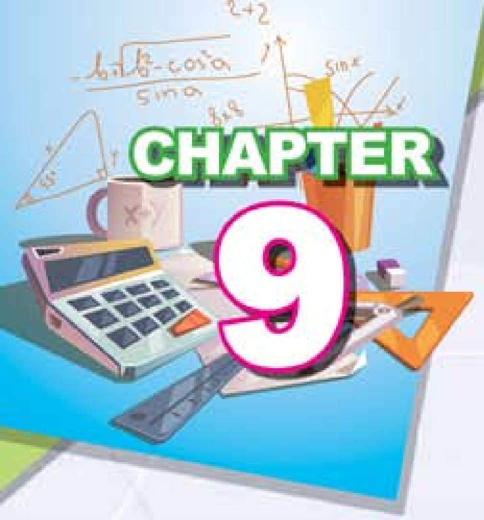


CHAPTER 9

Probability of Combined Events

You will learn

- Combined Events
- Dependent Events and Independent Events
- Mutually Exclusive Events and Non-Mutually Exclusive Events
- Application of Probability of Combined Events

The Malaysia National Football team had qualified for the 1972 Olympic Games in Munich. In 1974, the team won the bronze medal in the Asian Games held in Tehran. The success of the National team continued thereafter when it was qualified consecutively for the Asian Cups in 1976 and 1980. Malaysia won its first Suzuki Cup in 2010.

Do you know how a coach selects players to be defenders, midfielders and strikers in a football team?

Why Study This Chapter?

Apart from sports, the knowledge of probability is applied in the insurance industry to determine the amount of insurance premium. The knowledge of probability is also used in the fields of production and business, especially for risk management.

WORD BANK

- tree diagram — *gambar rajah pokok*
- probability — *kebarangkalian*
- dependent event — *peristiwa bersandar*
- mutually exclusive event — *peristiwa saling eksklusif*
- independent event — *peristiwa tak bersandar*
- non-mutually exclusive event — *peristiwa tidak saling eksklusif*
- sample space — *ruang sampel*

Walking Through Time

Blaise Pascal
(1623 – 1662)

Blaise Pascal was a French mathematician. He was also a physician, inventor and writer. Blaise Pascal invented the theory of probability together with Pierre de Fermat, a French mathematician. The theory of probability is an important mathematical foundation for statistics.

http://bt.sasbadi.com/m4243

9.1 Combined Events

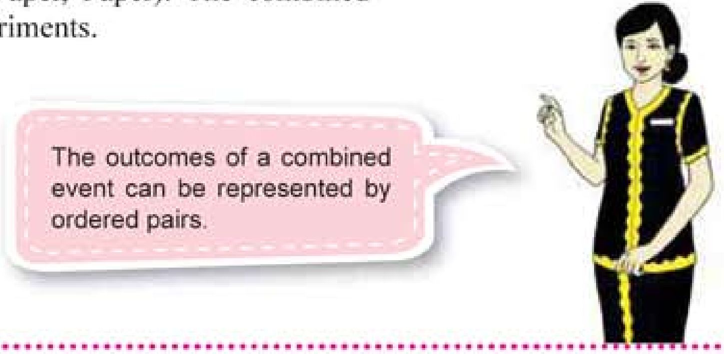
What are combined events?

Learning Standard

Describe combined events and list out the possible combined events.

In our daily lives, we need to make a lot of decisions based on uncertainties. For example, choosing the science or the art stream class or selecting what products to sell on the school entrepreneurship day. These decisions incur risks and we should be able to assess the risks before making any decisions. Probability is used to evaluate the uncertainties that are associated in the process of decision making.

The combined events are the combination of two or more events in an outcome. For example, the possible outcomes for two pupils playing "Rock-Paper-Scissors" are (Scissors, Rock), (Scissors, Paper), (Scissors, Scissors), (Rock, Scissors), (Rock, Paper), (Rock, Rock), (Paper, Scissors), (Paper, Rock) and (Paper, Paper). The combined events can result from one or more experiments.



Mind Stimulation 1

Aim: To list the possible outcomes of combined events

Material: Coins (10 sen, 20 sen and 50 sen), an empty box

Steps:

1. Pupils sit in pairs. Each pair is given a box containing three types of coins which are 10 sen, 20 sen and 50 sen coins.
2. Each pupil in the pair chooses a coin from the box at random. The values of the coins are recorded in the table below.
3. Return the coins to the box.
4. Repeat Steps 2 and 3 for 25 times.

(10, 20)				

MY MEMORY

A sample space is a set that contains all the possible outcomes of an experiment.

TIPS

(10, 20)

- 10: Coin chosen by the first pupil
- 20: Coin chosen by the second pupil

5. Write the sample space for the coin selection experiment for each pair of pupils.

$S = \{$ ______ $\}$

Discussion:

What is the possible number of outcomes in the activity?

From the activity in Mind Stimulation 1, it is found that:

The possible number of outcomes is $3 \times 2 = 6$.

In general,

$$n(S) = n(A) \times n(B)$$

$n(S)$ is all the possible number of outcomes, $n(A)$ and $n(B)$ represent the number of outcomes of event A and event B.

Example 1

Write the sample spaces for the combined events below.

(a) Five cards labelled with the letters "T, E, K, U, N" are put into a box. Two cards are taken out at random from the box one by one without replacement.

(b) Two coins are tossed (T and H representing tails and heads respectively).

Solution:

(a) {(T, E), (T, K), (T, U), (T, N), (E, T), (E, K), (E, U), (E, N), (K, T), (K, E), (K, U), (K, N), (U, T), (U, E), (U, K), (U, N), (N, T), (N, E), (N, K), (N, U)}

(b) {(T, T), (T, H), (H, T), (H, H)}

Self Practice 9.1a

Write the sample spaces for the combined events below.

1. Two books are chosen at random from a bookshelf that contains two history books (H), a geography book (G) and a mathematics book (M).

2. The children's gender for the family of two children.

3. A fair dice is rolled and a fair coin is tossed simultaneously.

4. Azhar (A) and Kai Meng (K) play a maximum of five badminton matches. The player that wins three sets is the winner.

9.2 Dependent Events and Independent Events

How do you differentiate between dependent and independent events?

Learning Standard

Differentiate between dependent and independent events.

Combined events can be categorised as dependent events and independent events.

Event A and event B are independent events if the occurrence of event A has no effect on the occurrence of event B and vice versa.

In other words, event A and event B are dependent events if the occurrence of event A affects the occurrence of event B.

Mind Stimulation 2

Aim: To differentiate dependent events and independent events

Steps:

1. Divide the class into groups.
2. Complete the table in the Activity Sheet below.

Activity Sheet:

Box P contains five cards labelled with the letters "R, U, A, N, G".

(a) Case I: Two cards are chosen at random from box P one by one without replacement. Write the probability of getting a consonant card on the first time and the second time in the table below.

(b) Case II: Two cards are chosen at random from box P one by one with replacement. The letter for the first card is recorded and it is returned to box P before the second card is chosen. Write the probability of getting a consonant card on the first time and the second time in the table below.

Case	Probability of getting a consonant card	
	First time	Second time
I		
II		

Discussion:

Why are the probabilities for the second time in case I and case II different? Discuss.

From the activity in Mind Stimulation 2, it is found that:

In case I, the first consonant card chosen is not returned to box P. The short of this first consonant card affected the probability of selecting the second consonant card.

Therefore,

The combined events of case I are dependent events.

In case II, the first consonant card chosen is returned to box P before the second card is chosen. The return of the first card results in the probability of selecting the second consonant card being the same as the probability of selecting the first consonant card. The probability of selecting the second consonant card is not affected by the probability of selecting the first consonant card.

Therefore,

The combined events of case II are independent events.

Example 2

Identify whether the following combined events are dependent events or independent events. Justify your answers.

(a) Obtain a tail twice when a fair coin is tossed twice.

(b) Obtain a tail in tossing a fair coin and obtained the number 4 in rolling a fair dice.

(c) Obtain two pens of the same colour when two pens are taken out one by one from a container that contains three red pens and two blue pens without replacement.

(d) Obtain two cards with the same letters when two cards are chosen at random from the cards labelled with the letters "B, A, I, K" one by one with replacement.

Solution:

(a) Independent events because the probability of getting a tail in the first toss does not affect the probability of getting a tail in the second toss.

(b) Independent events because the probability of getting a tail in tossing a fair coin does not affect the probability of getting the number 4 in rolling a fair dice.

(c) Dependent events because the probability of getting the first red pen affects the probability of getting the second red pen.

(d) Independent events because the probability of choosing the first card does not affect the probability of choosing the second card.

Self Practice 9.2a

Determine whether the following events are dependent events or independent events.

1. The pointer of a lucky wheel stops at the same sector twice consecutively.
2. The selection of two boys from a group of ten girls and fourteen boys at random.
3. Answer three objective questions with four options correctly if the answer of each question is chosen at random.
4. Box P contains two red cards and three black cards while box Q contains five red cards and six green cards. A card is chosen at random from box P and then put into box Q. After that, a card is chosen at random from box Q. Both cards chosen from box P and box Q are of the same colour.
5. Vincent and Bajat sit for a History test in school. Vincent and Bajat pass the History test.

How do you make and verify conjecture about the formula of probability of combined events?

Mind Stimulation 3

Aim: To make and verify conjecture about the formula of probability of combined events

Learning Standard

Make and verify conjecture about the formula of probability of combined events.

Steps:

1. Divide the class into groups.
2. Roll a fair dice and toss a fair coin at the same time.
3. Complete the table below by recording all the possible outcomes.

Dice	Coin	
	Tail (T)	Head (H)
1		
2		
3		
4		
5		
6		

Indicator

- Probability of event A, $P(A) = \frac{n(A)}{n(S)}$
- $0 \leq P(A) \leq 1$
- $P(A) = 0$ when event A will definitely not occur.
- $P(A) = 1$ when event A will definitely occur.

4. Based on the table above,
   (a) state the sample space for the above experiments.
   (b) state the probability, by listing all the possible outcomes of the combined events below.
      (i) Obtain an even number in rolling a dice and a tail in tossing a coin.

(ii) Obtain a prime number in rolling a dice and a head in tossing a coin.
(iii) Obtain a number less than 3 in rolling a dice and a tail in tossing a coin.

(c) Calculate the product of the probability of
(i) obtaining an even number in rolling a dice and a tail in tossing a coin.
(ii) obtaining a prime number in rolling a dice and a head in tossing a coin.
(iii) obtaining a number less than 3 in rolling a dice and a tail in tossing a coin.

Discussion:
Compare your answers in 4(b) and 4(c). What did you observe?

From the activity in Mind Stimulation 3, it is found that:

The probability of the intersection of two independent events A and B is equal to the product of the probability of event A and the probability of event B.

In general,

Multiplication rule of probability is $P(A \text{ and } B) = P(A) \times P(B)$

Example 3

Box F contains seven cards labelled with the letters "P, A, M, E, R, A, N" and box G contains five cards labelled with the numbers "3, 5, 6, 8, 11". A card is chosen at random from box F and box G respectively. Verify the conjecture about the formula of probability to get the letter "P" and an even number by listing all the possible outcomes.

Solution:

(a) Multiplication rule

P(getting a letter "P") $= \frac{1}{7}$

P(getting an even number) $= \frac{2}{5}$

P(getting a letter "P" and an even number) $= \frac{1}{7} \times \frac{2}{5}$

$= \frac{2}{35}$

List all the possible outcomes.

Possible outcomes $= \{(\text{P}, 6), (\text{P}, 8)\}$

$n(S) = 7 \times 5$
$= 35$

P(getting a letter "P" and an even number) $= \frac{2}{35}$

Therefore, it is shown that both methods give the same answer.

Indicator

$P(A \cap B \cap C)$
$= P(A) \times P(B) \times P(C)$

CHAPTER 9

Self Practice 9.2b

1. Two fair dice are rolled.

(a) Complete the following table by listing all the possible outcomes.

First dice	Second dice					
	1	2	3	4	5	6
1		(1, 2)	(1, 3)	(1, 4)		
2					(2, 5)	(2, 6)
3						
4						
5						
6						

(b) State $n(S)$ in this experiment.

(c) Verify the conjecture about the formula of probability of getting an odd number in the first dice and a prime number in the second dice by listing all the possible outcomes.

2. Kamal chooses two marbles randomly from a box which contains four red marbles, three yellow marbles and one green marble. The first marble is returned to the box before the second marble is chosen. Verify the conjecture about the formula of probability that two yellow marbles are chosen by listing all the possible outcomes.

3. Box A contains a red card and two yellow cards. Box B contains three red cards and a yellow card. Fauziah chooses a card from box A and box B respectively. Verify the conjecture about the formula of probability that Fauziah gets two yellow cards by listing all the possible outcomes.

How do you determine the probability of combined events for dependent events and independent events?

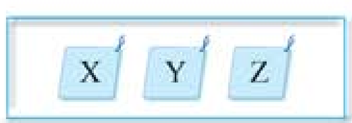
Learning Standard

Determine the probability of combined events for dependent and independent events.

Example 4

Box A and box B contain cards labelled with the numbers "3, 5, 7, 9" and the letters "X, Y, Z" respectively. A card is chosen randomly from box A and box B respectively.

Box A



Box B

Calculate the probability of getting a factor of 9 and the letter "Z".

CHAPTER 9

Solution:

$P(\text{A factor of 9}) = \frac{2}{4}$

$P(\text{The letter "Z"}) = \frac{1}{3}$

$P(\text{A factor of 9 and the letter "Z"}) = \frac{2}{4} \times \frac{1}{3}$

$= \frac{1}{6}$

Alternative Method

Factors of 9 and the letter "Z" = {(3, Z), (9, Z)}

$n(S) = 4 \times 3 = 12$

$P(\text{A factor of 9 and the letter "Z"}) = \frac{2}{12}$

$= \frac{1}{6}$

Example 5

A bag contains eight green marbles and a red marble. Two marbles are chosen randomly one by one from the bag without replacement. The colours of the marbles are recorded.

(a) Represent the above situation using a tree diagram.

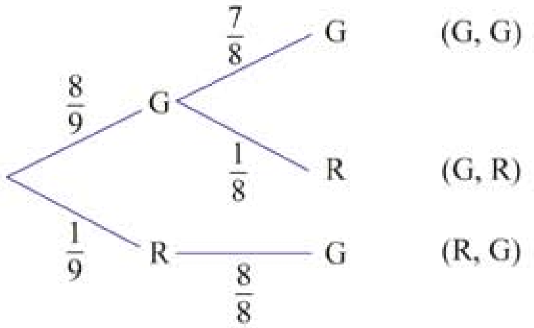
(b) Calculate the probability that

(i) the second marble is red.

(ii) both are green marbles.

Solution:

(a) **First marble** **Second marble** **Outcome**



MY MEMORY

A tree diagram displays all the possible outcomes of an event. Each branch in the tree diagram represents a possible outcome.

(b) (i) $P(\text{The second marble is red}) = \frac{8}{9} \times \frac{1}{8}$

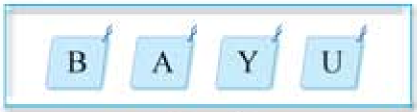
$= \frac{1}{9}$

(ii) $P(\text{Both are green marbles}) = \frac{8}{9} \times \frac{7}{8}$

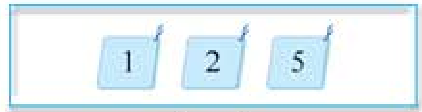
$= \frac{7}{9}$

Self Practice 9.2c

1. Box K and box L contain four cards labelled with the letters "B, A, Y, U" and three cards labelled with the numbers "1, 2, 5" respectively. A card is chosen at random from box K and box L respectively.



Box K



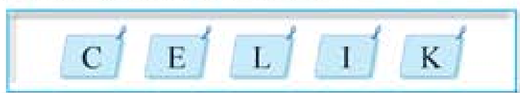
Box L

By listing all the possible outcomes, calculate the probability of getting a vowel and an even number.

2. A fair dice with four faces is labelled with "1, 2, 3, 4". The dice is rolled twice and the numbers that the dice lands on are recorded. By listing all the possible outcomes, calculate the probability of getting two odd numbers.

3. According to an investigation, the probability of rainfall on Mountain X in May is 0.45. Calculate the probability that Mountain X will have two consecutive rainy days in May.

4. Box T contains five cards labelled with the letters "C, E, L, I, K". Two cards are taken out randomly one by one from box T without replacement.



Box T

Calculate the probability of getting the first card labelled with a consonant and the second card labelled with a vowel.

5. A box contains twelve bulbs where two of the bulbs are burnt. Two bulbs are selected at random from the box. By sketching a tree diagram, calculate the probability of getting two burnt bulbs.

6. The following table shows the number of the Science and Mathematics Society members in SMK Didik Jaya.

Session	Number of members	
	Female	Male
Morning	146	124
Afternoon	82	96

Two members are selected randomly.

(a) from the male members. Calculate the probability that both members chosen are from the morning session. Give your answer correct to four significant figures.

(b) from the afternoon session. Calculate the probability that both members chosen are female. Give your answer correct to four significant figures.

9.3 Mutually Exclusive Events and Non-Mutually Exclusive Events

How do you differentiate between mutually exclusive and non-mutually exclusive events?

Learning Standard

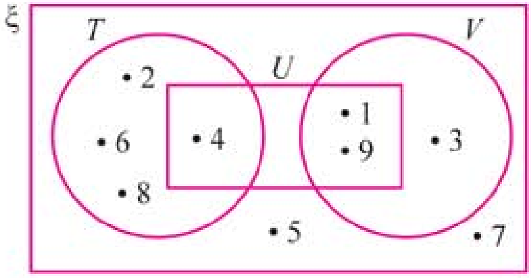
Differentiate between mutually exclusive and non-mutually exclusive events.

Some table tennis balls labelled from 1 to 9 are put in an empty basket. A pupil chooses a table tennis ball from the basket at random.

Let T is the event of getting an even number.
U is the event of getting a perfect square.
V is the event of getting a factor of 9.

The relationship between the three events, T, U and V, can be illustrated using a Venn diagram.

Based on the Venn diagram on the right, it is observed that event T and event V cannot happen at the same time. Thus, event T and event V are said to be mutually exclusive events. Event T and event U are non-mutually exclusive events as the table tennis ball labelled with 4 is the common outcome for both event T and event U. Are event U and event V mutually exclusive events? Discuss.



Mind Stimulation 4

Aim: To differentiate mutually exclusive events and non-mutually exclusive events

Steps:

1. Divide the class into groups.
2. Complete the following Activity Sheet.

Activity Sheet:

A pupil is chosen at random from your class. The following are event A to event F.

Event A: Pupils who are wearing spectacles.
Event B: Members of Girl Guides.
Event C: Boys.
Event D: Pupils who obtained Grade A in Mathematics test.
Event E: Pupils who love Mathematics.
Event F: Pupils who obtained Grade D in Mathematics test.

Mark ✓ for the mutually exclusive events or non-mutually exclusive events for the following combined events.

Combined event	Mutually exclusive events	Non-mutually exclusive events
Events A and B		
Events B and C		
Events B and D		
Events D and E		
Events E and F		
Events D and F		

CHAPTER 9

Discussion:
Are all the mutually exclusive events stated in the table? If they are not, please state the events.

From the activity in Mind Stimulation 4, it is found that:

For each combined event above, whether it is a mutually exclusive event or non-mutually exclusive event depends on the group of pupils in the respective classes. If there are only boys wearing spectacles, then event A and event B are mutually exclusive. However, for classes that have girls wearing spectacles, then event A and event B are non-mutually exclusive events.

In general,

A combined event A and B is known as a mutually exclusive event if there is no intersection between events A and B, $A \cap B = \phi$.

Example 6

A worker in a factory is chosen at random.
Given

A = A worker with wages less than RM2 500.
B = A worker that needs to pay income tax.
C = A worker who goes to work by car.

Determine whether the following pairs of events are mutually exclusive events or non-mutually exclusive events.

(a) A and B (b) A and C (c) B and C

INFO ZONE

Starting 2015, a person with an annual income of more than RM34 000 (after the deduction of EPF) will need to register an income tax file.

Lembaga Hasil Dalam Negeri, Updated: 16 March 2017

Solution:

(a) Event A and event B cannot occur together. Therefore, event A and event B are mutually exclusive.

(b) Event A and event C can occur together. Therefore, event A and event C are non-mutually exclusive.

(c) Event B and event C can occur together. Therefore, event B and event C are non-mutually exclusive.

Self Practice 9.3a

1. A fair dice is rolled. A list of events is given as follows.

   P is the event of getting a number that is greater than 4.
   Q is the event of getting an even number.
   R is the event of getting a perfect square.

   Determine whether the following pairs of events are mutually exclusive events or non-mutually exclusive events.

   (a) P and Q (b) P and R (c) Q and R

2. An egg is chosen at random from a farm.

K is the event of selecting a cracked egg.

L is the event of selecting a Grade A egg.

M is the event of selecting a Grade C egg.

Determine whether the following pairs of events are mutually exclusive events or non-mutually exclusive events.

(a) K and L (b) K and M (c) L and M

INFO ZONE

ASEAN countries: Malaysia, Brunei, Singapore, Cambodia, Indonesia, Vietnam, Myanmar, Philippines, Thailand, Laos.

Commonwealth countries: The countries that had been colonised by the British.

3. A tourist is selected randomly at Kuala Lumpur International Airport.

R is the event of selecting a tourist from an European country.

S is the event of selecting a tourist from an ASEAN country.

T is the event of selecting a tourist from a Commonwealth country.

Determine whether the following pairs of events are mutually exclusive events or non-mutually exclusive events.

(a) R and S (b) R and T (c) S and T

Learning Standard

Verify the formula of probability of combined events for mutually exclusive and non-mutually exclusive events.

How do you verify the formula of probability of combined events for mutually exclusive and non-mutually exclusive events?

Mind Stimulation 5

Aim: To verify the formula of probability of combined events for mutually exclusive and non-mutually exclusive events.

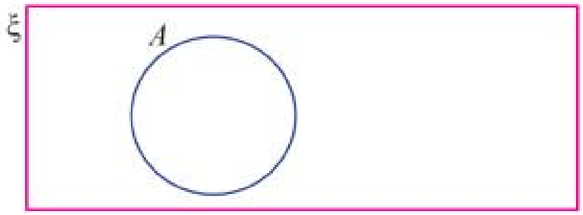
Steps:

1. Divide the class into groups.
2. Study the following case.

Fahmi holds an open house in conjunction of the Aidilfitri celebration.
80 guests come to the open house.
$\frac{2}{5}$ of the guests who come are Fahmi's colleagues.
There are 55 guests who come together with their family members. 18 of them are Fahmi's colleagues.
$\frac{1}{10}$ of the guests who come are Fahmi's childrens schoolmates.
All of Fahmi's children's schoolmates do not come with their family members.
A guest who comes to Fahmi's open house is selected at random.
A is the event that a guest chosen comes with family members.
B is the event that a guest chosen is Fahmi's colleague.
C is the event that a guest chosen is Fahmi's children's schoolmate.

CHAPTER 9

3. Complete the following Venn diagram to show the relationship between events A, B and C.



4. Based on the Venn diagram drawn, complete the following table.

	Probability			
(a)	$P(A) =$	$P(B) =$	$P(A$ and $B) =$	$P(A$ or $B) =$
(b)	$P(A) =$	$P(C) =$	$P(A$ and $C) =$	$P(A$ or $C) =$
(c)	$P(B) =$	$P(C) =$	$P(B$ and $C) =$	$P(B$ or $C) =$

Discussion:

1. Why do $P(A$ and $B)$, $P(A$ and $C)$ and $P(B$ and $C)$ need to be determined before calculating $P(A$ or $B)$, $P(A$ or $C)$ and $P(B$ or $C)$?
2. Based on the answers in (a), (b) and (c) in the above table, form an equation to relate all four probabilities for each (a), (b) and (c). Justify your answers.

From the activity in Mind Stimulation 5, it is found that:

1. $P(A$ and $B)$, $P(A$ and $C)$ and $P(B$ and $C)$ are identified first so that we can determine whether the combined events are mutually exclusive or non-mutually exclusive.
2. (a) The combined event A and B is non-mutually exclusive because $P(A \cap B) \neq 0$, then $P(A \text{ or } B) = P(A) + P(B) - P(A \cap B)$.
   (b) The combined event A and C and combined event B and C are both mutually exclusive because $P(A \cap C) = 0$ and $P(B \cap C) = 0$. Then, $P(A \text{ or } C) = P(A) + P(C)$ and $P(B \text{ or } C) = P(B) + P(C)$.

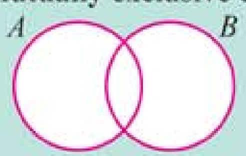
In general,

The **addition rule of probability** is
$P(A \cup B) = P(A) + P(B)$ or $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

MY MEMORY

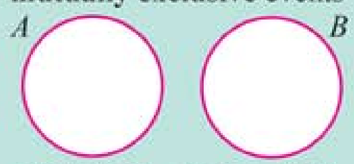
$P(A \text{ and } B) = P(A \cap B)$
$P(A \text{ or } B) = P(A \cup B)$

Events A and B are **non-mutually exclusive** events



$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

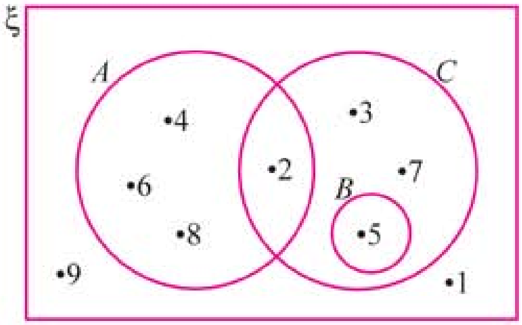
Events A and B are **mutually exclusive** events



$P(A \cup B) = P(A) + P(B)$

Example 7

The Venn diagram on the right shows the relationship between the universal set, ξ, A, B and C.



A number is chosen at random from the universal set, ξ. Verify the addition rule of probability for each of the following combined events.

(a) Obtaining an even number or a multiple of 5.
(b) Obtaining an even number or a prime number.

Solution:

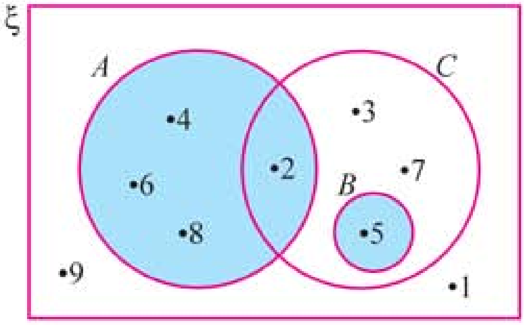
(a) $P(A \cup B) = \dfrac{n(A \cup B)}{n(S)}$

$= \dfrac{5}{9}$

$P(A) + P(B) = \dfrac{4}{9} + \dfrac{1}{9}$

$= \dfrac{5}{9}$

Hence, it is proven that $P(A \cup B) = P(A) + P(B)$.



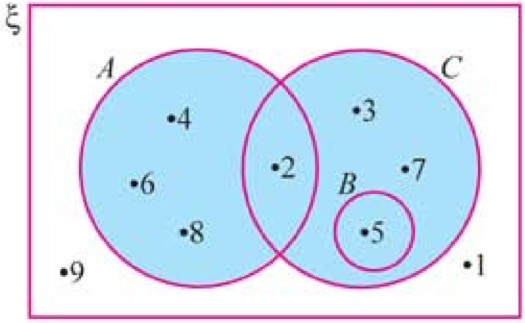
(b) $P(A \cup C) = \dfrac{n(A \cup C)}{n(S)}$

$= \dfrac{7}{9}$

$P(A) + P(C) - P(A \cap C) = \dfrac{4}{9} + \dfrac{4}{9} - \dfrac{1}{9}$

$= \dfrac{7}{9}$

Hence, it is proven that
$P(A \cup C) = P(A) + P(C) - P(A \cap C)$.



Example 8

Eight cards labelled with the numbers "4, 5, 6, 7, 8, 9, 10, 11" are put in a box. A card is chosen at random from the box.

A is the event of getting a number greater than 8.
B is the event of getting a prime number.
C is the event of getting an even number.

Verify the addition rule of probability for each of the following combined events by listing all the possible outcomes.

(a) $P(A \text{ or } B)$ (b) $P(A \text{ or } C)$ (c) $P(B \text{ or } C)$

Solution:

(a) $A \cap B = \{11\}$

$P(A) + P(B) - P(A \cap B) = \frac{3}{8} + \frac{3}{8} - \frac{1}{8}$
$= \frac{5}{8}$

$A = \{9, 10, 11\}, B = \{5, 7, 11\}$
$A \cup B = \{5, 7, 9, 10, 11\}$
$P(A \cup B) = \frac{5}{8}$

Hence, it is proven that $P(A) + P(B) - P(A \cap B) = P(A \cup B)$.

(b) $A \cap C = \{10\}$

$P(A) + P(C) - P(A \cap C) = \frac{3}{8} + \frac{4}{8} - \frac{1}{8}$
$= \frac{6}{8}$
$= \frac{3}{4}$

$A = \{9, 10, 11\}, C = \{4, 6, 8, 10\}$
$A \cup C = \{4, 6, 8, 9, 10, 11\}$
$P(A \cup C) = \frac{6}{8}$
$= \frac{3}{4}$

Hence, it is proven that $P(A) + P(C) - P(A \cap C) = P(A \cup C)$.

(c) $B \cap C = \{\ \}$

$P(B) + P(C) = \frac{3}{8} + \frac{4}{8}$
$= \frac{7}{8}$

$B = \{5, 7, 11\}, C = \{4, 6, 8, 10\}$
$B \cup C = \{4, 5, 6, 7, 8, 10, 11\}$
$P(B \cup C) = \frac{7}{8}$

Hence, it is proven that $P(B) + P(C) = P(B \cup C)$.

Self Practice 9.3b

1. Two fair dice are rolled simultaneously.

Q is the event that the total point obtained from the two dice is greater than 9.

R is the event that the product of the points obtained from the two dice is a multiple of 5.

S is the event that two equal points on the two dice are obtained.

Verify the addition rule of probability for each of the following combined events by listing all the possible outcomes.

(a) $P(Q \text{ or } R)$ (b) $P(Q \text{ or } S)$ (c) $P(R \text{ or } S)$

2. Two fair coins are tossed simultaneously.

J is the event of getting two tails.

K is the event of getting two heads.

L is the event of getting at least one tail.

Verify the addition rule of probability for each of the following combined events by listing all the possible outcomes.

(a) $P(J \text{ or } K)$ (b) $P(J \text{ or } L)$ (c) $P(K \text{ or } L)$

3. Seven cards labelled with the letters "B, A, H, A, G, I, A" are put in a box. A card is chosen at random.
L is the event of getting a vowel.
M is the event of getting a consonant.
N is the event of getting a letter "B".
(a) Draw a Venn diagram to represent the relationship between the events L, M and N.
(b) Verify the addition rule of probability for each of the following combined events by listing all the possible outcomes.
(i) $P(L \text{ or } M)$ (ii) $P(L \text{ or } N)$ (iii) $P(M \text{ or } N)$

INTERACTIVE ZONE

Is $P(A \cup B \cup C)$
$= P(A) + P(B) + P(C) - P(A \cap B) - P(A \cap C) - P(B \cap C) + P(A \cap B \cap C)$?
Discuss by using a Venn diagram.

How do you determine the probability of combined events for mutually exclusive and non-mutually exclusive events?

Learning Standard

Determine the probability of combined events for mutually exclusive and non-mutually exclusive events.

Example 9

Five cards labelled with the letters "C, I, N, T, A" are put in a box. A card is chosen at random. Calculate the probability that the card chosen is labelled with a consonant or letter "A".

Solution:

A card labelled with a consonant = {C, N, T}

A card labelled with letter "A" = {A}

$P(\text{A card labelled with a consonant or letter "A"}) = \frac{3}{5} + \frac{1}{5}$
$= \frac{4}{5}$

TIPS

For two mutually exclusive events,
$P(A \cup B) = P(A) + P(B)$

Example 10

In a banquet, the probabilities that Zalifah and Maran eat *cendol* are $\frac{5}{7}$ and $\frac{3}{5}$ respectively.

(a) Represent the probability that Zalifah and Maran eat *cendol* at the banquet using a Venn diagram.
(b) Calculate the probability that Zalifah or Maran eats *cendol* at the banquet.

Solution:

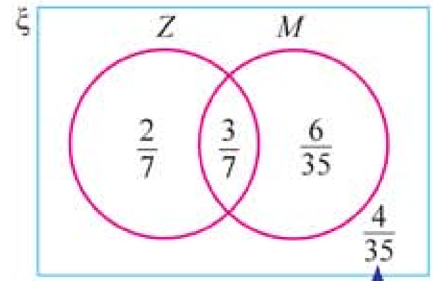
(a) $P(\text{Both Zalifah and Maran eat } cendol \text{ at the banquet})$
$= \frac{5}{7} \times \frac{3}{5}$
$= \frac{3}{7}$

$P(\text{Only Zalifah eats } cendol \text{ at the banquet}) = \frac{5}{7} - \frac{3}{7}$
$= \frac{2}{7}$

Malaysiaku

Cendol is a very popular dessert in Malaysia. The ingredients contain green droplets made of rice flour and *pandan* juice, and mixed together with ice, coconut milk and brown sugar.

P(Only Maran eats *cendol* at the banquet)
$= \frac{3}{5} - \frac{3}{7}$
$= \frac{6}{35}$



(b) P(Zalifah or Maran eats *cendol* at the banquet)
$= P(Z) + P(M) - P(Z \cap M)$
$= \frac{5}{7} + \frac{3}{5} - \frac{3}{7}$
$= \frac{31}{35}$

$1 - \frac{2}{7} - \frac{3}{7} - \frac{6}{35}$
$= \frac{4}{35}$

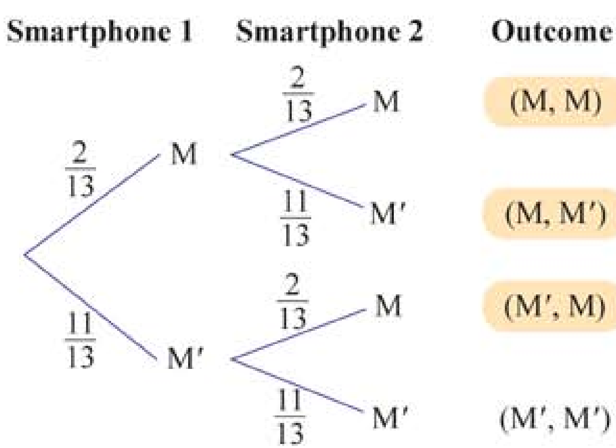
Alternative Method

P(Zalifah or Maran eats *cendol* at the banquet) $= \frac{2}{7} + \frac{3}{7} + \frac{6}{35}$
$= \frac{31}{35}$

Example 11

The probability that a smartphone manufactured by Jaya Factory has a display problem is $\frac{2}{13}$. Two smartphones are chosen at random. Draw a tree diagram to show all the possible outcomes. Hence, calculate the probability that at least one smartphone chosen has a display problem.

Solution:



M = Has a display problem
M′ = No display problem

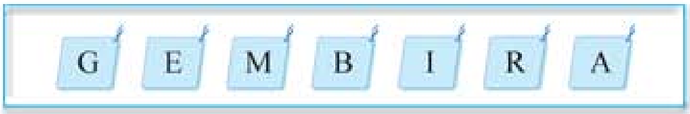
INTERACTIVE ZONE

Beside the given solution, what other methods can be used to solve Example 11? Discuss.

P(At least one smartphone has a display problem)
$= P(M, M) + P(M, M') + P(M', M)$
$= \left(\frac{2}{13} \times \frac{2}{13}\right) + \left(\frac{2}{13} \times \frac{11}{13}\right) + \left(\frac{11}{13} \times \frac{2}{13}\right)$
$= \frac{48}{169}$

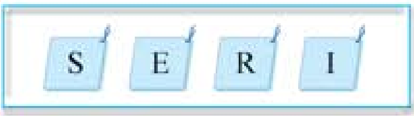
Self Practice 9.3c

1. Seven cards labelled with the letters "G, E, M, B, I, R, A" are put in a box. A card is chosen at random from the box.



By listing all the possible outcomes, calculate the probability that the card chosen is labelled with a vowel or letter "R".

2. Two boxes labelled as K and L contain four cards labelled with the letters "S, E, R, I" and three cards labelled with the numbers "4, 5, 6" respectively. A card is chosen at random from each box.



Box K

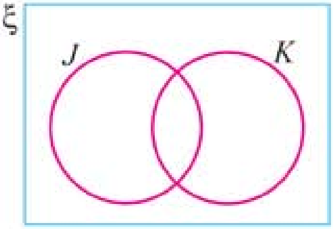
Box L

By listing all the possible outcomes, calculate the probability of getting a letter "S" from box K or a multiple of 3 from box L.

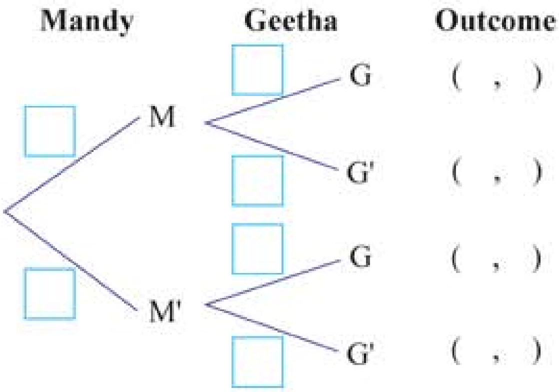
3. The probability of appointing Jessie as the chairman of the Finance Club (J) and the head of the sports house (K) are $\frac{3}{8}$ and $\frac{2}{9}$ respectively.

(a) Complete the Venn diagram on the right to represent the relationship between the probabilities of appointing Jessie as the chairman of the Finance Club and the head of the sports house.

(b) Calculate the probability of not appointing Jessie as the chairman of the Finance Club or the head of the sports house.



4. The Geography Club of SMK Cerdik organises a study group tour to Kota Kinabalu. The probability of Mandy and Geetha joining this tour are $\frac{4}{7}$ and $\frac{9}{14}$ respectively. Complete the following tree diagram. Hence, calculate the probability of either Mandy or Geetha joining this tour.



9.4 Application of Probability of Combined Events

How do you solve problems involving probability of combined events?

Learning Standard

Solve problems involving probability of combined events.

Example 12

A fair dice is rolled twice consecutively. If this experiment is carried out 540 times, how many times will at least one perfect square be obtained?

Solution:

Understanding the problem

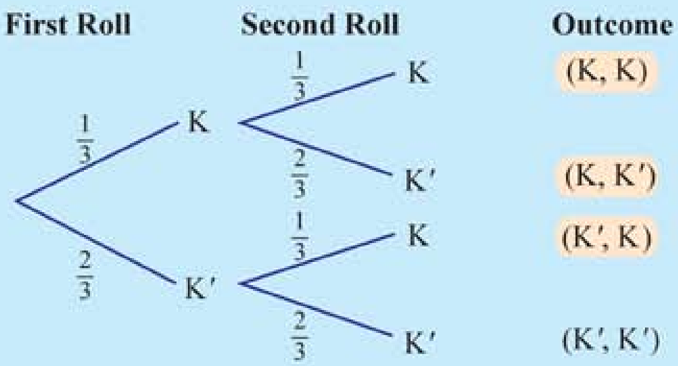
- Independent combined events
- Perfect squares = 1, 4
- At least one perfect square = (K, K), (K, K′) or (K′, K)

Planning a strategy

- Draw a tree diagram.
- $P(\text{perfect squares}) = \frac{2}{6} = \frac{1}{3}$
- Calculate $P[(K, K) \cup (K, K') \cup (K', K)]$
- $P[(K, K) \cup (K, K') \cup (K', K)] \times 540$ times

Implementing the strategy

K = Event of getting a perfect square
K' = Event of not getting a perfect square



$P[(K, K) \cup (K, K') \cup (K', K)]$

$= \left(\frac{1}{3} \times \frac{1}{3}\right) + \left(\frac{1}{3} \times \frac{2}{3}\right) + \left(\frac{2}{3} \times \frac{1}{3}\right)$

$= \frac{5}{9}$

$n(\text{at least one perfect square})$

$= \frac{5}{9} \times 540$

$= 300$ times

Checking Answer

$n(\text{at least one perfect square})$

$= \left(1 - \frac{2}{3} \times \frac{2}{3}\right) \times 540$

$= 300$

Conclusion

There are 300 times to obtain at least one perfect square.

Example 13

A box contains seven red marbles, five yellow marbles and three blue marbles. Two marbles are chosen randomly one by one from the box. If the first marble is blue, the blue marble is then returned to the box before a second marble is chosen. If the first marble is not blue, the marble is not returned to the box and then a second marble is chosen. Calculate the probability of getting two marbles of different colours.

Solution:

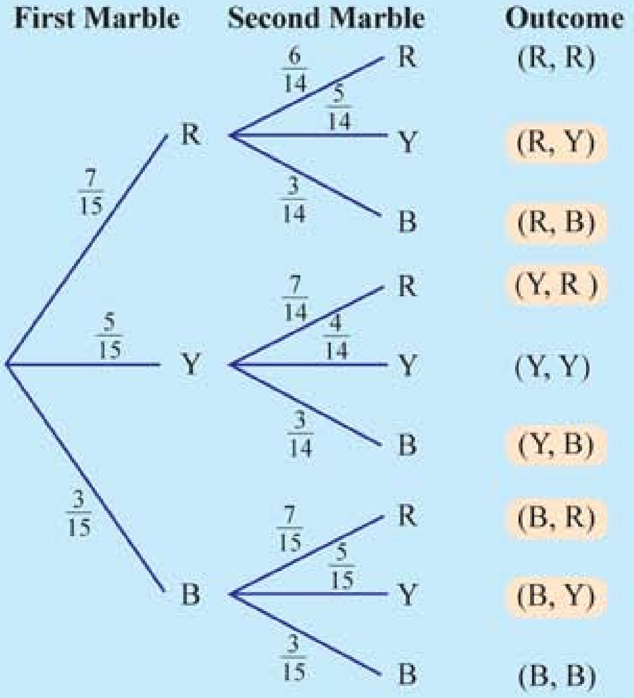
Understanding the problem

- Dependent combined events
- Total number of marbles = 15
- Two marbles of different colours = {(R, Y), (R, B), (Y, R), (Y, B), (B, R), (B, Y)}

Planning a strategy

- Draw a tree diagram.
- Calculate $P[(R, Y) \cup (R, B) \cup (Y, R) \cup (Y, B) \cup (B, R) \cup (B, Y)]$

Implementing the strategy



P(Marbles of different colours)

$= P(R, Y) + P(R, B) + P(Y, R) + P(Y, B) + P(B, R) + P(B, Y)$

$= \left(\frac{7}{15} \times \frac{5}{14}\right) + \left(\frac{7}{15} \times \frac{3}{14}\right) + \left(\frac{5}{15} \times \frac{7}{14}\right) + \left(\frac{5}{15} \times \frac{3}{14}\right) + \left(\frac{3}{15} \times \frac{7}{15}\right) + \left(\frac{3}{15} \times \frac{5}{15}\right)$

$= \frac{349}{525}$

Conclusion

The probability of getting two marbles of different colours is $\frac{349}{525}$.

Checking Answer

Complement method:

P(two marbles of different colours)

$= 1 - [P(R, R) + P(Y, Y) + P(B, B)]$

$= 1 - \left[\left(\frac{7}{15} \times \frac{6}{14}\right) + \left(\frac{5}{15} \times \frac{4}{14}\right) + \left(\frac{3}{15} \times \frac{3}{15}\right)\right]$

$= \frac{349}{525}$

Self Practice 9.4a

1. A study is carried out on the gender of the children from 16 000 families with two children. Estimate the number of families with at least one son in that study.

2. A box contains three yellow pens, five red pens and a black pen. Two pens are chosen at random from the box. Calculate the probability that both pens chosen are of the same colour.

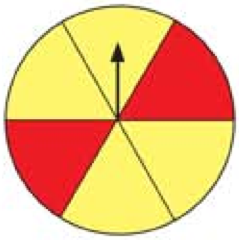
3. Jonathan enjoys watching the sunset on the beach. Jonathan has two options for either going to Pantai Jati or Pantai Cengal for two days.

   The probability that it will rain every evening at Pantai Jati is $\frac{19}{25}$.

   The probability that it will rain at Pantai Cengal depends on the previous day. If it does not rain on the previous day, the probability that it will rain in the evening is $\frac{5}{7}$. If it rains on the previous day, the probability it will rain in the evening is $\frac{2}{5}$.

   If the weather is good on both beaches on the previous day Jonathan departed, which beach should Jonathan choose so that he can enjoy watching the sunset on the beach for both evenings? Justify your answers.

4. Each customer of Naga Shop who spends more than RM200 will be given a chance to spin a lucky wheel that has six similar sectors. Two of the sectors are red and the rest are yellow.



Box	Number of cash vouchers	
	RM50	RM10
A	20	5
B	10	20

   If the pointer of the lucky wheel stops in the red sector, the customer has a chance to choose a cash voucher from box *A*. If the pointer of the lucky wheel stops in the yellow sector, the customer has a chance to choose a cash voucher from box *B*. The number of cash vouchers in box *A* and box *B* are shown in the table above.

   It is estimated that 450 customers of Naga Shop will spend more than RM200. Calculate the number of RM10 vouchers the Naga Shop needs to prepare.
   (The selected cash vouchers will be replaced with new vouchers by the shop so that the number of cash vouchers in both boxes remains the same.)

Comprehensive Practice

1. There are three blue coloured pencils and two green coloured pencils in a box. Two colour pencils are randomly selected one by one from the box without replacement. Write the sample space for the selected colour pencils.

2. A number is chosen at random from set $S = \{x : x \text{ is an integer}, 1 \leqslant x \leqslant 30\}$. Calculate the probability of
   (a) getting a multiple of 3 and a multiple of 12.
   (b) getting a factor of 20 or a factor of 8.

3. Two cards labelled with the numbers "77, 91" are put in box M and three cards labelled with the letters "R, I, A" are put in box N. A card is chosen at random from box M and box N respectively.
   (a) Complete the following table with all the possible outcomes.

Box M	**Box N**		
	R	I	A
77	(77, R)		
91		(91, I)	

   (b) By listing all the possible outcomes, calculate the probability that
      (i) a number card with the sum of its digits is greater than 10 and a consonant card are chosen.
      (ii) a number card with the sum of its digits is greater than 10 or a consonant card are chosen.

4. Two prefects are chosen at random from five prefects, where three of them are in Form Four and two are in Form Five. Draw a tree diagram to show all the possible outcomes. Then, calculate the probability that both prefects chosen are in the same form.

5. The probability of Kam Seng passing his Physics and Chemistry tests are 0.58 and 0.42 respectively. Calculate the probability that
   (a) Kam Seng passes both tests.
   (b) Kam Seng passes only one test.

6. Fatimah sends her resume for job application to three companies. The probabilities of Fatimah getting a job offer from companies X, Y and Z are $\frac{3}{5}$, $\frac{4}{9}$ and $\frac{5}{12}$ respectively. Calculate the probability of Fatimah getting a job offer from
   (a) any two companies.
   (b) at least one company.

7. It is given that event A and event B are two mutually exclusive events and $P(A) = \frac{1}{3}$.

(a) State the maximum value of $P(B)$.

(b) If $P(A \cup B) = \frac{7}{9}$, identify $P(B)$.

8. Box R contains five red marbles and seven green marbles while box T contains four red marbles and eight green marbles. A marble is randomly selected from box R. If the marble is red, that marble will be put into box T. If the marble is green, that marble will be returned to box R. Then, a marble will be randomly selected from box T. The colours of the selected marbles will be recorded.

(a) Calculate the probability of

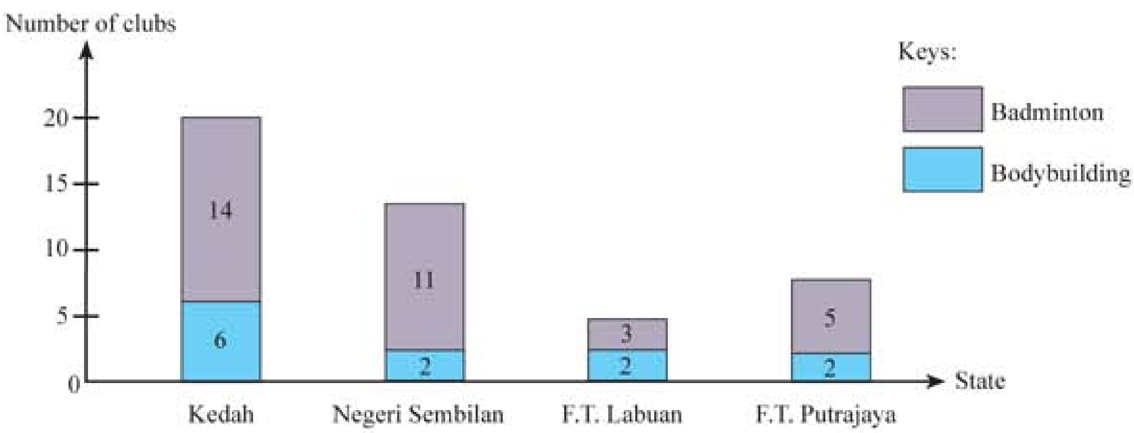
(i) selecting two red marbles.

(ii) selecting two marbles of different colours.

(b) Class 4 Amanah has 36 pupils. Each pupil is given an opportunity to select two marbles with the conditions stated above and a pupil who successfully selected two green marbles will be given a gift worth RM5. Estimate the cost of gifts needed.

9. Jacky has eight shirts and three of them are blue shirts. 40% of the shirts that Halim has are blue. $\frac{1}{5}$ of the shirts that Kumar has are blue. Jacky, Halim and Kumar each chooses a shirt at random to attend a meeting. Calculate the probability that two of them wear blue shirts.

10. The following bar chart shows the number of badminton clubs and bodybuilding clubs in Kedah, Negeri Sembilan, Federal Territory of Labuan and Federal Territory of Putrajaya.



A badminton club and a bodybuilding club in the four states are selected at random. Calculate the probability that

(a) both clubs selected are from Kedah.

(b) a club is selected each from the Federal Territories and Negeri Sembilan respectively.

11. The probability that Khaizan is involved in an accident in each round of a motorcycle race is 0.4. Khaizan has to quit the race if he is involved in an accident. The probability of Khaizan winning each round of the motorcycle race is 0.96 provided that he is not involved in any accident. Khaizan needs to complete three laps of the race track.

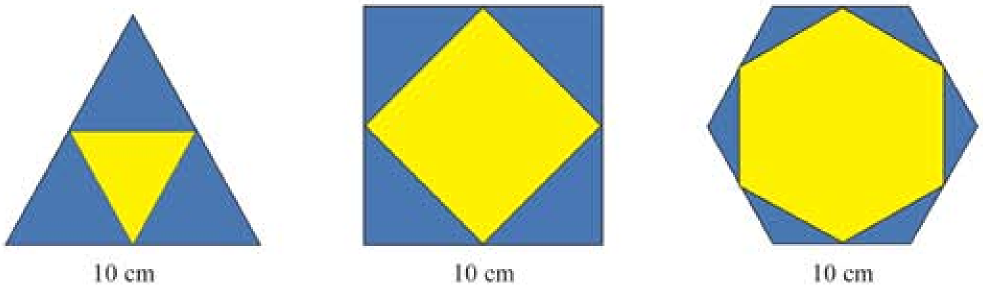
(a) Calculate the probability, correct to three decimal places, that

(i) Khaizan is the champion of the race.

(ii) Khaizan is unable to finish the race.

(b) Based on the answer in (a), is it advisable for Khaizan to encourage his younger brother to participate in a motorcycle race? State a moral value that you have learned to support your answer.

PROJECT



1. Construct three regular polygons as above. The polygons constructed have sides with equal lengths.
2. Carry out the activity stated below.

> A dart is thrown towards each of the above regular polygons. The experiment is carried out twenty times and the results of the dart that lands on the yellow and blue regions are recorded as (Y, Y, Y), (Y, B, Y).

3. Based on the recorded results, what is your conclusion? Elaborate the reasons to support your conclusion.
4. Further exploration: If you are the owner of a game booth on your school's Entrepreneurship Day, which of the regular polygons will you choose? State the reasons to support your choice.

CONCEPT MAP

Probability of Combined Events

Dependent Events and Independent Events

A and *B* are Dependent Events
- Event *A* affects the occurrence of event *B*.

Example:
Choose two cards from a box that contains cards labelled with the letters "B, A, I, K" without replacement.

A and *B* are Independent Events
- Event *A* does not affect the occurrence of event *B*.

Example:
A fair dice is rolled twice and "6" is obtained for two times.

Multiplication Rule of Probability

$P(A \cap B) = P(A) \times P(B)$

Example:
A fair dice is rolled twice and "6" is obtained for two times.

$P(\text{Two times of "6"}) = \frac{1}{6} \times \frac{1}{6}$

$= \frac{1}{36}$

Addition Rule of Probability

A and *B* are Mutually Exclusive Events if $A \cap B = \phi$,
$P(A \cup B) = P(A) + P(B)$

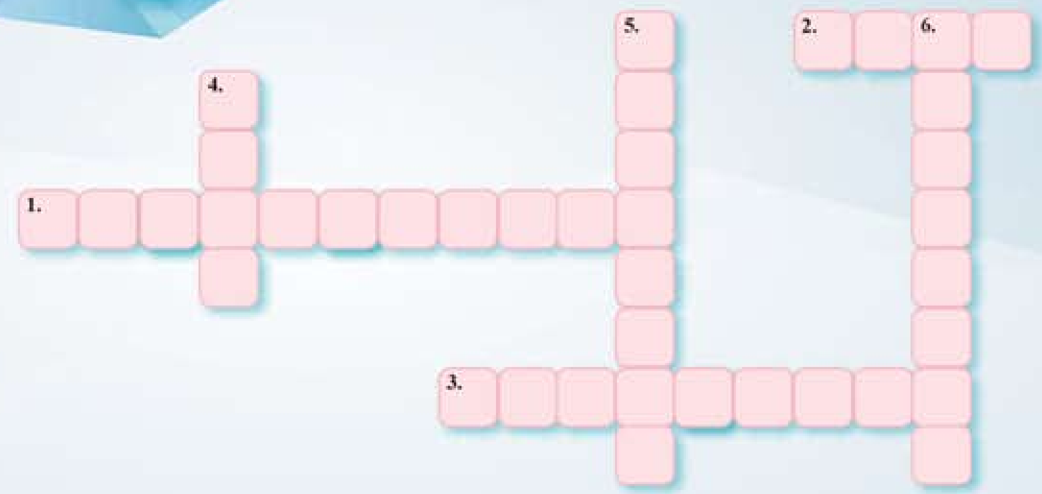
Example: $X = \{x : 1 \leqslant x \leqslant 10, x \in \mathbf{W}\}$
A number is chosen at random from set X.
Probability of choosing the number 2 or an odd number
$= P(\text{Number 2 or an odd number})$
$= \frac{1}{10} + \frac{5}{10}$
$= \frac{3}{5}$

A and *B* are Non-Mutually Exclusive Events if $A \cup B \neq \phi$,
$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

Example: $X = \{x : 1 \leqslant x \leqslant 10, x \in \mathbf{W}\}$
A number is chosen at random from set X.
Probability of choosing a prime number or an odd number
$= P(\text{A prime number or an odd number})$
$= \frac{4}{10} + \frac{5}{10} - \frac{3}{10}$
$= \frac{3}{5}$

Applications of Probability of Combined Events

Self Reflection



Across

1. *A* and *B* are two ______ events if the occurrence of event *A* does not affect the occurrence of event *B* and vice versa.
2. A ______ is rolled and a coin is tossed. The number of the outcomes is 12.
3. *A* and *B* are mutually ______ events meaning there is no intersection between events *A* and *B*.

Down

4. A number is chosen from $\{x : x \text{ is an integer and } 0 < x < 50\}$. K is the event of getting an ______ number and K is the event of getting an odd number. $P(K \cup L) = P(K) + P(L)$
5. $P(A \text{ and } B) = P(A)$ ______ $P(B)$
6. A ______ event is an outcome of the union or intersection of two or more events.

Mathematics Exploration

Bottle flipping is a game which involves throwing a plastic water bottle, either filled or partially filled with water so that the bottle rotates and then stands upright again.

Try to explore the factors that influence the likelihood of a successful bottle flipping.