CHAPTER 9

Straight Lines

What will you learn?

9.1 Straight Lines

Why do you learn this chapter?

- The concept of straight lines is widely used in the construction of various geometric shapes such as squares, triangles and kites.
- The concept of straight lines is used in engineering, architecture, construction, mapping, sciences, sports and so on.

Normally, every building is built vertically. Some buildings such as the Leaning Tower of Teluk Intan which was built in 1885, became inclined due to the soil structure. Although inclined and over 100 years old, the Leaning Tower of Teluk Intan is still standing strong and is a landmark of Teluk Intan. The leaning tower was declared a national heritage in 2015.

Exploring Era

Euclid was a Greek mathematician. He had conducted a lot of research about straight lines and geometry such that he was known as the founder of geometry.

The field of geometry was named Euclidean Geometry to commemorate Euclid's contributions to the fundamental principles of geometry.

http://bukutekskssm.my/Mathematics/F3/
ExploringEraChapter9.pdf

WORD BANK

straight line	*garis lurus*
parallel line	*garis selari*
vertical distance	*jarak mencancang*
horizontal distance	*jarak mengufuk*
gradient	*kecerunan*
axis	*paksi*
intercept	*pintasan*
simultaneous equation	*persamaan serentak*
intersection point	*titik persilangan*

9.1 Straight Lines

What is the equation of a straight line?

LEARNING STANDARD

Make connection between the equation, $y = mx + c$, and the gradient and y-intercept, and hence make generalisation about the equation of a straight line.

In Form 2, you have learnt how to draw the graph of linear function and non-linear functions by constructing a table of values of related functions.

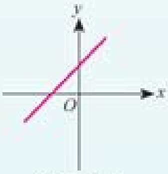


Diagram 1

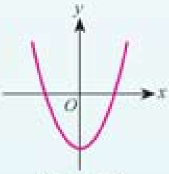


Diagram 2

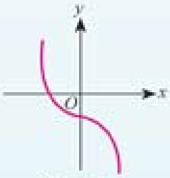


Diagram 3

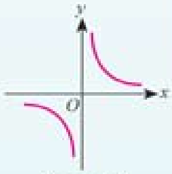


Diagram 4

Each of the above graph is drawn based on a specific function. The function is also an equation for the related graph.

Can you differentiate a graph of linear function and a graph of non-linear function? Discuss.

Brainstorming 1

In groups

Aim: To determine the relationship between equation $y = mx + c$ with gradient and y-intercept.

Materials: Graph paper, linear function cards.

Steps:

1. Get into four groups.
2. Each group is given a card written with two linear functions.

Group 1	*Group 2*	*Group 3*	*Group 4*
$y = 3x + 6$ $y = -2x - 4$	$y = 2x + 6$ $y = -4x + 8$	$y = 5x - 10$ $y = -3x + 9$	$y = 4x - 8$ $y = -2x + 2$

3. Complete the table of values below for each given function.

x	-3	-2	-1	0	1	2	3
y							

4. Based on the table of values, draw the graphs of the functions.

FLASHBACK

The gradient, m, of a straight line that connects two points (x_1, y_1) and (x_2, y_2)

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

or

$$m = -\frac{y\text{-intercept}}{x\text{-intercept}}$$

5. From the graph of the function, calculate the gradient and state the y-intercept.
6. Compare the values of gradient and y-intercept from the graph with the values in the function card.

Discussion:

1. Compare your findings in step 6 with linear function $y = mx + c$. What is your conclusion?
2. Present your findings. Are your findings the same as the other groups' findings?

From Brainstorming 1, it is found that:

(a) For a linear function, $y = mx + c$, m is the gradient and c is the y-intercept of the straight line.
(b) The graph of linear function, $y = mx + c$ is a straight line.

In general,

Equation of straight line

$$y = \underbrace{m}_{\text{Gradient}}x + \underbrace{c}_{y\text{-intercept}}$$

Brainstorming 2

In pairs

Aim: To produce a graph of linear function.

Materials: Dynamic software

Steps:

1. Start with *New sketch.*
2. Select *graph* icon.
3. Select *plot new function* and enter the required equation of straight line (Diagram 1).

New Function

g(x) = 2·x-3

2x - 3

The first graph of straight line: $y = 2x - 3$

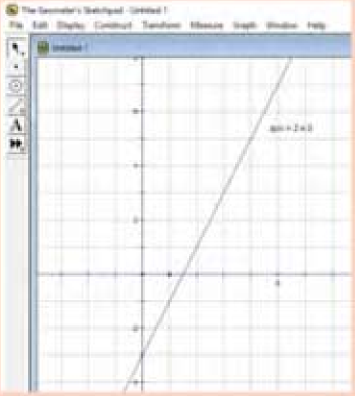


Diagram 1

4. Click *straightedge tool* and mark two points on the constructed graph of straight line.

5. Click *measure* and then click *slope* (Diagram 2).
The gradient value will be displayed (Diagram 3).

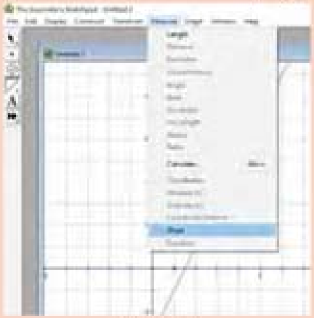


Diagram 2

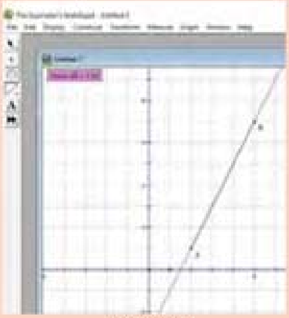


Diagram 3

6. Repeat steps 2 to 5 to draw and determine the gradient of the graph of straight line for function $y = -2x + 8$.
(Diagram 4)

The second graph of straight line: $y = -2x + 8$

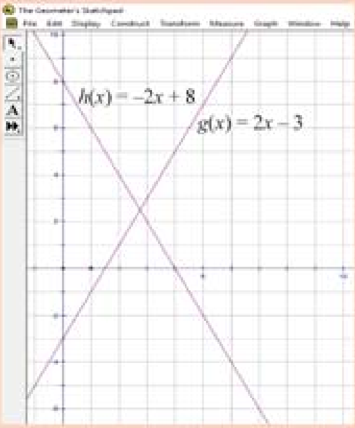
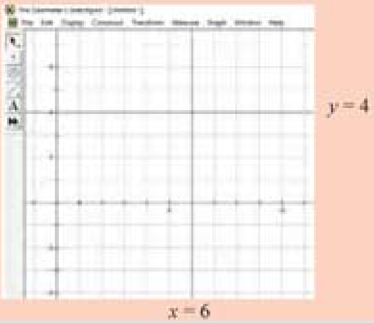


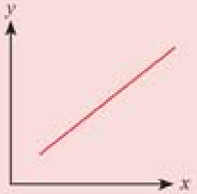
Diagram 4

7. **Straight lines that are parallel to the x-axis and y-axis.**
A displayed example of straight lines such as
(a) $y = 4$ (b) $x = 6$

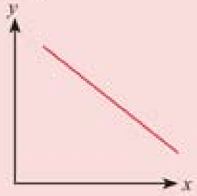


TIPS

Relationship between the value m and the form of a straight line graph.
If $m > 0$



If $m < 0$



Discussion:

1. Compare the forms of graph resulting from dynamic software with the forms of graph from Brainstorming 1.
2. Make a conclusion for the values of m and c of the equation of straight line in the form $y = mx + c$. Discuss the shape of the graph when
   (a) m is positive
   (b) m is negative
   (c) parallel to x-axis
   (d) parallel to y-axis

From Brainstorming 2, it is found that:

(a) The graph of linear function $y = mx + c$ is a straight line.
(b) The graph of function $y = h$ is a straight line parallel to x-axis.
(c) The graph of function $x = h$ is a straight line parallel to y-axis.

Example 1

Determine the gradient and y-intercept of the straight line

(a) $y = 2x + 9$
(b) $3y = –2x + 12$

Solution:

(a) Compare $y = 2x + 9$ with $y = mx + c$;
$m = 2$ and $c = 9$
Thus, gradient = 2 and y-intercept = 9

(b) Given $3y = –2x + 12$

Divide by 3 so that the coefficient of y is +1.

$$\xrightarrow{\div 3} \quad \frac{3y}{3} = -\frac{2x}{3} + \frac{12}{3}$$

$$y = -\frac{2}{3}x + 4$$

Compare $y = -\frac{2}{3}x + 4$ with $y = mx + c$;

$m = -\frac{2}{3}$ and $c = 4$

Thus, gradient = $-\frac{2}{3}$ and y-intercept = 4.

SMART MIND

What is the y-intercept of a straight line that passes through the origin?

QUIZ

What is the value of the gradient for the straight line
(a) $y = x$
(b) $y = -x$

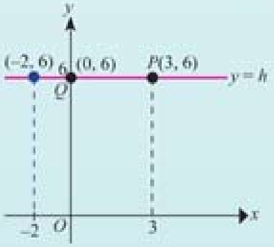
BULLETIN

In the equation $y = mx + c$, the coefficient of y is +1.

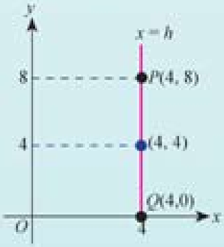
Example 2

State the value of h for the straight line graph below. State reasons for your answer.

(a)



(b)



Solution:

(a) $h = 6$ because the straight line $y = 6$ is always 6 units from the x-axis

(b) $h = 4$ because the straight line $x = 4$ is always 4 units from the y-axis.

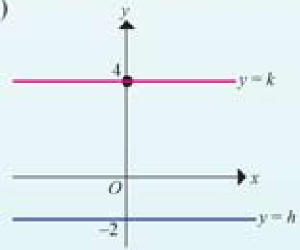
MIND TEST 9.1a

1. Determine the gradient and y-intercept of the following straight lines.

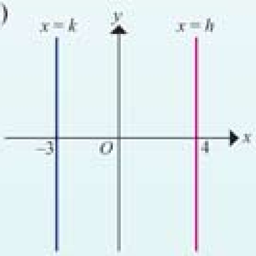
(a) $y = 3x + 5$
(b) $y = 2x - 7$
(c) $y = -x + 4$
(d) $2y = 8x + 6$
(e) $3y = -x + 18$
(f) $-4y = -2x + 5$

2. State the value of k and of h for each straight line graph given.

(a)



(b)



What is the relationship between the equations of straight lines in the form $ax + by = c$, $\frac{x}{a} + \frac{y}{b} = 1$ and $y = mx + c$?

LEARNING STANDARD

Investigate and interpret the equations of straight lines in other forms such as $ax + by = c$ and $\frac{x}{a} + \frac{y}{b} = 1$, and change to the form of $y = mx + c$, and vice versa.

Brainstorming 3

In groups

Aim: To determine the relationship between the equations of straight lines in the form of $ax + by = c$, $\frac{x}{a} + \frac{y}{b} = 1$ and $y = mx + c$.

Materials: Graph paper, straight line equation cards

Steps:

1. Get into four groups.
2. Each group is given a card with three equations of a straight line written on it.

Group 1	*Group 2*	*Group 3*	*Group 4*
$2x + 3y = 6$	$4x - 2y = -8$	$-3x + 4y = -12$	$-x - 4y = 4$
$\frac{x}{3} + \frac{y}{2} = 1$	$\frac{x}{(-2)} + \frac{y}{4} = 1$	$\frac{x}{4} + \frac{y}{(-3)} = 1$	$\frac{x}{(-4)} + \frac{y}{(-1)} = 1$
$y = -\frac{2}{3}x + 2$	$y = 2x + 4$	$y = \frac{3}{4}x - 3$	$y = -\frac{1}{4}x - 1$

3. Determine the corresponding value of y when $x = 0$ and the corresponding value of x when $y = 0$ for each equation.

Example:

x	0	3
y	2	0

$2x + 3y = 6$

When $x = 0$:

$$\begin{aligned} 2(0) + 3y &= 6 \\ 3y &= 6 \\ y &= 2 \end{aligned}$$

When $y = 0$:

$$\begin{aligned} 2x + 3(0) &= 6 \\ 2x &= 6 \\ x &= 3 \end{aligned}$$

4. Draw a straight line graph for each equation.
5. From the graph, state the x-intercept and y-intercept and determine the gradient of the graph.

TIPS

A straight line graph can be drawn by plotting at least two points.

TIPS

Gradient $= -\dfrac{y\text{-intercept}}{x\text{-intercept}}$

Discussion:

1. What is your conclusion about the relationship between the x-intercept with the y- intercept and the gradients of the three straight line graphs?
2. What is your conclusion about the relationship between the equations of straight line in different forms?

From Brainstorming 3, it is found that:

(a) The values of x-intercept and y-intercept and the gradient for these three straight lines are the same.

(b) Equations of straight line in the forms of $ax + by = c$, $\dfrac{x}{a} + \dfrac{y}{b} = 1$ and $y = mx + c$ produce the same straight line graph if the values of x-intercept and y-intercept are the same.

In general,

Straight line equation can also be written in the form of $\boldsymbol{ax + by = c}$ and $\boldsymbol{\dfrac{x}{a} + \dfrac{y}{b} = 1}$; $a \neq 0$ and $b \neq 0$

How do you change the equation of straight line in the form of $ax + by = c$, $\dfrac{x}{a} + \dfrac{y}{b} = 1$ to the form of $y = mx + c$ and vice versa?

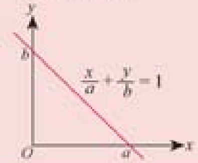
TIPS

For the straight line

$\dfrac{x}{a} + \dfrac{y}{b} = 1$,

a = x-intercept

b = y -intercept



Example 3

Change the equation of straight line below to the form of $\dfrac{x}{a} + \dfrac{y}{b} = 1$ and $y = mx + c$.

(a) $2x + 3y = 12$ (b) $3x - 5y = 15$

Solution:

(a) $2x + 3y = 12$

(i)

$$2x + 3y = 12$$

$$\boxed{\div 12} \rightarrow \left.\begin{aligned} \frac{2x}{12} + \frac{3y}{12} = \frac{12}{12} \end{aligned}\right\} \rightarrow$$

Divide by 12 to get a value of 1.

$$\frac{x}{6} + \frac{y}{4} = 1$$

(ii)

$$\begin{aligned} 2x + 3y &= 12 \\ 3y &= -2x + 12 \end{aligned}$$

$$\boxed{\div 3} \rightarrow \frac{3y}{3} = \frac{-2x}{3} + \frac{12}{3}$$

$$y = -\frac{2}{3}x + 4$$

Divide by 3 so that the coefficient of y is +1.

(b) $3x - 5y = 15$

(i)

$$3x - 5y = 15$$

$$\boxed{\div 15} \rightarrow \frac{3x}{15} - \frac{5y}{15} = \frac{15}{15}$$

$$\frac{x}{5} - \frac{y}{3} = 1$$

(ii)

$$3x - 5y = 15$$

$$-5y = -3x + 15$$

$$\boxed{\div (-5)} \rightarrow \frac{-5y}{(-5)} = \frac{-3x}{(-5)} + \frac{15}{(-5)}$$

$$y = \frac{3}{5}x - 3$$

DISCUSSION CORNER

Among the three forms of equation of straight line that you have learned, which is the easiest form to know the gradient value, the *y*-intercept value and the *x*-intercept value of a straight line? Discuss.

Example 4

Change the equation of straight line below to the form of $ax + by = c$ and $y = mx + c$.

(a) $\frac{x}{6} + \frac{y}{3} = 1$

(b) $-\frac{x}{2} + \frac{y}{4} = 1$

Solution:

(a) $\frac{x}{6} + \frac{y}{3} = 1$

(i)

$$\frac{x}{6} + \frac{y}{3} = 1$$

$$\frac{3x + 6y}{6(3)} = 1$$

Equate the denominator.

$$3x + 6y = 1(18)$$

$$3x + 6y = 18$$

$$x + 2y = 6$$

(ii)

$$\frac{x}{6} + \frac{y}{3} = 1$$

$$\frac{y}{3} = -\frac{x}{6} + 1$$

$$\boxed{\times 3} \rightarrow \frac{3y}{3} = \frac{-x(3)}{6} + 1(3)$$

$$y = -\frac{1}{2}x + 3$$

(b) $-\frac{x}{2} + \frac{y}{4} = 1$

(i)

$$-\frac{x}{2} + \frac{y}{4} = 1$$

$$\frac{-4x + 2y}{2(4)} = 1$$

$$-4x + 2y = 1(8)$$

$$-4x + 2y = 8$$

$$-2x + y = 4$$

(ii)

$$-\frac{x}{2} + \frac{y}{4} = 1$$

$$\frac{y}{4} = \frac{x}{2} + 1$$

$$\boxed{\times 4} \rightarrow \frac{4y}{4} = \frac{x(4)}{2} + 1(4)$$

$$y = 2x + 4$$

Multiply by 4 so that the coefficient *y* is +1.

TIPS

$-4x + 2y = 8$
can also be written as
$4x - 2y = -8$

QUIZ

What is the value of the gradient for the straight line $-\frac{x}{2} - \frac{y}{2} = 1$?

Example 5

Change the equation of the following straight lines to the form of $ax + by = c$ and $\frac{x}{a} + \frac{y}{b} = 1$.

(a) $y = -2x + 8$

(b) $y = 3x + 6$

Solution:

(a) $y = -2x + 8$

(i)

$$y = -2x + 8$$

$$2x + y = 8$$

(ii)

$$y = -2x + 8$$

$$2x + y = 8$$

$$\boxed{\div 8} \rightarrow \frac{2x}{8} + \frac{y}{8} = \frac{8}{8}$$

$$\frac{x}{4} + \frac{y}{8} = 1$$

(b) $y = 3x + 6$

(i)

$$y = 3x + 6$$

$$-3x + y = 6$$

(ii)

$$y = 3x + 6$$

$$-3x + y = 6$$

$$\boxed{\div 6} \rightarrow \frac{-3x}{6} + \frac{y}{6} = \frac{6}{6}$$

$$-\frac{x}{2} + \frac{y}{6} = 1$$

MIND TEST 9.1b

1. Write the equation of the following straight lines in the form of $\frac{x}{a} + \frac{y}{b} = 1$ and $y = mx + c$.

   (a) $3x - 4y = 24$ (b) $7x + 2y = 28$ (c) $5x - 3y = 15$ (d) $-2x + 3y = 9$

2. Write the equation of the following straight lines in the form of $ax + by = c$ and $y = mx + c$.

   (a) $\frac{x}{4} + \frac{y}{3} = 1$ (b) $-\frac{x}{3} + \frac{y}{6} = 1$ (c) $\frac{3x}{2} + \frac{y}{6} = 1$ (d) $\frac{2x}{3} - \frac{y}{4} = 1$

3. Write the equation of the following straight lines in the form of $ax + by = c$ and $\frac{x}{a} + \frac{y}{b} = 1$.

   (a) $y = 2x + 6$ (b) $y = 3x - 12$ (c) $y = -x + 5$ (d) $y = -2x - 4$

What is the relationship between the points on a straight line and the equation of the line?

LEARNING STANDARD

Investigate and make inference about the relationship between the points on a straight line and the equation of the line.

Diagram 1 and Diagram 2 show two straight lines drawn on a Cartesian plane based on the equation of straight lines $x + 2y = 4$ and $x - y = -3$.

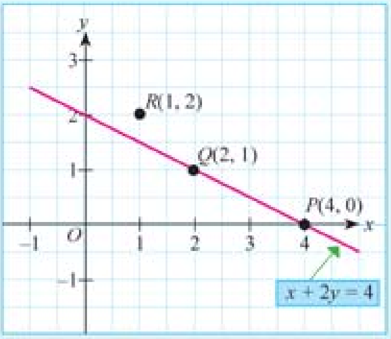


Diagram 1

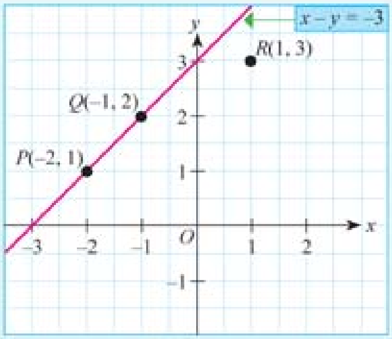


Diagram 2

Examine the position of points P, Q and R in Diagram 1 and Diagram 2. What can you say about the points P, Q and R and the straight line drawn?

(a) Diagram 1

$$\overbrace{x + 2y}^{\text{Left}} = \overbrace{4}^{\text{Right}}$$

(i) Substitute point $P(4, 0)$

Left: $x + 2y = 4 + 2(0) = 4$

Right: $= 4$

Equal

(ii) Substitute point $Q(2, 1)$

Left: $x + 2y = 2 + 2(1) = 4$

Right: $= 4$

Equal

(iii) Substitute point $R(1, 2)$

Left: $x + 2y = 1 + 2(2) = 5$

Right: $= 4$

Not Equal

(b) Diagram 2

$$\overbrace{x-y}^{\text{Left}} = \overbrace{-3}^{\text{Right}}$$

(i) Substitute point $P(-2, 1)$

Left:
$x - y$
$= -2 - 1$
$= -3$

Right:
$= -3$

Equal

(ii) Substitute point $Q(-1, 2)$

Left:
$x - y$
$= -1 - 2$
$= -3$

Right:
$= -3$

Equal

(iii) Substitute point $R(1, 3)$

Left:
$x - y$
$= 1 - 3$
$= -2$

Right:
$= -3$

Not Equal

From the above activity, it is found that:

(a) Points on a straight line or points that the straight line passes through will satisfy the equation of a straight line.

(b) Points that do not lie on a straight line will not satisfy the equation.

Example 6

1. Determine whether point P lies on the given straight line.

(a) $y = 3x + 2$, $P(2, 8)$

(b) $3x - 2y = 12$, $P(-4, 2)$

(c) $\frac{x}{3} + \frac{y}{2} = 1$, $P(6, -2)$

(d) $2y = -5x - 7$, $P(4, 3)$

Solution:

(a) $\underbrace{y}_{\text{Left}} = \underbrace{3x + 2}_{\text{Right}}$, $P(2, 8)$

Left:
$= 8$

Right:
$3x + 2$
$= 3(2) + 2$
$= 8$

Equal

Thus, $P(2, 8)$ lies on the straight line $y = 3x + 2$.

(b) $\underbrace{3x - 2y}_{\text{Left}} = \underbrace{12}_{\text{Right}}$, $P(-4, 2)$

Left:
$3x - 2y$
$= 3(-4) - 2(2)$
$= -16$

Right:
$= 12$

Not Equal

Thus, $P(-4, 2)$ does not lie on the straight line $3x - 2y = 12$

(c) $\underbrace{\frac{x}{3} + \frac{y}{2}}_{\text{Left}} = \underbrace{1}_{\text{Right}}$, $P(6, -2)$

Left:
$\frac{x}{3} + \frac{y}{2}$
$= \frac{(6)}{3} + \frac{(-2)}{2}$
$= 1$

Right:
$= 1$

Equal

Thus, $P(6, -2)$ lies on the straight line $\frac{x}{3} + \frac{y}{2} = 1$.

(d) $\underbrace{2y}_{\text{Left}} = \underbrace{-5x - 7}_{\text{Right}}$, $P(4, 3)$

Left:
$2y$
$= 2(3)$
$= 6$

Right:
$-5x - 7$
$= -5(4) - 7$
$= -27$

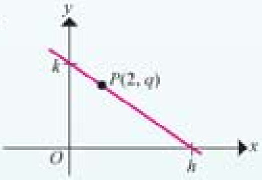
Not Equal

Thus, $P(4, 3)$ does not lie on the straight line $2y = -5x - 7$.

Example 7

The diagram shows a straight line $3x + 5y = 15$. Given that O is the origin, determine the value of

(a) h (b) k (c) q (d) gradient of the straight line $3x + 5y = 15$



Solution:

(a) h is the x-intercept.
Thus, $y = 0$

$$\begin{aligned} 3x + 5y &= 15 \\ 3(h) + 5(0) &= 15 \\ 3h &= 15 \\ h &= \frac{15}{3} \\ h &= 5 \end{aligned}$$

(b) k is the y-intercept.
Thus, $x = 0$

$$\begin{aligned} 3x + 5y &= 15 \\ 3(0) + 5(k) &= 15 \\ 5k &= 15 \\ k &= \frac{15}{5} \\ k &= 3 \end{aligned}$$

(c) $P(2, q)$ is a point on the straight line $3x + 5y = 15$.
Thus,

$$\begin{aligned} 3x + 5y &= 15 \\ 3(2) + 5(q) &= 15 \\ 6 + 5q &= 15 \\ 5q &= 15 - 6 \\ 5q &= 9 \\ q &= \frac{9}{5} \end{aligned}$$

(d) Gradient of the straight line $3x + 5y = 15$

$$m = -\frac{y\text{-intercept}}{x\text{-intercept}}$$

$$\text{Gradient} = -\frac{3}{5}$$

TIPS

- For points that lie on x-axis its value of y-coordinate is zero.
- For points that lie on y-axis its value of x-coordinate is zero.

FLASHBACK

Gradient, m

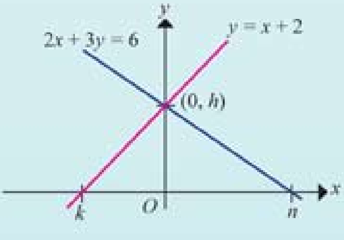
$$m = -\frac{y\text{-intercept}}{x\text{-intercept}}$$

SMART MIND

Do the coordinates $(-3, -3)$ lie on the straight line $y = x$?

MIND TEST 9.1c

1. Determine whether the following points lie on the straight line $y = 2x + 16$.
   (a) $M(-4, 3)$ (b) $N(1, 18)$ (c) $P(-8, 0)$ (d) $Q(-5, 8)$
2. Determine whether the following points lie on the straight line $2x + 3y = 12$.
   (a) $M(0, 4)$ (b) $N(3, -2)$ (c) $P(15, -6)$ (d) $Q(-4, 8)$
3. Determine whether the following points lie on the straight line $\frac{x}{2} + \frac{y}{3} = 1$.
   (a) $M(2, 0)$ (b) $N(-2, 12)$ (c) $P(4, -3)$ (d) $Q(0, 6)$
4. The diagram shows two straight lines, $y = x + 2$ and $2x + 3y = 6$. Given that O is the origin, determine the value of
   (a) h (b) k (c) n



What do you understand about the gradients of parallel lines?

LEARNING STANDARD

Investigate and make inference about the gradients of parallel lines.

You have learnt that the gradient of a straight line is the ratio of vertical distance to horizontal distance, and the corresponding angles of the parallel lines are equal.

Brainstorming 4

In pairs

Aim: To determine the relationship between gradients of straight lines with parallel lines.

Steps:

1. Examine the straight lines below that were drawn based on the equation of a straight line with the same gradient of $m = 2$.

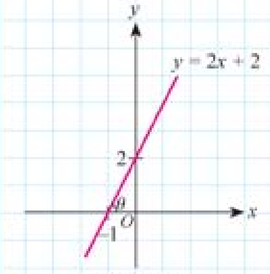


Diagram 1

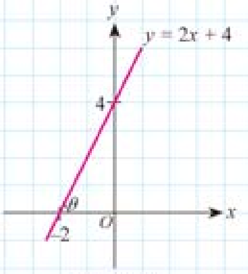


Diagram 2

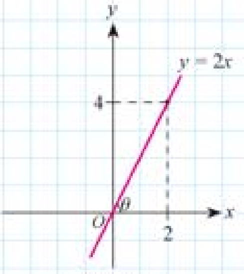


Diagram 3

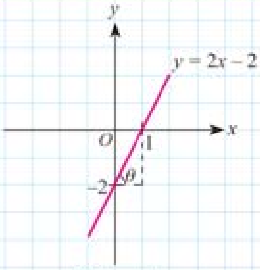


Diagram 4

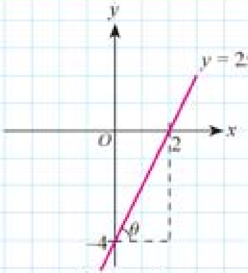
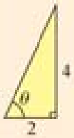
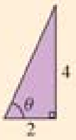
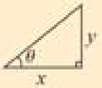


Diagram 5

2. Based on Diagram 1 to Diagram 5, calculate the value θ.

 Diagram 1	 Diagram 2	 Diagram 3	 Diagram 4	 Diagram 5
$\tan\theta = \frac{2}{1}$ $\theta = 63.43°$				

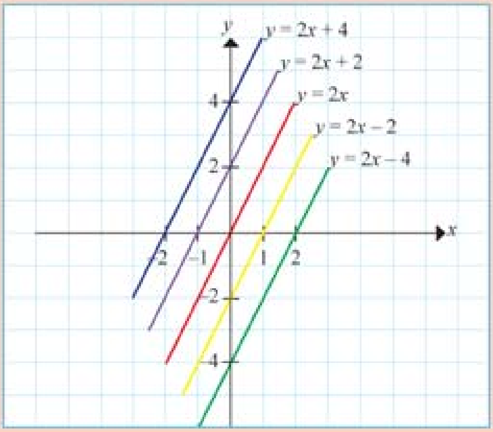
FLASHBACK



$\tan\theta = \frac{y}{x}$

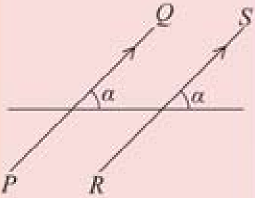
3. Are the values of θ for the five diagrams the same?

4. The graphs in Diagram 1 to Diagram 5 are combined as below.



TIPS

PQ and *RS* are parallel lines because they have corresponding angles.



Discussion:

1. What is the connection between the values of θ with the five straight lines above?
2. Are the straight lines $y = 2x + 4$, $y = 2x + 2$, $y = 2x$, $y = 2x - 2$ and $y = 2x - 4$ parallel? Why?
3. What are the connections between the gradients and the parallel lines?
4. Are your findings the same as those of the other groups?

From Brainstorming 4, it is found that:

The straight lines $y = 2x + 4$, $y = 2x + 2$, $y = 2x$, $y = 2x - 2$ and $y = 2x - 4$ are parallel because they have the same gradient, that is $m = 2$ and the same corresponding angle, that is 63.43°.

In general, **Straight lines** that have the same gradients are parallel.

Example 8

Determine whether the straight line $y = 3x + 5$ is parallel to the straight line $6x - 2y = 9$.

TIPS

To determine the gradient value of a straight line, change the equation of the given straight line to the form $y = mx + c$.

Solution:

$y = 3x + 5$

Compare with $y = mx + c$

Gradient = 3

$$\begin{aligned} 6x - 2y &= 9 \\ -2y &= -6x + 9 \\ \frac{-2y}{-2} &= \frac{-6x}{(-2)} + \frac{9}{(-2)} \\ y &= 3x - \frac{9}{2} \end{aligned}$$

Gradient = 3

Equal

The gradients of both straight lines are equal, thus $y = 3x + 5$ is parallel to $6x - 2y = 9$.

Example 9

Determine whether the straight line $y = 3x + 8$ is parallel to the straight line $6y = 3x - 9$.

Solution:

$y = 3x + 8$

Compare with $y = mx + c$

Gradient $= 3$

$$6y = 3x - 9$$
$$y = \frac{3x}{6} - \frac{9}{6}$$
$$y = \frac{1}{2}x - \frac{3}{2}$$
$$\text{Gradient} = \frac{1}{2}$$

Not equal

DISCUSSION CORNER

Will two parallel lines intersect? Discuss.

The gradients of both straight lines are not equal. So, $y = 3x + 8$ is not parallel to $6y = 3x - 9$.

Example 10

Given that the straight line $4x + 3y = 18$ is parallel to the straight line $2x + hy = 2$, calculate the value of h.

Solution:

If both straight lines are parallel, then the gradients are equal.

For $4x + 3y = 18$
$$3y = -4x + 18$$
$$y = -\frac{4}{3}x + 6$$
$$\text{Gradient} = -\frac{4}{3}$$

For $2x + hy = 20$
$$hy = -2x + 20$$
$$y = -\frac{2}{h}x + \frac{20}{h}$$
$$\text{Gradient} = -\frac{2}{h}$$

For,
$$-\frac{4}{3} = -\frac{2}{h}$$
$$h = 2 \times \frac{3}{4}$$
$$h = \frac{3}{2}$$

MIND TEST 9.1d

1. Determine whether the following pairs of straight lines are parallel.

(a) $3y = -6x + 3$ and $y + 2x = 14$

(b) $2x + 3y = 3$ and $2x + 6y = 12$

(c) $y = 2x + 1$ and $8x - 4y = 5$

(d) $y = -3x + 4$ and $9x + 2y = 12$

2. Determine the value of k for each of the following pairs of parallel lines.

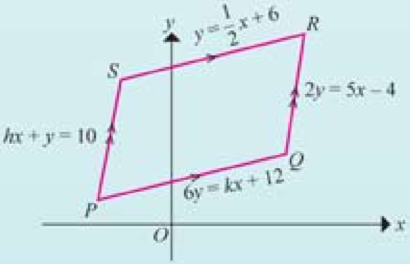
(a) $y = -3x + 4$ and $y + kx = 14$

(b) $kx + 2y = 7$ and $6x + 2y = 15$

(c) $8y = 5x + 1$ and $kx - 3y = 8$

(d) $3x + ky = 4$ and $2x + y = 3$

3. The diagram on the right shows a parallelogram $PQRS$. Given that the straight line PQ is parallel to SR and the straight line PS is parallel to QR, and O is the origin, calculate the values of h and k.



How do you determine the equation of a straight line?

LEARNING STANDARD

Determine the equation of a straight line.

The equation of a straight line $y = mx + c$ can be determined by the following steps:

1. Determine the value of gradient, m.
2. Determine a point which the straight line passes through or a point which lies on the straight line.
3. Substitute the gradient, m, the x-coordinate and y-coordinate from the point into the equation $y = mx + c$ to determine the value of c, that is, the y-intercept.

$$\underset{}{\text{y-coordinate value}} \longleftarrow y = \underset{\downarrow\atop\boxed{\text{Gradient value}}}{m}\overset{\boxed{\text{x-coordinate value}}\atop\uparrow}{x} + c$$

4. Substitute the gradient value and y-intercept value specified in the equation of the straight line $y = \boxed{m}x + \boxed{c}$.

Determine the equation of a straight line when the gradient and a point on the straight line are given

Example 11

Determine the equation of a straight line with a gradient of $\frac{1}{2}$ and passes through point $P(6, 8)$.

Solution:

$m = \frac{1}{2}$ Given $P(6, 8)$, thus $x = 6$, $y = 8$

Substitute the values of m, x and y into $y = mx + c$ to determine the value of c.

$8 = \frac{1}{2}(6) + c$

$8 = 3 + c$

$c = 8 - 3$

$c = 5$

Therefore, the equation of the straight line is $y = \frac{1}{2}x + 5$.

QUIZ

Determine the equation of a straight line with a gradient of 0 and passes through point $P(1, 5)$.

MIND TEST 9.1e

1. Determine the equation of a straight line with the given gradient and passes through point P given.

(a) Gradient = 2, $P(3, 7)$

(b) Gradient = –3, $P(-6, 4)$

(c) Gradient = $\frac{2}{3}$, $P(12, 5)$

(d) Gradient = $-\frac{1}{2}$, $P(4, -6)$

Determine the equation of a straight line that passes through two points

When two points on a straight line are given, the gradient of the straight line can be calculated. Hence the equation of the straight line can be determined.

Example 12

Determine the equation of a straight line that passes through point $P(-1, 5)$ and point $Q(2, -7)$.

Solution:

$$m = \frac{-7-5}{2-(-1)} = \frac{-12}{2+1} = \frac{-12}{3} = -4$$

For point $P(-1, 5)$, $x = -1$, $y = 5$.

Substitute the value of m, x and y into $y = mx + c$ to determine the value of c.

$5 = (-4)(-1) + c$
$5 = 4 + c$
$c = 5 - 4$
$c = 1$

Therefore, the equation of the straight line is $y = -4x + 1$.

TIPS

You can also substitute the value of point Q, where $x = 2$ and $y = -7$ and $m = -4$ into $y = mx + c$ to calculate the value of c and thus determine the equation of the straight line.

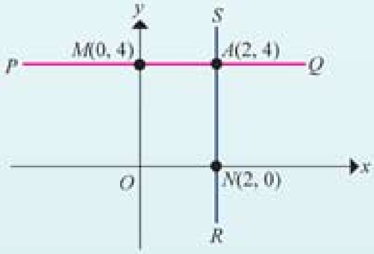
QUIZ

Determine the equation of a straight line that passes through the points $P(-4, 4)$ and $Q(5, -5)$.

Example 13

The diagram on the right shows straight lines PQ and RS. Given that straight line PQ is parallel to the x-axis and straight line RS is parallel to the y-axis, determine

(a) the equation of the straight line PQ
(b) the equation of the straight line RS



Solution:

(a) Gradient of straight line PQ with $A(2, 4)$ and $M(0, 4)$

$$m = \frac{4-4}{2-0} = \frac{0}{2} = 0$$

y-intercept = 4

Thus, the equation of the straight line PQ is

$y = 0(x) + 4$
$y = 4$

(b) Gradient of straight line RS with $A(2, 4)$ and $N(2, 0)$.

$$m = \frac{4-0}{2-2} = \frac{4}{0} = \text{Undefined}$$

The gradient of the straight line RS is undefined and is always 2 units from the y-axis.

Hence, the equation of the straight line RS is $x = 2$.

MIND TEST 9.1e

1. Determine the equation of the straight line that passes through the given pair of points.

(a) $K(0, 2)$, $L(6, 0)$	(b) $R(-2, 0)$, $S(0, 8)$	(c) $T(3, -1)$, $U(5, 7)$
(d) $G(-4, -2)$, $H(8, 6)$	(e) $M(-1, 3)$, $N(1, 5)$	(f) $P(-5, 3)$, $Q(4, -6)$

Determine the equation of a straight line which passes through a point and is parallel to a given straight line

By now you would know that if two straight lines are parallel, then the gradients of both lines are equal.

Example 14

The diagram below shows the straight line AB with equation $y = -2x + 6$. Determine the equation of a straight line parallel to AB and passes through point $P(5, 4)$.

Solution:

The equation of the straight line AB is $y = -2x + 6$, thus the gradient of AB is -2.
The straight line which passes through point P is parallel to AB, thus the gradient m of that line is -2.
Substitute the values of m, x and y into $y = mx + c$ to determine the value of c.

$$\begin{aligned} 4 &= (-2)(5) + c \\ 4 &= -10 + c \\ c &= 4 + 10 \\ c &= 14 \end{aligned}$$

Given $P(5, 4)$, thus $x = 5$ and $y = 4$.

Thus, the equation of the straight line parallel to AB and passes through the point P is $y = -2x + 14$.

Example 15

Determine the equation of a straight line parallel to the straight line $2x + 3y = 12$ and passes through point $G(6, 8)$.

Solution:

Given the equation of straight line $2x + 3y = 12$.

Thus, $3y = -2x + 12$

$$y = -\frac{2}{3}x + 4$$

Gradient of the straight line $= -\frac{2}{3}$.

The straight line which passes through point G is parallel to the straight line $2x + 3y = 12$.

Hence, the gradient of the straight line is $-\frac{2}{3}$.

Substitute the values of m, x and y into $y = mx + c$, and determine the value of c.

Thus,
$$\begin{aligned} 8 &= (-\frac{2}{3})(6) + c \\ 8 &= -4 + c \\ c &= 8 + 4 \\ c &= 12 \end{aligned}$$

Given $Q(6, 8)$, thus $x = 6$ and $y = 8$.

Hence, the equation of the straight line parallel to $2x + 3y = 12$ and passes through point G is $y = -\frac{2}{3}x + 12$.

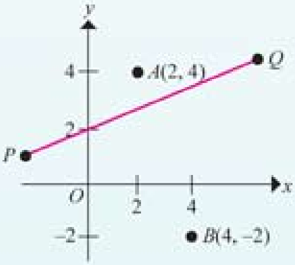
MIND TEST 9.1g

1. Determine the equation of a straight line that is parallel to the given straight line and passes through point P.

   (a) $y = 3x + 9$, $P(2, 7)$
   (b) $y = -2x + 7$, $P(-3, 4)$
   (c) $3x + 2y = 4$, $P(2, 6)$
   (d) $\frac{x}{2} + \frac{y}{3} = 1$, $P(-12, 9)$

2. The diagram on the right shows a straight line PQ. Given that the equation of the straight line PQ is $y = \frac{1}{3}x + 2$ and O is the origin, determine the equation of a straight line parallel to PQ and passes through

   (a) point $A(2, 4)$
   (b) point $B(4, -2)$
   (c) the origin



How do you determine the point of intersection of two straight lines?

LEARNING STANDARD

Determine the point of intersection of two straight lines.

The point of intersection of two straight lines can be determined by the following methods:

1. Drawing both straight line graphs on the same Cartesian plane and determine the point of intersection from the graphs.
2. Solving simultaneous equations using
   (a) substitution method
   (b) elimination method

REMINDER

The calculator should only be used for checking answers.

Example 16

Determine the point of intersection of the straight lines $2x + y = 5$ and $x + 2y = 1$.

Graphical method

(a) $2x + y = 5$

$y = -2x + 5$

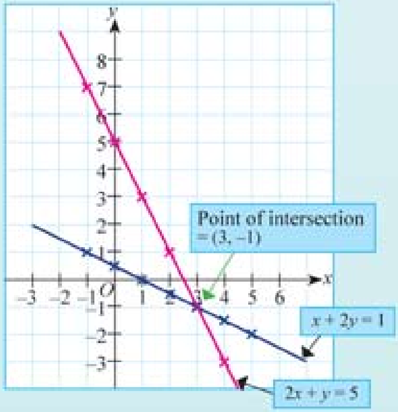
x	−1	0	1	2	3	4
y	7	5	3	1	−1	−3

(b) $x + 2y = 1$

$2y = -x + 1$

$y = -\frac{1}{2}x + \frac{1}{2}$

x	−1	0	1	2	3	4	5
y	1	$\frac{1}{2}$	0	$-\frac{1}{2}$	−1	$-\frac{3}{2}$	−2



From the graph, it is found that the point of intersection of the straight lines $2x + y = 5$ and $x + 2y = 1$ is $(3, -1)$.

Substitution method

$2x + y = 5$ -------------①

$x + 2y = 1$ -------------②

From ①, $y = 5 - 2x$--③

Substitute $y = 5 - 2x$ in ②,

$$\begin{aligned} x + 2(5 - 2x) &= 1 \\ x + 10 - 4x &= 1 \\ x - 4x &= 1 - 10 \\ -3x &= -9 \\ x &= 3 \end{aligned}$$

Substitute $x = 3$ in ③,

$$\begin{aligned} y &= 5 - 2(3) \\ y &= 5 - 6 \\ y &= -1 \end{aligned}$$

Thus, the point of intersection is $(3, -1)$.

Elimination method

$2x + y = 5$ -------------①

$x + 2y = 1$ -------------②

① × 2 $4x + 2y = 10$ ---------③

$x + 2y = 1$ ----------② (minus)

$$\begin{aligned} 3x &= 9 \\ x &= 3 \end{aligned}$$

Substitute $x = 3$ in ①,

$$\begin{aligned} 2(3) + y &= 5 \\ 6 + y &= 5 \\ y &= 5 - 6 \\ y &= -1 \end{aligned}$$

Thus, the point of intersection is $(3, -1)$.

Brainstorming 5

In pairs

Aim: To determine the coordinates of the intersection of two straight lines.

Materials: Dynamic software

Steps:

1. Start with *New Sketch* and click *Graph* next click *Show Grid.*
2. Click *Graph* again and select *Plot New Function* (Diagram 1).
3. Use *Plot New Function* to plot the intersection of the two straight lines.
4. Example: $y = x + 3$ and $y = -x + 5$.
5. Use *Arrow Tool* to select both straight line graphs. Click *Construct* and select *Intersection*.
6. Click *Measure* and select *Coordinates*. The intersection point A (1.00, 4.00) will be displayed (Diagram 2).
7. Repeat steps 1 to 6 for intersection of the other two straight lines.
   (a) $y = x + 2$ and $y = 2x + 4$ (Diagram 3)
   (b) $y = 4$ and $y = 3x - 2$ (Diagram 4)

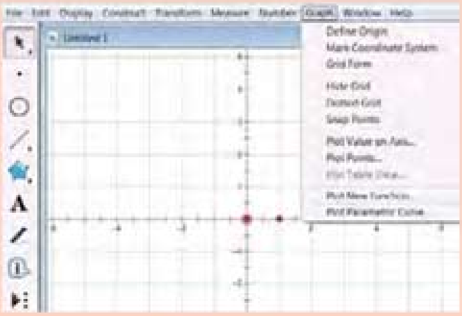


Diagram 1

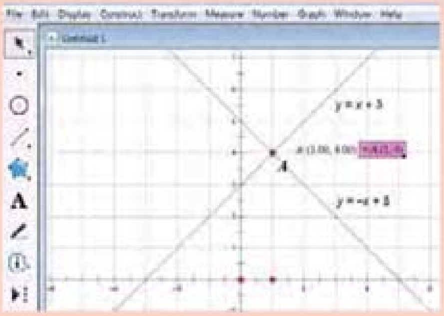


Diagram 2

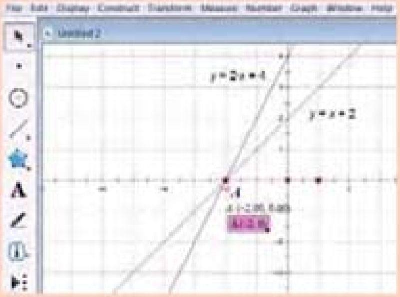


Diagram 3

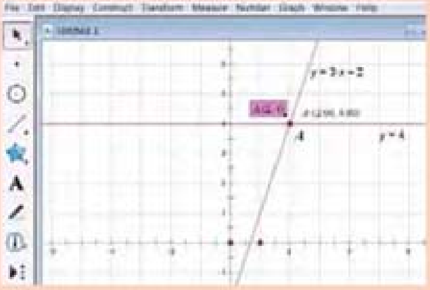


Diagram 4

Discussion:

What can you conclude from the results above?

From Brainstorming 5, it is found that:

(a) The point of intersection of two straight lines can be determined by plotting both straight lines on the Cartesian plane.

(b) Two straight lines that are not parallel intersect at only one point.

MIND TEST 9.1h

1. Determine the point of intersection of the following pairs of straight lines using the substitution method.

(a) $x = 3$, $2x + y = 10$ (b) $y = 4$, $3x - 2y = 7$

(c) $x + y = 5$, $2x - y = 4$ (d) $2x + y = 3$, $3x - 2y = 8$

2. Determine the point of intersection of the following pairs of straight lines using the elimination method.

(a) $x + y = 1$, $2x + y = -1$ (b) $x - y = -4$, $3x + y = 4$

(c) $x - y = -5$, $2x + 3y = -10$ (d) $2x - 3y = 5$, $3x + 2y = 14$

How do you solve problems involving straight lines?

LEARNING STANDARD

Solve problems involving straight lines.

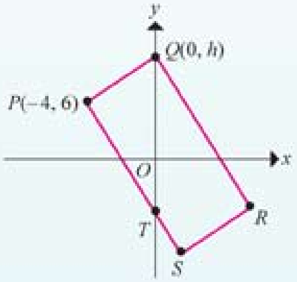
Example 17

The diagram on the right shows a parallelogram *PQRS*. Given that the gradient of *SR* is $\frac{1}{2}$ and y-intercept of straight line *PS* is -4, determine

(a) the value of h

(b) equation of straight line *PS*

(c) x-intercept for straight line *PS*



Solution:

Understanding the problem

- *PQRS* is a parallelogram.
- Gradient of *PQ* = gradient of *SR* = $\frac{1}{2}$
- y-intercept of *PS* is -4.

Planning a strategy

- The value of h can be determined by using the gradient that is, gradient of *PQ* = gradient of *SR* = $\frac{1}{2}$.
- The y-intercept of the straight line *PS* is -4. Thus, the coordinates of T are $(0, -4)$.
- The x-intercept of the straight line *PS* can be determined by substituting $y = 0$ into equation *PS*.

Implementing the strategy

(a) Gradient PQ = Gradient $SR = \frac{1}{2}$

$$\frac{h-6}{0-(-4)} = \frac{1}{2}$$
$$\frac{h-6}{4} = \frac{1}{2}$$
$$h - 6 = 2$$
$$h = 2 + 6$$
$$h = 8.$$

(b) Straight line *PS* passes through point $T(0, -4)$

Gradient $PS = \frac{-4-6}{0-(-4)} = \frac{-10}{4} = -\frac{5}{2}$

y-intercept of straight line *PS* is -4

Thus, equation of straight line *PS* is $y = -\frac{5}{2}x - 4$.

(c) Equation of straight line *PS* is $y = -\frac{5}{2}x - 4$

When $y = 0$

$$0 = -\frac{5}{2}x - 4$$
$$\frac{5}{2}x = -4$$
$$x = -\frac{8}{5}$$

x-intercept of the straight line *PS* is $-\frac{8}{5}$.

Making a conclusion

(a) The value of h is 8.

(b) The equation of the straight line *PS* is $y = -\frac{5}{2}x - 4$.

(c) x-intercept of the straight line *PS* is $-\frac{8}{5}$.

Example 18

Given straight lines $y = -\frac{1}{3}x + 3$ and $2x - y = 4$ intersect at point A, determine the coordinates of point A using the graphical method.

TIPS

A straight line can be drawn if its x-intercept and y-intercept are known.

Solution:

For the straight line $y = -\frac{1}{3}x + 3$,

(a) when $x = 0$,

$$y = -\frac{1}{3}(0) + 3$$
$$y = 3$$

y-intercept = 3

(b) when $y = 0$,

$$0 = -\frac{1}{3}(x) + 3$$
$$\frac{1}{3}x = 3$$
$$x = 9$$

x-intercept = 9

For the straight line $2x - y = 4$,

(a) when $x = 0$,

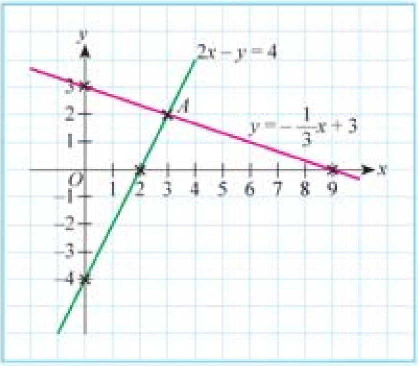
$$2(0) - y = 4$$
$$-y = 4$$
$$y = -4$$

y-intercept = -4

(b) when $y = 0$,

$$2x - (0) = 4$$
$$2x = 4$$
$$x = 2$$

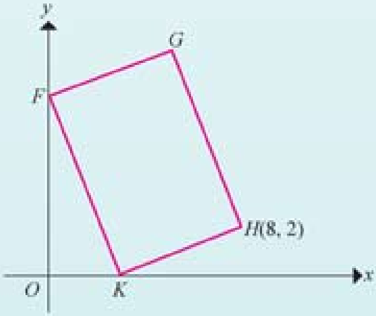
x-intercept = 2



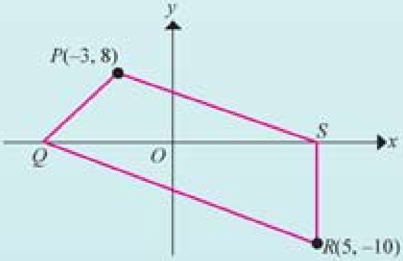
From the graph, it is found that the coordinates of A are (3, 2).

MIND TEST 9.1i

1. The diagram on the right shows a parallelogram $FGHK$. Given that O is the origin and point K is located on the x-axis, the equation of straight line FG is $2y = x + 20$, determine
   (a) the gradient of straight line FG
   (b) the y-intercept of straight line HK
   (c) the equation of straight line HK



2. In the diagram on the right, O is the origin and $PQRS$ is a trapezium where PS and QR are parallel. The straight line RS is parallel to the y-axis, and points Q and S are on the x-axis. Determine
   (a) the coordinates of S
   (b) the equation of straight line QR
   (c) the x-intercept of straight line QR



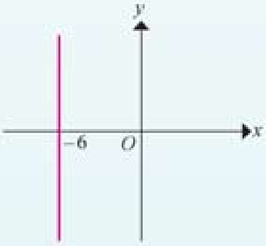
Dynamic Challenge

Test Yourself

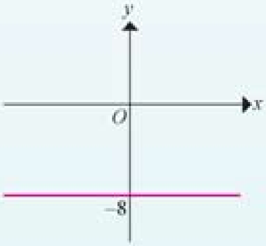
1. Given that $2x + 5y = 30$ is an equation of a straight line, determine
   (a) the x-intercept (b) the y-intercept (c) the gradient

2. State the equation of the straight line for each of the following diagrams.

(a)



(b)



3. Determine the equation of a straight line that has a gradient of 3 and passes through point $R(-4, 6)$.

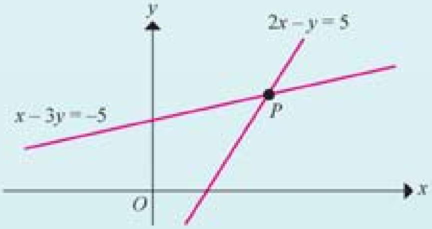
4. Determine the equation of a straight line that passes through point $P(-1, -2)$ and point $Q(3, 14)$.

5. Determine the equation of a straight line that passes through point $M(-3, 5)$ and is parallel to the straight line $6x + 2y = 18$.

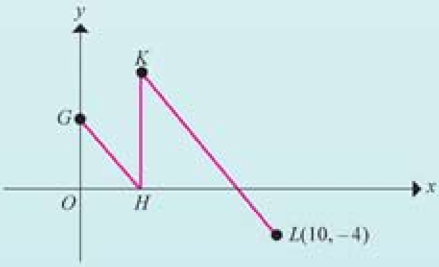
6. Determine the point of intersection of the straight lines $y = -8$ and $y = -4x + 12$.

Skills Enhancement

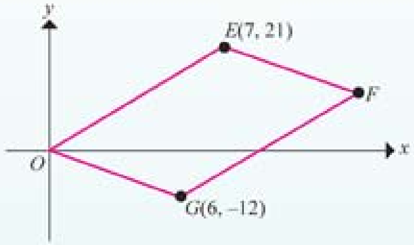
1. The diagram on the right shows two straight lines intersecting at point P. Given that O is the origin, determine the coordinates of P.



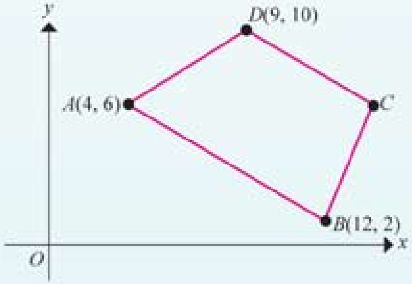
2. In the diagram on the right, GH, HK and KL are straight lines. Point H lies on the x-axis, GH is parallel to KL, and HK is parallel to the y-axis. Given the equation of GH is $2x + y = 6$,
   (a) state the equation of straight line HK
   (b) determine the equation of straight line KL and then state the x-intercept of KL



3. The diagram on the right shows a parallelogram *OEFG*. Given *O* is the origin, determine
   (a) the equation of straight line *OG*
   (b) the equation of straight line *EF*
   (c) the *x*-intercept of straight line *EF*

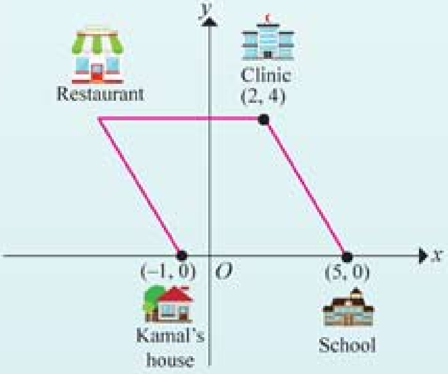


4. The diagram on the right shows a trapezium *ABCD* drawn on the Cartesian plane. Given *AB* is parallel to *DC*, determine
   (a) the equation of straight line *AB*
   (b) the equation of straight line *CD*
   (c) if the straight lines *AB* and *CD* intersect. State the reasons for your answer.

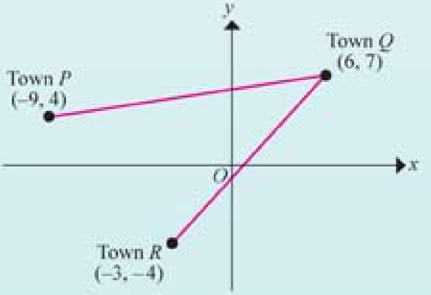


Self Mastery

1. The diagram on the right shows a parallelogram drawn on a Cartesian plane and it represents the locations of Kamal's house, the school, the clinic and the restaurant. Given that the scale is 1 unit = 1 km,
   (a) calculate the distance, in km, between Kamal's house and the school
   (b) determine the coordinates of the restaurant
   (c) calculate the distance, in km, between Kamal's house and the restaurant
   (d) determine the equation of the straight line connecting the school and the clinic

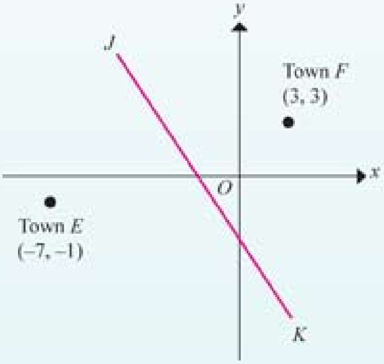


2. The diagram on the right shows the positions of town *P*, town *Q* and town *R* drawn on a Cartesian plane. Given that the scale is 1 unit = 2 km,
   (a) calculate the distance in km, between town *R* and the origin *O*
   (b) determine the equation of straight line connecting town *P* and town *Q*
   (c) calculate the nearest distance, in km, between town *P* and town *R*
   (d) calculate the time taken, in minutes, by Encik Mazlan to arrive at town *Q* if he drives from town *R* to town *Q* using the shortest route at an average speed of 50 km h^{-1}



3. The original height of a plant F is 9 cm. Its height is y cm after x days and is represented by the equation $y = \frac{3}{16}x + 9$. Plant G has the same growth rate as plant F. Plant G reaches a height of 15 cm after 8 days. Determine the equation to represent the height of plant G. Then, state the original height, in cm, of plant G.

4. JK is a straight road that passes through the midpoint between town E and town F.
   (a) The equation for the straight road JK is $y = -2x + k$, where k is a constant. Determine the value of k.
   (b) Another straight road GH with the equation $y = 2x + 17$ will be constructed. A traffic light will be installed at the junction of both roads JK and GH. Determine the coordinates of the traffic light.



TIPS

Solution by scale drawing is not accepted.

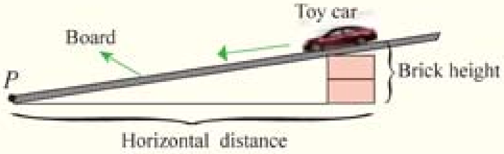
PROJECT

Title: Gradient and speed.

Material: Toy car, board, brick, long ruler and stopwatch.

Steps:

1. Place a board over two bricks as in the diagram below.



2. Measure the horizontal distance (fixed) and the height of the car from the ground. Calculate the gradient of the board and record it.

3. Release the toy car. Record the time, in seconds, for the toy car to reach point P.

4. Add the bricks one by one. Repeat steps 2 and 3.

5. What can you conclude about the relationship between the gradient of the board and the speed of the car?

CONCEPT MAP

Straight Lines

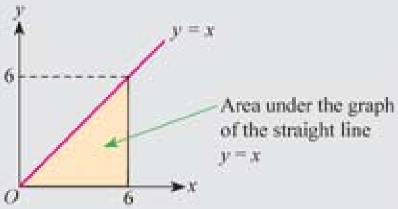
- Equation of a straight line
  $y = mx + c$
  m = gradient
  c = y-intercept
- Rewriting the equation of a straight line $y = mx + c$ in the form of $ax + by = c$ and $\frac{a}{x} + \frac{b}{y} = 1$ and vice versa.
- Point of intersection of two straight lines
- Equation of straight line parallel to the x-axis
  $y = k$; k = constant
  Equation of straight line parallel to the y-axis
  $x = h$; h = constant
- Gradients of parallel lines are equal

SELF-REFLECT

At the end of this chapter, I am able to:		🙂	☹
1.	Determine the gradient and y-intercept when the equation of the straight line in the form of $y = mx + c$ is given.		
2.	Determine the gradient and y-intercept when the equation of the straight line in the form of $ax + by = c$ is given.		
3.	Determine the gradient and y-intercept when the equation of the straight line in the form of $\frac{x}{a} + \frac{y}{b} = 1$ is given.		
4.	Determine whether a point lies on a given straight line.		
5.	Determine whether two straight lines are parallel.		
6.	Determine the equation of a straight line.		
7.	Determine the point of intersection of two straight lines.		
8.	Solve problems involving straight lines.		

EXPLORING MATHEMATICS

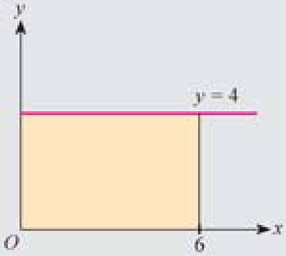
The area under a straight line can be determined if enough information is given.
For example, the area under the graph of the straight line $y = x$ for the range $0 \leqslant x \leqslant 6$ in the diagram on the right can be determined as follows:



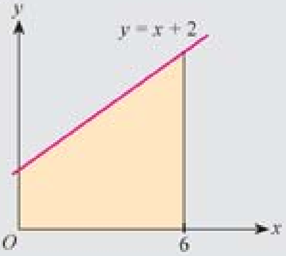
$$\begin{aligned}\text{The area under the graph} &= \frac{1}{2} \times \text{base} \times \text{height}\\ &= \frac{1}{2} \times 6 \text{ units} \times 6 \text{ units}\\ &= 18 \text{ unit}^2\end{aligned}$$

Worksheet

1.



2.



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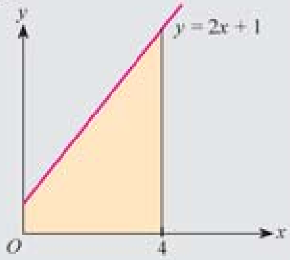

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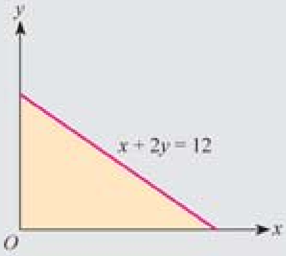
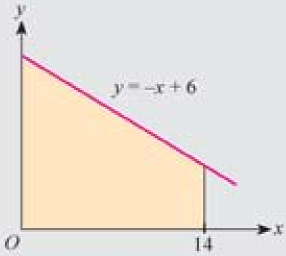
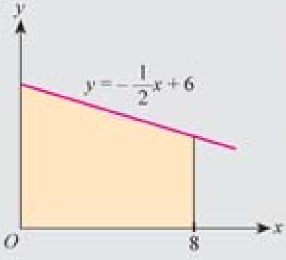


5.



6.



Steps:

1. Get into groups.
2. Calculate the area under each graph of straight line provided.
3. Present your group's findings.
4. Propose at least two other ways to determine the area under the graph of a straight line.