CHAPTER 1 Variation

What will you learn?

- Direct Variation
- Inverse Variation
- Combined Variation

Why study this chapter?

The concept of variation is commonly used in our daily life and in the fields of science and mathematics. For example, finding the distance travelled in terms of speed and given time, investigating the relation between the depth of the sea and the water pressure and others.

Do you know?

The symbol $\propto$ was introduced by William Emerson (1701-1782), an English mathematician, in 1768 in his book, The Doctrine of Fluxions.

For more information:

bit.do/DoYouKnowChap1

WORD BANK

constant	*pemalar*
variable	*pemboleh ubah*
combined variation	*ubahan bergabung*
direct variation	*ubahan langsung*
inverse variation	*ubahan songsang*
joint variation	*ubahan tercantum*

In scientific research, electricians conduct experiments to investigate the relation between two or more variables. For example, in an experiment on electric current, if the value of voltage increases, then the value of the current flowing through the circuit increases too and vice versa. In contrast, if the value of the resistance decreases, then the value of the current increases and vice versa. Do you know how the current (I), voltage (V) and resistance (R) relate to each other?

1.1 Direct Variation

What is the meaning of direct variation?

Learning Standard

Explain the meaning of direct variation.

5 cups of rice for 10 servings. 10 cups of rice for 20 servings.

A chef needs to decide the amount of ingredients to be used according to the number of servings. If the ingredients used increase, then the number of servings increases. On the contrary, if the ingredients used decrease, then the number of servings decreases too.

In our daily life, we often face situations that involve the relation of quantity changes. For example, the distance travelled and the taxi fare, the amount of interest earned over a period of time at a certain interest rate.

MIND MOBILISATION 1 Individual

Aim: To explain the meaning of direct variation.

Steps:

1. By browsing through the Bank Negara Malaysia website, find the latest rate of currency exchange of Malaysian Ringgit (RM) to Singapore Dollar ($), Thailand Baht (฿) and Japanese Yen (¥).
2. By using the rate of currency exchange, calculate the following values of Malaysian Ringgit to Singapore Dollar, Thailand Baht and Japanese Yen.

Malaysian Ringgit (RM)	10	20	30	40	50
Singapore Dollar ($)					
Thailand Baht (฿)					
Japanese Yen (¥)					

Discussion:

1. State the change
   (a) to Singapore Dollar, Thailand Baht and Japanese Yen when Malaysian Ringgit increases,
   (b) to Singapore Dollar, Thailand Baht and Japanese Yen when Malaysian Ringgit divides by four,
   (c) to Singapore Dollar, Thailand Baht and Japanese Yen when Malaysian Ringgit is doubled,
   (d) to Singapore Dollar, Thailand Baht and Japanese Yen when Malaysian Ringgit decreases by 50%.
2. What is the relation between the value of Malaysian Ringgit and the values of Singapore Dollar, Thailand Baht and Japanese Yen?

The results of Mind Mobilisation 1 show that the change in the value of Malaysian Ringgit, RM corresponds to the change in the values of Singapore Dollar, $, Thailand Baht, ฿ and Japanese Yen, ¥. The value of RM increases when the values of $, ฿ and ¥ increase, and the value of RM decreases when the values of $, ฿ and ¥ decrease.

This relation is known as a direct variation. In general,

Direct variation explains the relation between two variables, such that when variable y increases, then variable x also increases at the same rate and vice versa. This relation can be written as y varies directly as x.

Example 1

The total wage of a part time worker as a promoter varies directly as the number of hours he works. State the change on
(a) the total wage if the number of working hours is doubled,
(b) the total wage if the number of working hours decreases by 40%,
(c) the number of working hours if the total wage is halved.

Solution:

(a) The total wage is doubled.
(b) The total wage decreases by 40%.
(c) The number of working hours is halved.

Self Practice 1.1a

1. The value of resistance of a wire varies directly as its temperature. State the change on
   (a) the value of resistance if the temperature increases by 10%,
   (b) the value of resistance if the temperature decreases by half,
   (c) the temperature if the value of the resistance decreases by $\frac{1}{4}$.

2. Puan Wardina wants to buy green beans selling at RMx per kilogram. State the price if Puan Wardina buys
   (a) 500 g of green beans,
   (b) 2 kg of green beans.

3. The table below shows the relation between the time needed and the number of bottles of jam produced in a factory.

Time (minutes)	5	10	15	20	25
Number of bottles	10	20	30	40	50

State the change in the number of bottles of jam produced when
(a) the time is doubled,
(b) the time is halved.

What is the relation between two variables for a direct variation?

Learning Standard

Determine the relation between two variables for a direct variation.

MIND MOBILISATION 2 Group

Aim: To determine the relation between two variables for a direct variation.

Steps:

1. Divide the class into six groups.
2. Each group chooses a type of rate from the list below.
   (a) Rate of taxi fare by distance
   (b) Rate of parking by hours
   (c) Rate of heartbeat by minutes
   (d) Rate of toll by distance
   (e) Rate of data downloading (Mbps)
   (f) Rate of simple interest in savings per year
3. By browsing the internet, get information on the chosen rate and complete the table as shown below.
   For example, the rate of taxi fare is RM1.20 per km.

Distance travelled, x (km)	2	4	6	8	10
Fare, y (RM)					
$\frac{y}{x}$					

4. Draw the graph of y against x using a suitable scale based on the table.
5. Answer the questions in Discussion.
6. Present the results to the class.

Discussion:

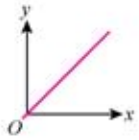
1. Based on the values of $\frac{y}{x}$, what conclusion can be made?
2. State the shape of the graph obtained.
3. What is the relation between variable y and variable x?

The results of Mind Mobilisation 2 show that for a direct variation,

(a) the value of $\frac{y}{x}$ is a constant,

(b) the graph of y against x is a straight line that passes through the origin,

(c) the variable y varies directly as the variable x.



Smart TiPS

The graph of y against x is a graph with variable y representing the vertical axis and variable x representing the horizontal axis.

In direct variation, quantity y is said to be varied directly as x if and only if $\frac{y}{x}$ is a constant, known as the constant of proportionality, k.

Info Bulletin

Constant is a fixed or unchanged quantity value.

Relation between the value of k and the concept of proportion:

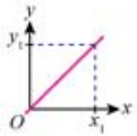
y	y_1	y_2	y_3	y_4
x	x_1	x_2	x_3	x_4

$$k = \frac{y_1}{x_1} = \frac{y_2}{x_2} = \frac{y_3}{x_3} = \frac{y_4}{x_4}$$

Relation between the value of k and the gradient of a straight line passes through the origin:

Gradient,

$$m = \frac{y_1}{x_1} = k$$



This relation is true only if the straight line passes through the origin.

Critical Mind

Does every gradient value represent the value of proportionality, k? Discuss.

When y varies directly as x, then the relation is written as $y \propto x$. From this relation, the value of constant of proportionality k can be determined, that is

$$k = \frac{y}{x}.$$

Smart TiPS

The relation of direct variation written as 'y varies directly as x' can also be written as 'x and y are directly proportional'.

For a **direct variation**, y varies directly as x can be written as

$$y \propto x \text{ (variation relation)}$$
$$y = kx \text{ (equation form)}$$

where k is a constant.

Smart TiPS

The symbol $\propto$ means 'is proportional to'.

MIND MOBILISATION 3 Group

Aim: To determine the relation between the variables y and x^2 for a direct variation.

Steps:

1. Given a circle with radius, x cm, and area, y cm^2.
2. Take turns to complete the table below in your group. (Use $\pi = \frac{22}{7}$)

Radius, x (cm)	3.5	7.0	10.5	14.0	17.5
Area of circle, y (cm^2)					
Square of radius, x^2 (cm^2)					
Cube of radius, x^3 (cm^3)					
$\frac{y}{x^2}$					
$\frac{y}{x^3}$					

3. Draw the graph of y against x^2 and the graph of y against x^3 by using approximate values.

Discussion:

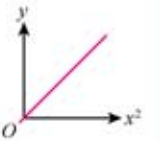
1. Compare the values of $\frac{y}{x^2}$ and $\frac{y}{x^3}$. What conclusion can be made?
2. Which graph is a straight line graph that passes through the origin?
3. What is the relation between variable y and variable x^2?

The results of Mind Mobilisation 3 show that for a direct variation,

(a) the value of $\frac{y}{x^2}$ is a constant. Hence, $k = \frac{y}{x^2}$,

(b) the graph of y against x^2 is a straight line that passes through the origin,

(c) variable y varies directly as variable x^2.



In general,

For a **direct variation**, y varies directly as x^n can be written as

$y \propto x^n$ (variation relation)

$y = kx^n$ (equation form)

where $n = 1, 2, 3, \frac{1}{2}, \frac{1}{3}$ and k is a constant.

The graph of y against x^n is a straight line passes through the origin where k is the gradient of the straight line.

Critical Mind

In Mind Mobilisation 3, what is the relation between the values of π and $\frac{y}{x^2}$?

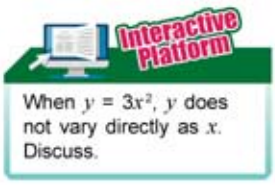
Smart TiPS

$y \propto x$ is true if and only if $n = 1$.

Example 2

A toy car moves from a state of rest. The distance travelled by the toy car, y, changes with time, t, as shown in the table below.

Time, t (s)	2	4	6	8	10	12
Distance, y (cm)	14	28	42	56	70	84



Determine whether y varies directly as t or t^2. Hence, write the relation in the form of variation.

Solution:

t	2	4	6	8	10	12
y	14	28	42	56	70	84
$\frac{y}{t}$	$\frac{14}{2} = 7$	$\frac{28}{4} = 7$	$\frac{42}{6} = 7$	$\frac{56}{8} = 7$	$\frac{70}{10} = 7$	$\frac{84}{12} = 7$
$\frac{y}{t^2}$	$\frac{14}{2^2} = 3.50$	$\frac{28}{4^2} = 1.75$	$\frac{42}{6^2} = 1.17$	$\frac{56}{8^2} = 0.88$	$\frac{70}{10^2} = 0.70$	$\frac{84}{12^2} = 0.58$

y varies directly as t because the value of $\frac{y}{t}$ is a constant. Hence, $y \propto t$.

y does not vary directly as t^2 because $\frac{y}{t^2}$ is not a constant.

Example 3

(a) By drawing the graph of y against x, determine whether y varies directly as x.

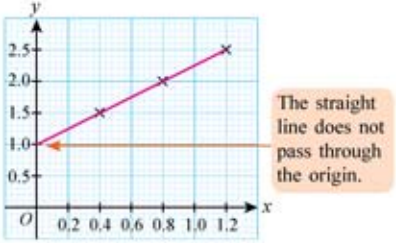
x	0.4	0.8	1.2
y	1.5	2.0	2.5

(b) By drawing the graph of y against x^2, determine whether y varies directly as x^2.

x	2	3	4	5
y	8	18	32	50

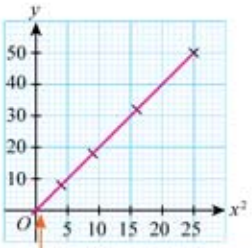
Solution:

(a)



Hence, y does not vary directly as x.

(b)



x^2	y
4	8
9	18
16	32
25	50

The straight line passes through the origin.

Hence, y varies directly as x^2.

Example 4

It is given $m = 0.8$ when $n = 0.125$. Express m in terms of n if
(a) m varies directly as n,
(b) m varies directly as the cube root of n.

Solution:

(a) $m \propto n$

$m = kn$ ← Write the relation in the form of equation.

$0.8 = k(0.125)$ ← Substitute the values of m and n into the equation to obtain the value of k.

$k = \frac{0.8}{0.125}$

$= 6.4$

Hence, $m = 6.4n$

(b) $m \propto \sqrt[3]{n}$

$m = k\sqrt[3]{n}$

$0.8 = k(\sqrt[3]{0.125})$

$k = \frac{0.8}{\sqrt[3]{0.125}}$

$= 1.6$

Hence, $m = 1.6\sqrt[3]{n}$

Example 5

The extension of a spring, x cm, varies directly as the mass of the load, w g, hanging on the string. It is given the extension of a spring is 3 cm when a load of 200 g is hung on the string. Express x in terms of w.

Solution:

$x \propto w$

$x = kw$ ← Write the relation in the form of equation.

$3 = k(200)$ ← Substitute the values of x and w into the equation to obtain the value of k.

$k = \frac{3}{200}$

$= 0.015$

Hence, $x = 0.015w$

Example 6

It is given y varies directly as x. If $y = 0.14$ when $x = 0.2$, calculate the value of
(a) y when $x = 5$,
(b) x when $y = 0.875$.

Solution:

$y \propto x$

$y = kx$

$0.14 = k(0.2)$

$k = \frac{0.14}{0.2}$

$= 0.7$

Hence, $y = 0.7x$

(a) When $x = 5$,

$y = 0.7(5)$

$= 3.5$

(b) When $y = 0.875$,

$0.875 = 0.7x$

$x = \frac{0.875}{0.7}$

$= 1.25$

Critical Mind

In the graph of Example 3(a), why does y not vary directly as x?

Smart Tips

(i) y varies directly as x, $y \propto x$

(ii) y varies directly as the square of x, $y \propto x^2$

(iii) y varies directly as the cube of x, $y \propto x^3$

(iv) y varies directly as the square root of x, $y \propto \sqrt{x}$

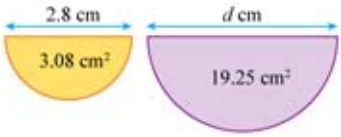
(v) y varies directly as the cube root of x, $y \propto \sqrt[3]{x}$

Historical Treasure

Robert Hooke (1635-1703) is a British scientist who introduced Hooke's Law in 1676. The law states that the extension of an elastic object is directly proportional to the force applied to stretch it, as long as it does not exceed the elastic limit.

Example 7

The area, L cm^2, of a semicircle varies directly as the square of its diameter, d cm. It is given that the area of the semicircle is 3.08 cm^2 when the diameter is 2.8 cm. Calculate the value of d when $L = 19.25$.



Solution:

$L \propto d^2$

$L = kd^2$

$3.08 = k(2.8)^2$

$k = \dfrac{3.08}{(2.8)^2}$

$= \dfrac{11}{28}$

Hence, $L = \dfrac{11}{28}d^2$

When $L = 19.25$,

$19.25 = \dfrac{11}{28}d^2$

$d^2 = \dfrac{19.25 \times 28}{11}$

$d = \sqrt{49}$

$= 7$ cm

Alternative Method:

Using the concept of proportion:

Given that $L_1 = 3.08$, $d_1 = 2.8$ and $L_2 = 19.25$

$\dfrac{L_1}{(d_1)^2} = \dfrac{L_2}{(d_2)^2}$

$\dfrac{3.08}{2.8^2} = \dfrac{19.25}{(d_2)^2}$

$(d_2)^2 = \dfrac{19.25 \times 2.8^2}{3.08}$

$d_2 = \sqrt{49}$

$= 7$ cm

Example 8

The period of oscillation, A seconds, of a simple pendulum varies directly as the square root of the length of its string, p cm. It is given that the 9 cm string of a simple pendulum oscillates for 1.2 seconds. Calculate the period of oscillation in seconds, if the length of the string is 25 cm.

Solution:

$A \propto \sqrt{p}$

$A = k\sqrt{p}$

$1.2 = k\sqrt{9}$

$k = \dfrac{1.2}{\sqrt{9}}$

$= 0.4$

Hence, $A = 0.4\sqrt{p}$

When $p = 25$,

$A = 0.4\sqrt{25}$

$= 2$ seconds

Alternative Method:

Using the concept of proportion:

Given that $A_1 = 1.2$, $p_1 = 9$ and $p_2 = 25$

$\dfrac{A_1}{\sqrt{p_1}} = \dfrac{A_2}{\sqrt{p_2}}$

$\dfrac{1.2}{\sqrt{9}} = \dfrac{A_2}{\sqrt{25}}$

$A_2 = \dfrac{1.2 \times \sqrt{25}}{\sqrt{9}}$

$= 2$ seconds

Self Practice 1.1b

1. The table below shows the values of two variables, x and y.

(a) Determine whether y varies directly as x or x^3. Then, write the relation in the form of variation.

x	1	2	3	4	5
y	2.5	5	7.5	10	12.5

(b) Determine whether y varies directly as x or $\sqrt{x}$. Then, write the relation in the form of variation.

x	4	9	25	36	49
y	0.6	0.9	1.5	1.8	2.1

2. A load is hanging on a spring. The table on the right shows the mass of the load, x g, with the extension of the spring, p cm. By drawing the graph of p against x, determine whether p varies directly as x.

Mass of load, x (g)	5	10	15	25	30
Extension of spring, p (cm)	1	2	3	5	6

3. It is given that $p = 32$ when $q = 4$. Express p in terms of q if
(a) p varies directly as q^3,
(b) p varies directly as the square root of q.

4. The salary, RMx, obtained by a worker varies directly as the total time he works, t hours. It is given a worker received RM112 after he worked for 14 hours. Write an equation that relates x and t.

5. Given that $y = 1.8$ when $x = 0.6$, calculate the value of y when $x = 5$ if
(a) $y \propto x$,
(b) $y \propto x^2$.

6. It is given that s varies directly as $t^{\frac{1}{3}}$. If $s = 1.2$ when $t = 27$, calculate the value of
(a) s when $t = 64$,
(b) t when $s = 0.28$.

7. The number of words typed, a, by Saiful varies directly as the time he types, t minutes. If Saiful types 270 words in 6 minutes, calculate the time he needs to spend to type 675 words.

8. An object falls from a height, h m, varies directly as the square of its time, t s on planet Q. Given that the object falls from the height of 5 m in 2 s, calculate the time taken in seconds, for the object to fall from a height of 45 m on the planet.

9. It is given that the volume of the paint, x litres, varies directly as the area of the wall, d m^2. If 3 litres of paint can paint 36 m^2 of wall,
(a) express the equation in terms of x and d,
(b) calculate the volume of the paint in litres, needed to paint a wall of 9 m in height and 5 m in width.

What is the relation between three or more variables for a given joint variation?

Learning Standard

Determine the relation between three or more variables for a given joint variation.

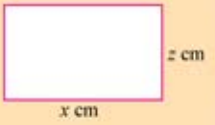
In Form 3, you have learnt the calculation for simple interest, I that involves P, r and t. The situation above shows an example of joint variation. A **joint variation** is a direct variation in which one variable varies as a product of two or more variables.

MIND MOBILISATION 4 Individual

Aim: To determine the relation between three variables for a joint variation.

Steps:

1. The diagram on the right shows a rectangle with x cm in length and z cm in width. It is given that the area of the rectangle is y cm^2. Complete the table below.



(A) If z is a constant				
Length, x (cm)	2	3	4	5
Width, z (cm)	6	6	6	6
Area, y (cm^2)				

(B) If x is a constant				
Length, x (cm)	2	2	2	2
Width, z (cm)	6	5	4	3
Area, y (cm^2)				

(C) If x, z and y are variables				
Length, x (cm)	2	3	4	5
Width, z (cm)	8	6	3	2
Area, y (cm^2)				
$\frac{y}{xz}$				

Discussion:

1. What is the relation between y and x if z is a constant?
2. What is the relation between y and z if x is a constant?
3. What is the relation between y, x and z if three of them are variables?

The results of Mind Mobilisation 4 show that y varies directly as x, and y varies directly as z. Hence, y varies jointly as x and z, that is $y \propto xz$.

In general,

For a **joint variation**, y varies jointly as x^m and z^n can be written as

$y \propto x^m z^n$ (variation relation)

$y = kx^m z^n$ (equation form)

where

$m = 1, 2, 3, \frac{1}{2}, \frac{1}{3}$,

$n = 1, 2, 3, \frac{1}{2}, \frac{1}{3}$ and

k is a constant.

Smart TiPS

$y \propto xz$ is the combination of two relations $y \propto x$ and $y \propto z$.

Critical Mind

Discuss the relation of the equation of simple interest, $I = Prt$.

Example 9

Write the relation using the symbol $\propto$ and in equation form for each of the following.

(a) p varies directly as q and $\sqrt{r}$.

(b) y varies directly as the square of w and the cube of x.

(c) The volume of a prism, V varies directly as the area of cross-section, A and its height, h.

(d) The mass, w of a cylindrical metal rod varies directly as its length, p and the square of the diameter of its base, d.

Smart TiPS

$y \propto xz$ can be read as

- y varies directly as x and z
- y varies jointly as x and z
- y is jointly proportional to x and z

Solution:

(a) $p \propto q\sqrt{r}$ ← $p \propto q$ and $p \propto \sqrt{r}$

$p = kq\sqrt{r}$

(b) $y \propto w^2x^3$ ← $y \propto w^2$ and $y \propto x^3$

$y = kw^2x^3$

(c) $V \propto Ah$ ← $V \propto A$ and $V \propto h$

$V = kAh$

(d) $w \propto pd^2$ ← $w \propto p$ and $w \propto d^2$

$w = kpd^2$

Example 10

Given that $x \propto y^2z$, express x in terms of y and z if $x = 6$ when $y = 3$ and $z = 5$.

Solution:

$x \propto y^2z$

$x = ky^2z$ ← Write the relation in the form of equation.

$6 = k(3)^2(5)$ ← Substitute the values of x, y and z into the equation to obtain the value of k.

$k = \frac{6}{(3)^2(5)}$

$= \frac{2}{15}$

Hence, $x = \frac{2}{15}y^2z$

Example 11

The gravitational potential energy, E Joules, of an object varies directly as its mass, m kg, the gravitational acceleration, g m s^{-2} and its position at a height of h m from the ground. Given that $E = 197$ Joules when $m = 4$ kg, $g = 9.81$ m s^{-2} and $h = 5$ m, write an equation that relates E, m, g and h.

Solution:

$E \propto mgh$

$E = kmgh$ ← Write the relation in the form of equation.

$197 = k(4)(9.81)(5)$ ← Substitute the values of E, m, g and h into the equation to obtain the value of k.

$$k = \frac{197}{(4)(9.81)(5)}$$

$= 1$

Hence, $E = mgh$

Info Bulletin

(i) Gravitational potential energy is the energy stored within an object, due to the object's position.

(ii) The gravitational acceleration, g, is a constant. The energy is stored due to Earth's gravitational pull towards an object. The value of g differs between celestial bodies. For example, the value of g on Earth is 9.81 m s^{-2} and on the moon is 1.62 m s^{-2}.

Example 12

Three quantities, S, T and U vary as shown in the table on the right. It is given that S varies directly as T and the cube root of U. Calculate the values of x and y.

S	6	x	50
T	0.8	1.2	40
U	27	125	y

Solution:

$S \propto T\sqrt[3]{U}$

$S = kT\sqrt[3]{U}$

$S = 6$ when $T = 0.8$ and $U = 27$,

$6 = k(0.8)(\sqrt[3]{27})$

$$k = \frac{6}{(0.8)(\sqrt[3]{27})}$$

$= 2.5$

Hence, $S = 2.5T\sqrt[3]{U}$

When $T = 1.2$ and $U = 125$,

$x = 2.5(1.2)(\sqrt[3]{125})$

$= 15$

When $S = 50$ and $T = 40$,

$50 = 2.5(40)(\sqrt[3]{y})$

$$\sqrt[3]{y} = \frac{50}{(2.5)(40)}$$

$= 0.5$

$y = 0.5^3$

$= 0.125$

Alternative Method:

Using the concept of proportion:

Given that $S_1 = 6$, $T_1 = 0.8$, $U_1 = 27$ and $S_2 = x$, $T_2 = 1.2$, $U_2 = 125$

$$\frac{S_1}{T_1\sqrt[3]{U_1}} = \frac{S_2}{T_2\sqrt[3]{U_2}}$$

$$\frac{6}{(0.8)(\sqrt[3]{27})} = \frac{x}{(1.2)(\sqrt[3]{125})}$$

$$x = \frac{(6)(1.2)(\sqrt[3]{125})}{(0.8)(\sqrt[3]{27})}$$

$= 15$

Given that $S_1 = 6$, $T_1 = 0.8$, $U_1 = 27$ and $S_2 = 50$, $T_2 = 40$, $U_2 = y$

$$\frac{S_1}{T_1\sqrt[3]{U_1}} = \frac{S_2}{T_2\sqrt[3]{U_2}}$$

$$\frac{6}{(0.8)(\sqrt[3]{27})} = \frac{50}{(40)(\sqrt[3]{y})}$$

$$\sqrt[3]{y} = \frac{(50)(0.8)(\sqrt[3]{27})}{(40)(6)}$$

$= 0.5$

$y = 0.5^3$

$= 0.125$

Self Practice 1.1c

1. Write the relation by using the symbol $\propto$ for each of the following.
   (a) s varies directly as t and u.
   (b) v varies directly as w^2 and x.
   (c) a varies directly as the cube of b and the square root of c.
   (d) The area of the curved surface, A cm^2, of a cylinder varies directly as the radius of its base, r cm and its height, h cm.

2. Calculate the constant, k for each of the following.
   (a) p varies directly as q^3 and $r^{\frac{1}{3}}$. Given that $p = 5.184$ when $q = 1.2$ and $r = 216$.
   (b) p varies directly as q, r and the square of s. Given that $p = \frac{1}{3}$ when $q = \frac{1}{5}$, $r = \frac{3}{2}$ and $s = \frac{1}{3}$.

3. Given that $y = 6$ when $x = 0.64$ and $z = 5$, express y in terms of x and z if
   (a) y varies directly as $\sqrt{x}$ and z,
   (b) y varies directly as x and the square of z.

4. The price of a metal rod, RMx, varies directly as the length, p cm and the square of the radius, r cm. If a metal rod with a length of 150 cm and a radius of 3 cm is sold at the price of RM27, write an equation that relates x with p and r.

5. It is given that G varies directly as H and the square root of M. If $G = 42$ when $H = 7$ and $M = 16$, calculate
   (a) the value of G when $H = 4$ and $M = 81$,
   (b) the value of M when $G = 18$ and $H = 20$.

6. The table below shows the changes in three quantities. It is given that P varies directly as the cube of Q, and R. Calculate the values of x and y.

P	86.4	x	1.215
Q	1.2	2	y
R	10	0.4	9

7. The kinetic energy, E joules, of an object varies directly as the mass, w kg and the square of speed, v m s^{-1}, of the object. It is given that the kinetic energy of a 3-kg mass object moves with a speed of 12 m s^{-1} is 216 Joules. Calculate the speed in m s^{-1}, of an object if the mass and the kinetic energy are 5 kg and 640 Joules respectively.

8. The volume of a cone, V cm^3, varies directly as the height, h cm and the square of its base radius, r cm. A cone with a height of 21 cm and a radius of 6 cm has a volume of 792 cm^3. Calculate the volume in cm^3, of a cone with a height of 14 cm and a radius of 15 cm.

How to solve problems involving direct variation?

Learning Standard

Solve problems involving direct variation.

Historical Treasure

The relation between the volume and the temperature of a fixed mass of gas at a fixed pressure is first researched by Jacques Charles.

Example 13

Charles' Law states that for a fixed mass of gas, the volume, V cm^3, of the gas is directly proportional to its absolute temperature, T Kelvin, if the pressure of the gas is fixed. It is given that a container contains 30 cm^3 of gas at a temperature of 30°C.

(a) Express V in terms of T.

(b) Calculate the volume in cm^3, of the gas if the temperature changes to −11°C.
[Formula for the conversion of temperature in degree Celsius to Kelvin: x°C = $(273 + x)$ K]

Solution:

Understanding the problem

$V \propto T$

$V = 30$ when $T = (273 + x)$K

Devising a strategy

(a) Write the direct variation in the form of equation.

(b) Substitute the value of T into the equation and hence, calculate the volume of the gas.

Implementing the strategy

(a) $V \propto T$

$V = kT$

$V = k(273 + x)$

$30 = k(273 + 30)$

$k = \frac{30}{303}$

$= \frac{10}{101}$

$V = \frac{10}{101}T$

(b) $V = \frac{10}{101}T$

$V = \frac{10}{101}(273 - 11)$

$= 25.94$ cm^3

Making a conclusion

(a) $V = \frac{10}{101}T$

(b) $V = 25.94$ cm^3

Example 14

Madam Soon saves her money in a savings account. It is given that the interest, I, received varies directly as the principal, p, and the period in year, t of the savings. Madam Soon receives an interest of RM200 when she saves RM4 000 for two years.

(a) Calculate the period of the savings for Madam Soon to receive an interest of RM650 with a principal of RM5 200.

(b) Madam Soon wants to get the same amount of interest but reduce the period of her savings in (a). Does she need to increase or decrease her principal? Explain your answer.

Solution:

Understanding the problem

$I \propto pt$

$I = 200$ when $p = 4\ 000$ and $t = 2$

Devising a strategy

(a) Write the direct variation in the form of equation and calculate the value of t when $I = 650$ and $p = 5\ 200$.
(b) Understand the changes for every variable in the equation.

Making a conclusion

(a) 5 years.
(b) The principal has to be increased to get the same amount of interest if the period of the savings is reduced. This is because the interest received is directly proportional to the product of the principal and the period.

Implementing the strategy

(a) $I = kpt$

$200 = k(4\ 000)(2)$

$k = \dfrac{200}{(4\ 000)(2)}$

$= 0.025$

$\therefore I = 0.025pt$

When $I = 650$ and $p = 5\ 200$,

$650 = (0.025)(5\ 200)t$

$t = \dfrac{650}{(0.025)(5\ 200)}$

$= 5$ years

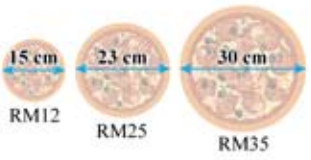
(b) To maintain the value of I in the equation $I = 0.025pt$,
- when p decreases, t increases
- when p increases, t decreases

Self Practice 1.1d

1. Lee fills a tank with water using a rubber hose at 8:00 a.m. At 11:00 a.m., Lee discovers that the tank is filled with 48% of water.
   (a) Write an equation that relates the volume of water, V, that is filled into the tank with the time taken, t.
   (b) At what time will the tank be fully filled with water?

2. Aminah wants to cut a few triangles from a piece of card. It is given that the area of the triangles cut, A cm^2, varies directly as its base, x cm, and its height, y cm. At the beginning, she cuts a triangle with $A = 14$, $x = 7$ and $y = 4$.
   (a) Write the relation between A with x and y.
   (b) Aminah plans to cut the second triangle with the value of base increased by 20% and the value of height decreased by 10%. What is the percentage of changes in the area for the second triangle?

3. A pizza shop sells three sizes of pizza at different prices as shown in the diagram on the right. Does the price of pizza, RMp, vary directly as the surface area, A cm^2, of the pizza? If it does not, which size of pizza is worth buying?



1.2 Inverse Variation

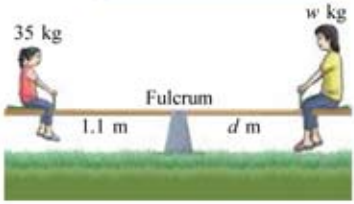
What is the meaning of inverse variation?

Learning Standard

Explain the meaning of inverse variation.

While on a seesaw, the heavier person will be at a lower position whereas the lighter person will be at a higher position. Do you know how to balance the seesaw?

To balance a seesaw, the heavier one has to sit closer to the fulcrum of the seesaw, or the lighter one has to sit further away from the fulcrum. Observe the diagram on the right. The relation between the person's mass, w, and the distance between the person and the fulcrum, d, is said to be varied inversely. When w increases, d decreases, and vice versa.



MIND MOBILISATION 5 Individual

Aim: To explain the meaning of inverse variation.

Steps:

1. Understand the following situation and then answer questions in the Discussion.
   A school hall has a few doors. The table below shows the relation between the number of doors opened, x, and the time taken, y, for a group of pupils to leave the hall.

Number of doors opened, x	2	3	4	5	6
Time taken, y (minutes)	24	16	12	9.6	8

Discussion:

1. If the number of doors opened increases, does the time taken for the pupils to leave the hall increase or decrease?
2. If the number of doors opened decreases, does the time taken for the pupils to leave the hall increase or decrease?
3. What is the relation between the number of doors opened and the time taken for the pupils to leave the hall?

The results of Mind Mobilisation 5 show that when the number of doors opened increases, the time taken for the pupils to leave the hall decreases. Similarly, when the number of doors opened decreases, the time taken for the pupils to leave the hall increases. The change in the number of doors opened leads to an opposite implication to the time taken for the pupils to leave the hall.

This relation is known as inverse variation. In general,

In **inverse variation**, variable y increases when the variable x decreases at the same rate, and vice versa. This relation can be written as y varies inversely as x.

Example 15

Chia Ming takes part in a 42-km marathon. The table below shows the relation between the time taken by Chia Ming and his average speed.

Time, t (hours)	4	5	6	7	8
Average speed, v (km/h)	10.50	8.40	7.00	6.00	5.25

State the change in average speed if the time taken
(a) increases by two times,
(b) decreases by 1.5 times.

Solution:

(a)

t	v
4	10.50
8	5.25

$\times 2$ (from 4 to 8), $\div 2$ (from 10.50 to 5.25)

When the time increases two times, then the average speed decreases two times.

(b)

t	v
6	7.00
4	10.50

$\div 1.5$ (from 6 to 4), $\times 1.5$ (from 7.00 to 10.50)

When the time decreases 1.5 times, then the average speed increases 1.5 times.

Self Practice 1.2a

1. The table on the right shows the relation between the number of workers and the number of days needed to install tiles in a house. State the change in the number of days if the number of workers
   (a) is multiplied by two,
   (b) decreases by half.

Number of workers	Number of days
2	12
4	6
6	4
8	3

2. Cikgu Farid has a sum of money to buy gifts for the winners of a Mathematics quiz. If the price of a gift is RM10, then Cikgu Farid can buy 10 gifts. State the number of gifts that can be bought if the price of a gift
   (a) increases by two times,
   (b) decreases by 50%.

What is the relation between two variables for an inverse variation?

Learning Standard

Determine the relation between two variables for an inverse variation.

MIND MOBILISATION 6 Group

Aim: To determine the relation between two variables for an inverse variation.

Steps:

A piece of cardboard has an area of 1.44 m². The cardboard will be cut into y small square cards with an area of x m² each.

1. Open the worksheet by scanning the QR code to do this activity.

	A	B	C	D	E	F
1	Area of cardboard =	1.44 m²				
2						
3	Area of small square card, x (m²)	0.02				
4	Number of small square cards, y	72				
5	xy	1.44				
6	$\frac{1}{x}$	50				
7						

Scan the QR code or visit bit.do/WSChap1 to obtain the worksheet.

2. Insert the area of the small card, $x = 0.04$ in coloured cell and click Enter. Repeat the same step for the value of x equals to 0.09, 0.16 and 0.36. Observe the values of y, xy and $\frac{1}{x}$ for each value of x.
3. Draw the graph of y against x and the graph of y against $\frac{1}{x}$ using a suitable scale based on the obtained values.

Discussion:

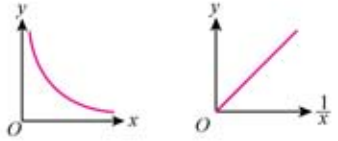
1. Compare the values of xy. What conclusion can be made on the values of xy?
2. What are the shapes of the graphs obtained?
3. What is the relation between y and x?

The results of Mind Mobilisation 6 show that for an inverse variation,

(a) the value of xy is a constant. Hence, $k = xy$,

(b) the graph of y against x is a hyperbola and the graph of y against $\frac{1}{x}$ is a straight line that starts from the origin,

(c) y varies inversely as x.



When variable y varies inversely as variable x, the value of xy is a constant that is represented with k.

Relation between the value of k and the concept of proportion:

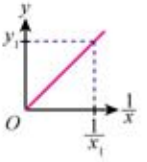
y	y_1	y_2	y_3	y_4
x	x_1	x_2	x_3	x_4

$k = x_1y_1 = x_2y_2 = x_3y_3 = x_4y_4$

Relation between the value of k and the gradient of the straight line that starts from the origin for the graph of y against $\frac{1}{x}$:

Gradient,

$$m = \frac{y_1}{\frac{1}{x_1}}$$
$$= x_1y_1$$
$$= k$$



This relation is true only if the straight line starts from the origin.

Smart TiPS

The relation of inverse variation written as 'y varies inversely as x' can also be written as 'x and y are inversely proportional'.

Hence, we can relate the two variables x and y in the form of equation, that is

$$xy = k$$
$$y = \frac{k}{x}.$$

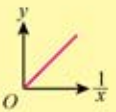
For an **inverse variation**, y varies inversely as x can be written as

$$y \propto \frac{1}{x} \text{ (variation relation)}$$
$$y = \frac{k}{x} \text{ (equation form)}$$

where k is a constant.

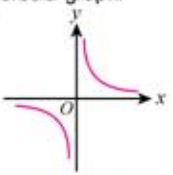
(a) The graph of y against x is a hyperbola.

(b) The graph of y against $\frac{1}{x}$ is a straight line that starts from the origin ($x \neq 0$).



MEMORY BOX

$y = \frac{a}{x}$, $x \neq 0$ is a reciprocal function with a hyperbola graph.



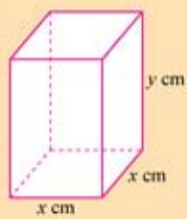
MIND MOBILISATION 7 2 2 Group

Aim: To determine the relation between the variables y and x^2 for an inverse variation.

Steps:

1. It is given that the volume of a cuboid with a square base is 180 cm^3. The table below shows the relation between the length of sides of the base and the height of the cuboid. Complete the table below.

Length of sides of the base, x (cm)	2	3	4	5	6
Height, y (cm)					
xy					
x^2y					
$\frac{1}{x^2}$					



2. Draw the graph of y against x and the graph of y against $\frac{1}{x^2}$ using a suitable scale based on the table.

Discussion:

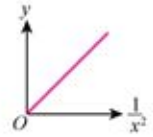
1. Compare the values of xy and x^2y. What conclusion can be made?
2. Which graph is a straight line graph that starts from the origin?
3. What is the relation between y and x^2?

The results of Mind Mobilisation 7 show that for an inverse variation,

(a) the value of x^2y is a constant. Hence, $k = x^2y$,

(b) the graph of y against $\frac{1}{x^2}$ is a straight line that starts from the origin,

(c) y varies inversely as x^2.



In general,

For an **inverse variation**, y varies inversely as x^n can be written as

$y \propto \frac{1}{x^n}$ (variation relation)

$y = \frac{k}{x^n}$ (equation form)

where $n = 1, 2, 3, \frac{1}{2}, \frac{1}{3}$ and k is a constant.

The graph of y against $\frac{1}{x^n}$ is a straight line that starts from the origin ($x \neq 0$) where k is the gradient of the straight line.

When $y = \frac{1}{x^2}$, y does not vary directly as x. Discuss this statement.

Smart TIPS

$y \propto \frac{1}{x}$ is true if and only if $n = 1$.

Example 16

It is given that the time needed, t, to assemble a piece of furniture varies inversely as the number of workers, x. The table below shows the relation between x and t.

Number of workers, x	2	3	4	5	6
Time needed, t (minutes)	180	120	90	72	60

By calculating the values of xt and x^2t, determine whether t varies inversely as x or x^2. Hence, write the relation by using the symbol $\propto$.

Solution:

x	2	3	4	5	6
t	180	120	90	72	60
xt	$2(180) = 360$	$3(120) = 360$	$4(90) = 360$	$5(72) = 360$	$6(60) = 360$
x^2t	$2^2(180) = 720$	$3^2(120) = 1080$	$4^2(90) = 1440$	$5^2(72) = 1800$	$6^2(60) = 2160$

The value of xt is a constant, whereas the value of x^2t is not a constant. Hence, t varies inversely as x, that is $t \propto \frac{1}{x}$.

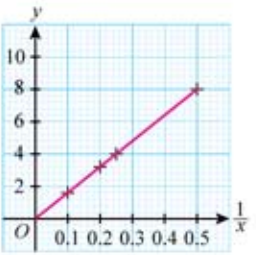
Example 17

Two quantities, x and y, vary based on the table on the right. By using a suitable scale, draw the graph of y against $\frac{1}{x}$ and show that y varies inversely as x.

x	2	4	5	10
y	8	4	3.2	1.6

Solution:

$\frac{1}{x}$	0.5	0.25	0.2	0.1
y	8	4	3.2	1.6



The graph of y against $\frac{1}{x}$ shows a straight line that starts from the origin. Hence, y varies inversely as x.

Example 18

It is given that $x = 0.25$ when $y = 3$. Express y in terms of x if
(a) y varies inversely as x,
(b) y varies inversely as the square root of x.

Solution:

(a) $y \propto \frac{1}{x}$

$y = \frac{k}{x}$ ← Write the relation in the form of equation.

$3 = \frac{k}{0.25}$ ← Substitute the values of y and x into the equation to obtain the value of k.

$k = 3(0.25)$
$= 0.75$

Hence, $y = \frac{0.75}{x}$

(b) $y \propto \frac{1}{\sqrt{x}}$

$y = \frac{k}{\sqrt{x}}$

$3 = \frac{k}{\sqrt{0.25}}$

$k = 3(\sqrt{0.25})$
$= 1.5$

Hence, $y = \frac{1.5}{\sqrt{x}}$

Smart TiPS

(i) y varies inversely as x,
$y \propto \frac{1}{x}$

(ii) y varies inversely as the square of x,
$y \propto \frac{1}{x^2}$

(iii) y varies inversely as the cube of x,
$y \propto \frac{1}{x^3}$

(iv) y varies inversely as the square root of x,
$y \propto \frac{1}{\sqrt{x}}$

(v) y varies inversely as the cube root of x,
$y \propto \frac{1}{\sqrt[3]{x}}$

Example 19

The force of gravity, F, varies inversely as the square of the distance between two objects, d. It is given that the force of gravity between two objects is 15 N when the distance between them is 1.2 cm. Write an expression of F in terms of d.

Solution:

$$F \propto \frac{1}{d^2}$$

$$F = \frac{k}{d^2}$$ ← Write the relation between F and d in the form of equation.

$$15 = \frac{k}{(1.2)^2}$$ ← Substitute the values of F and d into the equation to obtain the value of k.

$$k = 15(1.2)^2$$
$$= 21.6$$

Hence, $F = \frac{21.6}{d^2}$

Historical Treasure

Sir Isaac Newton (1642-1727), a famous English physicist, published Newton's Laws of Gravity in his book Philosophiae Naturalis Principia Mathematica in 1687.

Example 20

It is given that p varies inversely as q. If $p = 2$ when $q = 7$, calculate the value of p when $q = 1.6$.

Solution:

$$p \propto \frac{1}{q}$$

$$p = \frac{k}{q}$$

$$2 = \frac{k}{7}$$

$$k = 2(7)$$
$$= 14$$

Hence, $p = \frac{14}{q}$

When $q = 1.6$,

$$p = \frac{14}{1.6}$$
$$= 8.75$$

Alternative Method:

Using the concept of proportion:
Given that $p_1 = 2$, $q_1 = 7$, $q_2 = 1.6$

$$p_1 q_1 = p_2 q_2$$
$$2 \times 7 = p_2 \times 1.6$$
$$p_2 = \frac{2 \times 7}{1.6}$$
$$= 8.75$$

Self Practice 1.2b

1. The table below shows the values of two variables, x and y.

(a) Determine whether y varies inversely as x. If yes, write the relation in the form of variation.

x	1	$1\frac{1}{2}$	2	$2\frac{1}{2}$	3
y	6	4	3	2	1

(b) Determine whether y varies inversely as x^2. If yes, write the relation in the form of variation.

x	1	2	3	4	5
y	3.6	0.9	0.4	0.225	0.144

2. The table below shows the current, I (Ampere, A) flows in a circuit with its resistance, R (Ohm, Ω).

Resistance, R (Ω)	42	35	30	21	15
Current, I (A)	5	6	7	10	14

By plotting the graph of I against $\frac{1}{R}$ using a suitable scale, determine whether I varies inversely as R.

3. It is given that $g = 0.15$ when $h = 8$. Express g in terms of h if
(a) g varies inversely as h,
(b) g varies inversely as h^2,
(c) g varies inversely as the cube root of h.

4. Given that $y = 0.5$ when $x = 16$, calculate the value of y when $x = 0.04$ if

(a) $y \propto \frac{1}{x}$

(b) $y \propto \frac{1}{x^3}$

(c) $y \propto \frac{1}{x^{\frac{1}{2}}}$

(d) $y \propto \frac{1}{\sqrt[3]{x}}$

5. Calculate the values of s and t for each of the following relations.

(a) y varies inversely as x.

x	3	s	5
y	2	1.5	t

(b) y varies inversely as the square root of x.

x	$\frac{9}{4}$	s	0.04
y	8	2.4	t

6. A company prints a certain number of books every day. The table below shows the number of printing machines in operation, M and the time needed, T to print the books. It is given that T varies inversely as M.

M	6	8	q
T	10	p	4

(a) Express T in terms of M.
(b) Determine the values of p and q.

7. The period of oscillation, T, of a simple pendulum varies inversely as the square root of gravitational acceleration, g. In an experiment, the period of oscillation is 1.01 seconds when the gravitational acceleration is 9.85 m s^{-2}. Express T in terms of g.

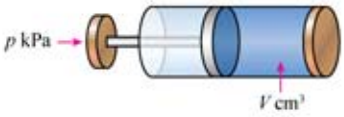
How to solve problems involving inverse variation?

Learning Standard

Solve problems involving inverse variation.

Example 21

According to Boyle's Law, the gas pressure, p, of a mass of gas is inversely proportional to the volume of the gas, V, if the temperature of the gas is fixed. The diagram on the right shows the gas trapped in a cylinder. When the volume inside the cylinder is 80 cm³, the gas pressure is 190.25 kPa. Calculate the volume of the gas in cm³, when the gas pressure inside the cylinder is 121.76 kPa.



Solution:

Understanding the problem

- p is inversely proportional to V
- When $V = 80$ cm³, $p = 190.25$ kPa

Devising a strategy

Determine the relation between p and V in the form of equation. Hence, calculate the value of V when $p = 121.76$ kPa.

Implementing the strategy

$$p \propto \frac{1}{V}$$

$$p = \frac{k}{V}$$

When $V = 80$, $p = 190.25$,

$$190.25 = \frac{k}{80}$$

$$k = 190.25 \times 80$$

$$= 15\ 220$$

Hence, $p = \dfrac{15\ 220}{V}$

When $p = 121.76$,

$$121.76 = \frac{15\ 220}{V}$$

$$V = \frac{15\ 220}{121.76}$$

$$= 125 \text{ cm}^3$$

Making a conclusion

When $p = 121.76$ kPa, $V = 125$ cm³

Self Practice 1.2c

1. The table on the right shows the resistance of a length of wire varies as the radius of its cross-section.
   (a) Determine whether the resistance, R, of the wire varies inversely as r^2.
   (b) Calculate the radius in mm, of the wire if the resistance is 25 Ω.

Resistance, R (Ω)	1	4	9	16
Radius of cross-section, r (mm)	1.2	0.6	0.4	0.3

2. The number of cubes, b, produced from a fixed quantity of metal varies inversely as the cube of its sides, p cm. If $b = 16$ when $p = 1.5$, calculate the value of p when $b = 250$.

3. The number of oscillations, A of a simple pendulum varies inversely as the square root of the length of the pendulum, p cm, in a fixed period. Given that the number of oscillations is 9 when the length of the pendulum is 36 cm, calculate the length of the simple pendulum if the number of oscillations is 15.

The results of Mind Mobilisation 8 show that y varies directly as x, and y varies inversely as z. Therefore, y varies directly as x and inversely as z, that is $y \propto \frac{x}{z}$.

In general,

For a **combined variation**, y varies directly as x^m and inversely as z^n can be written as

$y \propto \frac{x^m}{z^n}$ (variation relation)

$y = \frac{kx^m}{z^n}$ (equation form)

where $m = 1, 2, 3, \frac{1}{2}, \frac{1}{3}$, $n = 1, 2, 3, \frac{1}{2}, \frac{1}{3}$ and k is a constant.

Example 22

It is given that y varies directly as the square of x and inversely as the square root of z. If $y = 8$ when $x = 4$ and $z = 36$, express y in terms of x and z.

Solution:

$$y \propto \frac{x^2}{\sqrt{z}}$$

$$y = \frac{kx^2}{\sqrt{z}}$$ ← Write the relation between y, x and z in the form of equation.

$$8 = \frac{k(4)^2}{\sqrt{36}}$$ ← Substitute the values of y, x and z into the equation to obtain the value of k.

$$k = \frac{8\sqrt{36}}{4^2}$$
$$= 3$$

Hence, $y = \frac{3x^2}{\sqrt{z}}$

Smart TiPS

(i) y varies directly as w and the cube of x, and inversely as the cube root of z,
$$y \propto \frac{wx^3}{\sqrt[3]{z}}$$

(ii) y varies directly as the product of w^2 and $\sqrt{x}$, and inversely as z^3,
$$y \propto \frac{w^2\sqrt{x}}{z^3}$$

Example 23

The table below shows the values of P, Q and R. It is given that P varies directly as the cube of Q and inversely as R. Calculate the values of x and y.

P	4	3.6	0.081
Q	2	6	y
R	0.6	x	2.7

1.3 Combined Variation

What is the relation between three or more variables for a combined variation?

Learning Standard

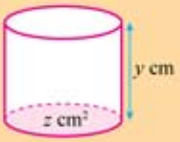
Determine the relation between three or more variables for a combined variation.

MIND MOBILISATION 8 Group

Aim: To determine the relation between three variables for a combined variation which involves direct variation and inverse variation.

Steps:

1. The formula for the volume of a cylinder is the product of its base area and its height. If x, y and z represent the volume, height and base area of the cylinder respectively, complete the table below.



(A) If z is fixed				
Volume, x (cm³)	100	200	300	400
Height, y (cm)				
Base area, z (cm²)	50	50	50	50
$\frac{x}{y}$				

(B) If x is fixed				
Volume, x (cm³)	120	120	120	120
Height, y (cm)				
Base area, z (cm²)	40	60	80	100
yz				

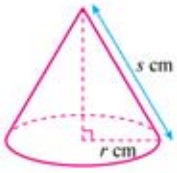
(C) If x, y and z are variables				
Volume, x (cm³)	80	180	320	500
Height, y (cm)				
Base area, z (cm²)	20	30	40	50
$\frac{yz}{x}$				

Discussion:

1. What is the value of $\frac{x}{y}$? Hence, state the relation between y and x if z is a constant.
2. What is the value of yz? Hence, state the relation between y and z if x is a constant.
3. What is the value of $\frac{yz}{x}$? Hence, state the relation between y, x and z if three of them are variables.

10. The electric current, I (Ampere) varies directly as the power, P (Watt) and inversely as the voltage, V (Volt) for an electrical appliance. It is given that a hairdryer with 550 W of power and 240 V of voltage uses an electric current of 2.2 A. Calculate the electric current used by a fan with 75 W of power and 240 V of voltage.

11. The area of the curved surface, A cm^2, of a cone varies directly as the radius of its base, r cm, and the slant height, s cm. It is given that $A = 88$ cm^2 when $r = 3.5$ cm and $s = 8$ cm.



(a) Calculate the value of A when $r = 5$ cm and $s = 9.8$ cm.

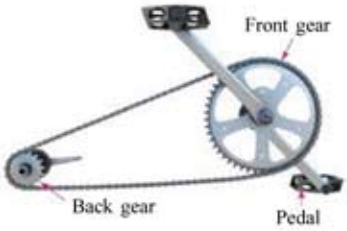
(b) What is the change in the area of the curved surface if the slant height decreases and the base radius is fixed?

12. It is given that Y varies directly as X and inversely as W. If $Y = 0.9$ when $X = 18$ and $W = 5$, calculate

(a) the value of W when $Y = 20$ and $X = 6$,

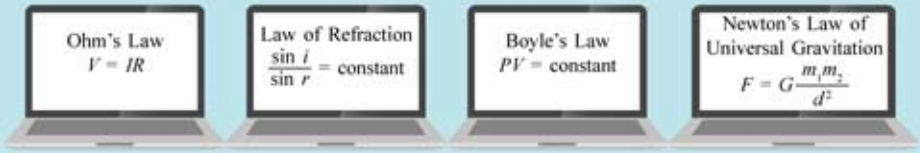
(b) the percentage of the change of Y when X increases by 10% and W decreases by 20%.

13. The speed of a bicycle, S, varies directly as the number of revolutions per minute of the bicycle pedal, P, and the number of the front gear teeth, d, and inversely as the number of back gear teeth, b. Santhami rides a bicycle at a speed of 26.4 km per hour. The bicycle pedal moves at 75 revolutions per minute with 40 front gear teeth and 20 back gear teeth. Describe the change in speed of Santhami's bicycle if he pedals at 90 revolutions per minute.



EXPLORING MATHEMATICS

The variation concept is widely used in science. Some examples are:



In groups, find information from the internet on the quantities represented by the variables for the above formulae and how the concept of variation is applied in these laws. Briefly describe the usage of the formulae. Then, conduct further investigations on the application of the concept of variation in the field of finance, environment and social science. Present your findings to share information with other groups.

Solution:

$P \propto \frac{Q^3}{R}$

$P = \frac{kQ^3}{R}$

$4 = \frac{k(2)^3}{0.6}$

$k = \frac{4(0.6)}{2^3} = \frac{3}{10}$

Hence, $P = \frac{3Q^3}{10R}$

When $P = 3.6$, $Q = 6$ and $R = x$,

$P = \frac{3Q^3}{10R}$

$3.6 = \frac{3(6)^3}{10x}$

$x = \frac{3(6)^3}{10(3.6)}$

$= 18$

When $P = 0.081$, $Q = y$ and $R = 2.7$,

$P = \frac{3Q^3}{10R}$

$0.081 = \frac{3(y)^3}{10(2.7)}$

$y = \sqrt[3]{\frac{0.081(10)(2.7)}{3}}$

$= 0.9$

Self Practice 1.3a

1. Write in the form of variation and in the form of equation for each of the following combined variations.
   (a) w varies directly as the cube root of v and inversely as the square of x.
   (b) F varies directly as G and H^3, and inversely as $\sqrt{t}$.
   (c) The acceleration of an object, A varies directly as the distance travelled, s, and inversely as the square of the time taken, t, by the object.

2. The time taken, t hours, to arrange chairs in a hall varies directly as the number of chairs, c, and inversely as the number of workers involved, p. It is given that 5 workers used 2 hours to arrange 1 000 chairs. Express t in terms of c and p.

3. It is given that M varies directly as the square of N and inversely as P. If $M = 4.8$ when $N = 6$ and $P = 1.5$, calculate
   (a) the value of P when $M = 0.8$ and $N = 2.4$,
   (b) the value of N when $M = 19$ and $P = 3.8$.

4. The table on the right shows the changes in three quantities T, e and f. It is given that T varies inversely as the square root of e and the cube of f. Calculate the values of a and b.

T	5	a	0.256
e	1.44	36	b
f	2	0.4	5

How to solve problems involving combined variation?

Learning Standard

Solve problems involving combined variation.

Example 24

The pressure, p N m^{-2}, on the tyre of a wheelbarrow varies directly as the mass of the wheelbarrow, m kg, and inversely as the surface area of the tyre touching the ground, l m^2. It is given that the pressure on the tyre is 45 000 N m^{-2} when the mass of the wheelbarrow is 90 kg and the surface area of the tyre touching the ground is 0.02 m^2.

(a) Calculate the value of p when $m = 120$ and $l = 0.5$.

(b) What can be done to reduce the pressure on the tyre if the mass of the wheelbarrow is fixed?

Solution:

Understanding the problem

- p varies directly as m and inversely as l
- When $m = 90$ and $l = 0.02$, $p = 45\ 000$

Devising a strategy

(a) Determine the relation between p, m and l in the form of equation. Hence, substitute the values of $m = 120$ and $l = 0.5$ into the equation to calculate the value of p.
(b) Understand the changes of every variable in the equation.

Implementing the strategy

(a) $p \propto \frac{m}{l}$

$p = \frac{km}{l}$

$45\ 000 = \frac{k(90)}{0.02}$

$k = \frac{(0.02)(45\ 000)}{90}$

$= 10$

Hence, $p = \frac{10m}{l}$

When $m = 120$, $l = 0.5$,

$p = \frac{(10)(120)}{0.5}$

$= 2\ 400\ \text{N m}^{-2}$

(b) If the value of m is fixed,
- p decreases, when l increases
- p increases, when l decreases

Making a conclusion

(a) $p = 2\ 400\ \text{N m}^{-2}$
(b) Use a broader tyre because the pressure on the tyre reduces when the surface area of the tyre touching the ground increases, and the mass is fixed.

Self Practice 1.3b

1. Mr Kamal wants to install rectangular tiles in his bedroom. The number of tiles needed, J, varies inversely as the length, p m, and width, l m, of the tiles used. Mr Kamal needs 120 pieces of tiles if the tile is 0.4 m in length and 0.5 m in width.
(a) Calculate the number of tiles needed if the length is 0.2 m and the width is 0.3 m.
(b) If the area of the tile increases, what is the change in the number of tiles needed?

2. The average number of daily phone calls, C, between two cities varies directly as the populations of the two cities, P_1 and P_2, and inversely as the square of the distance, j, between the two cities. The distance between city A and city B is 210 km. The average number of daily phone calls between the two cities is 15 750 and the populations of city A and city B are 105 000 and 220 500 respectively. Give your answer to the nearest whole number, calculate
(a) the distance between city P and city Q if the populations are 83 400 and 62 000 respectively and the average number of daily phone calls is 19 151,
(b) the population of city J if the population of city K is 1 100 000 with the distance between the two cities is 351 km. The average number of daily phone calls between city J and city K is 18 857.

Summary Arena

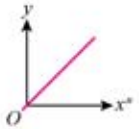
VARIATION

Direct Variation

y varies directly as x^n, for

$n = 1, 2, 3, \frac{1}{2}, \frac{1}{3}$

- $y \propto x^n$
- $y = kx^n$ where k is a constant and $k = \frac{y}{x^n}$
- The graph of y against x^n is a straight line that passes through the origin

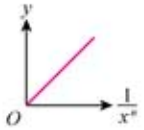


Inverse Variation

y varies inversely as x^n, for

$n = 1, 2, 3, \frac{1}{2}, \frac{1}{3}$

- $y \propto \frac{1}{x^n}$
- $y = \frac{k}{x^n}$ where k is a constant and $k = yx^n$
- The graph of y against $\frac{1}{x^n}$ is a straight line that starts from the origin ($x \neq 0$).



y varies inversely as x^n

Joint Variation

A direct variation where a variable varies as the product of two or more variables.

- y varies jointly as x^m and z^n, for

  $m = 1, 2, 3, \frac{1}{2}, \frac{1}{3}$ and

  $n = 1, 2, 3, \frac{1}{2}, \frac{1}{3}$

- $y \propto x^m z^n$
- $y = kx^m z^n$ where k is a constant and $k = \frac{y}{x^m z^n}$

Combined Variation

Involves a combination of direct variation or joint variation, and inverse variation.

- For $p = 1, 2, 3, \frac{1}{2}, \frac{1}{3}$, $q = 1, 2, 3, \frac{1}{2}, \frac{1}{3}$ and $r = 1, 2, 3, \frac{1}{2}, \frac{1}{3}$

  (i) $y \propto \frac{x^p}{z^q}$

  $y = \frac{kx^p}{z^q}$

  (ii) $y \propto \frac{w^p x^q}{z^r}$

  $y = \frac{kw^p x^q}{z^r}$

  where k is a constant.

Reflection

At the end of this chapter, I can

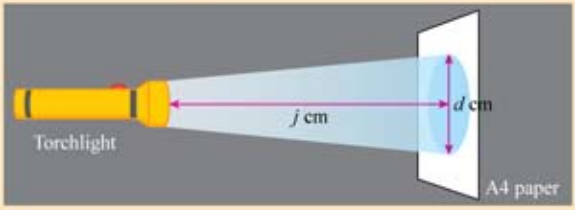
explain the meaning of direct variation.		
determine the relation between two variables for a direct variation.		
determine the relation between three or more variables for a given joint variation.		
solve problems involving direct variation.		
explain the meaning of inverse variation.		
determine the relation between two variables for an inverse variation.		
solve problems involving inverse variation.		
determine the relation between three or more variables for a combined variation.		
solve problems involving combined variation.		

MINI PROJECT

Materials: Torchlight, A4 paper and measurement tape

Steps:

1. Carry out this project in groups. Paste the A4 paper and place the torchlight facing the paper in a dark classroom as shown in the diagram below.



2. With a different distance of the torchlight from the A4 paper, j, for example 10 cm, 20 cm, 30 cm, ..., record the diameter of the light image, d, formed on the paper. Construct a table to record your data.
3. Represent the data obtained using a suitable line graph.
4. Write a formula to show the relation between d and j. Then, make a conclusion on the diameter of the light image formed and the distance of the torchlight from the paper.
5. How can these findings be applied in your daily life?

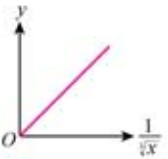
Extensive Practice

Scan the QR code or visit bit.do/QuizE01 for interactive quiz

UNDERSTAND

1. Write the variation relation for each of the following.
   (a) w is directly proportional to the cube of x.
   (b) a varies directly as b and inversely as the cube of c.
   (c) p varies directly as q and the square root of r.
   (d) The distance travelled, s m, by a bicycle varies directly as its acceleration, a m s^{-2}, and the square of the time taken, t s.

2. The diagram on the right shows the graph of y against $\frac{1}{\sqrt[3]{x}}$. Write the relation between y and $\sqrt[3]{x}$ by using the symbol $\propto$.



3. Write the following relations in sentences.
   (a) $y \propto xz$ (b) $e \propto \frac{1}{f}$ (c) $p \propto \frac{\sqrt[3]{q}}{r}$ (d) $n \propto \frac{pq^2}{\sqrt{r}}$

4. State whether each of the following has a relation of y varies directly as x.
   (a) $x - y = 0$ (b) $y + 3 = x$ (c) $xy = 10$ (d) $\frac{x}{y} = 0.5$

MASTERY

5. Calculate the constant, k for each of the following.
   (a) L varies directly as the cube of m. $L = 16.384$ when $m = 3.2$.
   (b) h varies directly as a and the square of b. $h = 96$ when $a = 18$ and $b = 4$.
   (c) P varies directly as q^2 and r, and inversely as $\sqrt[3]{s}$. $P = 17.01$ when $q = 4.5$, $r = 9$ and $s = 3\,375$.

6. It is given that m varies inversely as n and p. If $m = 6$ when $n = 0.4$ and $p = 5$, write an equation that relates m, n and p.

7. It is given that $f \propto g^2h$ and $f = 24$ when $g = 4$ and $h = 5$. Calculate the value of g when $f = 5.88$ and $h = 10$.

8. The table on the right shows the changes in three quantities. It is given that y varies directly as x and inversely as the square root of z. Calculate the values of m and n.

y	4	m	51
x	0.3	6	1.7
z	3.24	225	n

CHALLENGE

9. P varies inversely as Q and $Q = 3R - 2$. It is given that $P = 0.02$ when $R = 4$.
   (a) Express P in terms of Q.
   (b) Calculate the value of R when $P = 5$.