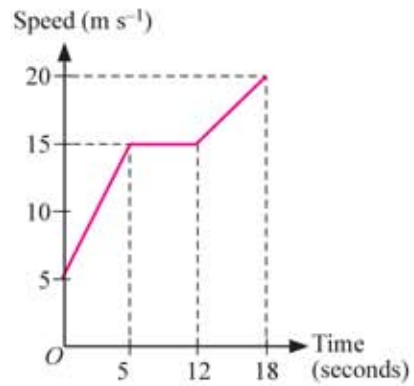


2. The speed-time graph shows the motion of a particle for a period of 18 seconds.

(a) Calculate the acceleration, in m s^{-2}, of the particle for the last 6 seconds.

(b) Calculate the total distance, in metres, travelled by the particle in the period of 18 seconds.

(c) Describe the motion of the particle when it moves at a uniform speed.

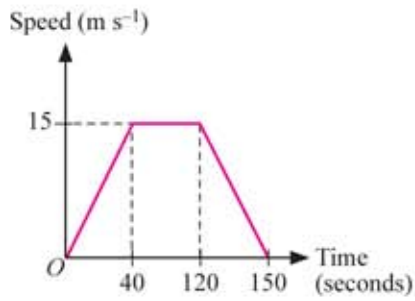


3. Encik Merisat visits his friend who lives in the same housing estate by car. The speed-time graph shows the journey of Encik Merisat to his friend's house.

(a) Calculate the rate of change of speed , in m s^{-2}, of his car for the first 20 seconds.

(b) Calculate the distance, in metres, travelled at a uniform speed.

(c) Describe Encik Merisat's journey for the period of 2.5 minutes.



How do you solve problems involving speed-time graphs?

Example 16

Learning Standard

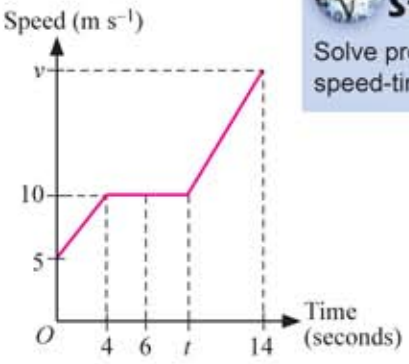
Solve problems involving speed-time graphs.

The speed-time graph shows the motion of a car for a period of 14 seconds. Calculate

(a) the average speed, in m s^{-1}, for the first 6 seconds.

(b) the value of t, if the distance travelled by the car for the first 4 seconds is half the distance travelled at a uniform speed.

(c) the value of v, if the acceleration for the last 2 seconds is 3.5 m s^{-2}.



Solution:

Understanding the problem

(a) Average speed for the first 6 seconds.

(b) Value of t, that is the time when moving at a uniform speed.

(c) Value of v, that is the final speed when the acceleration is 3.5 m s^{-2}.

Planning a strategy

(a) Average speed $= \dfrac{\text{Total distance}}{\text{Total time}}$

(b) Distance for the first 4 seconds $= \dfrac{1}{2}$ (distance travelled at a uniform speed)

(c) Acceleration $= \dfrac{\text{Change of speed}}{\text{Change in time}}$

Carrying out strategy

(a) Total distance travelled for the first 6 seconds

$$= \left[\frac{1}{2} \times 4 \times (5 + 10)\right] + [(6 - 4) \times 10]$$
$$= (30 + 20) \text{ m}$$
$$= 50 \text{ m}$$

$$\text{Average speed} = \frac{50 \text{ m}}{6 \text{ s}}$$
$$= \frac{25}{3} \text{ m s}^{-1}$$

(b) Area of trapezium $= \dfrac{1}{2}$ (area of rectangle)

$$\frac{1}{2} \times 4 \times (5 + 10) = \frac{1}{2} \times (t - 4) \times 10$$
$$30 = 5t - 20$$
$$50 = 5t$$
$$t = 10$$

(c) Acceleration $= 3.5$ m s^{-2}

$$\frac{v - 10}{14 - 10} = 3.5$$
$$\frac{v - 10}{4} = 3.5$$
$$v - 10 = 14$$
$$v = 24$$

Conclusion

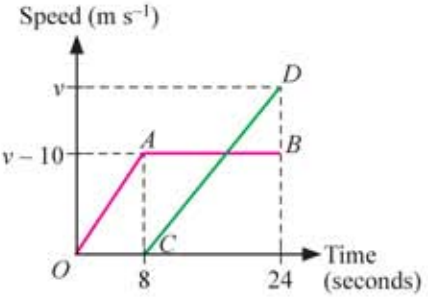
(a) The average speed for the first 6 seconds is $\dfrac{25}{3}$ m s^{-1}.

(b) $t = 10$

(c) $v = 24$

Example 17

The speed-time graph shows the motion of two vehicles. Graph OAB represents the motion of Encik Zabadi's car and graph CD represents the motion of a taxi driven by Mr Low. The difference between the distance travelled by the car and the taxi in the period of 24 seconds is 160 m. Calculate the value of v.



Solution:

Understanding the problem

- Value of v, that is the final speed of the taxi in the period of 16 seconds.

Planning a strategy

- Distance travelled by the car – distance travelled by the taxi = 160 m

Carrying out strategy

Distance travelled by the car (OAB) – distance travelled by the taxi (CD) = 160

$$\left[\frac{1}{2}\times(v-10)\times(24+16)\right]-\left[\frac{1}{2}\times(24-8)\times(v)\right]=160$$

$$\left[\frac{1}{2}\times(v-10)\times 40\right]-\left[\frac{1}{2}\times 16\times v\right]=160$$

$$20v-200-8v=160$$

$$12v=360$$

$$v=30$$

Checking Answer

- Distance travelled by the car $=\frac{1}{2}\times(30-10)\times(24+16)=400$ m
- Distance travelled by the taxi $=\frac{1}{2}\times 16\times 30=240$ m
- Difference in distance $=400-240=160$ m

Conclusion

Value of $v = 30$

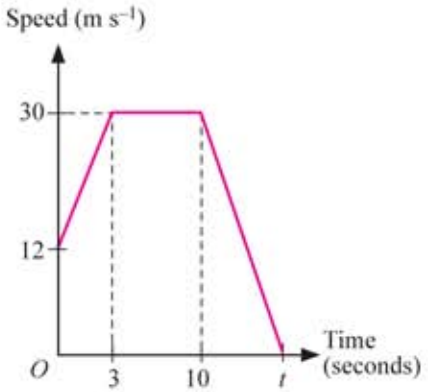
Example 18

The speed-time graph shows the motion of a van for a period of t seconds. Calculate

(a) the rate of change of speed, in m s^{-2}, for the first 3 seconds.

(b) the distance, in metres, travelled for the first 10 seconds.

(c) the value of t, if the magnitude of the rate of change of speed after 10 seconds is the same as the magnitude of the rate of change of speed for the first 3 seconds.



Solution:

(a) Rate of change of speed $=\dfrac{(30-12)\text{ m s}^{-1}}{(3-0)\text{ s}}$

$= 6\text{ m s}^{-2}$

(b) Distance travelled $= \left[\frac{1}{2} \times 3 \times (12 + 30)\right] + [(10 - 3) \times 30]$

$= (63 + 210)$ m

$= 273$ m

(c) Acceleration after 10 seconds = acceleration for the first 3 seconds

$$-\left[\frac{(0 - 30)\text{ m s}^{-1}}{(t - 10)\text{ s}}\right] = \frac{(30 - 12)\text{ m s}^{-1}}{(3 - 0)\text{ s}}$$

$$-\left(\frac{-30}{t - 10}\right) = \frac{18}{3}$$

The rate of change of speed is in the same direction (magnitude).

$$\frac{30}{t - 10} = 6$$

$$\frac{30}{6} = t - 10$$

$$t = 15$$

INFO ZONE

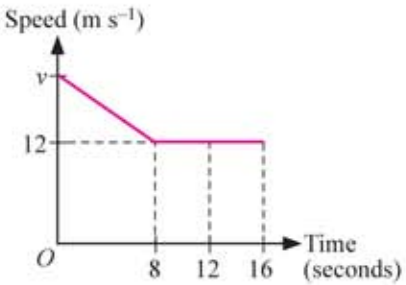
Magnitude
- distance travelled in a certain direction.

Self Practice 7.2d

1. The speed-time graph shows the motion of Dion Johan's car for a period of 16 seconds. Calculate

(a) the distance travelled, in metres, at a uniform speed.

(b) the value of v, if the average speed of the car for the first 12 seconds is 14 m s^{-1}.

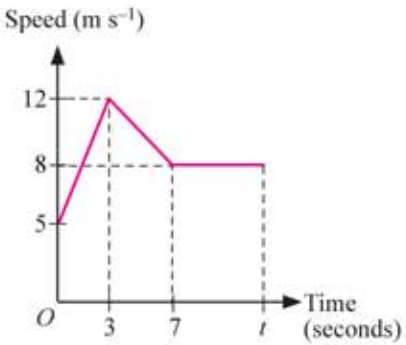


2. The speed-time graph shows the motion of the motorcycle ridden by Abit Lusang for a period of t seconds. Calculate

(a) the deceleration of the motion in m s^{-2}.

(b) the distance, in metres, when the rate of change of speed is positive.

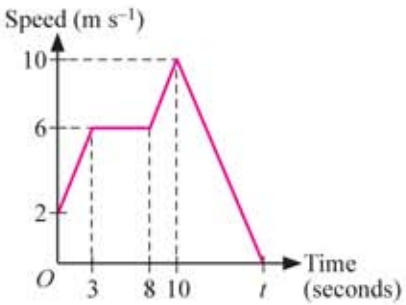
(c) the value of t, if the total distance in t seconds is 121.5 m.



3. The speed-time graph shows the motion of a car for a period of t seconds. Calculate

(a) the total distance, in metres, when the rate of change of speed is positive.

(b) the value of t, if the magnitude of the rate of change of speed from the 8th second till the 10th second is the same as the magnitude of the rate of change of speed after 10 seconds.



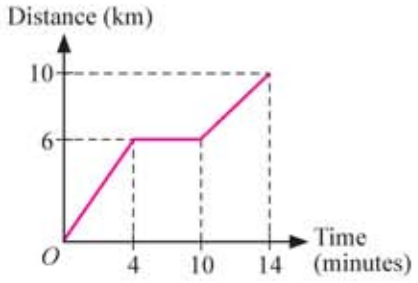
Comprehensive Practice

1. The distance-time graph shows the motion of an express bus for a period of 14 minutes. Calculate

(a) the duration when the bus is stationary.

(b) the rate of change in distance, in km h^{-1}, of the bus for the last 4 minutes.

(c) the average speed, in km h^{-1}, of the bus for the period of 14 minutes.

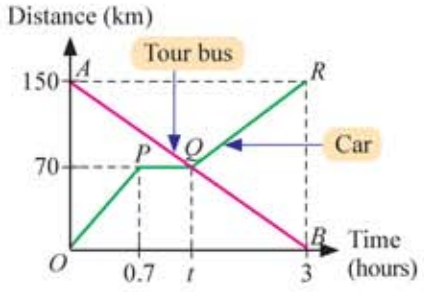


2. A car and a tour bus travel for 150 km in 3 hours. The distance-time graph shows the motion of the car and the tour bus. Calculate

(a) the rate of change in distance, in km h^{-1}, of the car for the first 24 minutes.

(b) the value of t.

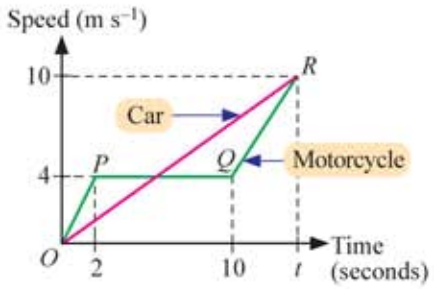
(c) the rate of change in distance, in km h^{-1}, of the car for the last 80 km.



3. The speed-time graph shows the motion of a car and a motorcycle. Calculate

(a) the duration when the motorcycle travels at a uniform speed.

(b) the value of t, if the distances travelled by the car and the motorcycle are the same for the period of t seconds.

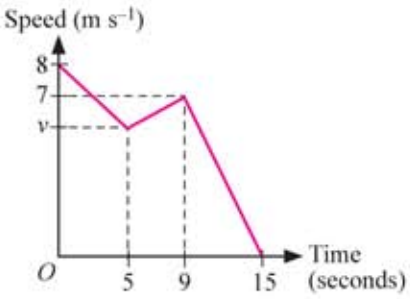


4. The speed-time graph shows the motion of a particle for a period of 15 seconds. Calculate

(a) the rate of change of speed, in m s^{-2}, of a particle for the last 6 seconds.

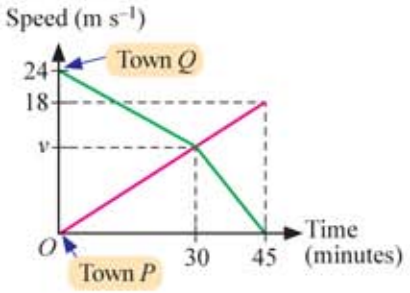
(b) the value of v, if the ratio of the distance travelled in the first 5 seconds to the last 6 seconds is 5 : 3.

(c) the average speed, in km h^{-1}, of the particle for the period of 15 seconds.



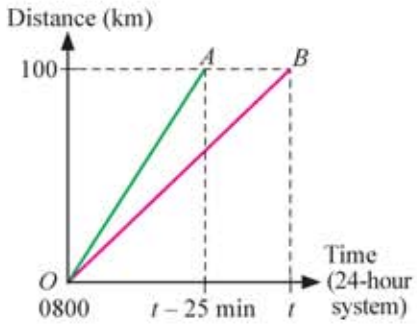
5. The speed-time graph shows the motion of two cars for a period of 45 minutes. Puan Nisha drives from Town P to Town Q and Puan Farah drives in the opposite direction. Calculate

(a) the value of v, if the rate of change of speed of Puan Farah's car for the first 30 minutes is the same as the acceleration of Puan Nisha's car for the period of 45 minutes.



(b) the distance from Town Q, in km, when the two vehicles meet.
(c) the time, in minutes, taken by Puan Nisha to reach Town Q if the acceleration of her car does not change.

6. The distance-time graph shows the motion of two cars for a distance of 100 km. Graph OA represents the motion of the car driven by Mr Lee at an average speed of v km h^{-1} and graph OB represents the motion of the car driven by Encik Dollah at an average speed of $(v - 20)$ km h^{-1}. Calculate



(a) the value of v, if the difference between the time taken by Mr Lee and Encik Dollah to reach the destination is 25 minutes.
(b) the time, in the 24-hour system, Mr Lee reaches his destination.

7. (a) (i) Diagram 1 shows the distance-time graph of car A for a period of t seconds. It is given that the average speed of car A is 25 m s^{-1}. Calculate the value of t.

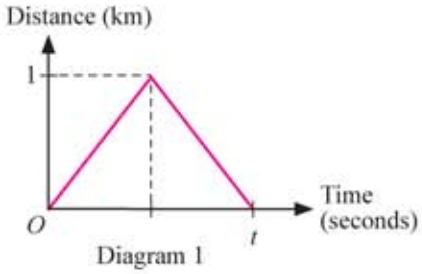


Diagram 1

(ii) Describe the motion of car A for the period of t seconds.

(b) Diagram 2 shows the speed-time graph of car B. It is given that the uniform speed, v, of car B is the same as the average speed of car A and both vehicles travel the same distance. If the values of t, in seconds, for both graphs are the same, calculate the duration, in minutes, during which car B moves at a uniform speed.

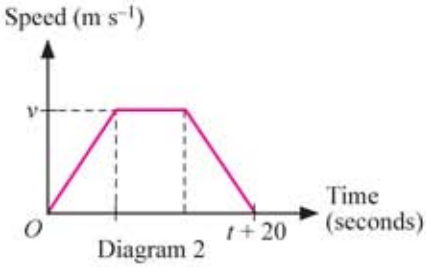


Diagram 2

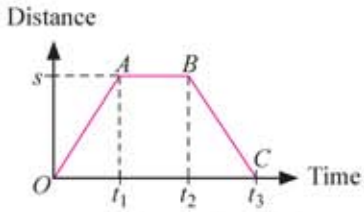
PROJECT

1. Collect information about the various speed limits in the area where you stay.
2. Record the speed limits according to specific areas. For example, 30 km h^{-1} at the surrounding areas of a school.
3. What is the implication if a driver does not follow the speed limit?
4. Prepare a report with photos and present your findings using multimedia.

CONCEPT MAP

Graphs of Motion

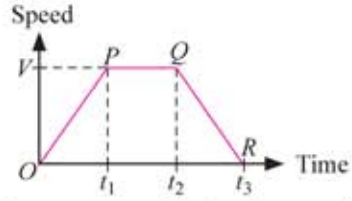
Distance-Time Graphs



- Gradient = $\frac{\text{Change in distance}}{\text{Change in time}}$

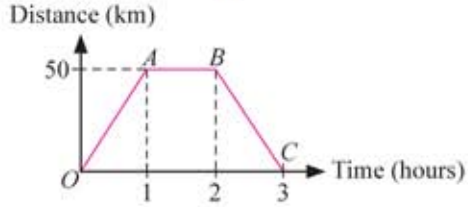
  Rate of change in distance with respect to time = speed
- OA ⇨ positive gradient
  (motion towards destination)
- AB ⇨ zero gradient
  (stationary)
- BC ⇨ negative gradient
  (motion towards origin)
- Positive and negative speeds indicate opposite direction of motion.

Speed-Time Graphs



- Distance travelled = Area under graph
- Gradient = $\frac{\text{Change of speed}}{\text{Change in time}}$

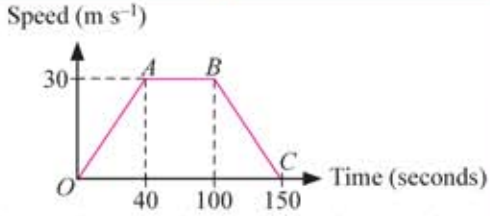
  Rate of change of speed with respect to time = acceleration
- OP ⇨ positive gradient
  ⇨ speed increases
  ⇨ acceleration
- PQ ⇨ zero gradient
  ⇨ no change of speed
  ⇨ uniform speed
- QR ⇨ negative gradient
  ⇨ speed decreases
  ⇨ deceleration



(a) Speed of the object in the first hour or in the last hour

$= \frac{50 - 0}{1 - 0} = 50 \text{ km h}^{-1}$

(b) The object is stationary for 1 hour (AB)



(a) Acceleration in the first 40 seconds

$= \frac{30 - 0}{40 - 0} = 0.75 \text{ m s}^{-2}$

(b) Acceleration in the last 50 seconds

$= \frac{0 - 30}{150 - 100} = -0.6 \text{ m s}^{-2}$

(c) Uniform speed for 1 minute (AB)

Average speed = $\frac{\text{Total distance}}{\text{Total time}}$

Self Reflection

Instructions

1. Fill in the boxes with the correct answers using capital letters.
2. Fill in the mystery word puzzle according to the numbered boxes.
3. How do you feel with the completed mystery word puzzle?

(a) $\dfrac{\text{Change in distance}}{\text{Change in time}} =$ ☐☐☐☐☐

(b) For a distance-time graph, zero gradient means the state of ☐☐☐☐[4]☐☐☐☐[1]

(c) $\dfrac{\text{Change of speed}}{\text{Change in } ☐☐[6]☐☐} =$ Acceleration

(d) For a speed-time graph, negative gradient means ☐☐☐☐[3]☐☐☐☐☐[5]

(e) A uniform speed of a motion occurs when the value of the ☐☐[2]☐☐☐☐☐ is zero in a speed-time graph.

(f) Average [7]☐☐☐☐☐ $= \dfrac{\text{Total distance}}{\text{Total time}}$

Mystery Word Puzzle

[4]☐ [2]☐[6]☐ [6]☐[2]☐[3]☐[2]☐[1]☐[7]☐[4]☐[2]☐[5]☐

Mathematics Exploration

1. Divide the class into groups.
2. Each group is required to collect information about high-speed vehicles that travel on the surface of the earth, in the air and outer space from various sources.
3. The information obtained should include the history of the invention, applications as well as side effects.
4. Exhibit your group's findings at the Mathematics Corner for the benefits of others.