CHAPTER 8

Lines and Angles

What will you learn?

- Lines and Angles
- Angles related to Intersecting Lines
- Angles related to Parallel Lines and Transversals

Why study this chapter?

Lines and angles constitute the basic knowledge in geometry and construction. Discuss the objects around you that involve the knowledge of lines and angles.

Lines and angles are used in various fields. In engineering, lines and angles provide the visual beauty and stability to the structure of a building. In visual arts, lines and angles produce stunning patterns. How are lines and angles used in photography?

Euclid of Alexandria (325 B.C. – 265 B.C.) was a mathematician who had contributed much to geometry. He wrote a set of books entitled 'The Element'. In the books, Euclid defined points and lines as the fundamentals in geometrical constructions.

Euclid of Alexandria

For more information:

http://goo.gl/mhn3oT

Word Link

• intersecting lines	• *garis bersilang*
• transversal	• *garis rentas lintang*
• parallel lines	• *garis selari*
• perpendicular line	• *garis serenjang*
• congruency	• *kekongruenan*
• perpendicular bisector	• *pembahagi dua sama serenjang*
• angle bisector	• *pembahagi dua sama sudut*
• adjacent angles	• *sudut bersebelahan*
• vertically opposite angles	• *sudut bertentang bucu*
• angle of elevation	• *sudut dongak*
• conjugate angles	• *sudut konjugat*
• interior angles	• *sudut pedalaman*
• complementary angles	• *sudut pelengkap*
• supplementary angles	• *sudut penggenap*
• reflex angle	• *sudut refleks*
• alternate angles	• *sudut selang-seli*
• corresponding angles	• *sudut sepadan*
• angle of depression	• *sudut tunduk*
• line segment	• *tembereng garis*

Open the folder downloaded from page vii for the audio of Word Link.

8.1 Lines and Angles

Lines and angles exist in our surroundings. For instance, coconut leaves which resemble lines at certain angles exhibit the beauty of art in nature.

▶ **What are congruency of line segments and congruency of angles?**

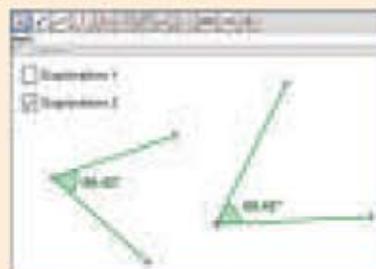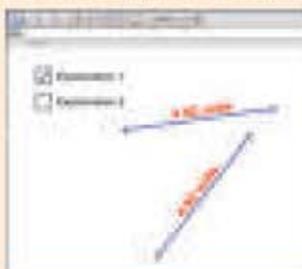## Exploration Activity 1

Aim: To determine congruency of line segments and congruency of angles.

Instruction: • Explore by yourself before the lesson begins and discuss in groups of four during the lesson.
• Open the folder downloaded from page vii.

1. Open the file *Congruent segments angles.ggb* using *GeoGebra*.

2. Click and drag the points in both explorations displayed.
3. Discuss with your friends what you have observed.

From the results of Exploration Activity 1, it is found that
(i) in Exploration 1, the two line segments displayed always have the same length. Line segments having the same length are known as **congruent line segments**.
(ii) in Exploration 2, the two angles displayed always have the same size. Angles having the same size are known as **congruent angles**.

A line segment is denoted using capital letters at both ends of the segment. For instance,
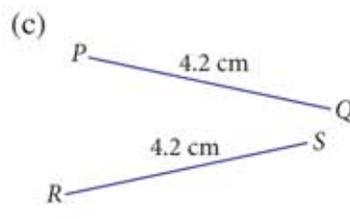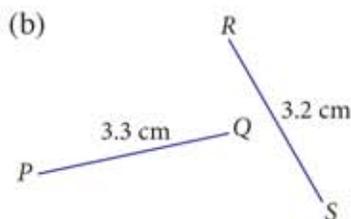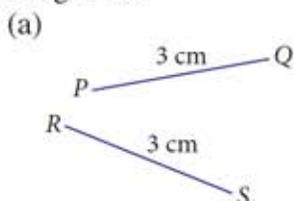
P ———————— 6 cm ———————— Q
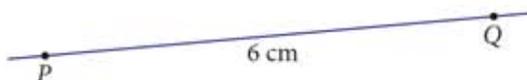
Thus, $PQ = 6$ cm.

An angle is denoted using the symbol '∠' and capital letters at the vertex and at the ends of the two arms of the angle. For instance,
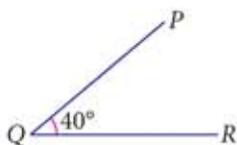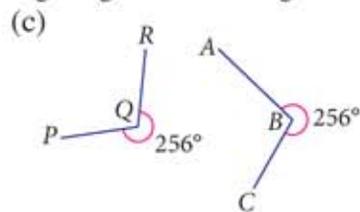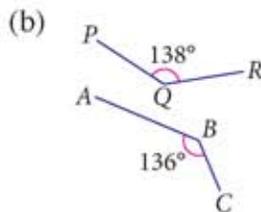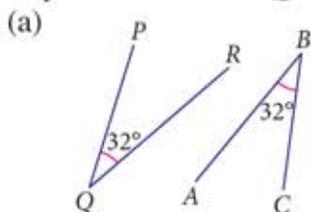

Thus, $\angle PQR = 40°$ or $\angle RQP = 40°$
or $P\hat{Q}R = 40°$ or $R\hat{Q}P = 40°$
or $\angle Q = 40°$

Self Practice   8.1a

1. Explain whether line segments PQ and RS in each of the following diagrams are congruent:

   (a)

   (b)

   (c)

2. Explain whether $\angle PQR$ and $\angle ABC$ in each of the following diagrams are congruent:

   (a)

   (b)

   (c)

▶ How do you estimate and measure the length of a line segment and the size of an angle?

The length of a line segment can be estimated by comparing the length with objects of a known length. For instance, if an eraser is known to have a length of 4 cm, then the length of line AB below can be estimated as approximately 10 cm.
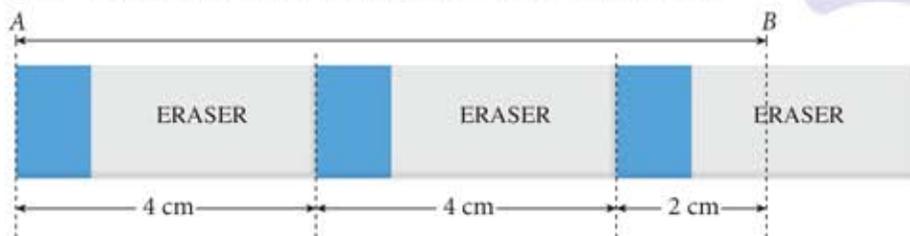
The length of a line segment can be measured more accurately by using a ruler.
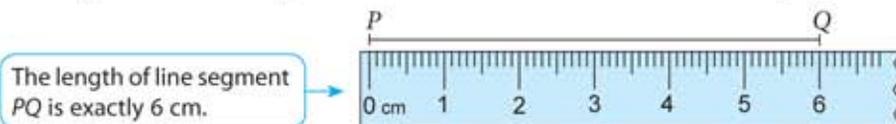
The length of line segment PQ is exactly 6 cm.

The size of an angle can be estimated by recognising the right angle first. An angle that appears more than a right angle has an angle greater than 90°. An angle that appears less than a right angle has an angle less than 90°. For instance,

Right angle = 90°   An angle less than 90°   An angle greater than 90°
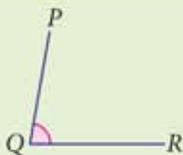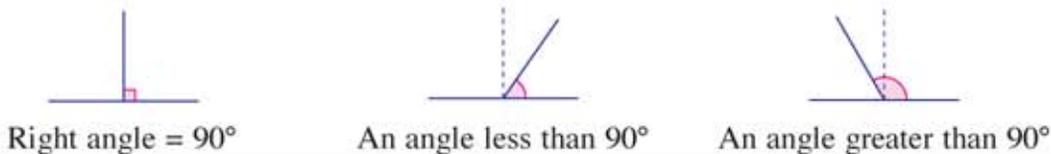
∠*PQR* appears slightly less than a right angle. Thus, ∠*PQR* can be estimated at about 80°.

From what you have learnt, the size of an angle can be measured more accurately using a protractor.

Size of ∠*ABC* is exactly 135°.
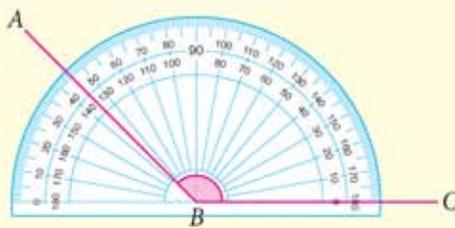
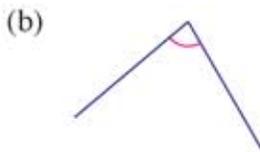## Self Practice 8.1b

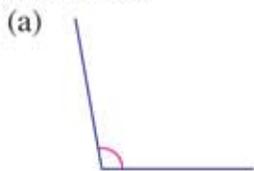1. Estimate the length of each of the following line segments. Explain how you would do the estimation.
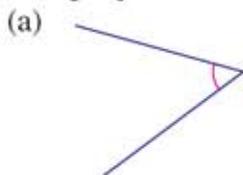
   (a)

   (b)

2. Measure each of the following line segments.

   (a)

   (b)

3. Estimate the size of each of the following angles. Explain how you would do the estimation.

   (a)

   (b)

   (c)

4. Using a protractor, measure each of the following angles accurately:

   (a)

   (b)

   (c)

▶ What are the properties of the angle on a straight line, a reflex angle and the angle of one whole turn?

Flashback 🔊

Explain the meanings of an acute angle and an obtuse angle.

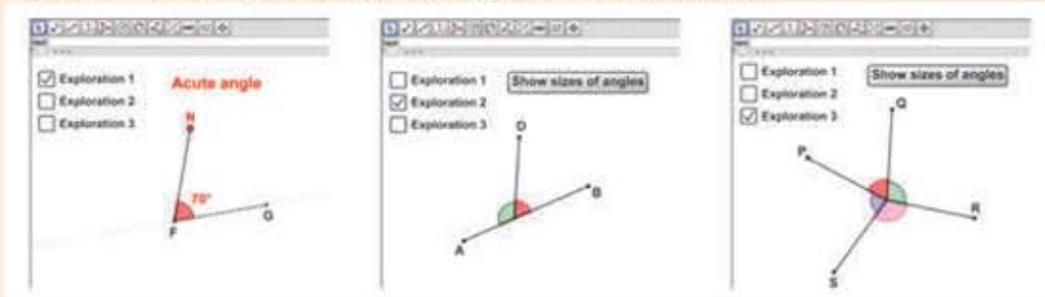# Exploration Activity 2 🔴 Flip Classroom

Aim: To explain the properties of the angle on a straight line, a reflex angle and the angle of one whole turn.

Instruction: • Explore by yourself before the lesson begins and discuss in groups of four during the lesson.
• Open the folder downloaded from page vii.

1. Open the file *Angles straight line.ggb* using *GeoGebra*.

2. Select 'Exploration 1'. Click and drag the red point and observe the change of the angle displayed.
   (i) Click and drag the red point so that the angle is on a straight line. What is the size of the angle displayed?
   (ii) Click and drag the red point so that the angle goes through one whole turn. What is the size of the angle displayed?

3. Click and drag the black points to change the position of the angle for further exploration.

4. Discuss with your friends, then compare and explain the meanings of the angle on a straight line, a reflex angle and the angle of one whole turn.

5. Select 'Exploration 2'.

6. Explain how you would determine the sum of the angles displayed.

7. Click and drag the black points and click on the button displayed on the screen for further exploration.

8. Select 'Exploration 3' and repeat Steps 6 and 7.

9. Discuss with your friends and state all the conclusions that can be made regarding your explorations.

From the results of Exploration Activity 2, it is found that
(i) the angle on a straight line is 180°. Thus, the sum of angles on a straight line is 180°.
(ii) a reflex angle is an angle with a size more than 180° and less than 360°.
(iii) the angle of one whole turn is 360°. Thus, the sum of angles at a point is 360°.

Self Practice   8.1c

1. For each of the following angles, state whether the angle is an angle on a straight line, a reflex angle or an angle of one whole turn.
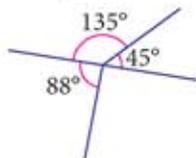   (a)   (b)   (c)
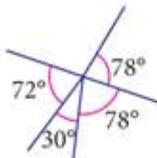
2. Copy each of the following diagrams. Identify and mark (i) the angle on a straight line, (ii) the reflex angle and (iii) the angle of one whole turn. Explain how you obtained your answers.
   (a)   (b)
   135°   78°
   45°   72°
   88°   30°   78°

▶ **What are the properties of complementary angles, supplementary angles and conjugate angles?**
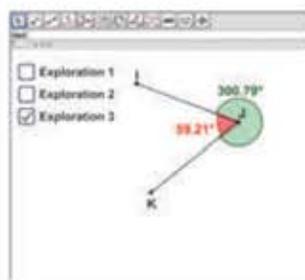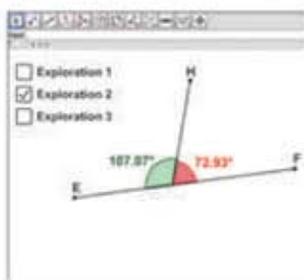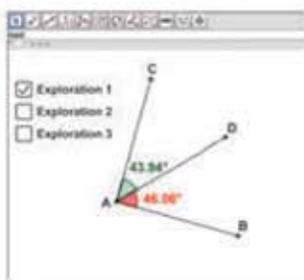
Exploration Activity 3

Aim: To describe the properties of complementary angles, supplementary angles and conjugate angles.

Instruction: • Explore by yourself before the lesson begins and discuss in groups of four during the lesson.
• Open the folder downloaded from page vii.

1. Open the file *Complementary angles.ggb* using *GeoGebra*.

2. Select 'Exploration 1'. Determine the sum of the two angles displayed. What do you observe?

3. Click and drag the black points and repeat Step **2**. Discuss with your friends and state a conclusion.

4. Repeat Steps **2** and **3** for 'Exploration 2' and 'Exploration 3'.

5. State the conclusions that you can make.

From the results of Exploration Activity 3, it is found that

(i)   in Exploration 1, the sum of the two angles is always 90°.
       These angles are known as **complementary angles**.

(ii) in Exploration 2, the sum of the two angles is always 180°.
       These angles are known as **supplementary angles**.

(iii) in Exploration 3, the sum of the two angles is always 360°.
   These angles are known as **conjugate angles**.

Self Practice   8.1d

1. Explain whether each of the following statements is TRUE or FALSE.
    (a) 60° is the complement of 30°.
    (b) 125° is the supplement of 45°.
    (c) 300° and 60° are conjugate angles.
    (d) 142° and 38° are supplementary angles.
    (e) If angle A is the conjugate of angle B, then $A = 360° - B$.

▶ How do you solve problems?

Example 1

In the diagram, p and 54° are complementary angles. It is given that p and q are supplementary angles and the angle conjugate to r is 260°. Calculate the values of p, q, r and s.
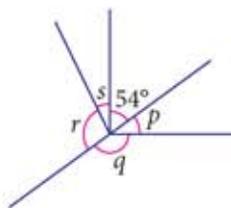
LEARNING STANDARDS

Solve problems involving complementary angles, supplementary angles and conjugate angles.

Solution

$p + 54° = 90°$ ← Complementary angles
Thus, $p = 90° - 54°$
$\qquad = 36°$

$p + q = 180°$ ← Supplementary angles
$36° + q = 180°$
Thus, $q = 180° - 36°$
$\qquad = 144°$

$r + 260° = 360°$ ← Conjugate angles
Thus, $r = 360° - 260°$
$\qquad = 100°$

$s + 100° + 144° + 36° + 54° = 360°$ ← The angle of one whole turn
$\qquad\qquad\qquad s + 334° = 360°$
$\qquad\qquad\qquad\qquad s = 360° - 334°$
$\qquad\qquad\qquad\qquad\quad = 26°$

1. In the diagram, a and $46°$ are complementary angles. It is given that a and b are supplementary angles and the angle conjugate to c is $283°$. Calculate the values of a, b, c and d.
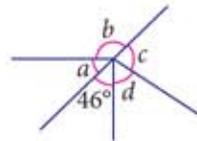
2. It is given that p and q are supplementary angles such that $p > q$. If the difference between p and q is $52°$, calculate the values of p and q.

⏻ How do you perform a geometrical construction?

Logos, house plans or technical designs are drawings that require accurate measurements. These drawings can be done with the help of compasses and straight edge tools only, any geometrical tool such as set squares, rulers or by using geometry software.

The work of using geometrical tools or geometry software to do drawings of accurate measurements is known as a **geometrical construction**.

LEARNING STANDARDS

Construct
(i) line segments,
(ii) perpendicular bisectors of line segments,
(iii) perpendicular line to a straight line,
(iv) parallel lines
and explain the rationale of construction steps.

(i) Line segments

A section of a straight line with a fixed length is known as a **line segment**.

SMART TIPS

Use a sharp pencil while performing geometrical construction and do not erase all construction lines.

Example 2

Construct the line segment AB with a length of 8 cm using only a pair of compasses and a ruler.

Solution

Let's Discuss

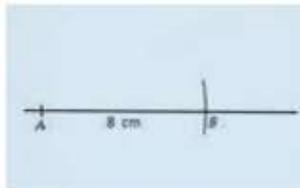Why is the construction of a line segment using only a ruler less accurate? Explain.

❶ Draw a line and mark point A on the line.

❷ Measure a distance of 8 cm on the compasses.

❸ From point A, with the distance of 8 cm on the compasses, mark point B on the line.

CHAPTER 8

(ii) Perpendicular bisectors

If line AB is perpendicular to line segment CD and divide CD into two parts of equal length, then line AB is known as the **perpendicular bisector** of CD.
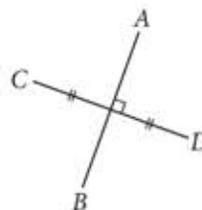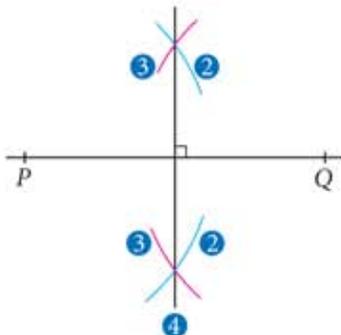

Example 3

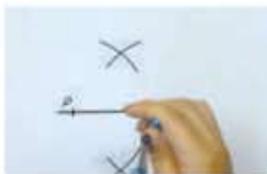Construct the perpendicular bisector of line segment PQ using only a pair of compasses and a ruler.
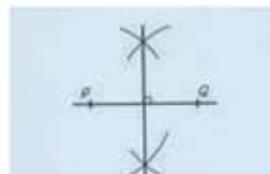

Solution

| ① Adjust and set the compasses to a distance slightly more than half the length of line PQ.	② Construct two arcs from P, one above and one below PQ.	③ Without altering the distance on the compasses, construct two arcs from Q, one above and one below PQ.	④ Draw a line joining the points of intersection of the arcs constructed in Steps **2** and **3**.

In the construction of perpendicular bisector,
(a) if the distance measured on the compasses is less than half the length of PQ, what will happen? Discuss.
(b) is the perpendicular bisector of PQ, which is RS, the axis of symmetry of triangle PQR? Discuss and explain.
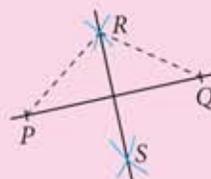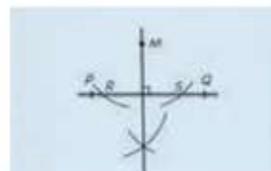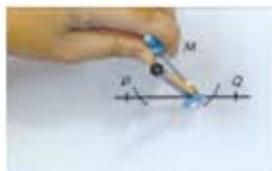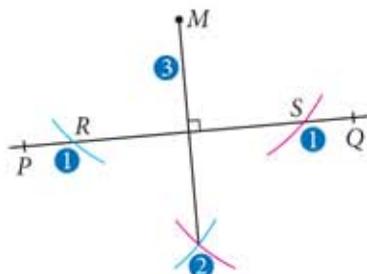

(iii) Perpendicular line to a straight line

Example 4

Using only a pair of compasses and a ruler, construct the perpendicular line from point M to the straight line PQ.

•M

Flashback

If a line is perpendicular to line PQ, then the line is known as **perpendicular line** to line PQ.

P ———————— *Q*

Solution

Let's Discuss

Referring to the construction of a perpendicular line in Example 4, what is the relationship between the construction methods of a perpendicular line and a perpendicular bisector?
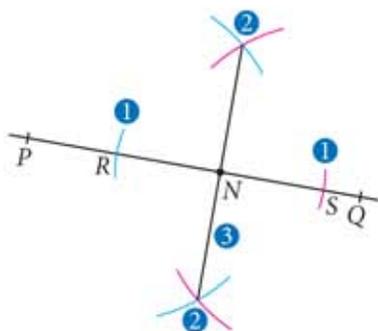

① With the same distance on the compasses from M, construct two arcs from M intersecting PQ. Label the two points of intersection as R and S.

② With the same distance on the compasses from R and from S, construct an arc from R and from S respectively so that the arcs intersect.

③ Draw a line joining M and the point of intersection of the arcs.

Scan the QR Code or visit **https://youtu.be/Raz8EGk4x0Y** to watch the video of the construction of a perpendicular line to a straight line passing through a point. Then, explain the rationale of the construction steps.
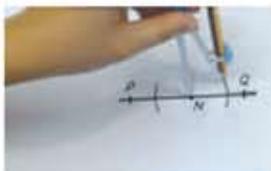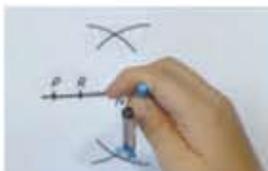
Example 5

Using only a pair of compasses and a ruler, construct the perpendicular line to PQ passing through point N.

Solution

① With the same distance on the compasses, construct two arcs from N intersecting PQ. Label the two points of intersection as R and S.

② Widen the compasses to a distance more than NR or NS, and with the same distance from R and from S on the compasses, construct two arcs above and below PQ respectively so that the arcs intersect.

③ Draw a line joining N and the points of intersection of the arcs constructed in Step **2**.

Scan the QR Code or visit **https://youtu.be/FoxcZU8P7JA** to watch the video of the construction of a perpendicular line to a straight line passing through a point on the straight line. Then, explain the rationale of the construction steps.

Scan the QR Code or visit **https://youtu.be/Kow8PWTwIRM** to watch the video of the construction of a perpendicular line to a straight line using a set square. Hence, construct the perpendicular line in Example 4 and Example 5 by using only a set square and a ruler.

(iv) Parallel lines

Example 6

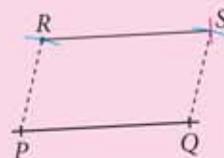Using only a pair of compasses and a ruler, construct the line that is parallel to PQ passing through point R.

$\overset{\bullet}{R}$

Flashback

Lines that will never meet even when they are extended are known as **parallel lines**.

Solution

Let's Discuss
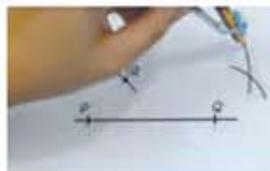
Explain how the construction of parallel lines relates to the construction of parallelogram $PQSR$.

① With the distance on the compasses equals to the length of PR, construct an arc from P passing through R.

② Keeping the same distance on the compasses, construct an arc from Q.

③ With the distance on the compasses equals to the length of PQ, construct an arc from R intersecting the arc constructed in Step **2**.

④ Draw a line joining R and the point of intersection constructed in Step **3**.

Scan the QR Code or visit **https://youtu.be/ Xwa44YIHcCg** to watch the video of the construction of parallel lines. Then, explain the rationale of the construction steps.

Scan the QR Code or visit **https:// youtu.be/J8Im4Rz5poc** to watch the video of the construction of parallel lines using a set square. Hence, construct the parallel lines in Example 6 by using only a set square and a ruler.

1. Construct each of the following line segments:
   (a) $AB = 6$ cm   (b) $PQ = 5.4$ cm   (c) $RS = 7.3$ cm

2. Copy and construct the perpendicular bisector of each of the following line segments:
   (a)

   P   Q

   (b)

   A   B

3. For each of the following, copy and construct the perpendicular line to line segment PQ passing through point M.
   (a)

   P   Q

   M

   (b)

   P   M   Q

4. For each of the following, copy and construct a line parallel to line PQ passing through point M.
   (a)

   P   Q

   M

   (b)

   M

   P   Q

▶ How do you construct angles and angle bisectors?

(i) Constructing an angle of 60°

Other than using a protractor to construct an angle, an **angle of 60°** can be constructed using only a pair of compasses and a ruler based on the **concept of interior angles of an equilateral triangle**. The construction of three line segments of equal length forms a triangle with **interior angles of 60°**.
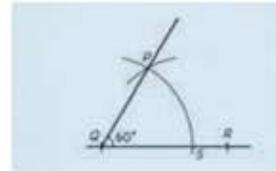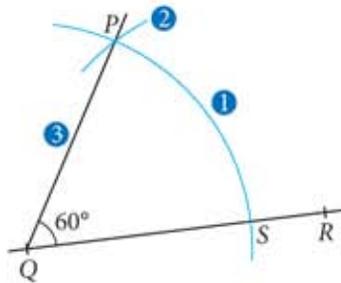
LEARNING STANDARDS

Construct angles and angle bisectors, and explain the rationale of construction steps.

Example   7

Using only a pair of compasses and a ruler, construct line PQ so that $\angle PQR = 60°$.

Q   R

Solution

① Construct a long arc from Q intersecting QR. Mark the point of intersection as S.

② Keeping the same distance on the compasses, construct an arc from S intersecting the arc constructed in Step **1**. Mark the point of intersection as P.

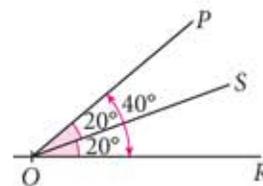③ Draw a line joining point P and point Q. Thus, $\angle PQR = 60°$.

Scan the QR Code or visit **https://youtu.be/urXMY5G2-6Y** to watch the video of the construction of an angle of 60°. Then, explain the rationale of the construction steps.

(ii) Angle bisectors

If a line divides an angle into two equal angles, the line is known as an **angle bisector**.

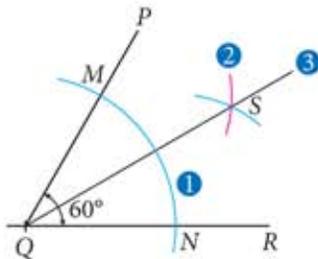For instance, line OS is the angle bisector of $\angle POR$.

Example   8

Using only a pair of compasses and a ruler, construct the angle bisector of $\angle PQR$.
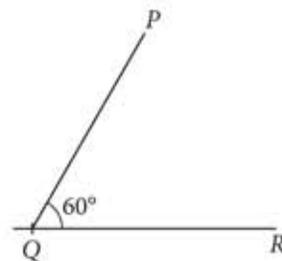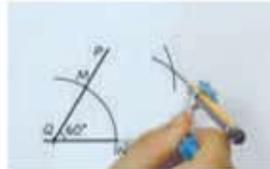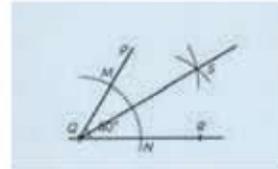
Solution

① Construct a long arc from Q intersecting both arms QP and QR. Mark the two points of intersection as M and N.

② With the same distance on the compasses, construct an arc from M and from N respectively so that the arcs intersect. Mark the point of intersection as S.

③ Draw a line joining point Q and point S. QS is the angle bisector of $\angle PQR$.

It is found that $\angle PQS = \angle SQR = 30°$. We have actually constructed an angle of $30°$. Beginning with the construction of an angle of $60°$ and the knowledge of an angle bisector, we can construct other angles accurately using only a pair of compasses and a ruler. For instance,

120°

90°

15°
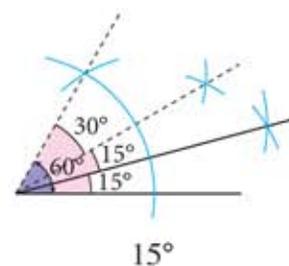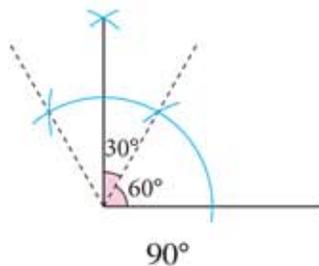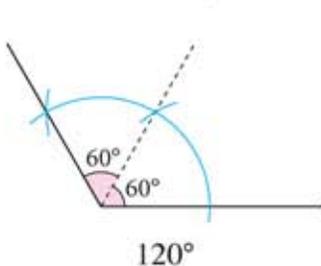
Let's Discuss

(a) What is the rationale for the construction of angles of 120°, 90° and 15° as shown above?
(b) Discuss and state other angles that can be constructed by using only a pair of compasses and a ruler.

Self Practice 8.1g

1. For each of the following angles, explain briefly how you would construct the angle using only a pair of compasses and a ruler. Then, construct the angle.
   (a) 90°   (b) 45°   (c) 75°   (d) 105°

1. Determine whether each of the following statements is TRUE or FALSE.
   (a) A reflex angle is greater than the angle on a straight line.
   (b) The sum of an obtuse angle and an acute angle yields a reflex angle.
   (c) Two adjacent angles on a straight line are supplementary angles.
   (d) The sum of two conjugate angles is the angle of one whole turn.
   (e) If m and n are conjugate angles, then either m or n is a reflex angle but not both of them.

2. It is given that p and q are congruent angles and also complementary angles. State the values of p and q.

3. It is given that $PQ = 7$ cm. If PQ and RS are congruent line segments, state the length of line segment RS.

4. If p and q are supplementary angles and the difference between p and q is $30°$, find the values of p and q such that $p > q$.

5. If p and q are conjugate angles and the size of q is four times that of p, find the values of p and q.

6. (a) Using only a ruler and a pair of compasses,
      (i) copy the given line segment PQ and construct a triangle PQR beginning from the line PQ such that $QR = 6$ cm and $\angle RPQ = 60°$,
      (ii) then, construct the perpendicular line to PR passing through point Q.
   (b) Based on your construction in (a), measure the perpendicular distance from Q to line PR.

7. The diagram shows a triangle ABC. Using only a ruler and a pair of compasses,
   (a) copy the given line segment AB and construct triangle ABC according to the given measurements beginning from straight line AB.
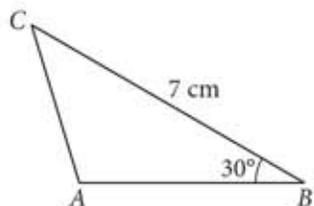
   (b) then, construct the perpendicular bisector of line AC. The perpendicular bisector intersects line AC at point P and intersects line BC at point Q. Measure $\angle PQC$.

8.2 Angles related to Intersecting Lines

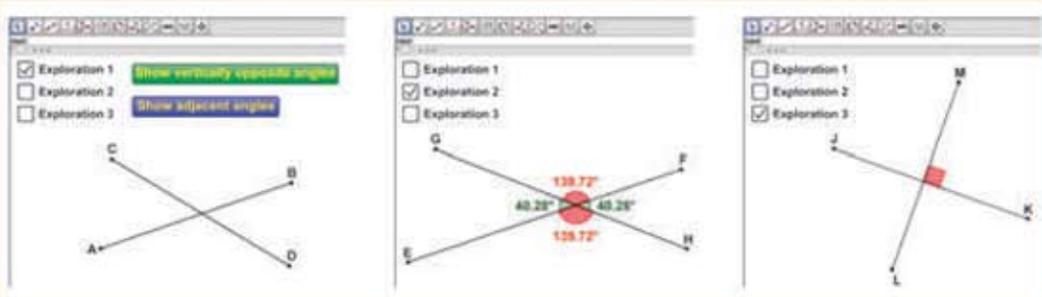▶ **What are vertically opposite angles and adjacent angles at intersecting lines?**

Exploration Activity 4

Aim: To explore angles at intersecting lines.

Instruction: • Explore by yourself before the lesson begins and discuss in groups of four during the lesson.
• Open the folder downloaded from page vii.

1. Open the file *Angles intersecting lines.ggb* using *GeoGebra*.

2. Select 'Exploration 1'. The screen displayed shows two intersecting lines.

3. Click at 'Show vertically opposite angles' and 'Show adjacent angles'.

4. Discuss with your friends and explain the meanings of vertically opposite angles and adjacent angles at intersecting lines. Then, state another pair of vertically opposite angles and another pair of adjacent angles.

5. Select 'Exploration 2'. Click and drag the black points and observe the displayed angles.

6. Discuss with your friends and explain the properties of vertically opposite angles and adjacent angles at intersecting lines.

7. Select 'Exploration 3'. The screen displayed shows two lines perpendicular to each other. What conclusion can you make about the angles at the intersecting lines?

8. Discuss with your friends and make an overall conclusion about angles at intersecting lines.
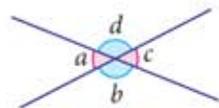
From the results of Exploration Activity 4, it is found that

(i) **vertically opposite angles** at intersecting lines are equal.

(ii) the sum of **adjacent angles** at intersecting lines is $180°$.

$$\angle a = \angle c \qquad \angle a + \angle d = 180° \qquad \angle c + \angle b = 180°$$
$$\angle b = \angle d \qquad \angle d + \angle c = 180° \qquad \angle b + \angle a = 180°$$

(iii) if two intersecting lines are perpendicular to each other, then all angles at the intersecting lines have the same size of $90°$.

Example 9

The diagram shows two intersecting lines. Identify and state

(a) the vertically opposite angles.

(b) the adjacent angles at the intersecting lines.
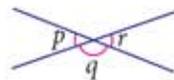
Solution

(a) Angles p and r are the vertically opposite angles.

(b) Angles p and q, q and r are the adjacent angles at the intersecting lines.

Self Practice 8.2a

1. For each of the following diagrams, identify and state
   (i) the vertically opposite angles.
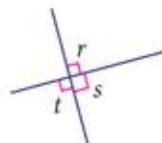   (ii) the adjacent angles at the intersecting lines.
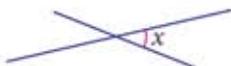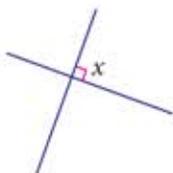
   (a)   (b)   (c)
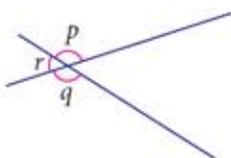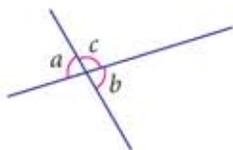
2. Copy each of the following diagrams, mark and label
   (i) the angle y if angles x and y are vertically opposite angles.
   (ii) the angle z if angles x and z are adjacent angles at the intersecting lines.
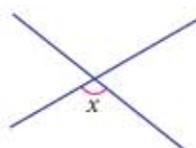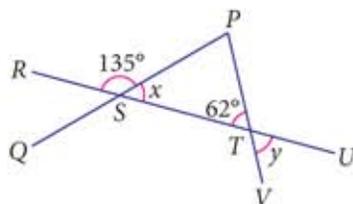
   (a)   (b)   (c)

▶ How do you determine the values of the angles at intersecting lines?

Example 10

In the diagram, PSQ, $RSTU$ and PTV are straight lines. Find the values of x and y.

LEARNING STANDARDS

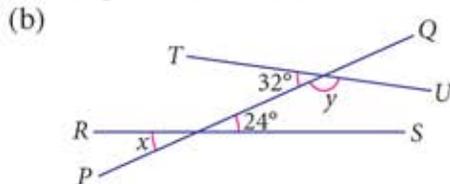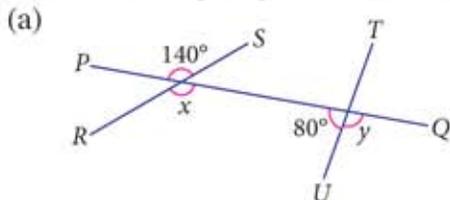Determine the values of angles related to intersecting lines, given the values of other angles.

Solution

$x + 135° = 180°$ ← Adjacent angles at intersecting lines

$\quad\quad x = 180° - 135°$

$\quad\quad\quad = 45°$

$y = 62°$ ← Vertically opposite angles

Self Practice 8.2b

1. In the following diagrams, PQ, RS and TU are straight lines. Find the values of x and y.

(a)

(b)

▶ How do you solve problems?

Example 11

In the diagram, POR and TOQ are straight lines. Find the values of x and y.

Solution

$\angle POT = 72°$

$x + 60° = 72°$ ← $\angle POT$ and $\angle QOR$ are vertically opposite angles.

$\quad\quad x = 72° - 60°$

$\quad\quad\quad = 12°$

$\angle TOR + 72° = 180°$ ← $\angle QOR$ and $\angle TOR$ are adjacent angles at intersecting lines.

$\quad\quad \angle TOR = 180° - 72°$

$\quad\quad\quad\quad = 108°$

$\quad\quad y + y = 108°$

$\quad\quad\quad 2y = 108°$

$\quad\quad\quad\quad y = 54°$

Self Practice 8.2c

1. In the diagram, POR and QOT are straight lines. Find the values of x and y.
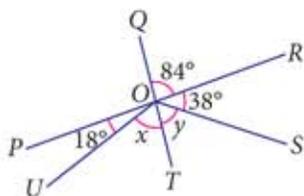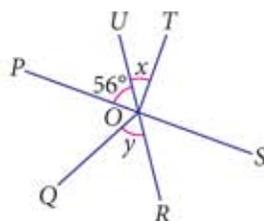

2. In the diagram, QOS and ROU are straight lines. OT is the bisector of $\angle UOS$. $\angle POQ$ and $\angle QOR$ are complementary angles. Find the values of x and y.
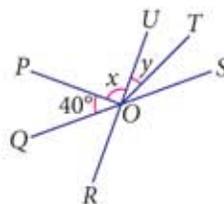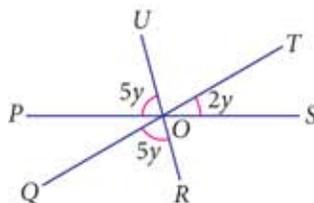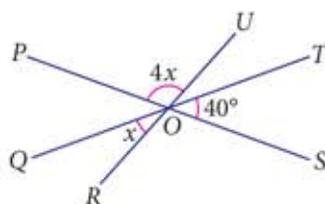
1. In the diagram, *POS* and *UOR* are straight lines. *OQ* is the bisector of $\angle POR$. $\angle POU$ and $\angle UOT$ are complementary angles. Find the values of x and y.

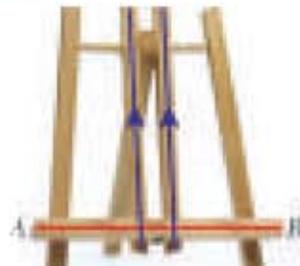2. In the diagram, *POS*, *QOT* and *UOR* are straight lines. Find the value of y.

3. In the diagram, *POS*, *QOT* and *ROU* are straight lines. Find the value of x.

8.3 Angles related to Parallel Lines and Transversals

▶ **What are parallel lines and transversals?**

Parallel lines can be denoted using arrow heads '→' as shown in the photograph.

A straight line that intersects two or more straight lines is known as a **transversal**. For instance, straight line *AB* shown in the photograph is a transversal.
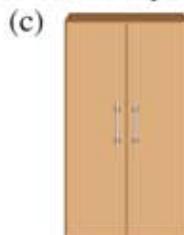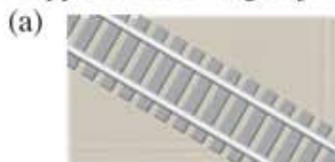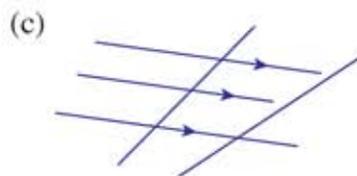
Self Practice **8.3a**

1. Copy the following objects. Draw and mark parallel lines on the copied objects.
   (a)
   (b)
   (c)

2. Copy each of the following diagrams and label the transversal as *AB*. Then draw another transversal and label the line as *PQ*.

(a)

(b)

(c)
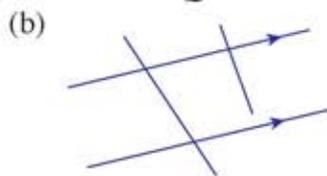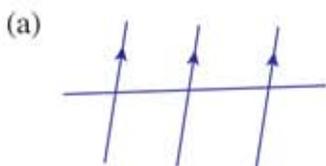
3. For each of the following diagrams, determine if the line labelled as *PQ* is a transversal or not, and make justification.

(a) P

Q

(b) P

Q

(c) P

Q

▶ **What are corresponding angles, alternate angles and interior angles?**
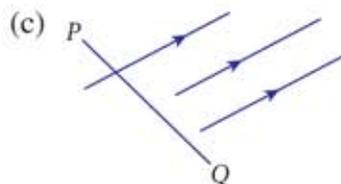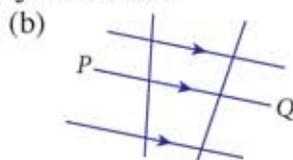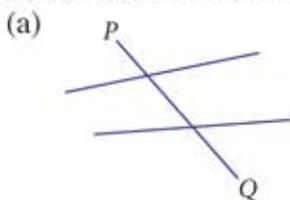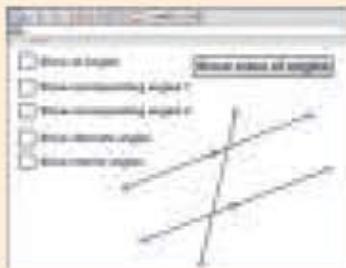
Exploration Activity 5

Aim: To explore angles related to parallel lines.

Instruction: • Explore by yourself before the lesson begins and discuss in groups of four during the lesson.
• Open the folder downloaded from page vii.

1. Open the file *Angles parallel lines.ggb* using *GeoGebra*.

2. Click and drag the points on the screen displayed and identify the parallel lines and the transversal.

3. Select 'Show corresponding angles 1' and 'Show corresponding angles 2' to explore the properties of corresponding angles.

4. Click at 'Show sizes of angles' and drag the points on the screen displayed if necessary.

5. State another pair of corresponding angles.

6. Discuss with your friends and explain the properties of corresponding angles.

7. Select 'Show alternate angles' and 'Show interior angles' and continue exploring alternate angles and interior angles.

8. State another pair of alternate angles and another pair of interior angles.

9. Discuss with your friends and explain the properties of alternate angles and interior angles.

10. State the conclusions that can be made from your explorations.
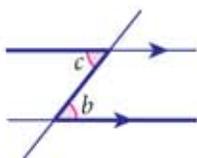
CHAPTER

8

From the results of Exploration Activity 5, it is found that when a transversal intersects two or more parallel lines,
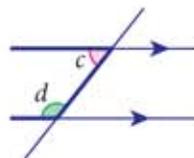
(i) the corresponding angles are equal.
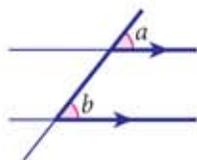
Thus, $\angle a = \angle b$

(ii) the alternate angles are equal.
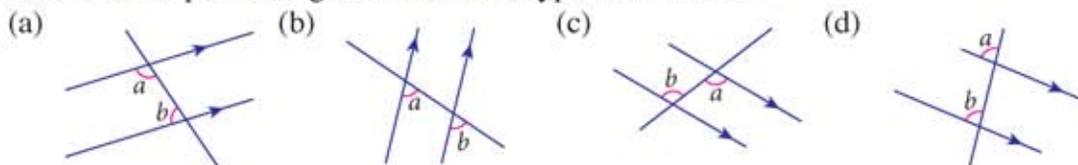

Thus, $\angle b = \angle c$

(iii) the sum of the interior angles is 180°.

Thus, $\angle c + \angle d = 180°$

Self Practice   **8.3b**

1. Copy each of the following diagrams. Explain and state whether the two angles labelled as a and b are corresponding angles, alternate angles or interior angles. Then label another pair of angles of the same type with c and d.

   (a)

   (b)

   (c)

   (d)

2. Copy each of the following diagrams and label
   (i)   angle p if a and p are corresponding angles.
   (ii) angle q if b and q are alternate angles.
   (iii) angle r if c and r are interior angles.

   (a)

   (b)

▶ **How do you determine whether two straight lines are parallel?**

Example   12

In each of the following diagrams, determine whether straight line PQ and straight line RS are parallel.
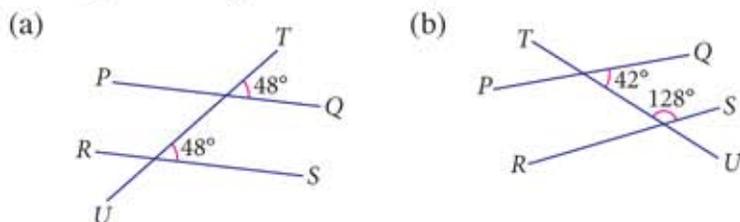
   (a)

   (b)

CHAPTER 8

Solution

(a) PQ and RS are parallel. ← ⟨The corresponding angles, 48°, are equal.⟩

(b) $42° + 128° = 170°$
$$\neq 180°$$ ← ⟨The sum of the interior angles should be 180° if PQ and RS are parallel.⟩

Thus, PQ and RS are not parallel.

Self Practice **8.3c**

1. For each of the following diagrams, determine whether line PQ and line RS are parallel.
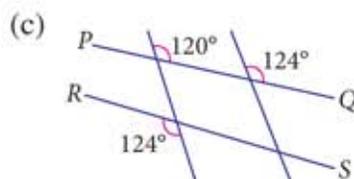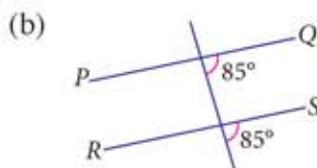
(a)

(b)

(c)

▶ How do you determine the values of angles related to parallel lines and transversals?

Example 13

In the diagram, PQ, RS and TU are parallel lines. Find the values of a, b and c.
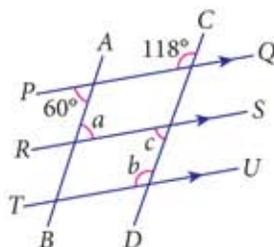
Solution

$a = 60°$
↑ ⟨Alternate angles⟩

$b = 118°$
↑ ⟨Corresponding angles⟩

$b + c = 180°$ ← ⟨Interior angles⟩
$118° + c = 180°$
$c = 180° - 118°$
$= 62°$

Self Practice **8.3d**

1. In the diagram, EF and GH are straight lines. Find the values of a, b, c and d.

2. In the diagram, find the values of a and b.
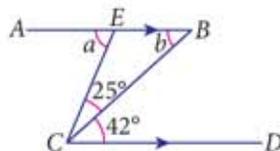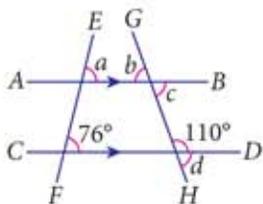
▶ What are angles of elevation and depression?

The concept of angles is very important in land surveying. A land surveyor uses angles to determine the distance. The two types of angles used are the angle of elevation and the angle of depression.

Exploration **Activity 6**

Aim: To recognise the angle of elevation and the angle of depression.

Instruction: • Explore by yourself before the lesson begins and discuss in groups of four during the lesson.
• Open the folder downloaded from page vii.

1. Open the file *Angles of elevation and depression.ggb* using *GeoGebra*.

2. Select 'Angle of elevation'. The screen displayed shows point *A* and point *B* at different levels.

3. Click at the button on the screen to explore the angle of elevation.

4. Observe the animation on the screen and explain in general the meaning of the angle of elevation.

5. Click and drag point *A* or point *B* to different positions and click the button on the screen for further exploration about the angle of elevation.

6. Select 'Angle of depression' and repeat Steps 3 to 5 to explore the angle of depression.

7. Discuss with your friends the angle of elevation and the angle of depression.
   (a) How are the positions of *A* and *B* related to the angle of elevation and the angle of depression?
   (b) If *A* is at a higher level than *B* or vice versa, explain the angle of elevation or the angle of depression between *A* and *B*.

8. Discuss with your friends and state the conclusions that can be made about the angle of elevation and the angle of depression.
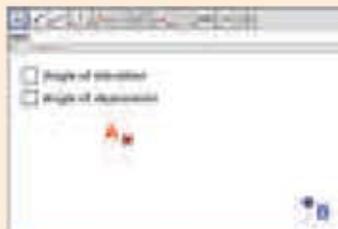
From the results of Exploration Activity 6, it is found that
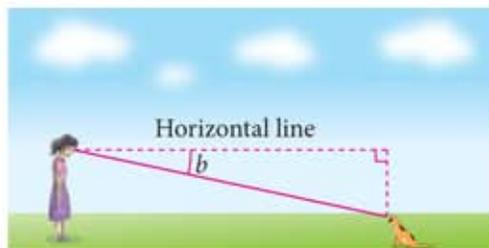
(i) When a person sees an object which is at a higher level, the angle between the line of sight and the horizontal line is known as the **angle of elevation**.

Angle a is the angle of elevation of the bird from the sight of Jefri.

(ii) When a person sees an object which is at a lower level, the angle between the line of sight and the horizontal line is known as the **angle of depression**.

Angle b is the angle of depression of the cat from the sight of Kim.

Let's Discuss

When we figure out the angle of elevation and the angle of depression between two objects at different levels,
(a) is the angle of elevation always the same as the angle of depression?
(b) what is the relation between the angle of elevation and the angle of depression?

Let's Discuss

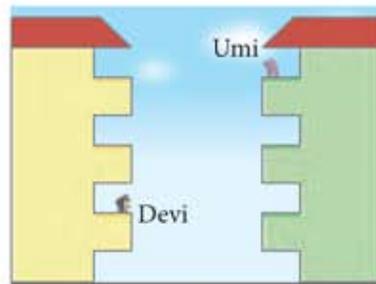"The angle of elevation and the angle of depression are always acute angles."
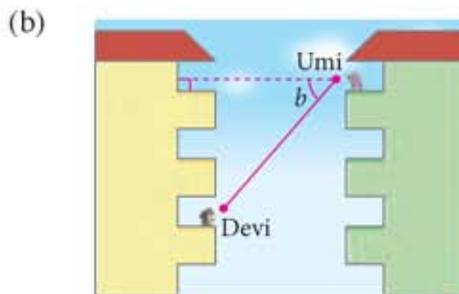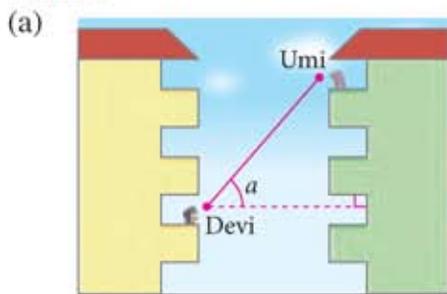Is this statement true? Discuss.

Example 14

The diagram shows the positions of Devi and Umi at two buildings on a horizontal ground. Draw and label
(a) angle a to denote the angle of elevation of Umi from Devi.
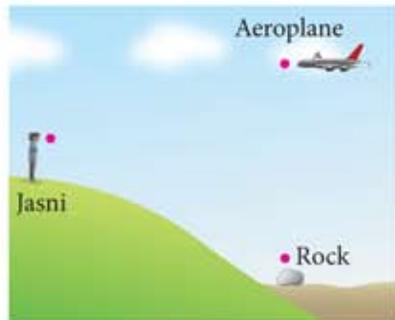(b) angle b to denote the angle of depression of Devi from Umi.

Solution

(a)

(b)

CHAPTER 8

1. Jasni is standing on top of a hill observing an aeroplane and a rock. Represent the situation by drawing and labelling
   (a) angle a which represents the angle of elevation of the aeroplane from Jasni.
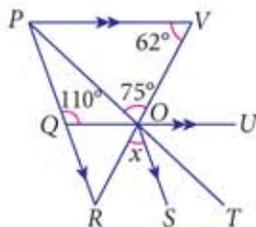   (b) angle b which represents the angle of depression of the rock from Jasni.

How do you solve problems?

Example 15

In the diagram, POT, QOU, PQR and VOR are straight lines.
(a) Find the value of x.
(b) Assuming PV is a horizontal line and $\angle PVO$ is the angle of depression of O from V, find the angle of elevation of P from O.

LEARNING STANDARDS

Solve problems involving angles related to parallel lines and transversals.

Solution

(a) $\angle QOS = \angle PQO$ ← Alternate angles
           $= 110°$
    $\angle QOR = \angle PVR$ ← Corresponding angles
           $= 62°$
    Thus, $x = 110° - 62°$
           $= 48°$

(b) $\angle QOV + 62° = 180°$ ← The sum of the interior angles
    $\angle QOV = 180° - 62°$
           $= 118°$
    $\angle POQ = 118° - 75°$
           $= 43°$
    Thus, the angle of elevation of P from O is $43°$.

Career in Mathematics

Photographers and videographers use the knowledge of lines and angles to help them capture photographs and record videos skilfully.

1. In the diagram, PQR and QTU are straight lines.
   (a) Find the values of x and y.
   (b) Assuming WV is a horizontal line, find the angle of elevation of V from T.

Mastery Q **8.3**

Open the folder downloaded from page vii for extra questions of Mastery Q 8.3.

1. In the diagram, *PQR* and *UTS* are straight lines. Find the values of *x* and *y*.

2. In the diagram, *PQRS* is a straight line. Find the values of *x* and *y*.

3. In the diagram, *QRST* is a straight line. Find the values of *x* and *y*.

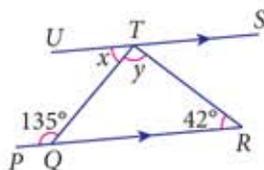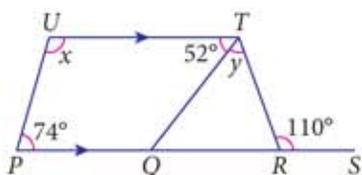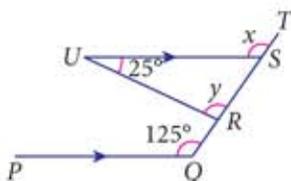4. The diagram shows a hexagonal wooden frame *PQRSTU* resting on a horizontal ground.
   (a) State the angle of elevation of *Q* from *O*.
   (b) State the angle of depression of *T* from *O*.
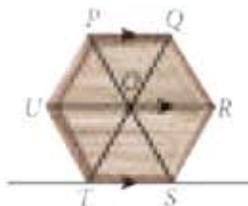
5. In the diagram, *VUTS* and *PQR* are straight lines.
   (a) Find the values of *x* and *y*.
   (b) Find the reflex angle *QWU*.
   (c) Assuming *VUTS* is a horizontal line, find the angle of depression of *R* from *T*.
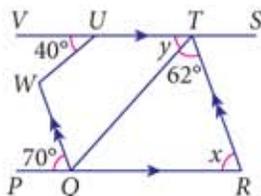
6. The diagram shows the positions of Kamal, Cindy, Adila and David in two condominiums. The angle of elevation of Cindy from David is 15°, the angle of elevation of Adila from Cindy is 18° and the angle of depression of Adila from Kamal is 40°. Find the values of *x* and *y*.
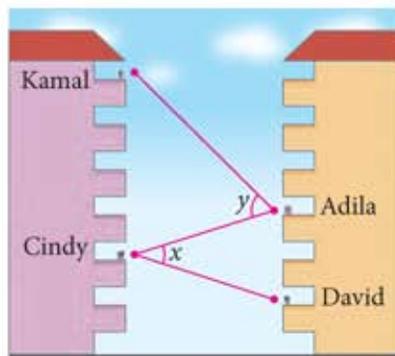
SUMMARY

LINES AND ANGLES

Line segment	Perpendicular bisector	Perpendicular line	Parallel lines	Construction of 60°	Angle bisector

The angle on a straight line	Reflex angle	The angle of one whole turn	Complementary angles	Supplementary angles	Conjugate angles
180°		360°			

Angles related to intersecting lines

- Vertically opposite angles
  $\angle a = \angle c$ and $\angle b = \angle d$
- Adjacent angles at intersecting lines
  $\angle a + \angle d = 180°$
  $\angle d + \angle c = 180°$
  $\angle c + \angle b = 180°$
  $\angle b + \angle a = 180°$

Angles related to parallel lines and transversals

Transversal

- Corresponding angles
  $\angle p = \angle s$
- Alternate angles
  $\angle r = \angle s$
- Interior angles
  $\angle q + \angle s = 180°$

- Angle of elevation

  Angle of elevation

- Angle of depression

  Angle of depression

| At the end of this chapter, I can...	Very good	Work harder
determine and explain the congruency of line segments and angles.		
estimate and measure the size of line segments and angles.		
recognise, compare and explain the properties of angles on a straight line, reflex angles, and one whole turn angles.		
describe the properties and solve problems involving complementary angles, supplementary angles and conjugate angles.		
construct line segments, perpendicular bisectors of line segments, perpendicular line to a straight line, parallel lines and explain the rationale of construction steps.		
construct angles and angle bisectors and explain the rationale of construction steps.		
identify, explain and draw vertically opposite angles and adjacent angles at intersecting lines, including perpendicular lines.		
determine the values of angles and solve problems involving angles related to intersecting lines.		
recognise, explain and draw parallel lines, transversals, corresponding angles, alternate angles and interior angles.		
determine whether two straight lines are parallel.		
determine the values of angles and solve problems involving angles related to parallel lines and transversals.		
recognise and represent angles of elevation and angles of depression in real-life situations.		

CHAPTER 8

Let's PRACTISE

Test Yourself

1. In the diagram, x and $25°$ are complementary angles. Find the values of x and y.
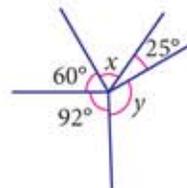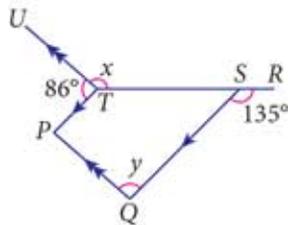
2. The diagram shows a straight line *POR*. Find the values of *x* and *y*.
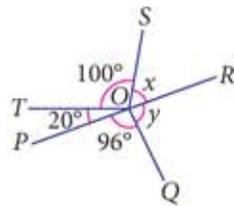
3. Using only a pair of compasses and a ruler, construct a line segment *AB* with a length of 8 cm. Then construct point *C* such that $\angle ABC = 60°$ and *BC* = 5 cm. Then, construct the perpendicular line from *C* to *AB*.
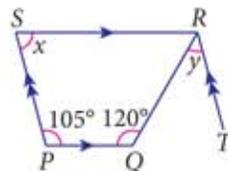
Self Mastery

4. *TSR* is a straight line as shown in the diagram. Find the values of *x* and *y*.

5. If *x* and *y* are supplementary angles and *x* : *y* = 2 : 3, find the values of *x* and *y*.

6. (a) In the diagram, find the values of *x* and *y*.
   (b) Hence, using only a pair of compasses and a ruler, construct the trapezium *PQRS* such that *PQ* = 4 cm and *PS* = 6 cm.
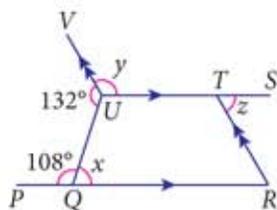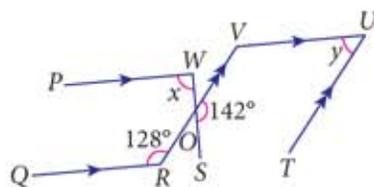

Challenge Yourself

7. *PS* and *QT* are straight lines as shown in the diagram. Find the values of *x* and *y*.
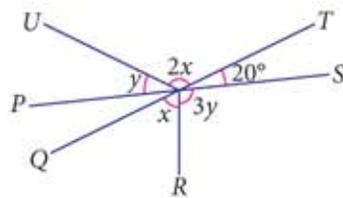

8. In the diagram, *UTS* and *PQR* are straight lines. Find the values of *x*, *y* and *z*.

9. The diagram shows two straight lines, *WOS* and *ROV*. Find the values of *x* and *y*.

10. The diagram shows the positions of three friends. Hui Li is on a hot-air balloon, Kamala is on a horizontal ground and Rizal is on top of a hill. The angle of depression of Kamala from Hui Li is 78°. Based on the information given in the diagram, find
  (a) the angle of depression of Rizal from Hui Li.
 (b) the angle of elevation of Rizal from Kamala.
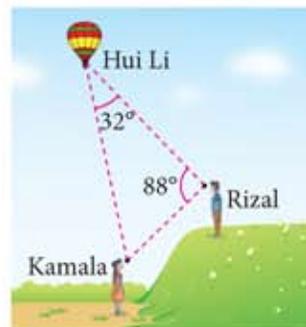
ASSIGNMENT

Hafeeza is assigned to take photographs during her elder sister's wedding. Explain how the knowledge of lines like parallel lines and perpendicular lines as well as the knowledge of angles like angle of elevation and angle of depression can help Hafeeza solve problems like positioning the cameras, choice of lenses, perspective issue and composition issue of subjects in the photographs and others. Write a report and present your finding during the lesson.

Exploring MATHEMATICS

Open the folder downloaded from page vii. Open the file *Angle of 120.ggb* using *GeoGebra*. The screen displayed shows the construction of 120° using only ⊙ ⊿ ⟋ in *GeoGebra*. Explain the rationale behind the construction.

Using only ⟋ ⊙ ⟋ ⟋ ⟋ in *GeoGebra*, and beginning with a given line segment *AB*, construct each of the following:

(a) The perpendicular bisector of *AB*.
(b) The line perpendicular to *AB* passing through a given point.
(c) An angle of 30° such that *AB* is one of the arms of the angle.
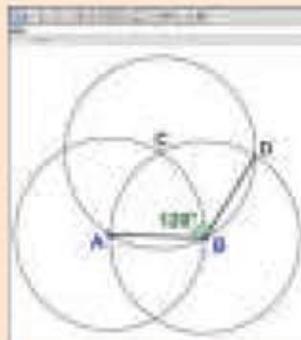
Present your work in class during the lesson by explaining the rationale behind the constructions that you have done.