CHAPTER 2 Matrices

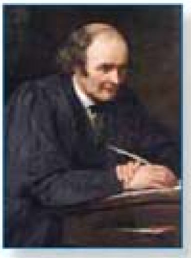
What will you learn?

- Matrices
- Basic Operation on Matrices

Why study this chapter?

Matrices are used in the field of mathematics to represent and solve problems in algebra, statistic and geometry. In daily life, it can be used by a purchasing manager to record the amount of inventory in his business.

Do you know?



Arthur Cayley (1821-1895) was an English mathematician who developed the theory of matrices from the aspect of algebra in his work A Memoir on the Theory of Matrices. Cayley created matrices when he was researching the theory of transformation. Two American mathematicians, Benjamin Peirce (1809-1880) and Charles S. Peirce (1839-1914) worked together with Cayley in the development of algebraic matrices.

For more information:

bit.do/DoYouKnowChap2

WORD BANK

matrix	*matriks*
row matrix	*matriks baris*
identity matrix	*matriks identiti*
column matrix	*matriks lajur*
square matrix	*matriks segi empat sama*
rectangular matrix	*matriks segi empat tepat*
zero matrix	*matriks sifar*
inverse matrix	*matriks songsang*
scalar multiplication	*pendaraban skalar*
determinant	*penentu*
order	*peringkat*
element	*unsur*

According to Digital Report 2018, the total number of internet users has reached 25.08 million people, that is 79% of the residents in Malaysia. Nowadays, the society prefers to carry out daily activities such as shopping and making payment for services through the internet. Data is created in this process and supports the growth of big data.

Big data refers to data that is very large, complex and difficult to process with regular database management. This set of data can help the management teams in various fields to make better decisions. In the business field, big data is analysed to reduce cost and time, develop new products and strategise business plans. The process of data analysis largely involves matrices.

2.1 Matrices

How to represent the information from real situations in the form of matrices?

Learning Standard

Represent information from real situations in the form of matrices.

Kedai Elektrik Sinar Jaya records the sales of three types of fans in a spreadsheet. The diagram below shows the in-store and online sales of the fans for the months of March to May. How can these data be arranged?

	A	B	C	D	E	F	G	H	I	J	K	L
1		The sales of the fans in March				The sales of the fans in April				The sales of the fans in May		
2		Fan				Fan				Fan		
3		Stand	Ceiling	Wall		Stand	Ceiling	Wall		Stand	Ceiling	Wall
4	In-store	16	18	11		20	15	9		15	21	10
5	Online	5	10	4		7	12	5		10	24	10
6												
7		The total sales in March and April				The difference in sales between April and May						
8		Fan				Fan						
9		Stand	Ceiling	Wall		Stand	Ceiling	Wall				
10	In-store	36	33	20		-5	6	1				
11	Online	12	22	9		3	12	5				
12												

The data for the month of March is represented in the form of table. It can also be represented in the form of matrix as shown below.

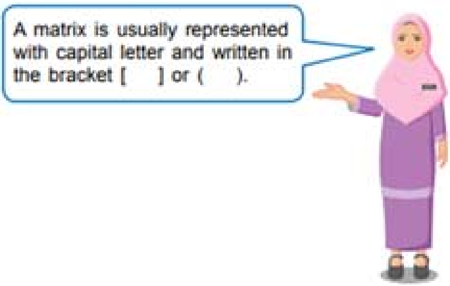
	Fan		
	Stand	Ceiling	Wall
In-store	16	18	11
Online	5	10	4

Table form

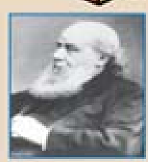
$$\begin{array}{c c}
\text{rows} \rightarrow & \begin{pmatrix} 16 & 18 & 11 \\ 5 & 10 & 4 \end{pmatrix} \\
 & \uparrow \quad \uparrow \quad \uparrow \\
 & \text{columns}
\end{array}$$

Matrix form

A **matrix** is a set of numbers arranged in rows and columns to form a rectangular or a square array.



Historical Treasure



James Joseph Sylvester (1814-1897) was the first mathematician that used the term matrix in 1850.

Example 1

Represent the following information in the form of matrices.

(a) The table below shows the recommended daily calorie intake for females by category.

	Child	Teenager	Adult	Senior Citizen
Recommended daily calorie intake (kcal)	1 700	2 100	2 000	1 800

(b) The table below shows the medal collection of Malaysian contingent for three events in the 29th SEA Games.

	Gold	Silver	Bronze
Diving	13	5	1
Pencak Silat	10	2	4
Athletics	8	8	9

(c) In Test 1, Samad scores 76 marks for Bahasa Melayu, 82 marks for Mathematics and 72 marks for History, while Hamid scores 80 marks for Bahasa Melayu, 88 marks for Mathematics and 70 marks for History.

Info Bulletin

Calories are a measure of the value of energy in food. The caloric content of food depends on the content of carbohydrates, proteins and fats in it. For example, 1 g of protein equals 4 kcal.

My Malaysia

In the 29th SEA Games that was held in Kuala Lumpur in 2017, the Malaysian contingent was crowned the overall champion with 145 golds.

Solution:

(a) $\text{row 1} \rightarrow \underbrace{\begin{bmatrix} 1700 & 2100 & 2000 & 1800 \end{bmatrix}}$ or $\begin{array}{c} \begin{bmatrix} 1700 \\ 2100 \\ 2000 \\ 1800 \end{bmatrix} \\ \uparrow \\ \text{column 1} \end{array}$

This is a row matrix. A row matrix has only one row.

This is a column matrix. A column matrix has only one column.

(b)
$$\begin{array}{cc} \begin{array}{c} \text{row 1} \rightarrow \\ \text{row 2} \rightarrow \\ \text{row 3} \rightarrow \end{array} & \begin{bmatrix} 13 & 5 & 1 \\ 10 & 2 & 4 \\ 8 & 8 & 9 \end{bmatrix} \\ & \begin{array}{ccc} \uparrow & \uparrow & \uparrow \\ \text{column 1} & \text{column 2} & \text{column 3} \end{array} \end{array} \quad \text{or} \quad \begin{bmatrix} 13 & 10 & 8 \\ 5 & 2 & 8 \\ 1 & 4 & 9 \end{bmatrix}$$

These are square matrices. A square matrix has the same number of rows and columns.

Critical Mind

Is [6] a square matrix? Explain your answer.

(c)
$$\begin{array}{cc} \begin{array}{c} \text{row 1} \rightarrow \\ \text{row 2} \rightarrow \\ \text{row 3} \rightarrow \end{array} & \begin{bmatrix} 76 & 80 \\ 82 & 88 \\ 72 & 70 \end{bmatrix} \\ & \begin{array}{cc} \uparrow & \uparrow \\ \text{column 1} & \text{column 2} \end{array} \end{array} \quad \text{or} \quad \begin{array}{cc} \begin{array}{c} \text{row 1} \rightarrow \\ \text{row 2} \rightarrow \end{array} & \begin{bmatrix} 76 & 82 & 72 \\ 80 & 88 & 70 \end{bmatrix} \\ & \begin{array}{ccc} \uparrow & \uparrow & \uparrow \\ \text{column 1} & \text{column 2} & \text{column 3} \end{array} \end{array}$$

These are rectangular matrices. A rectangular matrix has different number of rows and columns.

Self Practice 2.1a

1. A fifth generation (5G) mobile technology exhibition is attended by 857 teenagers, 3 180 adults and 211 senior citizens. Represent the information in the form of matrix.

2. The table on the right shows the average Air Pollution Index (API) in Putrajaya, Jerantut and Sandakan for three days. Represent the information in the form of matrix.

	Monday	Tuesday	Wednesday
Putrajaya	53	52	50
Jerantut	20	21	20
Sandakan	47	48	46

3. The table below shows the average number of books read by the pupils in Program Nilam in SMK Setia for 2019.

	Form 3	Form 4	Form 5
Bahasa Melayu	20	18	15
English	12	10	11

Represent the information above in the form of matrix.

How to determine the order of a matrix, hence identify certain elements in a matrix?

Learning Standard

Determine the order of a matrix, hence identify certain elements in a matrix.

Order of a matrix can be determined by counting the number of rows followed by the number of columns of the matrix. For example,

$$\begin{array}{r c}
\begin{array}{r} \text{row 1} \longrightarrow \\ \text{row 2} \longrightarrow \end{array} & \begin{bmatrix} 16 & 18 & 11 \\ 5 & 10 & 4 \end{bmatrix}_{2 \times 3} \\
 & \begin{array}{ccc} \uparrow & \uparrow & \uparrow \\ \text{column 1} & \text{column 2} & \text{column 3} \end{array}
\end{array}$$

This matrix has 2 rows and 3 columns. Therefore, it is a matrix with order 2×3 and can be read as "matrix 2 by 3".

Matrix with m rows and n columns has the order $m \times n$ and is read as "matrix m by n".

Every number in the matrix is known as an **element** of the matrix. For example, the element at the 2nd row and 3rd column for the matrix $\begin{bmatrix} 16 & 18 & 11 \\ 5 & 10 & 4 \end{bmatrix}$ is 4.

Capital letter is used to denote a matrix, for example $A = \begin{bmatrix} 16 & 18 & 11 \\ 5 & 10 & 4 \end{bmatrix}$, and the element at the 2nd row and 3rd column can be represented as a_{23}, for example $a_{23} = 4$.

In general, the element at the i^{th} row and j^{th} column in matrix A can be represented as

$$a_{ij}$$

i^{th} row $\quad$ j^{th} column

Hence, $A = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \cdots & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{bmatrix}$

Info Bulletin

a_{21} is read as "a two one".

Info Bulletin

A row matrix has the order $1 \times n$ while a column matrix has the order $m \times 1$.

Example 2

It is given that three matrices, $P = [3 \quad -7 \quad 9]$, $Q = \begin{bmatrix} 4 \\ 5 \end{bmatrix}$ and $R = \begin{bmatrix} 1 & 5 & -2 & 0 \\ 3 & 7 & 2 & 8 \\ -4 & 11 & 6 & 1 \\ 9 & 3 & -1 & 5 \end{bmatrix}$. Determine

(a) the order of each matrix,

(b) the element

(i) at the first row and 3rd column of matrix P, p_{13},

(ii) at the 2nd row and first column of matrix Q, q_{21},

(iii) at the 3rd row and 4th column of matrix R, r_{34}.

Solution:

(a) The order of matrix P is 1×3.

The order of matrix Q is 2×1.

The order of matrix R is 4×4.

$$\begin{array}{cccc} \text{row 1} \rightarrow & [\,3 & -7 & 9\,] \\ & \uparrow & \uparrow & \uparrow \\ & \text{column 1} & \text{column 2} & \text{column 3} \end{array}$$

This matrix has 1 row and 3 columns.

(b) (i) p_{13} of matrix P is 9.

(ii) q_{21} of matrix Q is 5.

(iii) r_{34} of matrix R is 1.

Example 3

Given that matrix $D = \begin{bmatrix} -2 & 5 \\ 0 & 4 \\ 1 & 9 \end{bmatrix}$, determine

(a) the order of the matrix, (b) the elements d_{11}, d_{21} and d_{32}.

Solution:

(a) 3×2

(b) $d_{11} = -2$ ← d_{11} is the element at the first row and first column.

$d_{21} = 0$ ← d_{21} is the element at the 2nd row and first column.

$d_{32} = 9$ ← d_{32} is the element at the 3rd row and 2nd column.

Historical Treasure

Cuthbert Edmund Cullis (1875-1954) was an English mathematician who introduced brackets for matrices in 1913. Cullis used the notation $A = [a_{ij}]$ to represent the element at i^{th} row and j^{th} column in a matrix.

Self Practice 2.1b

1. Determine the order of the following matrices.

(a) $\begin{bmatrix} 15 & -8 \end{bmatrix}$ (b) $\begin{bmatrix} 6 \\ 9 \end{bmatrix}$ (c) $\begin{bmatrix} 4 & -1 & 7 \\ 8 & 0 & 2 \\ 5 & 11 & 3 \end{bmatrix}$ (d) $\begin{bmatrix} 12 & 9 & 1 \\ 5 & 10 & 7 \end{bmatrix}$

2. For each of the following matrices, determine
(i) the order of the matrix,
(ii) the element at the 2nd row and 2nd column,
(iii) the element at the 3rd row and first column.

(a) $\begin{bmatrix} 1 & 5 \\ -6 & 0 \\ 9 & 12 \end{bmatrix}$ (b) $\begin{bmatrix} 2 & -3 & 5 \\ -1 & 16 & 0 \\ 9 & 1 & 8 \end{bmatrix}$

3. Given that matrix $F = \begin{bmatrix} -8 & 14 & 2 \\ 7 & 3 & -5 \end{bmatrix}$, determine the order of matrix F. Hence, identify the elements f_{13}, f_{22} and f_{11}.

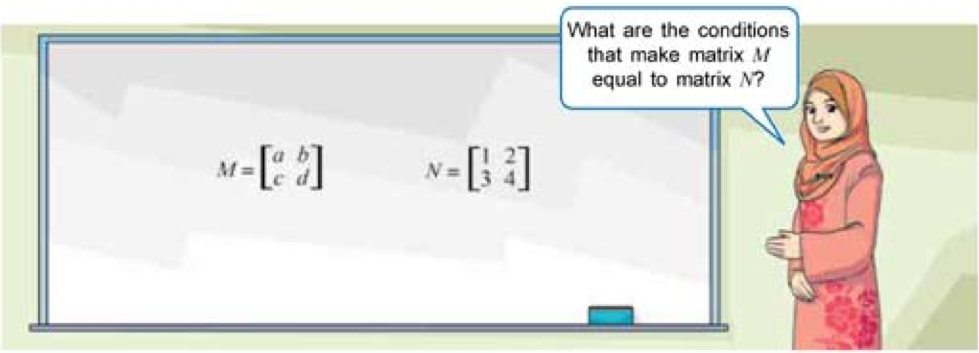
4. Given that matrix $B = \begin{bmatrix} 1 & -16 \\ 20 & 4 \end{bmatrix}$, calculate the value of $b_{12} + b_{21}$.

How to determine whether two matrices are equal?

Learning Standard

Determine whether two matrices are equal.

Look at the two matrices below.



It is found that matrix M and matrix N have the same order, which is 2×2 and if each corresponding element is equal, which is $a = 1$, $b = 2$, $c = 3$ and $d = 4$, then matrix M and matrix N are **equal** and can be written as $M = N$.

> $M = N$ if and only if both the matrices have the same order and each corresponding element is equal.

Example 4

Determine whether each of the following pairs of matrices is equal. Give your reason.

(a) $A = \begin{bmatrix} 2 & 11 \\ 1 & 3 \end{bmatrix}$ and $B = \begin{bmatrix} 2 & 11 \\ 1 & 3 \end{bmatrix}$

(b) $C = [3 \quad 9]$ and $D = \begin{bmatrix} 3 \\ 9 \end{bmatrix}$

(c) $E = \begin{bmatrix} 8 & 3 \\ -7 & 0 \end{bmatrix}$ and $F = \begin{bmatrix} 8 & -7 \\ 3 & 0 \end{bmatrix}$

(d) $G = \begin{bmatrix} 0.5 & -2 \\ 6 & 0.8 \\ -1 & 12 \end{bmatrix}$ and $H = \begin{bmatrix} \frac{1}{2} & -2 \\ 6 & \frac{4}{5} \\ -1 & 12 \end{bmatrix}$

Solution:

(a) $A = B$ because both the matrices have the same order and each corresponding element is equal.

(b) $C \neq D$ because both the matrices do not have the same order. The order of C is 1×2 while the order of D is 2×1.

(c) $E \neq F$ because the corresponding elements are different.

(d) $G = H$ because both the matrices have the same order and each corresponding element is equal.

Example 5

It is given that matrix $P = \begin{bmatrix} x & 7 \\ 0 & 5 - 3z \end{bmatrix}$ and matrix $Q = \begin{bmatrix} 5 & y + 1 \\ 0 & 2z \end{bmatrix}$. Determine the values of x, y and z if $P = Q$.

Solution:

$P = Q$, hence all the corresponding elements are equal.

$x = 5$, $\quad 7 = y + 1$, $\quad 5 - 3z = 2z$

$y = 7 - 1 \qquad 5 = 5z$

$y = 6 \qquad z = 1$

Self Practice 2.1c

1. Determine whether each of the following pairs of matrices is equal.

(a) $\begin{bmatrix} 4 \\ 3 \end{bmatrix}$ and $\begin{bmatrix} 4 \\ 3 \end{bmatrix}$

(b) $\begin{bmatrix} 0.1 & 6 \\ -1 & 1.5 \end{bmatrix}$ and $\begin{bmatrix} \frac{1}{10} & 6 \\ -1 & \frac{3}{2} \end{bmatrix}$

(c) $\begin{bmatrix} 12 \\ -7 \end{bmatrix}$ and $[12 \quad -7]$

(d) $\begin{bmatrix} 0 & 9 \\ 8 & 1 \end{bmatrix}$ and $\begin{bmatrix} 0 & 8 \\ 9 & 1 \end{bmatrix}$

2. Given that $P = Q$, calculate the values of x, y and z.

(a) $P = \begin{bmatrix} 6 & 0 \\ 3 & y \\ 2z - 3 & -5 \end{bmatrix}$ and $Q = \begin{bmatrix} x & 0 \\ 3 & 2 \\ -2 & -5 \end{bmatrix}$

(b) $P = \begin{bmatrix} 10 & -1 \\ 6y + 5 & 3z + 4 \end{bmatrix}$ and $Q = \begin{bmatrix} 5x & -1 \\ 2y - 9 & -4x \end{bmatrix}$

2.2 Basic Operation on Matrices

How to add and subtract matrices?

Learning Standard

Add and subtract matrices.

The sales of the fans in March

	Fan		
	Stand	Ceiling	Wall
In-store	16	18	11
Online	5	10	4

The sales of the fans in April

	Fan		
	Stand	Ceiling	Wall
In-store	20	15	9
Online	7	12	5

The sales of the fans in May

	Fan		
	Stand	Ceiling	Wall
In-store	15	21	10
Online	10	24	10

The total sales in March and April

	Fan		
	Stand	Ceiling	Wall
In-store	36	33	20
Online	12	22	9

The difference in sales between April and May

	Fan		
	Stand	Ceiling	Wall
In-store	-5	6	1
Online	3	12	5

Observe the spreadsheet above that shows the sales of fans in Kedai Elektrik Sinar Jaya. The sales of fans in the months of March, April and May can be represented with matrix $P = \begin{bmatrix} 16 & 18 & 11 \\ 5 & 10 & 4 \end{bmatrix}$, matrix $Q = \begin{bmatrix} 20 & 15 & 9 \\ 7 & 12 & 5 \end{bmatrix}$ and matrix $R = \begin{bmatrix} 15 & 21 & 10 \\ 10 & 24 & 10 \end{bmatrix}$ respectively.

The total sales in March and April can be obtained by adding the matrix P and matrix Q, that is

$$\begin{bmatrix} 16 & 18 & 11 \\ 5 & 10 & 4 \end{bmatrix} + \begin{bmatrix} 20 & 15 & 9 \\ 7 & 12 & 5 \end{bmatrix} = \begin{bmatrix} 36 & 33 & 20 \\ 12 & 22 & 9 \end{bmatrix}.$$

The difference in sales between April and May can also be determined by performing the subtraction of matrix R and matrix Q, that is

$$\begin{bmatrix} 15 & 21 & 10 \\ 10 & 24 & 10 \end{bmatrix} - \begin{bmatrix} 20 & 15 & 9 \\ 7 & 12 & 5 \end{bmatrix} = \begin{bmatrix} -5 & 6 & 1 \\ 3 & 12 & 5 \end{bmatrix}.$$

Addition and subtraction of matrices can only be performed on the matrices with the **same order**.

Each **corresponding element** is added or subtracted to obtain a single matrix with the same order.

For matrix $A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$ and matrix $B = \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix}$,

$A + B = \begin{bmatrix} a_{11} + b_{11} & a_{12} + b_{12} \\ a_{21} + b_{21} & a_{22} + b_{22} \end{bmatrix}$ and $A - B = \begin{bmatrix} a_{11} - b_{11} & a_{12} - b_{12} \\ a_{21} - b_{21} & a_{22} - b_{22} \end{bmatrix}$.

Example 6

Determine whether addition and subtraction can be performed on the following pairs of matrices. Give your reason.

(a) $A = \begin{bmatrix} 2 & -5 & 3 \\ 8 & 11 & 7 \end{bmatrix}$ and $B = \begin{bmatrix} 9 & 6 \\ 4 & -1 \end{bmatrix}$

(b) $C = \begin{bmatrix} 1 & 12 \end{bmatrix}$ and $D = \begin{bmatrix} 0 & -4 \end{bmatrix}$

(c) $E = \begin{bmatrix} 15 & -4 \\ -1 & 0 \end{bmatrix}$ and $F = \begin{bmatrix} 2 & -5 \\ 13 & 0 \end{bmatrix}$

(d) $G = \begin{bmatrix} 10 \\ -3 \end{bmatrix}$ and $H = \begin{bmatrix} 8 & 7 \\ 16 & 1 \end{bmatrix}$

Solution:

(a) No because matrix A and matrix B have different orders.
(b) Yes because matrix C and matrix D have the same order.
(c) Yes because matrix E and matrix F have the same order.
(d) No because matrix G and matrix H have different orders.

Example 7

It is given that matrix $C = \begin{bmatrix} 10 & -8 & 4 \\ 6 & -11 & 7 \end{bmatrix}$, matrix $D = \begin{bmatrix} 14 & -2 & 1 \\ -3 & 5 & 9 \end{bmatrix}$, matrix $P = \begin{bmatrix} -2 & \frac{5}{6} & 6 \\ 7.4 & -13 & 5 \\ 1 & 9 & \frac{1}{4} \end{bmatrix}$

and matrix $Q = \begin{bmatrix} 18 & \frac{1}{3} & -7 \\ 2.5 & -8 & 3 \\ 12 & 0 & 0.4 \end{bmatrix}$. Calculate

(a) $C + D$

(b) $P - Q$

Solution:

(a)
$$C + D = \begin{bmatrix} 10 & -8 & 4 \\ 6 & -11 & 7 \end{bmatrix} + \begin{bmatrix} 14 & -2 & 1 \\ -3 & 5 & 9 \end{bmatrix}$$
$$= \begin{bmatrix} 10 + 14 & -8 + (-2) & 4 + 1 \\ 6 + (-3) & -11 + 5 & 7 + 9 \end{bmatrix}$$

Add corresponding elements

$$= \begin{bmatrix} 24 & -10 & 5 \\ 3 & -6 & 16 \end{bmatrix}$$

(b)
$$P - Q = \begin{bmatrix} -2 & \frac{5}{6} & 6 \\ 7.4 & -13 & 5 \\ 1 & 9 & \frac{1}{4} \end{bmatrix} - \begin{bmatrix} 18 & \frac{1}{3} & -7 \\ 2.5 & -8 & 3 \\ 12 & 0 & 0.4 \end{bmatrix}$$
$$= \begin{bmatrix} -2 - 18 & \frac{5}{6} - \frac{1}{3} & 6 - (-7) \\ 7.4 - 2.5 & -13 - (-8) & 5 - 3 \\ 1 - 12 & 9 - 0 & \frac{1}{4} - 0.4 \end{bmatrix}$$

Subtract corresponding elements

$$= \begin{bmatrix} -20 & \frac{1}{2} & 13 \\ 4.9 & -5 & 2 \\ -11 & 9 & -0.15 \end{bmatrix}$$

i-Technology

A scientific calculator can be used to perform the addition and subtraction of matrices. Scan the QR code or visit bit.do/VideoE201 to watch the related video.

Example 8

Given that matrix $D = \begin{bmatrix} 2x - 1 & -3 \\ -12 & 5 + y \end{bmatrix}$, matrix $E = \begin{bmatrix} x & 2 \\ 7 & y \end{bmatrix}$ and $D + E = \begin{bmatrix} 8 & -1 \\ -5 & 13 \end{bmatrix}$, calculate the values of x and y.

Info Bulletin

The matrices that are expressed in the form of equation is known as matrix equation.
For example, $A + B = C$.

Solution:

$$D + E = \begin{bmatrix} 8 & -1 \\ -5 & 13 \end{bmatrix}$$

$$\begin{bmatrix} 2x - 1 & -3 \\ -12 & 5 + y \end{bmatrix} + \begin{bmatrix} x & 2 \\ 7 & y \end{bmatrix} = \begin{bmatrix} 8 & -1 \\ -5 & 13 \end{bmatrix}$$

$$\begin{bmatrix} 2x - 1 + x & -3 + 2 \\ -12 + 7 & 5 + y + y \end{bmatrix} = \begin{bmatrix} 8 & -1 \\ -5 & 13 \end{bmatrix}$$

$$\begin{bmatrix} 3x - 1 & -1 \\ -5 & 5 + 2y \end{bmatrix} = \begin{bmatrix} 8 & -1 \\ -5 & 13 \end{bmatrix}$$

Equal matrices

$3x - 1 = 8$ and $5 + 2y = 13$

Compare the corresponding elements

$3x = 9$ $\quad$ $2y = 8$

$x = 3$ $\quad$ $y = 4$

Hence, $x = 3$ and $y = 4$

Example 9

Given that $F + \begin{bmatrix} 16 \\ -3 \end{bmatrix} - \begin{bmatrix} 7 \\ 10 \end{bmatrix} = \begin{bmatrix} -2 \\ 3 \end{bmatrix}$, calculate matrix F.

Solution:

$$F + \begin{bmatrix} 16 \\ -3 \end{bmatrix} - \begin{bmatrix} 7 \\ 10 \end{bmatrix} = \begin{bmatrix} -2 \\ 3 \end{bmatrix}$$

$$F = \begin{bmatrix} -2 \\ 3 \end{bmatrix} - \begin{bmatrix} 16 \\ -3 \end{bmatrix} + \begin{bmatrix} 7 \\ 10 \end{bmatrix}$$

$$= \begin{bmatrix} -11 \\ 16 \end{bmatrix}$$

Checking Answer

Substitute matrix $F = \begin{bmatrix} -11 \\ 16 \end{bmatrix}$ into the equation.

$$\begin{bmatrix} -11 \\ 16 \end{bmatrix} + \begin{bmatrix} 16 \\ -3 \end{bmatrix} - \begin{bmatrix} 7 \\ 10 \end{bmatrix}$$

$$= \begin{bmatrix} -11 + 16 - 7 \\ 16 + (-3) - 10 \end{bmatrix}$$

$$= \begin{bmatrix} -2 \\ 3 \end{bmatrix}$$

Example 10

The table on the right shows the stock records of Form 4 textbooks of Science, Mathematics and Economy in SMK Taman Suria. Calculate the closing stock of each type of textbook.

	Science	Mathematics	Economy
Opening stock	326	335	82
New books received	56	47	15
Lost and damaged books	32	26	11

Solution:

Closing stock = Opening stock + New books received – Lost and damaged books

$= [326 \quad 335 \quad 82] + [56 \quad 47 \quad 15] - [32 \quad 26 \quad 11]$

$= [350 \quad 356 \quad 86]$

Hence, the closing stock of Science, Mathematics and Economy textbooks are 350, 356 and 86 respectively.

Self Practice 2.2a

1. Determine whether addition and subtraction can be performed on each of the following pairs of matrices.

(a) $\begin{bmatrix} -5 & 9 \\ 1 & 0 \end{bmatrix}$ and $\begin{bmatrix} 3 & 7 \\ 6 & 1 \end{bmatrix}$

(b) $\begin{bmatrix} 13 & -1 & 11 \\ -2 & 8 & 4 \end{bmatrix}$ and $\begin{bmatrix} 4 & -16 & 7 \\ 1 & 5 & 0 \\ 3 & 2 & 8 \end{bmatrix}$

(c) $\begin{bmatrix} 10 \\ -3 \end{bmatrix}$ and $\begin{bmatrix} 12 & -7 \end{bmatrix}$

(d) $\begin{bmatrix} 2 & -9 \end{bmatrix}$ and $\begin{bmatrix} 1 & 6 \end{bmatrix}$

2. Given that matrix $P = \begin{bmatrix} 12 & 1 \\ -3 & 4 \end{bmatrix}$, matrix $Q = \begin{bmatrix} 8 & -2 \\ 0 & 5 \end{bmatrix}$ and matrix $R = \begin{bmatrix} 6 & 3 \\ 7 & -1 \end{bmatrix}$, calculate

(a) $P - Q + R$

(b) $P + Q - R$

3. Solve each of the following.

(a) $\begin{bmatrix} 12 & 10 & 1 \\ -4 & 0 & -7 \end{bmatrix} + \begin{bmatrix} 1 & -1 & 9 \\ 2 & 8 & 3 \end{bmatrix}$

(b) $\begin{bmatrix} 18 & -3 \\ -7 & 15 \end{bmatrix} - \begin{bmatrix} 11 & 5 \\ -1 & 2 \end{bmatrix}$

(c) $\begin{bmatrix} -1 \\ 6 \end{bmatrix} + \begin{bmatrix} 4 \\ 9 \end{bmatrix} - \begin{bmatrix} 19 \\ -3 \end{bmatrix}$

(d) $\begin{bmatrix} 2 & 8 \\ 1 & 5 \end{bmatrix} + \begin{bmatrix} 14 & 6 \\ -1 & 15 \end{bmatrix}$

4. Given that $\begin{bmatrix} 3a + 2 \\ 9 - b \end{bmatrix} + \begin{bmatrix} 4 \\ 2b \end{bmatrix} = \begin{bmatrix} 15 \\ -8 \end{bmatrix}$, calculate the values of a and b.

5. Given that matrix $S = \begin{bmatrix} 4x + 1 & -5 \\ 6 - y & x \end{bmatrix}$, matrix $T = \begin{bmatrix} x & 6 \\ 7 & 3y \end{bmatrix}$ and $S - T = \begin{bmatrix} 10 & -11 \\ -2 & z \end{bmatrix}$, calculate the values of x, y and z.

6. Given that $\begin{bmatrix} 3 & -4 \\ 1 & 0 \\ -6 & 7 \end{bmatrix} + \begin{bmatrix} -7 & 2 \\ 9 & 6 \\ 10 & 8 \end{bmatrix} - V = \begin{bmatrix} 11 & -4 \\ -1 & 5 \\ 6 & 9 \end{bmatrix}$, calculate matrix V.

7. Mr Gopal has two stores, A and B. The tables below show the income and expenses for the sales of food and drink in both stores for the month of June.

Income		
	Food	Drink
Store A	RM2 650	RM1 890
Store B	RM1 560	RM910

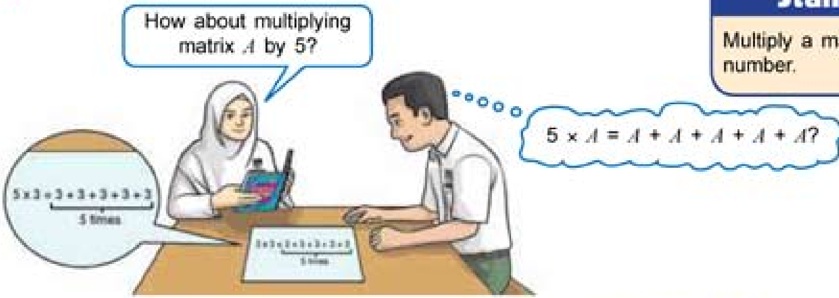
Expenses		
	Food	Drink
Store A	RM930	RM850
Store B	RM540	RM260

Calculate the total profit earned by Mr Gopal from each of his stores in the month of June. Show your calculation in the form of matrices.

[It is given that profit = income – expense]

CHAPTER 2

How to multiply a matrix by a number?



Learning Standard

Multiply a matrix by a number.

Multiplication of a matrix by a number is a process of **repeated addition** of the matrix. If matrix A is multiplied by a number n, then matrix A can be added to the same matrix A repeatedly for n times, that is

$$nA = \underbrace{A + A + \ldots + A}_{n \text{ times}}.$$

This formula means that each element in matrix A is added to the same element repeatedly for n times. Therefore, to multiply a matrix by a number, multiply every element in the matrix with the number.

It is given that matrix $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ and n is a number.

Hence, $nA = n\begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} na & nb \\ nc & nd \end{bmatrix}$.

n is known as a scalar.

Info Bulletin

When a matrix is multiplied by a real number, that real number is called a scalar.

Multiplication of a matrix by a number is known as **scalar multiplication**.

Example 11

Given that $D = \begin{bmatrix} -5 & 4 \\ 2 & 1 \end{bmatrix}$, calculate

(a) $3D$

(b) $-\frac{1}{2}D$

Solution:

(a)
$$\begin{aligned} 3D &= 3\begin{bmatrix} -5 & 4 \\ 2 & 1 \end{bmatrix} \\ &= \begin{bmatrix} 3(-5) & 3(4) \\ 3(2) & 3(1) \end{bmatrix} \\ &= \begin{bmatrix} -15 & 12 \\ 6 & 3 \end{bmatrix} \end{aligned}$$

Multiply all elements by 3

(b)
$$\begin{aligned} -\frac{1}{2}D &= -\frac{1}{2}\begin{bmatrix} -5 & 4 \\ 2 & 1 \end{bmatrix} \\ &= \begin{bmatrix} \left(-\frac{1}{2}\right)(-5) & \left(-\frac{1}{2}\right)(4) \\ \left(-\frac{1}{2}\right)(2) & \left(-\frac{1}{2}\right)(1) \end{bmatrix} \\ &= \begin{bmatrix} \frac{5}{2} & -2 \\ -1 & -\frac{1}{2} \end{bmatrix} \end{aligned}$$

MIND MOBILISATION 1 Group

Aim: To explore the laws of arithmetic operations in the addition and subtraction of matrices.

Steps:

1. Divide the class into groups of 4 pupils.
2. Determine the result of the addition and subtraction in the **Activity Sheet** below.

Activity Sheet:

$A = \begin{bmatrix} 2 & 7 \\ 6 & 11 \end{bmatrix}$, $B = \begin{bmatrix} -4 & 3 \\ 5 & 8 \end{bmatrix}$, $C = \begin{bmatrix} 9 & 2 \\ 10 & -1 \end{bmatrix}$, $O = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

(a)

Commutative Law	
$A + B$	$B + A$
$A - B$	$B - A$

(b)

Distributive Law	
$h(A + B)$	$hA + hB$
$h(A - B)$	$hA - hB$

(c)

Associative Law	
$(A + B) + C$	$A + (B + C)$
$(A - B) - C$	$A - (B - C)$

(d)

Addition and Subtraction of Zero Matrix	
$A + O$	$A - O$

Discussion:

Based on the results obtained in each of the tables above, what conclusion can be made? What is the relation between the process of addition and subtraction of matrices with the laws of arithmetic operations?

The results of Mind Mobilisation 1 show that;

(a) $A + B = B + A$. Addition of matrices obeys the **Commutative Law**.
$A - B \neq B - A$. Subtraction of matrices does not obey the Commutative Law.

(b) $h(A + B) = hA + hB$, $h(A - B) = hA - hB$.
Addition and subtraction of matrices obey the **Distributive Law**.

(c) $(A + B) + C = A + (B + C)$. Addition of matrices obeys the **Associative Law**.
$(A - B) - C \neq A - (B - C)$. Subtraction of matrices does not obey the Associative Law.

(d) A matrix with all its elements equal to zero is called **zero matrix**, for example $\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$. Addition and subtraction of matrix A with zero matrix, O are:
$A + O = A$ and $A - O = A$

Info Bulletin

Example of zero matrix:
$O_{1 \times 2} = [0 \quad 0]$
$O_{2 \times 3} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$

Example 12

Given that $P = \begin{bmatrix} 7 & 9 \\ -3 & 8 \\ 6 & 12 \end{bmatrix}$ and $Q = \begin{bmatrix} 2 & -3 \\ 1 & 5 \\ 0 & 4 \end{bmatrix}$, calculate $3(P - Q)$.

Solution:

$$3(P - Q) = 3\left(\begin{bmatrix} 7 & 9 \\ -3 & 8 \\ 6 & 12 \end{bmatrix} - \begin{bmatrix} 2 & -3 \\ 1 & 5 \\ 0 & 4 \end{bmatrix}\right)$$

Perform subtraction inside the bracket

$$= 3\begin{bmatrix} 5 & 12 \\ -4 & 3 \\ 6 & 8 \end{bmatrix}$$

Multiply all the elements by 3

$$= \begin{bmatrix} 15 & 36 \\ -12 & 9 \\ 18 & 24 \end{bmatrix}$$

Interactive Platform

Show that
$3P - 3Q = 3(P - Q)$

i-Technology

A scientific calculator can be used to perform the scalar multiplication of matrices. Scan the QR code or visit bit.do/VideoE202 to watch the related video.

Example 13

(a) Given that $\frac{1}{2}\begin{bmatrix} 4 \\ 12 \end{bmatrix} - \begin{bmatrix} x \\ -3 \end{bmatrix} = \begin{bmatrix} 5 \\ y \end{bmatrix}$, calculate the values of x and y.

(b) Given that $4R + \begin{bmatrix} 9 & 0 \\ 2 & -3 \end{bmatrix} = \begin{bmatrix} -3 & 4 \\ 10 & 1 \end{bmatrix}$, calculate matrix R.

Solution:

(a)
$$\frac{1}{2}\begin{bmatrix} 4 \\ 12 \end{bmatrix} - \begin{bmatrix} x \\ -3 \end{bmatrix} = \begin{bmatrix} 5 \\ y \end{bmatrix}$$
$$\begin{bmatrix} 2 \\ 6 \end{bmatrix} - \begin{bmatrix} x \\ -3 \end{bmatrix} = \begin{bmatrix} 5 \\ y \end{bmatrix}$$
$$\begin{bmatrix} 2 - x \\ 6 - (-3) \end{bmatrix} = \begin{bmatrix} 5 \\ y \end{bmatrix}$$

Compare the corresponding elements.

$2 - x = 5$, $6 - (-3) = y$

$x = -3$ $\quad$ $y = 9$

(b)
$$4R + \begin{bmatrix} 9 & 0 \\ 2 & -3 \end{bmatrix} = \begin{bmatrix} -3 & 4 \\ 10 & 1 \end{bmatrix}$$
$$4R = \begin{bmatrix} -3 & 4 \\ 10 & 1 \end{bmatrix} - \begin{bmatrix} 9 & 0 \\ 2 & -3 \end{bmatrix}$$
$$4R = \begin{bmatrix} -12 & 4 \\ 8 & 4 \end{bmatrix}$$
$$R = \frac{1}{4}\begin{bmatrix} -12 & 4 \\ 8 & 4 \end{bmatrix}$$

$4R = A$
$R = \frac{A}{4}$
$= \frac{1}{4}A$

$$= \begin{bmatrix} -3 & 1 \\ 2 & 1 \end{bmatrix}$$

Example 14

The average number of vehicles per day in a parking area for 5 working days is represented in Table X. Table Y represents the average number of vehicles per day on weekends.

	Car	Motorcycle
Covered	42	8
Non-covered	20	11

Table X

	Car	Motorcycle
Covered	25	5
Non-covered	12	3

Table Y

Calculate the number of vehicles parked at the parking area in a week.

Solution:

$$5X + 2Y = 5\begin{bmatrix} 42 & 8 \\ 20 & 11 \end{bmatrix} + 2\begin{bmatrix} 25 & 5 \\ 12 & 3 \end{bmatrix} \leftarrow \text{5 working days + 2 days during the weekend}$$

$$= \begin{bmatrix} 210 & 40 \\ 100 & 55 \end{bmatrix} + \begin{bmatrix} 50 & 10 \\ 24 & 6 \end{bmatrix}$$

$$= \begin{bmatrix} 260 & 50 \\ 124 & 61 \end{bmatrix}$$

Self Practice 2.2b

1. Determine the product of each of the following matrices.

(a) $3\begin{bmatrix} -7 \\ 2 \end{bmatrix}$ (b) $0.6\begin{bmatrix} 11 & 5 \end{bmatrix}$ (c) $\frac{1}{4}\begin{bmatrix} 12 & -20 \\ -6 & 16 \\ 9 & 1 \end{bmatrix}$

(d) $-2\begin{bmatrix} 0.4 & 8 \\ -9 & 2.5 \end{bmatrix}$ (e) $1.2\begin{bmatrix} 10 & -1 & 11 \\ 3 & 7 & -5 \end{bmatrix}$ (f) $-\frac{1}{20}\begin{bmatrix} 100 \\ -90 \\ -20 \end{bmatrix}$

2. Solve each of the following operations.

(a) $5\begin{bmatrix} 1 & 2 \\ 3 & -1 \\ 0 & -4 \end{bmatrix} + \begin{bmatrix} 7 & 5 \\ 6 & 1 \\ -1 & 8 \end{bmatrix} - \frac{1}{2}\begin{bmatrix} 10 & 2 \\ 9 & -4 \\ -6 & 14 \end{bmatrix}$ (b) $6\begin{bmatrix} -1 \\ 4 \end{bmatrix} - 0.5\begin{bmatrix} 8 \\ 14 \end{bmatrix} - 2\begin{bmatrix} 1 \\ -3 \end{bmatrix}$

(c) $7\begin{bmatrix} 3 & -2 & 1 \end{bmatrix} - \frac{1}{3}\begin{bmatrix} 21 & 6 & -9 \end{bmatrix}$ (d) $0.2\begin{bmatrix} 10 & -25 \\ -6 & 8 \end{bmatrix} + \frac{1}{5}\begin{bmatrix} 15 & 20 \\ -5 & 2.5 \end{bmatrix}$

3. Given that matrix $E = \begin{bmatrix} 9 & 6 \\ 2 & 11 \end{bmatrix}$, matrix $F = \begin{bmatrix} -7 & 22 \\ 3 & 4 \end{bmatrix}$ and matrix $G = \begin{bmatrix} -1 & 10 \\ -8 & 5 \end{bmatrix}$, show that $(E + F) + G = E + (F + G)$.

4. Given that matrix $P = \begin{bmatrix} \frac{1}{2} \\ -0.7 \end{bmatrix}$, matrix $Q = \begin{bmatrix} 4 \\ -5 \end{bmatrix}$ and matrix $O = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$, calculate $P - 1.4Q + O$.

5. Given that $4\begin{bmatrix} 2 & 3 \\ -5 & b \end{bmatrix} - \begin{bmatrix} -4 & 9 \\ c & 0.1 \end{bmatrix} = \frac{3}{5}\begin{bmatrix} a & 5 \\ -35 & 1.5 \end{bmatrix}$, calculate the values of a, b and c.

6. Given that $\begin{bmatrix} -10 & 9 \end{bmatrix} - 2X + 5\begin{bmatrix} 2 & 1 \end{bmatrix} = \begin{bmatrix} 3 & 8 \end{bmatrix}$, calculate matrix X.

7. The shoe shop owned by Encik Jamal sells adult and children shoes. Table 1 shows the stock for each type of shoes at the beginning of a week while Table 2 shows the sales of each type of shoes of that week.

	Female	Male
Adult	85	70
Children	110	98

Table 1

	Female	Male
Adult	33	24
Children	42	40

Table 2

Calculate the closing inventory of each type of shoes at the end of the week. Show your calculation in the form of matrices.

How to multiply two matrices?

Learning Standard

Multiply two matrices.

Based on the previous situation, the sales of fans in the month of March can be represented with a matrix as follows:

$$P = \begin{array}{c} \\ \\ \end{array}\begin{array}{c} \begin{array}{ccc} \text{Stand} & \text{Ceiling} & \text{Wall} \end{array} \\ \begin{bmatrix} 16 & 18 & 11 \\ 5 & 10 & 4 \end{bmatrix} \end{array} \begin{array}{l} \\ \text{In-store} \\ \text{Online} \end{array}$$

The commissions for selling each unit of the stand fans, ceiling fans and wall fans are RM25, RM30 and RM20 respectively. How do you calculate the total commission earned from the in-store and online sales?

Total commission earned from the in-store sales	Total commission earned from the online sales
= (16 × RM25) + (18 × RM30) + (11 × RM20) = RM1 160	= (5 × RM25) + (10 × RM30) + (4 × RM20) = RM505

The total commission earned can be calculated in the form of a matrix. If the commission for selling each unit of the fans is represented in the form of column matrix, that is $K = \begin{bmatrix} 25 \\ 30 \\ 20 \end{bmatrix}$, then the total commission earned from the in-store and online sales can be written in the form of matrix as follows.

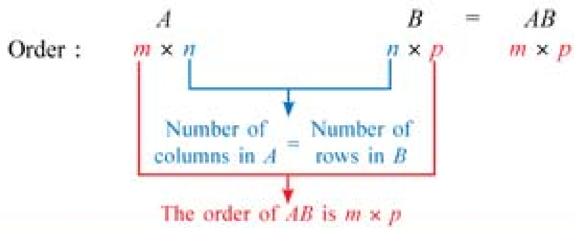
$$\begin{aligned} PK &= \begin{bmatrix} 16 & 18 & 11 \\ 5 & 10 & 4 \end{bmatrix}_{2 \times 3} \begin{bmatrix} 25 \\ 30 \\ 20 \end{bmatrix}_{3 \times 1} \\ &= \begin{bmatrix} 16(25) + 18(30) + 11(20) \\ 5(25) + 10(30) + 4(20) \end{bmatrix} \\ &= \begin{bmatrix} 1160 \\ 505 \end{bmatrix}_{2 \times 1} \end{aligned}$$

Critical Mind

Assume K is a row matrix. Perform the multiplication on matrices, PK. Is the multiplication possible?

PK is known as the multiplication of matrix P with matrix K and $\begin{bmatrix} 1160 \\ 505 \end{bmatrix}$ is the product of the two matrices.

In general, to multiply two matrices, A and B, the number of columns in matrix A must be the same with the number of rows in matrix B. The number of rows in matrix A and the number of columns in matrix B become the order of the product of the two matrices, AB.



If matrix A has an order of $m \times n$ and matrix B has an order of $n \times p$, then the multiplication AB can be performed and the order of AB is $m \times p$.

Example 15

It is given that matrix $C = \begin{bmatrix} 8 \\ 3 \end{bmatrix}$ and matrix $D = \begin{bmatrix} 1 & 2 \\ -2 & 5 \end{bmatrix}$. Determine whether multiplication CD and DC can be performed. If yes, state the order of the product of the matrices.

Solution:

Order of matrix $C = 2 \times 1$, order of matrix $D = 2 \times 2$

$$\underset{C}{2 \times 1} \qquad \underset{D}{2 \times 2}$$

Not the same

$$\underset{D}{2 \times 2} \qquad \underset{C}{2 \times 1}$$

Same

Order of DC

Multiplication CD cannot be performed because the number of columns in matrix C is not the same with the number of rows in matrix D.

Multiplication DC can be performed because the number of columns in matrix D is the same with the number of rows in matrix C. The order of DC is 2×1.

Example 16

It is given that matrix $A = \begin{bmatrix} 2 & 3 \\ 1 & 5 \end{bmatrix}$ and matrix $B = \begin{bmatrix} 6 & -7 \\ -2 & 1 \end{bmatrix}$. Calculate AB.

Solution:

$$\begin{bmatrix} 2 & 3 \\ 1 & 5 \end{bmatrix}\begin{bmatrix} 6 & -7 \\ -2 & 1 \end{bmatrix} = \begin{bmatrix} (2)(6) + (3)(-2) & \boxed{} \\ \boxed{} & \boxed{} \end{bmatrix} = \begin{bmatrix} 6 & \\ & \end{bmatrix}$$

The element at row 1 and column 1

$$\begin{bmatrix} 2 & 3 \\ 1 & 5 \end{bmatrix}\begin{bmatrix} 6 & -7 \\ -2 & 1 \end{bmatrix} = \begin{bmatrix} 6 & (2)(-7) + (3)(1) \\ \boxed{} & \boxed{} \end{bmatrix} = \begin{bmatrix} 6 & -11 \\ & \end{bmatrix}$$

The element at row 1 and column 2

$$\begin{bmatrix} 2 & 3 \\ 1 & 5 \end{bmatrix}\begin{bmatrix} 6 & -7 \\ -2 & 1 \end{bmatrix} = \begin{bmatrix} 6 & -11 \\ (1)(6) + (5)(-2) & \boxed{} \end{bmatrix} = \begin{bmatrix} 6 & -11 \\ -4 & \end{bmatrix}$$

The element at row 2 and column 1

$$\begin{bmatrix} 2 & 3 \\ 1 & 5 \end{bmatrix}\begin{bmatrix} 6 & -7 \\ -2 & 1 \end{bmatrix} = \begin{bmatrix} 6 & -11 \\ -4 & (1)(-7) + (5)(1) \end{bmatrix} = \begin{bmatrix} 6 & -11 \\ -4 & -2 \end{bmatrix}$$

The element at row 2 and column 2

$$\begin{aligned} \text{Hence, } AB &= \begin{bmatrix} 2 & 3 \\ 1 & 5 \end{bmatrix}\begin{bmatrix} 6 & -7 \\ -2 & 1 \end{bmatrix} \\ &= \begin{bmatrix} (2)(6) + (3)(-2) & (2)(-7) + (3)(1) \\ (1)(6) + (5)(-2) & (1)(-7) + (5)(1) \end{bmatrix} \\ &= \begin{bmatrix} 6 & -11 \\ -4 & -2 \end{bmatrix} \end{aligned}$$

Smart TIPS

$$\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}_{2 \times 2} \begin{bmatrix} b_{11} \\ b_{21} \end{bmatrix}_{2 \times 1}$$
$$= \begin{bmatrix} (a_{11} \times b_{11}) + (a_{12} \times b_{21}) \\ (a_{21} \times b_{11}) + (a_{22} \times b_{21}) \end{bmatrix}$$
$$= \begin{bmatrix} c_{11} + d_{11} \\ e_{21} + f_{21} \end{bmatrix}$$
$$= \begin{bmatrix} g_{11} \\ h_{21} \end{bmatrix}_{2 \times 1}$$

Example 17

It is given that matrix $E = \begin{bmatrix} 5 & -1 & 0 \\ -4 & 8 & 7 \end{bmatrix}$, matrix $F = \begin{bmatrix} -2 \\ 7 \end{bmatrix}$ and matrix $G = [4 \quad 3]$. Calculate

(a) GE (b) FG (c) GF

Solution:

(a) $$GE = [4 \quad 3]_{1 \times 2} \begin{bmatrix} 5 & -1 & 0 \\ -4 & 8 & 7 \end{bmatrix}_{2 \times 3}$$
$$= [4(5) + 3(-4) \quad 4(-1) + 3(8) \quad 4(0) + 3(7)]$$
$$= [8 \quad 20 \quad 21]_{1 \times 3}$$

The product is a matrix with order 1×3

(b) $$FG = \begin{bmatrix} -2 \\ 7 \end{bmatrix}_{2 \times 1} [4 \quad 3]_{1 \times 2}$$
$$= \begin{bmatrix} (-2)(4) & (-2)(3) \\ 7(4) & 7(3) \end{bmatrix}$$
$$= \begin{bmatrix} -8 & -6 \\ 28 & 21 \end{bmatrix}_{2 \times 2}$$

The product is a matrix with order 2×2

(c) $$GF = [4 \quad 3]_{1 \times 2} \begin{bmatrix} -2 \\ 7 \end{bmatrix}_{2 \times 1}$$
$$= [4(-2) + 3(7)]$$
$$= [13]_{1 \times 1}$$

The product is a matrix with order 1×1

Example 18

It is given that matrix $K = \begin{bmatrix} 3 & 2 \\ -1 & 5 \\ 4 & -2 \end{bmatrix}$ and matrix $L = \begin{bmatrix} -4 & 2 \\ 0 & 1 \end{bmatrix}$. Calculate

(a) L^2 (b) KL^2 (c) L^3

Solution:

(a) $$L^2 = LL$$
$$= \begin{bmatrix} -4 & 2 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} -4 & 2 \\ 0 & 1 \end{bmatrix}$$
$$= \begin{bmatrix} 16 + 0 & -8 + 2 \\ 0 + 0 & 0 + 1 \end{bmatrix}$$
$$= \begin{bmatrix} 16 & -6 \\ 0 & 1 \end{bmatrix}$$

Smart TIPS

Always check the order of the matrix before performing multiplication. For example, when a matrix with order $m \times n$ is multiplied with a matrix with order $n \times p$, it produces a matrix with order $m \times p$.

Critical Mind

Why $FG \neq GF$?

i-Technology

A scientific calculator can be used to perform the multiplication of two matrices. Scan the QR code or visit bit.do/VideoE203 to watch the related video.

Info Bulletin

Given that $L = \begin{bmatrix} -4 & 2 \\ 0 & 1 \end{bmatrix}$,

$L^2 \neq \begin{bmatrix} (-4)^2 & 2^2 \\ 0^2 & 1^2 \end{bmatrix}$.

$L^2 = LL$,

$L^3 = L^2L = LL^2$.

(b) KL^2

$$= \begin{bmatrix} 3 & 2 \\ -1 & 5 \\ 4 & -2 \end{bmatrix} \begin{bmatrix} 16 & -6 \\ 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 48 + 0 & -18 + 2 \\ -16 + 0 & 6 + 5 \\ 64 + 0 & -24 + (-2) \end{bmatrix}$$

$$= \begin{bmatrix} 48 & -16 \\ -16 & 11 \\ 64 & -26 \end{bmatrix}$$

(c) L^3

$$= L^2L$$

$$= \begin{bmatrix} 16 & -6 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} -4 & 2 \\ 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} -64 + 0 & 32 + (-6) \\ 0 + 0 & 0 + 1 \end{bmatrix}$$

$$= \begin{bmatrix} -64 & 26 \\ 0 & 1 \end{bmatrix}$$

Critical Mind

Try to calculate LL^2.

Mathematics is fun!

Scan the QR code or visit bit.do/MatrixCalculator to use a matrix calculator.

Example 19

Given that $[6x \quad -5]\begin{bmatrix} 2 & 3 \\ 1 & 4 \end{bmatrix} = [7 \quad 1 - y]$, calculate the values of x and y.

Solution:

$$[6x \quad -5]\begin{bmatrix} 2 & 3 \\ 1 & 4 \end{bmatrix} = [7 \quad 1 - y]$$

$$[6x \quad -5]\begin{bmatrix} 2 & 3 \\ 1 & 4 \end{bmatrix} = [(6x)(2) + (-5)(1) \quad (6x)(3) + (-5)(4)]_{1 \times 2}$$
$$= [12x - 5 \quad 18x - 20]$$

Hence, $[12x - 5 \quad 18x - 20] = [7 \quad 1 - y]$.

Compare the corresponding elements.

$$12x - 5 = 7 \qquad \text{and} \qquad 18x - 20 = 1 - y$$
$$12x = 12 \qquad\qquad 18(1) - 20 = 1 - y$$
$$x = 1 \qquad\qquad -2 = 1 - y$$
$$y = 3$$

Critical Mind

It is given that matrix $P = [a \quad b + 1]$ and matrix $Q = \begin{bmatrix} c - 1 & 2 \\ -3 & 4d \end{bmatrix}$.

Calculate

(i) PQ
(ii) Q^2

Example 20

The table below shows the share units purchased by Khairil and Mahmud.

	Share A (unit)	Share B (unit)
Khairil	5 000	4 000
Mahmud	2 000	6 000

It is given that the prices of 1 unit of share A and 1 unit of share B during the purchase are RM1.50 and RM0.82 respectively. Calculate the total investment of Khairil and the total investment of Mahmud.

Solution:

$$\begin{bmatrix} 5000 & 4000 \\ 2000 & 6000 \end{bmatrix} \begin{bmatrix} 1.50 \\ 0.82 \end{bmatrix} = \begin{bmatrix} 7500 + 3280 \\ 3000 + 4920 \end{bmatrix}$$
$$= \begin{bmatrix} 10780 \\ 7920 \end{bmatrix}$$

The total investments of Khairil and Mahmud are RM10 780 and RM7 920 respectively.