

# CHAPTER 9

## Basic Polygons



### What will you learn?

- Polygons
- Properties of Triangles and the Interior and Exterior Angles of Triangles
- Properties of Quadrilaterals and the Interior and Exterior Angles of Quadrilaterals

### Why study this chapter?

As the basic knowledge about the uses of polygons in the field of building construction. Discuss other fields that involve the use of polygons.



Putra Mosque is one of the main landmarks in Putrajaya and a tourist attraction because of the beautiful and magnificent designs. The combination of the various polygons exhibits the uniqueness in the artwork.



How are polygons used in the artwork of buildings? What types of polygons are used?



## Walking through Time

Carl Friedrich Gauss (1777 – 1855) was a German mathematician who contributed much in the field of mathematics. One of his major discoveries was introducing the techniques to construct a 17-sided polygon using only a pair of compasses and a straight edge.



Carl Friedrich Gauss

For more information:



<https://goo.gl/Tvk2d3>

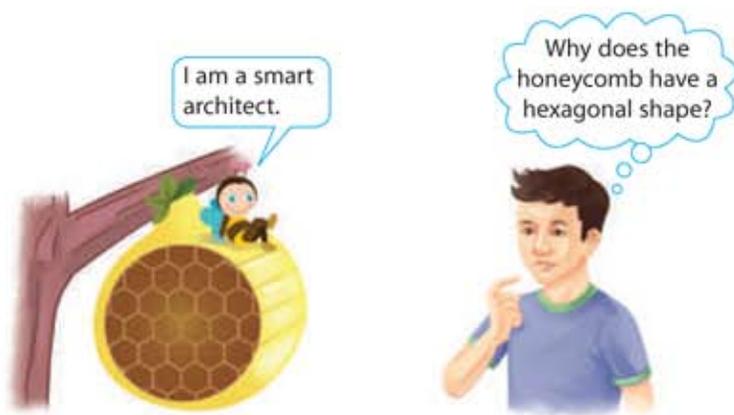
### Word Link



- |                  |                            |
|------------------|----------------------------|
| • vertex         | • <i>bucu</i>              |
| • conjecture     | • <i>konjektur</i>         |
| • kite           | • <i>layang</i>            |
| • diagonal       | • <i>pepenjuru</i>         |
| • rhombus        | • <i>rombus</i>            |
| • square         | • <i>segi empat sama</i>   |
| • parallelogram  | • <i>segi empat selari</i> |
| • rectangle      | • <i>segi empat tepat</i>  |
| • triangle       | • <i>segi tiga</i>         |
| • side           | • <i>sisi</i>              |
| • quadrilateral  | • <i>sisi empat</i>        |
| • obtuse angle   | • <i>sudut cakah</i>       |
| • interior angle | • <i>sudut pedalaman</i>   |
| • exterior angle | • <i>sudut peluaran</i>    |
| • right angle    | • <i>sudut tegak</i>       |
| • acute angle    | • <i>sudut tirus</i>       |
| • trapezium      | • <i>trapezium</i>         |

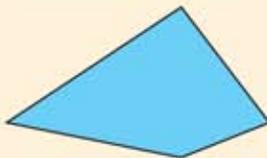
Open the folder downloaded from page vii for the audio of Word Link.

## 9.1 Polygons



### Flashback

A polygon is an enclosed figure on a plane bounded by 3 or more straight sides.



▶ What is the relationship between the number of sides, vertices and diagonals of a polygon?

### Exploration Activity 1



### LEARNING STANDARDS

State the relationship between the number of sides, vertices and diagonals of polygons.

**Aim:** To explore the number of sides, vertices and diagonals of a polygon.

**Instruction:**

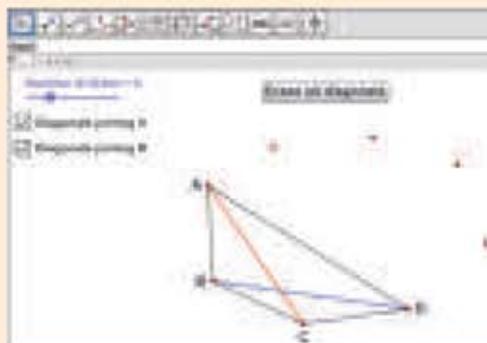
- Explore by yourself before the lesson begins and discuss in groups of four during the lesson.
- Open the folder downloaded from page vii.

1. Open the file *Diagonals of polygon.ggb* using *GeoGebra*.

2. Click and drag the slider 'Number of Sides' to change the number of sides of the polygon displayed.

3. Click at the checkboxes to display the diagonals of polygon.

4. Record the number of sides, the number of vertices and the number of diagonals displayed.



5. Open the spreadsheet file *Table of diagonals of polygon.xls* and type all the values obtained in Step 4 in the cells provided.

Polygon with vertices	Number of sides	Number of vertices	Number of diagonals joining						Total number of diagonals
			A (red)	B (blue)	C (green)	D (purple)	E (orange)	F (pink)	
ABC	3	3							0
ABCD	4	4							2
ABCDE	5	5							5
ABCDEF	6	6							9
ABCDEFG	7	7							14
ABCDEFGH	8	8							20
	9	9							27
	10	10							35

- Based on the table in the spreadsheet, explain the relationship between the number of sides and the number of vertices of a polygon.
- Discuss with your friends the relationship between the number of sides and the number of diagonals of a polygon.
- Complete the table in the spreadsheet for a polygon of 9 sides and of 10 sides.

From the results of Exploration Activity 1, it is found that for a polygon

- number of vertices = number of sides
- number of diagonals can be determined according to the following steps.

Identify the number of sides of the polygon.

Minus 3 from the number of sides. Let the result be  $m$ .

The value  $m$  is multiplied by 2. Thus we get  $2m$ .

The value  $2m$  is added to all the integers with values from less than  $m$  to 1.

### Example 1

Find the number of vertices and the number of diagonals of a polygon with 10 sides.

#### Solution

Number of vertices = Number of sides  
= 10

$$10 - 3 = 7$$

Thus, the number of diagonals =  $2(7) + 6 + 5 + 4 + 3 + 2 + 1$   
= 35

### SMART TIPS

The number of diagonals of a polygon having  $n$  sides can also be calculated by using the following formula.

Number of diagonals  
=  $\frac{n(n-3)}{2}$



[https://youtu.be/3Zkh0\\_aCebU](https://youtu.be/3Zkh0_aCebU)

### Self Practice 9.1a

- Find the number of vertices and the number of diagonals of a polygon with
  - 6 sides,
  - 9 sides,
  - 12 sides,
  - 20 sides.

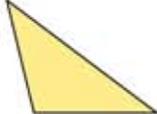
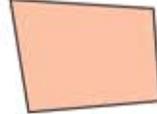
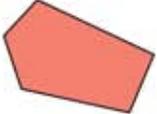
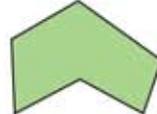
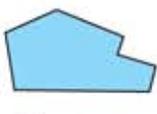
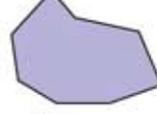
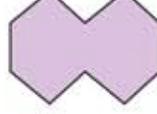
## ▶ How do you draw, label and name a polygon?



### LEARNING STANDARDS

Draw polygons, label vertices of polygons and name the polygons based on the labelled vertices.

A polygon is named according to the number of its sides.

<b>Name of polygon</b>				
	Triangle	Quadrilateral	Pentagon	Hexagon
<b>Number of sides</b>	3	4	5	6
<b>Name of polygon</b>				
	Heptagon	Octagon	Nonagon	Decagon
<b>Number of sides</b>	7	8	9	10

A polygon can be drawn according to the following steps.

Identify the number of sides of the polygon.

Mark points equal in number to the number of sides.

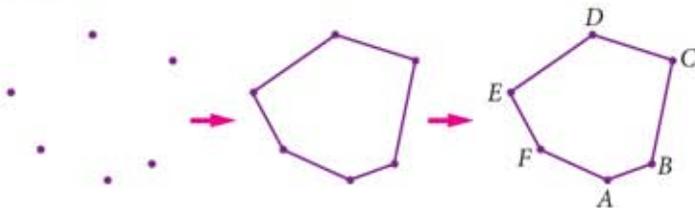
Join all the points with straight lines to form a closed figure.

Label the vertices and name the polygon.

### Example 2

Draw a polygon with six sides. Then label and name the polygon.

**Solution**



Thus, the polygon is a hexagon  $ABCDEF$ .

### SMART TIPS

The vertices of a polygon are usually labelled in alphabetical order and the polygon is named either clockwise or anticlockwise of the vertices.

Make sure that when drawing a polygon, do not mark two or more points that can form a straight line.



## Self Practice 9.1b

1. Draw each of the following polygons according to the given number of sides, then label and name the polygon.  
(a) 5 sides                      (b) 8 sides                      (c) 10 sides

## Mastery Q 9.1



Open the folder downloaded from page vii for extra questions of Mastery Q 9.1.

1. For each of the following, state whether the statement is TRUE or FALSE.  
(a) A polygon with 11 sides has 11 vertices.  
(b) A polygon with 12 sides has 54 diagonals.
2. Draw a polygon with 8 sides. Then label and name the polygon. Finally, based on the results from Exploration Activity 1, draw all the diagonals systematically.

## 9.2 Properties of Triangles and the Interior and Exterior Angles of Triangles

### What are the properties of a triangle?

### Exploration Activity 2

**Aim:** To explore the geometric properties of a triangle.

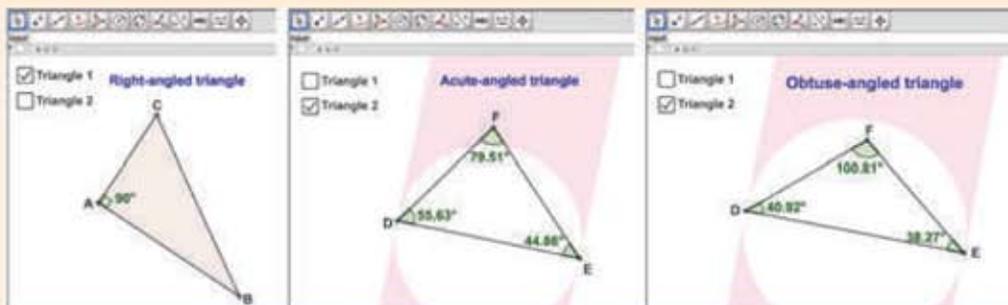
- Instruction:**
- Explore by yourself before the lesson begins and discuss in groups of four during the lesson.
  - Open the folder downloaded from vii.



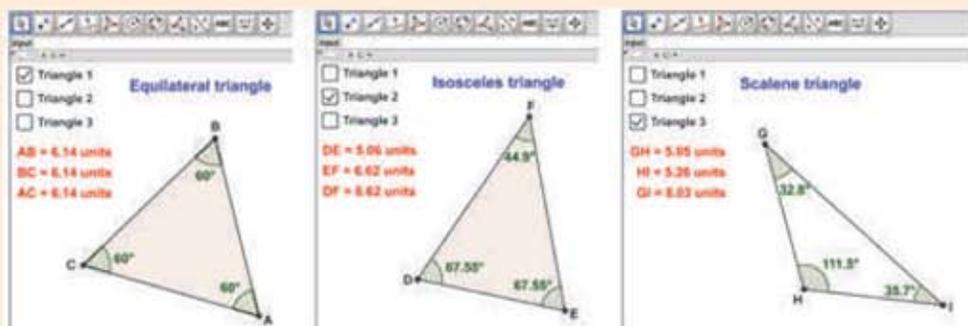
### LEARNING STANDARDS

Recognise and list geometric properties of various types of triangles. Hence classify triangles based on geometric properties.

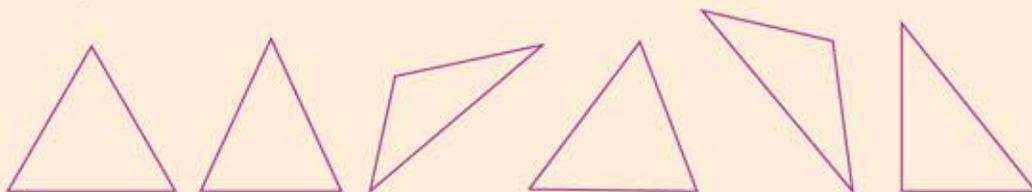
1. Open the file *Triangles properties of angles.ggb* using *GeoGebra*.



- Click at Triangle 1. Click and drag points  $A$ ,  $B$  and  $C$  to change the triangle displayed. State the properties of the right-angled triangle.
- Click at Triangle 2. Click and drag points  $D$ ,  $E$  and  $F$  to change the triangle displayed. Observe the change of types of triangle when point  $F$  lies in the pink region or in the white region. State the properties of the acute-angled triangle and the obtuse-angled triangle.
- Open the file *Triangles properties of sides.ggb* using *GeoGebra*.



- Click at Triangle 1. Click and drag points  $A$  and  $B$  to change the triangle displayed. Observe the change in the length of the sides and the interior angles of the triangle.
- Repeat the exploration in Step 5 for Triangle 2 and Triangle 3.
- State the properties of an equilateral triangle, an isosceles triangle and a scalene triangle.
- Discuss with your friends the properties of the various types of triangles.
- Open and print the file *Triangles axes of symmetry.pdf*. Cut out the triangles in the printout.



- By folding each of the triangles, or otherwise, explain how you can find the number of axes of symmetry for each type of triangle.

From the results of Exploration Activity 2, it is found that triangles can be classified based on the geometric properties of their interior angles or the length of their sides.

Triangles in Table (a) are classified according to the geometric properties of their sides.

Table (a)

Type of triangle	Equilateral triangle	Isosceles triangle	Scalene triangle
Number of axes of symmetry	3	1	None
Geometric properties	<ul style="list-style-type: none"> <li>All the sides are of the same length.</li> <li>Every interior angle is <math>60^\circ</math>.</li> </ul>	<ul style="list-style-type: none"> <li>Two of the sides have the same length.</li> <li>The two base angles are of the same size.</li> </ul>	<ul style="list-style-type: none"> <li>All the sides are of different lengths.</li> <li>All the interior angles are of different sizes.</li> </ul>

Triangles in Table (b) are classified according to the geometric properties of their angles.

Table (b)

Type of triangle	Acute-angled triangle	Obtuse-angled triangle	Right-angled triangle
Geometric properties	<ul style="list-style-type: none"> <li>Every interior angle is an acute angle.</li> </ul>	<ul style="list-style-type: none"> <li>One of the interior angles is an obtuse angle.</li> </ul>	<ul style="list-style-type: none"> <li>One of the interior angles is a right angle (<math>90^\circ</math>).</li> </ul>

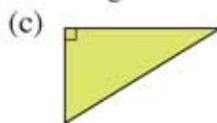
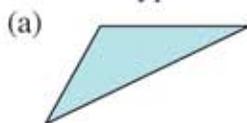
### Let's Discuss

Discuss and explain whether each of the following statements is TRUE or FALSE.

- An equilateral triangle is also an isosceles triangle.
- An isosceles triangle could be an obtuse-angled triangle.
- A right-angled triangle could be an isosceles triangle.
- A right-angled triangle may have axes of symmetry.

### Self Practice 9.2a

1. State the type of triangles for each of the following.



**▶ How do you determine the interior angles and exterior angles of a triangle?**

**LEARNING STANDARDS**

Make and verify conjectures about  
 (i) the sum of interior angles,  
 (ii) the sum of interior angle and adjacent exterior angle,  
 (iii) the relation between exterior angle and the sum of the opposite interior angles of a triangle.

**Exploration Activity 3**

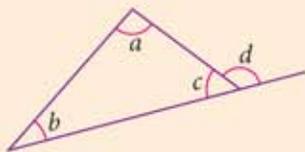


**Aim:** To explore the interior angles and exterior angles of a triangle.

**Instruction:**

- Explore by yourself before the lesson begins and discuss in groups of four during the lesson.
- Open the folder downloaded from page vii.

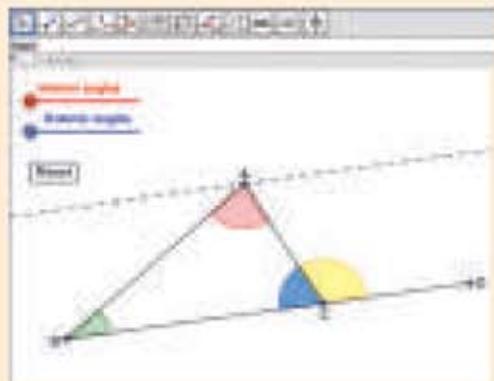
1. Copy and complete the following table to make conjectures about the sum of angles shown in the diagram. Then continue the exploration to verify the conjectures.



Sum of angles	Conjecture
$a + b + c$	
$c + d$	
$a + b$	

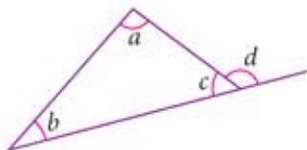
(Notes: To make a conjecture means to make a guess based on observations.)

2. Open the file *Interior and exterior angles of triangle.ggb* using *GeoGebra*.
3. Click and drag the slider 'Interior angles' towards the right. Explain what you observe.
4. Click at 'Reset' or click and drag the slider back to its original position.
5. Click and drag points *A*, *B* or *C* to change the shape of the triangle and repeat Step 3.
6. Discuss with your friends your findings.
7. Repeat Steps 3 to 6 for slider 'Exterior angles'.
8. Consider the angle on a straight line, explain the relationship between the interior angle of a triangle (the angle in blue colour) and its adjacent exterior angle (the angle in yellow colour). Explain all the conclusions arrived at.



From the results of Exploration Activity 3, it is found that for a triangle,

- the sum of all the interior angles is  $180^\circ$ .
- the sum of an interior angle and its adjacent exterior angle is  $180^\circ$ .
- an exterior angle is the sum of two opposite interior angles.



$$\begin{aligned} a + b + c &= 180^\circ \\ c + d &= 180^\circ \\ d &= a + b \end{aligned}$$

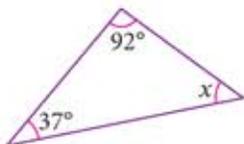
### Flashback

The angle on a straight line is  $180^\circ$ .

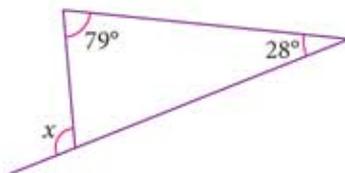
### Example 3

Find the value of  $x$  in each of the following diagrams.

(a)



(b)



### Solution

$$\begin{aligned} \text{(a)} \quad 92^\circ + 37^\circ + x &= 180^\circ \\ 129^\circ + x &= 180^\circ \\ x &= 180^\circ - 129^\circ \\ &= 51^\circ \end{aligned}$$

The sum of all the interior angles is  $180^\circ$ .

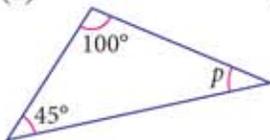
$$\begin{aligned} \text{(b)} \quad x &= 79^\circ + 28^\circ \\ &= 107^\circ \end{aligned}$$

The exterior angle is the sum of two opposite interior angles.

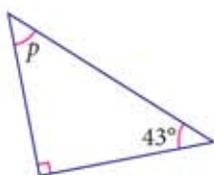
### Self Practice 9.2b

1. Find the value of  $p$  in each of the following triangles.

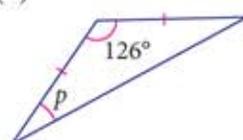
(a)



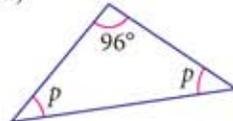
(b)



(c)

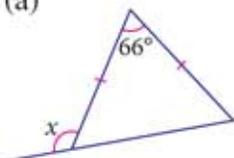


(d)

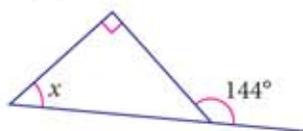


2. Find the value of  $x$  in each of the following diagrams.

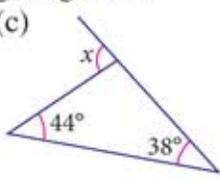
(a)



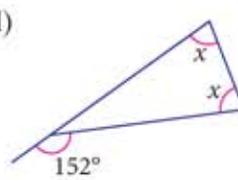
(b)



(c)



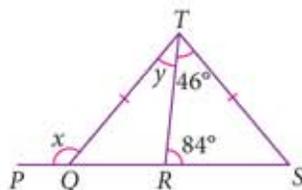
(d)



## ▶ How do you solve problems?

### Example 4

In the diagram,  $PQRS$  is a straight line. Calculate the values of  $x$  and  $y$ .



### Solution

$$\angle RST + 84^\circ + 46^\circ = 180^\circ \leftarrow \text{Sum of the interior angles of } \triangle RST.$$

$$\begin{aligned} \angle RST &= 180^\circ - 84^\circ - 46^\circ \\ &= 50^\circ \end{aligned}$$

$$\angle TQS = 50^\circ \leftarrow \text{Angles at the base of an isosceles triangle.}$$

$$\begin{aligned} x + 50^\circ &= 180^\circ \leftarrow \text{Sum of the interior angle and its adjacent exterior angle.} \\ x &= 180^\circ - 50^\circ \\ &= 130^\circ \end{aligned}$$

$$\angle QTS + 50^\circ + 50^\circ = 180^\circ \leftarrow \text{Sum of the interior angles of } \triangle QTS.$$

$$\begin{aligned} \angle QTS &= 180^\circ - 50^\circ - 50^\circ \\ &= 80^\circ \end{aligned}$$

$$\begin{aligned} y + 46^\circ &= 80^\circ \\ y &= 80^\circ - 46^\circ \\ &= 34^\circ \end{aligned}$$

### LEARNING STANDARDS

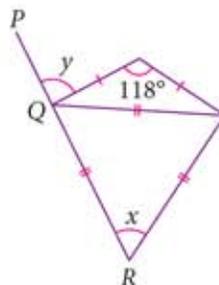
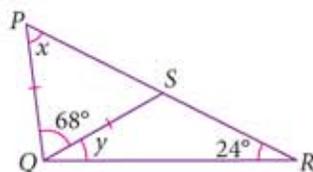
Solve problems involving triangles.

### Did You Know?

The symbol  $\Delta$  is used to denote a triangle.

### Self Practice 9.2c

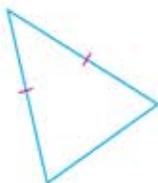
- In the diagram,  $PSR$  is a straight line. Calculate the values of  $x$  and  $y$ .
- In the diagram,  $PQR$  is a straight line. Calculate the values of  $x$  and  $y$ .



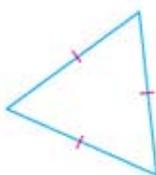


1. For each of the following triangles, state the number of axes of symmetry.

(a)



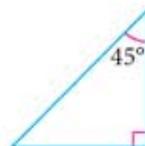
(b)



(c)

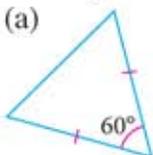


(d)

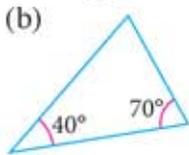


2. Identify the type of triangle for each of the following triangles.

(a)



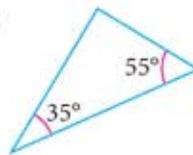
(b)



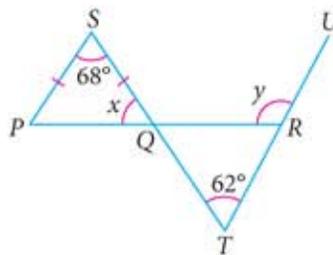
(c)



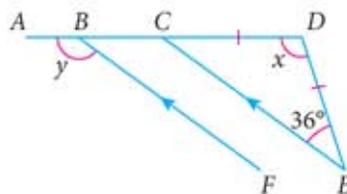
(d)



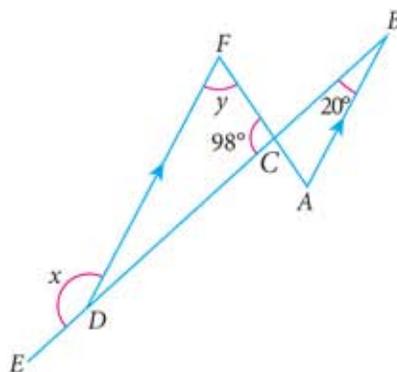
3. In the diagram,  $PQR$ ,  $SQT$  and  $TRU$  are straight lines. Calculate the values of  $x$  and  $y$ .



4. In the diagram,  $ABCD$  is a straight line. Calculate the values of  $x$  and  $y$ .



5. In the diagram,  $BCDE$  and  $ACF$  are straight lines. Calculate the values of  $x$  and  $y$ .



## 9.3 Properties of Quadrilaterals and the Interior and Exterior Angles of Quadrilaterals

**▶ What are the properties of a quadrilateral?**



### LEARNING STANDARDS

Describe the geometric properties of various types of quadrilaterals. Hence classify quadrilaterals based on the geometric properties.

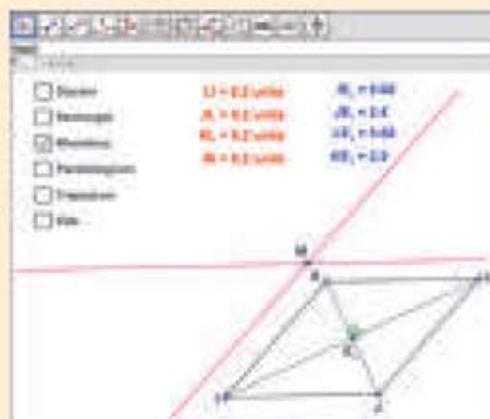
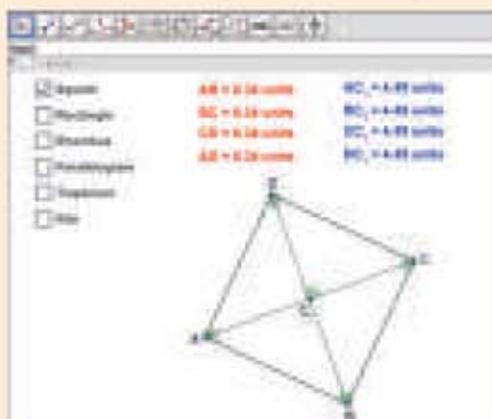
### Exploration Activity 4



**Aim:** To explore the geometric properties of a quadrilateral.

- Instruction:**
- Explore by yourself before the lesson begins and discuss in groups of four during the lesson.
  - Open the folder downloaded from page vii.

1. Open the file *Quadrilaterals geometric properties.ggb* using *GeoGebra*.

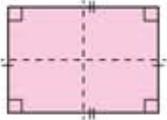
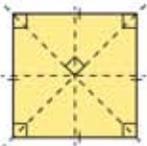
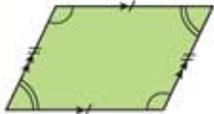
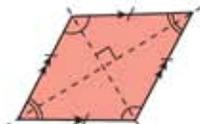
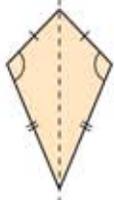


- Click the checkbox for the first type of quadrilateral. Click and drag the vertices of the quadrilateral to change the dimensions of the quadrilateral. Explain the properties of the quadrilateral.
- Repeat the exploration in Step 2 for all the other types of quadrilaterals.
- Discuss with your friends the geometric properties of the various types of quadrilaterals.
- Open and print the file *Quadrilaterals axes of symmetry.pdf*. Cut out the quadrilaterals in the printout.



- By folding each of the quadrilaterals, or otherwise, explain how you can find the number of axes of symmetry for each type of quadrilateral.

The following table shows the types of quadrilaterals and their geometric properties.

Type of quadrilateral	Number of axes of symmetry	Geometric properties
Rectangle 	2	<ul style="list-style-type: none"> <li>The opposite sides are parallel and of equal length.</li> <li>All of its interior angles are <math>90^\circ</math>.</li> <li>The diagonals are of equal length and are bisectors of each other.</li> </ul>
Square 	4	<ul style="list-style-type: none"> <li>All the sides are of equal length.</li> <li>The opposite sides are parallel.</li> <li>All of its interior angles are <math>90^\circ</math>.</li> <li>The diagonals are of equal length and are perpendicular bisectors of each other.</li> </ul>
Parallelogram 	None	<ul style="list-style-type: none"> <li>The opposite sides are parallel and of equal length.</li> <li>The opposite angles are equal.</li> <li>The diagonals are bisectors of each other.</li> </ul>
Rhombus 	2	<ul style="list-style-type: none"> <li>All the sides are of equal length.</li> <li>The opposite sides are parallel.</li> <li>The opposite angles are equal.</li> <li>The diagonals are perpendicular bisectors of each other.</li> </ul>
Trapezium 	None	<ul style="list-style-type: none"> <li>Only one pair of opposite sides is parallel.</li> </ul>
Kite 	1	<ul style="list-style-type: none"> <li>Two pairs of adjacent sides are of equal length.</li> <li>One pair of opposite angles is equal.</li> <li>One of the diagonals is the perpendicular bisector of the other.</li> <li>One of the diagonals is the angle bisector of the angles at the vertices.</li> </ul>

### Self Practice 9.3a

- Explain the common properties of a rhombus and a square.
- Explain the geometric properties of a rectangle compared to a parallelogram.

### Let's Discuss

Discuss and explain whether each of the following statements is TRUE or FALSE.

- A square is also a rhombus.
- A trapezium may have axes of symmetry.

**▶ How do you determine the interior angles and the exterior angles of a quadrilateral?**

**LEARNING STANDARDS**

Make and verify the conjectures about

- the sum of interior angles of a quadrilateral,
- the sum of interior angle and adjacent exterior angle of a quadrilateral, and
- the relationship between the opposite angles in a parallelogram.

**Exploration Activity 5**

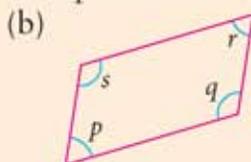


**Aim:** To explore the interior angles and the exterior angles of a quadrilateral.

**Instruction:**

- Explore by yourself before the lesson begins and discuss in groups of four during the lesson.
- Open the folder downloaded from page vii.

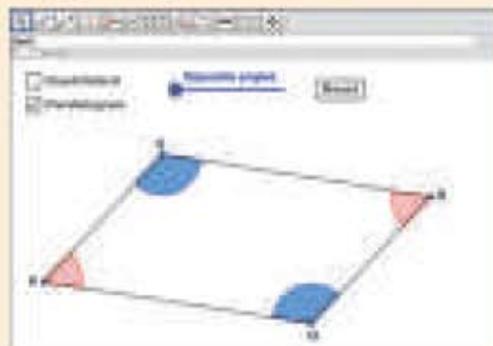
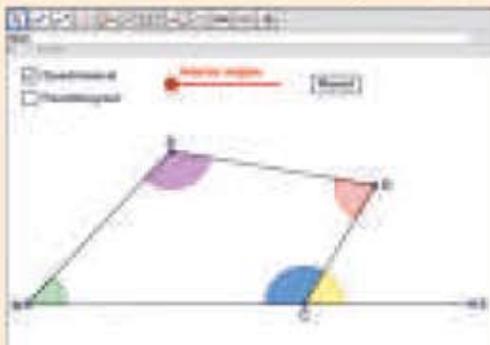
1. Copy and complete the following table to make conjectures about quadrilaterals and parallelograms. Then continue the exploration to verify the conjectures.



Sum of angles	Conjecture
$a + b + c + d$	
$c + e$	

Angles	Conjecture
$p$ and $r$	
$q$ and $s$	

2. Open the file *Interior angles of quadrilateral.ggb* using *GeoGebra*.



- Then, select 'Quadrilateral' to display a quadrilateral with its interior and exterior angles.
- Click and drag the slider 'Interior angles' towards the right. Hence, explain what you observe.
- Click at 'Reset' or drag the slider back to its original position.
- Click and drag points A, B, C or D to change the shape of the quadrilateral and repeat Step 4.
- Discuss with your friends your findings.

8. Consider the angle on a straight line, explain the relationship between the interior angle of a quadrilateral (the angle in blue colour) and its adjacent exterior angle (the angle in yellow colour).
9. Select 'Parallelogram' to display the interior angles of a parallelogram.
10. Click and drag the slider 'Opposite angles' towards the right. Explain what you observe.
11. Click at 'Reset' or drag the slider back to its original position.
12. Click and drag points  $P$ ,  $Q$  or  $S$  to change the shape of the parallelogram and repeat Step 10.
13. Explain all the conclusions arrived at.

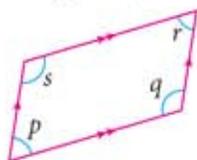
From the results of Exploration Activity 5, it is found that

- (a) the sum of the interior angles of a quadrilateral is  $360^\circ$ .
- (b) the sum of an interior angle of a quadrilateral and its adjacent exterior angle is  $180^\circ$ .
- (c) the opposite angles in a parallelogram (or rhombus) are equal.



$$a + b + c + d = 360^\circ$$

$$c + e = 180^\circ$$



$$p = r$$

$$q = s$$



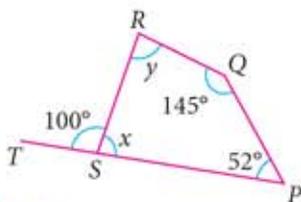
Scan the QR Code or visit <https://youtu.be/p7IPCvwE0vY> to watch a video about the sum of the interior angles of a quadrilateral.



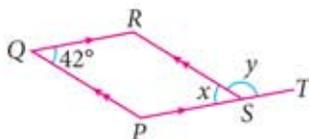
### Example 5

In each of the following diagrams,  $PST$  is a straight line. Calculate the values of  $x$  and  $y$ .

(a)



(b)



### Solution

(a)  $x + 100^\circ = 180^\circ$   
 $x = 180^\circ - 100^\circ$   
 $= 80^\circ$

Sum of the interior angle and its adjacent exterior angle is  $180^\circ$ .

$y + 80^\circ + 52^\circ + 145^\circ = 360^\circ$   
 $y + 277^\circ = 360^\circ$   
 $y = 360^\circ - 277^\circ$   
 $= 83^\circ$

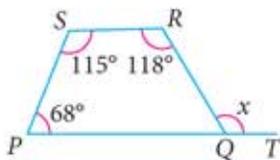
Sum of the interior angles of a quadrilateral is  $360^\circ$ .

(b)  $x = 42^\circ$   
 $y + 42^\circ = 180^\circ$   
 $y = 180^\circ - 42^\circ$   
 $= 138^\circ$

Opposite angles in a parallelogram are equal.

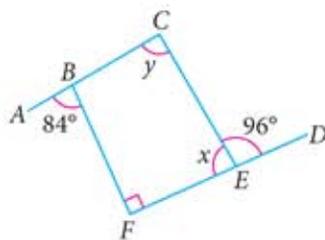
## Self Practice 9.3b

1.



In the diagram,  $PQT$  is a straight line. Calculate the value of  $x$ .

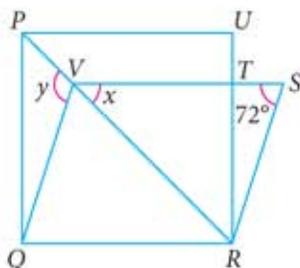
2.



In the diagram,  $ABC$  and  $DEF$  are straight lines. Calculate the values of  $x$  and  $y$ .

### ▶ How do you solve problems involving quadrilaterals?

#### Example 6



In the diagram,  $PQRU$  is a square and  $QRSV$  is a parallelogram.  $PVR$  is a straight line. Find the values of  $x$  and  $y$ .

#### Solution

$$\angle PQR = 90^\circ \text{ dan } PQ = QR$$

$$\text{Thus, } \angle QRV = \frac{90^\circ}{2}$$

$$x = 45^\circ \leftarrow x = \angle QRV$$

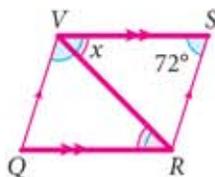
$$\angle QVS + 72^\circ = 180^\circ$$

$$\begin{aligned} \angle QVS &= 180^\circ - 72^\circ \\ &= 108^\circ \end{aligned}$$

$$\text{Therefore, } \angle QVR + x = 108^\circ$$

$$\begin{aligned} \angle QVR &= 108^\circ - 45^\circ \\ &= 63^\circ \end{aligned}$$

$$\begin{aligned} \text{Hence, } y &= 180^\circ - 63^\circ \\ &= 117^\circ \end{aligned}$$



#### LEARNING STANDARDS

Solve problems involving quadrilaterals.

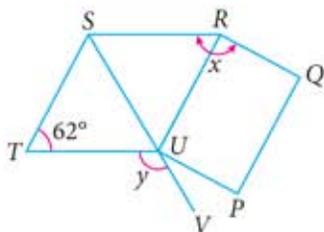


#### Career in Mathematics

An architect uses the knowledge of polygons to design a building.

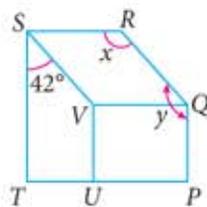
**Self Practice 9.3c**

1.



In the diagram,  $PQRU$  is a rectangle and  $RSTU$  is a rhombus.  $SUV$  is a straight line. Calculate the values of  $x$  and  $y$ .

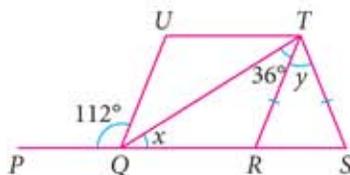
2.



In the diagram,  $PQVU$  is a rectangle,  $QRSV$  is a parallelogram and  $STUV$  is a trapezium. Calculate the values of  $x$  and  $y$ .

**▶ How do you solve problems involving the combinations of triangles and quadrilaterals?**

**Example 7**



In the diagram,  $QRTU$  is a parallelogram and  $PQRS$  is a straight line. Find the values of  $x$  and  $y$ .

**Solution**

$\angle QRT = 112^\circ$  ←  $\angle QRT$  and  $112^\circ$  are corresponding angles.

$x + 112^\circ + 36^\circ = 180^\circ$  ← Sum of the interior angles of  $\triangle QRT$ .

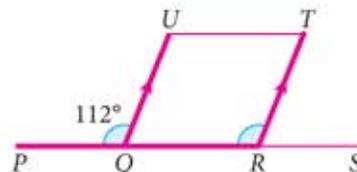
$$\begin{aligned} x &= 180^\circ - 112^\circ - 36^\circ \\ &= 32^\circ \end{aligned}$$

$\angle TRS + 112^\circ = 180^\circ$  ← Sum of the interior angle and its adjacent exterior angle.

$$\begin{aligned} \angle TRS &= 180^\circ - 112^\circ \\ &= 68^\circ \end{aligned}$$

$y + 68^\circ + 68^\circ = 180^\circ$  ← Sum of the interior angles of  $\triangle RST$ .

$$\begin{aligned} y &= 180^\circ - 68^\circ - 68^\circ \\ &= 44^\circ \end{aligned}$$

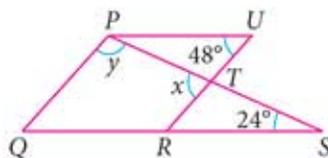


**LEARNING STANDARDS**

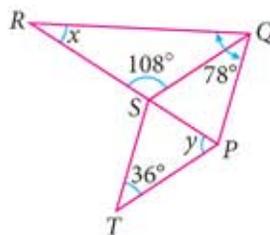
Solve problems involving the combinations of triangles and quadrilaterals.

**Self Practice 9.3d**

1. In the diagram,  $PQRU$  is a parallelogram.  $PTS$ ,  $QRS$  and  $RTU$  are straight lines. Find the values of  $x$  and  $y$ .



2. In the diagram,  $PQST$  is a parallelogram.  $PSR$  is a straight line. Find the values of  $x$  and  $y$ .



**Mastery Q 9.3**

Open the folder downloaded from page vii for extra questions of Mastery Q 9.3.

- State the similarities and differences between the geometric properties of a parallelogram and a rhombus.
- Using suitable mathematical notation, copy and denote all the geometric properties of the following quadrilaterals.

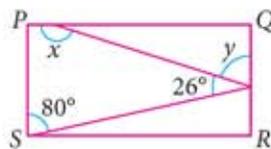
(a)



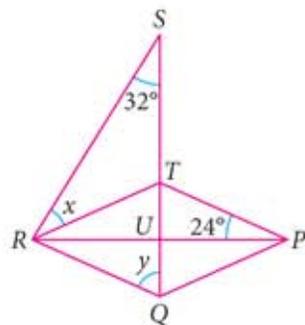
(b)



3. In the diagram,  $PQRS$  is a rectangle. Find the values of  $x$  and  $y$ .

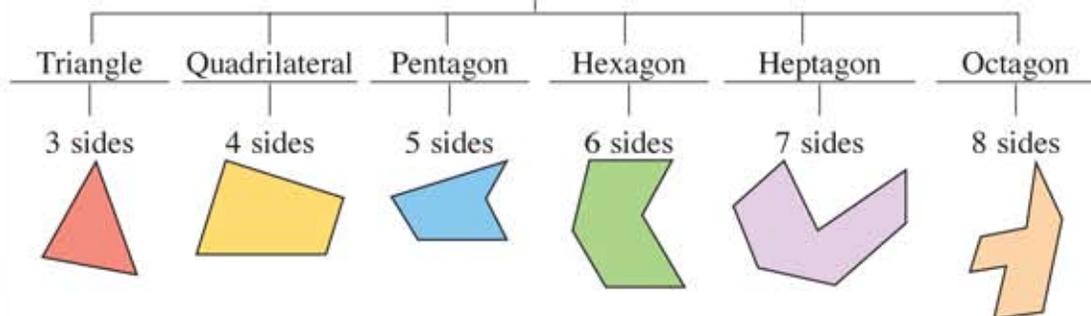


4. In the diagram,  $PQRT$  is a rhombus.  $STUQ$  and  $PUR$  are straight lines. Find the values of  $x$  and  $y$ .



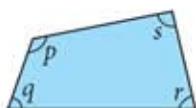
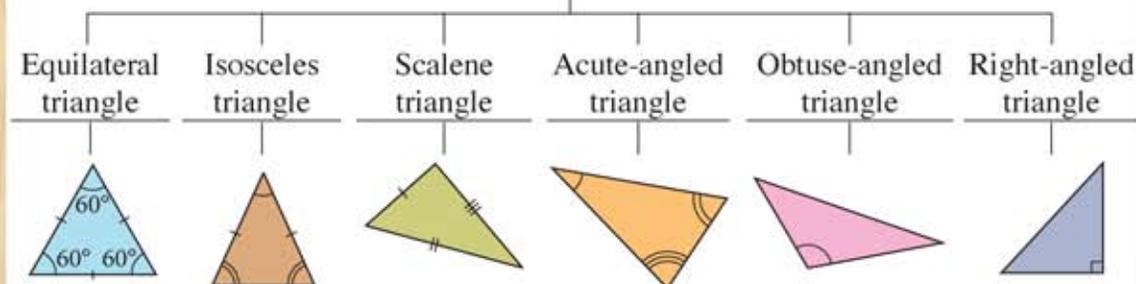
# SUMMARY

## Polygons



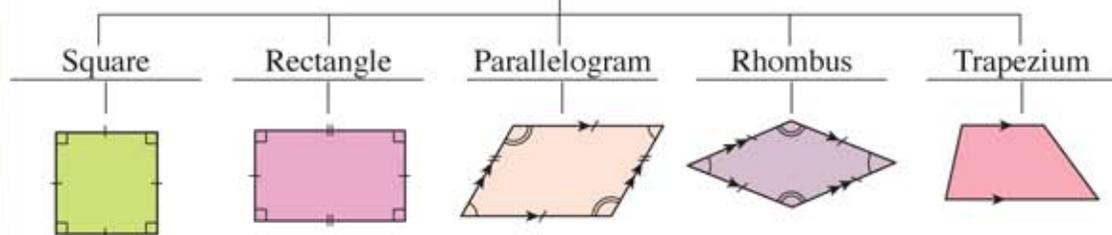
$$a + b + c = 180^\circ$$

## Triangles



$$p + q + r + s = 360^\circ$$

## Quadrilaterals



## At the end of this chapter, I can...



state the relationship between the number of sides, vertices and diagonals of polygons.

draw polygons, label vertices of polygons and name the polygons based on the labelled vertices.

recognise and list geometric properties of various types of triangles. Hence, classify triangles based on geometric properties.

make and verify conjectures about  
 (i) the sum of interior angles,  
 (ii) the sum of interior angle and adjacent exterior angle,  
 (iii) the relation between exterior angle and the sum of the opposite interior angles of a triangle.

solve problems involving triangles.

describe the geometric properties of various types of quadrilaterals. Hence classify quadrilaterals based on geometric properties.

make and verify the conjectures about  
 (i) the sum of interior angles of a quadrilateral,  
 (ii) the sum of interior angle and adjacent exterior angle of a quadrilateral, and  
 (iii) the relationship between the opposite angles in a parallelogram.

solve problems involving quadrilaterals.

solve problems involving the combinations of triangles and quadrilaterals.



## Let's PRACTISE

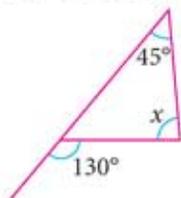
### Test Yourself

1. Mark  for a TRUE statement and  for a FALSE statement.

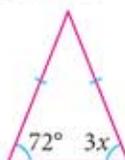
- A right-angled triangle has an axis of symmetry if one of the interior angles is  $45^\circ$ .
- If the axis of symmetry of an equilateral triangle  $PQR$  passes through vertex  $P$ , then the axis of symmetry is the angle bisector of the angle at  $P$ .
- A diagonal of a rectangle is the perpendicular bisector of the other diagonal.
- A square and a rhombus are quadrilaterals and their diagonals intersect at right angles.

2. Determine the types of
- quadrilaterals which have two axes of symmetry.
  - triangles which do not have an axis of symmetry.
  - quadrilaterals with all the sides having the same length.
  - quadrilaterals with all the interior angles being  $90^\circ$ .
3. Find the value of  $x$  in each of the following diagrams.

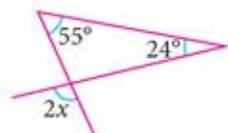
(a)



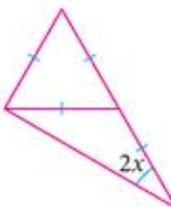
(b)



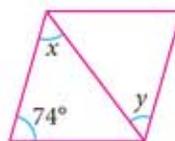
(c)



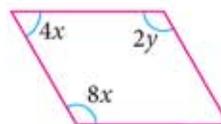
(d)

**Self Mastery**

4. The diagram shows a rhombus. Find the values of  $x$  and  $y$ .



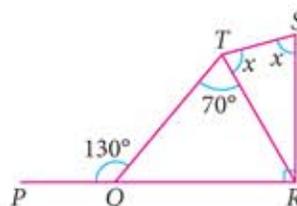
5. A parallelogram is shown in the diagram. Find the values of  $x$  and  $y$ .



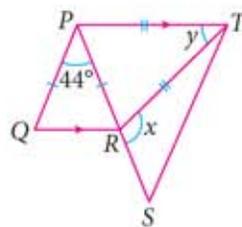
6. Four angles of a quadrilateral are in the ratio of 3 : 4 : 5 : 6. Explain how you would calculate the biggest angle of the quadrilateral.

**Challenge Yourself**

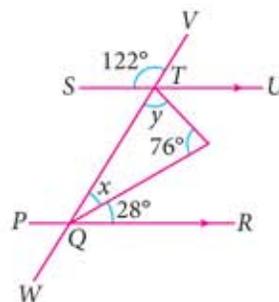
7. In the diagram,  $PQR$  is a straight line. Find the value of  $x$ .



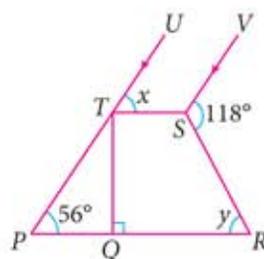
8.  $PRS$  is a straight line as shown in the diagram. Find the values of  $x$  and  $y$ .



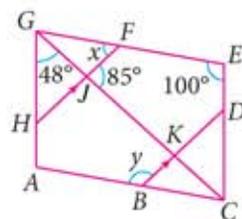
9.  $PQR$ ,  $STU$  and  $VTQW$  are straight lines. Find the values of  $x$  and  $y$ .



10.  $PRST$  is a trapezium.  $PQR$  and  $PTU$  are straight lines. Find the values of  $x$  and  $y$ .



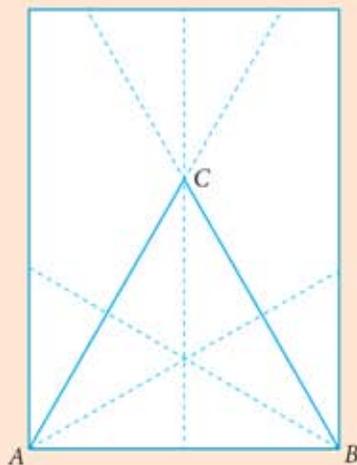
11.  $ACEG$  is a parallelogram.  $BKD$ ,  $HJF$  and  $CKJG$  are straight lines. Find the values of  $x$  and  $y$ .



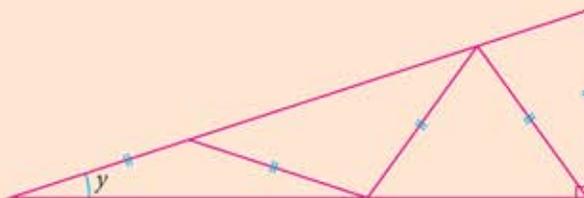
## ASSIGNMENT

You have been asked to do a study regarding the use of polygons in your school buildings. Write a report to explain how the use of polygons have enhanced the visual beauty of your school.

- A** The diagram shows a piece of A4-size paper. Without using any geometrical tools but only by folding the paper, explain the rationale behind the folding to obtain an equilateral triangle  $ABC$ . (Hint: The folding lines have been drawn in the diagram as a guide.)

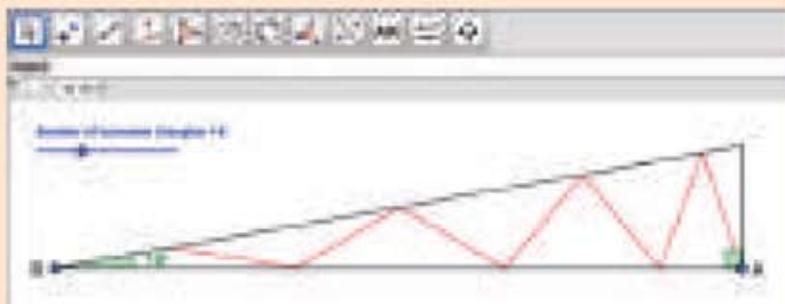


- B**



In the diagram, four isosceles triangles are inscribed in a right-angled triangle. Explain how you would find the value of  $y$ .

Next, open the file *Isosceles triangles inscribed.ggb* from the folder downloaded from page vii using *GeoGebra*.



Click and drag the slider 'Number of isosceles triangles', point  $A$  and point  $B$  to change the information displayed. With the help of the information, explain the relationship between the value of  $y$  and the number of isosceles triangles that can be drawn.