Self Practice 2.2c

1. Given that four matrices, $P = \begin{bmatrix} 3 & 6 \\ -1 & 2 \end{bmatrix}$, $Q = \begin{bmatrix} 7 \\ 9 \end{bmatrix}$, $R = [4 \quad 8 \quad 5]$ and $S = \begin{bmatrix} 0 & -6 & 1 \\ 3 & 11 & -2 \end{bmatrix}$, determine whether the following multiplication of matrices can be performed. If yes, state the order of the product of the pairs of matrices.
   (a) PQ (b) QR (c) RS
   (d) SP (e) PS (f) QP

2. It is given that four matrices, $T = \begin{bmatrix} 1 & 3 & 4 \\ -2 & 2 & -1 \end{bmatrix}$, $U = \begin{bmatrix} 0 & -4 \\ -3 & 5 \\ 1 & 2 \end{bmatrix}$, $V = \begin{bmatrix} -6 \\ 2 \end{bmatrix}$ and $W = \begin{bmatrix} 2 & 1 \\ 3 & -4 \end{bmatrix}$.
   Calculate
   (a) TU (b) UW (c) UV
   (d) WV (e) W^2 (f) W^3

3. Given that $\begin{bmatrix} -1 & x \\ y & 3 \end{bmatrix}\begin{bmatrix} 4 \\ 7 \end{bmatrix} = \begin{bmatrix} 31 \\ 29 \end{bmatrix}$, calculate the values of x and y.

4. Given that $\begin{bmatrix} 9 & r \\ 5 & -2 \end{bmatrix}\begin{bmatrix} 4 & s \\ 6 & 1 \end{bmatrix} = \begin{bmatrix} 15 & 14.5 \\ 8 & 8 \end{bmatrix}$, calculate the values of r and s.

5. Given that $G = \begin{bmatrix} p & 5 \\ 1 & -4 \end{bmatrix}$ and $H = \begin{bmatrix} -6 & 7 \\ 3 & 0 \end{bmatrix}$, calculate the values of p, q and r if
   (a) $GH = \begin{bmatrix} 3 & 2q \\ -18 & 3p + r \end{bmatrix}$ (b) $G^2 = \begin{bmatrix} r & -25 \\ -5 & 7q \end{bmatrix}$
   (c) $HG = \begin{bmatrix} -11 & 2.5q \\ \frac{p + 3r}{2} & 5p \end{bmatrix}$ (d) $H^2 = \begin{bmatrix} 57 & 6p \\ 1.2q & \frac{7r}{5} \end{bmatrix}$

6. Mr Koh rents a booth in Educational Expo to sell three types of goods as shown in the table below.

	Goods A	Goods B	Goods C
First day	40	28	36
Second day	42	36	30
Third day	35	25	42

   It is given that the profits of one item sold for selling goods A, B and C are RM5, RM8 and RM6 respectively. Calculate the total profit earned by Mr Koh every day. Show your calculation in the form of matrices.
   [It is given that total profit = sales of goods A × profit of goods A
   + sales of goods B × profit of goods B
   + sales of goods C × profit of goods C]

What are the characteristics of identity matrix?

$$a \times 1 = a$$
$$1 \times a = a$$

Learning Standard

Explain the characteristics of identity matrix.

When 1 is multiplied with any number, a, its product is a. When a matrix is multiplied with matrix A, its product is matrix A. Then the first matrix is the identity matrix. What are the characteristics of an identity matrix?

MIND MOBILISATION 2 Group

Aim: To determine the identity matrix.

Steps:

1. Divide the class into groups of 4 pupils.
2. Copy the **Activity Sheet** below and take turn to complete it.

Activity Sheet:

	Matrix A	Matrix B	AB	BA
(a)	$\begin{bmatrix} 5 & -2 \end{bmatrix}$	$\begin{bmatrix} 1 \\ 0 \end{bmatrix}$		
(b)	$\begin{bmatrix} 2 & 3 \\ 4 & -1 \end{bmatrix}$	$\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$		
(c)	$\begin{bmatrix} 2 & 3 \\ 4 & -1 \end{bmatrix}$	$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$		
(d)	$\begin{bmatrix} 2 & 3 \\ 4 & -1 \end{bmatrix}$	$\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$		
(e)	$\begin{bmatrix} -1 & 2 & 3 \\ 0 & 4 & 1 \\ 5 & 3 & -2 \end{bmatrix}$	$\begin{bmatrix} 1 & 1 & 1 \\ 0 & 0 & 0 \\ 1 & 1 & 1 \end{bmatrix}$		
(f)	$\begin{bmatrix} -1 & 2 & 3 \\ 0 & 4 & 1 \\ 5 & 3 & -2 \end{bmatrix}$	$\begin{bmatrix} 0 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 1 & 0 \end{bmatrix}$		
(g)	$\begin{bmatrix} -1 & 2 & 3 \\ 0 & 4 & 1 \\ 5 & 3 & -2 \end{bmatrix}$	$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$		

Discussion:

1. Which matrix B when multiplied with matrix A will result in matrix A?
2. What are the elements in matrix B? How are the elements positioned in matrix B?

The results of Mind Mobilisation 2 show that;

(a) Matrix B in the form of $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ or $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ when multiplied with matrix A will result in matrix A, that is $AB = BA = A$.

(b) Elements in the matrices B consist of 0 and 1 only with element 1 placed along the diagonal from the top left corner to the bottom right corner and the other elements are zero.

Smart TIPS

Identity matrix, I with order $n \times n$:

$$I = \begin{bmatrix} 1 & 0 & 0 & \cdots & 0 \\ 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & 1 \end{bmatrix}$$

Matrix $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ or $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ is known as identity matrix and is denoted by I.

Identity matrices are square matrices. **Identity matrix**, I, when multiplied with a matrix A, will result in matrix A.

$$AI = IA = A$$

Info Bulletin

Arrange element 1 along the diagonal from the top left corner to the bottom right corner and the other elements are zero. The diagonal is called main diagonal. This matrix is called a diagonal matrix.

Example 21

Write the identity matrix based on the order given below.

(a) 1×1 (b) 2×2

(c) 4×4 (d) 5×5

Solution:

(a) $[1]$

(b) $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

(c) $\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

(d) $\begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$

Critical Mind

Identity matrix is a diagonal matrix. Is the diagonal matrix an identity matrix? Discuss.

Self Practice 2.2d

1. Which of the following matrices are identity matrix? If it is not an identity matrix, give your reason.

(a) $[0 \quad 1]$

(b) $\begin{bmatrix} 0 & 0 \\ 1 & 1 \end{bmatrix}$

(c) $\begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 0 \\ 0 & 1 & 0 \end{bmatrix}$

(d) $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

(e) $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

(f) $\begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 0 \\ 1 & 0 & 1 \end{bmatrix}$

2. It is given that matrix $C = \begin{bmatrix} -1 & 3 \\ 2 & 5 \end{bmatrix}$ and matrix $D = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$. Show that matrix D is an identity matrix.

3. It is given that matrix $S = \begin{bmatrix} 7 & 2 \\ 6 & 3 \end{bmatrix}$ and matrix $T = \begin{bmatrix} 3 & 1 \\ -5 & 4 \end{bmatrix}$. Calculate

(a) $SI + TI$ (b) $(IS)T$ (c) $4IT - I^2$ (d) $(S - I)I$

What is the meaning of inverse matrix?

Learning Standard

Explain the meaning of inverse matrix and hence determine the inverse matrix for a 2 × 2 matrix.

$$\begin{bmatrix} 2 & 1 \\ 5 & 3 \end{bmatrix}\begin{bmatrix} 3 & -1 \\ -5 & 2 \end{bmatrix} = \begin{bmatrix} 3 & -1 \\ -5 & 2 \end{bmatrix}\begin{bmatrix} 2 & 1 \\ 5 & 3 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$AB = BA = I$$

If multiplication of matrix A and matrix B produces an identity matrix, I, then matrix B is the inverse of matrix A and vice versa.

MIND MOBILISATION 3 Group

Aim: To determine the inverse matrix.

Steps:

1. Divide the class into groups of 4 pupils.
2. Each pupil chooses a piece of matrix A card and a piece of matrix B card as shown below.

$A = \begin{bmatrix} 1 & 3 \\ 2 & 7 \end{bmatrix}$ $A = \begin{bmatrix} 4 & -3 \\ 3 & -2 \end{bmatrix}$ $A = \begin{bmatrix} 2 & 0 \\ 6 & -1 \end{bmatrix}$ $A = \begin{bmatrix} 3 & 2 \\ 2 & 1 \end{bmatrix}$

$B = \begin{bmatrix} 7 & -3 \\ -2 & 1 \end{bmatrix}$ $B = \begin{bmatrix} -2 & 3 \\ -3 & 4 \end{bmatrix}$ $B = \begin{bmatrix} -1 & 6 \\ 0 & 2 \end{bmatrix}$ $B = \begin{bmatrix} -1 & 2 \\ 2 & -3 \end{bmatrix}$

3. Pupils perform the multiplication AB and BA. The product is recorded in the table as shown on the right.
4. Pupils exchange matrix B card with others in the group. Step **3** is repeated.
5. Pupils discuss their results in groups.

Matrix A	Matrix B	AB	BA

Discussion:

1. Which two matrices result in identity matrix after performing multiplication?
2. What is the conclusion about the relationship between the two matrices?

The results of Mind Mobilisation 3 show that

$$\begin{bmatrix} 1 & 3 \\ 2 & 7 \end{bmatrix}\begin{bmatrix} 7 & -3 \\ -2 & 1 \end{bmatrix} = \begin{bmatrix} 7 & -3 \\ -2 & 1 \end{bmatrix}\begin{bmatrix} 1 & 3 \\ 2 & 7 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 4 & -3 \\ 3 & -2 \end{bmatrix}\begin{bmatrix} -2 & 3 \\ -3 & 4 \end{bmatrix} = \begin{bmatrix} -2 & 3 \\ -3 & 4 \end{bmatrix}\begin{bmatrix} 4 & -3 \\ 3 & -2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 3 & 2 \\ 2 & 1 \end{bmatrix}\begin{bmatrix} -1 & 2 \\ 2 & -3 \end{bmatrix} = \begin{bmatrix} -1 & 2 \\ 2 & -3 \end{bmatrix}\begin{bmatrix} 3 & 2 \\ 2 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

The pairs of matrices above are inverse matrices of one another.

Info Bulletin

A^{-1} is read as "inverse matrix of A".

$$A^{-1} \neq \frac{1}{A}$$

Multiplication of matrix A and **inverse matrix** of A, A^{-1}, will result in identity matrix, I.

$$AA^{-1} = A^{-1}A = I$$

Example 22

Determine whether the following matrix is the inverse matrix of $\begin{bmatrix} 4 & 1 \\ 7 & 2 \end{bmatrix}$. Explain your answer.

(a) $\begin{bmatrix} 4 & 1 \\ -7 & 2 \end{bmatrix}$

(b) $\begin{bmatrix} 2 & -1 \\ -7 & 4 \end{bmatrix}$

Solution:

(a) $\begin{bmatrix} 4 & 1 \\ 7 & 2 \end{bmatrix}\begin{bmatrix} 4 & 1 \\ -7 & 2 \end{bmatrix} = \begin{bmatrix} 9 & 6 \\ 14 & 11 \end{bmatrix}$ ← The product is not an identity matrix

$\begin{bmatrix} 4 & 1 \\ -7 & 2 \end{bmatrix}$ is not the inverse matrix of $\begin{bmatrix} 4 & 1 \\ 7 & 2 \end{bmatrix}$ because the product of these two matrices is not an identity matrix.

(b) $\begin{bmatrix} 4 & 1 \\ 7 & 2 \end{bmatrix}\begin{bmatrix} 2 & -1 \\ -7 & 4 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ ← The product is an identity matrix

$\begin{bmatrix} 2 & -1 \\ -7 & 4 \end{bmatrix}\begin{bmatrix} 4 & 1 \\ 7 & 2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ ← The product is an identity matrix

$\begin{bmatrix} 2 & -1 \\ -7 & 4 \end{bmatrix}$ is the inverse matrix of $\begin{bmatrix} 4 & 1 \\ 7 & 2 \end{bmatrix}$ because the product of these two matrices is an identity matrix.

Info Bulletin

Inverse matrix only exists in the form of square matrix because both AA^{-1} and $A^{-1}A$ equal to I. However, not all the square matrices have inverse matrices.

Self Practice 2.2e

1. Determine whether the following matrices are inverse matrices of one another.

(a) $\begin{bmatrix} 5 & 4 \\ 3 & 2 \end{bmatrix}, \begin{bmatrix} 2 & -4 \\ -3 & 5 \end{bmatrix}$

(b) $\begin{bmatrix} 1 & 3 \\ 1 & 4 \end{bmatrix}, \begin{bmatrix} \frac{1}{2} & 3 \\ 1 & 2 \end{bmatrix}$

(c) $\begin{bmatrix} 1 & 2 \\ 4 & 9 \end{bmatrix}, \begin{bmatrix} 1 & -2 \\ -4 & 9 \end{bmatrix}$

(d) $\begin{bmatrix} -2 & 3 \\ -5 & 7 \end{bmatrix}, \begin{bmatrix} 7 & -3 \\ 5 & -2 \end{bmatrix}$

How to determine the inverse matrix for a 2 × 2 matrix?

MIND MOBILISATION 4 Group

Aim: To derive the formula used to determine an inverse matrix for 2 × 2 matrix.

Steps:

1. Divide the class into groups of 4 pupils.
2. Copy and complete the following **Activity Sheet** by following the given instructions.

Activity Sheet:

It is given that $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ and $A^{-1} = \begin{bmatrix} p & q \\ r & s \end{bmatrix}$.

Instruction	Steps	
Multiply matrix A and A^{-1}	$AA^{-1} = \begin{bmatrix} ap + br & \square \\ \square & \square \end{bmatrix}$	
Form 4 equations from $AA^{-1} = I$	$\begin{bmatrix} ap + br & \square \\ \square & \square \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ (i) $ap + br = 1$ (ii) $\square$ (iii) $\square$ (iv) $\square$	
With substitution method, express p, q, r and s in terms of a, b, c and d	Use the equations in (i) and (iii), express p and r in terms of a, b, c and d.	Use the equations in (ii) and (iv), express q and s in terms of a, b, c and d.
	$p = \dfrac{d}{ad - bc}$ $r = \square$	$q = \square$ $s = \square$
Write matrix A^{-1} in terms of a, b, c and d	$A^{-1} = \begin{bmatrix} \dfrac{d}{ad - bc} & \square \\ \square & \square \end{bmatrix}$	
Write A^{-1} as scalar multiplication	$A^{-1} = \dfrac{1}{\square}\begin{bmatrix} d & \square \\ \square & \square \end{bmatrix}$	

3. Write all the steps on a piece of paper and paste it on the wall. Pupils give comments by attaching sticky notes on the findings of other groups.

Discussion:

What is the formula used to determine an inverse matrix?

The results of Mind Mobilisation 4 show that

(a) the scalar is $\dfrac{1}{ad - bc}$,

(b) the element a_{11} is d, element a_{12} is $-b$, element a_{21} is $-c$ and element a_{22} is a.
Note that the position of a and d are exchanged while b and c are multiplied by -1.

Given matrix $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$, the inverse matrix, A^{-1} can be obtained using the formula as follows:

$$A^{-1} = \frac{1}{ad - bc}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix} \text{ where } ad - bc \neq 0$$

Determinant for matrix A, $|A|$ is $ad - bc$ where $ad - bc \neq 0$. Therefore inverse matrix, A^{-1} exists. Inverse matrix does not exist when $ad - bc = 0$.

Historical Treasure

The term determinant is introduced in 1801 by Carl Gauss (1777-1855), a German mathematician.

Example 23

For each of the following matrices, determine whether the inverse matrix exists. If yes, calculate the inverse matrix.

(a) $A = \begin{bmatrix} 1 & 2 \\ 4 & 8 \end{bmatrix}$ (b) $B = \begin{bmatrix} 3 & 5 \\ 2 & 4 \end{bmatrix}$

Solution:

(a) $ad - bc = 1(8) - 2(4)$
$= 8 - 8$
$= 0$

Determine the existence of inverse matrix with matrix determinant, $ad - bc$

$|A| = 0$. Hence, A^{-1} does not exist.

(b) $ad - bc = 3(4) - 5(2)$
$= 12 - 10$
$= 2$
$\neq 0$

$|B| \neq 0$. Hence, B^{-1} exists.

$$B^{-1} = \frac{1}{3(4) - 5(2)}\begin{bmatrix} 4 & -5 \\ -2 & 3 \end{bmatrix}$$

$$= \frac{1}{2}\begin{bmatrix} 4 & -5 \\ -2 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} 2 & -\frac{5}{2} \\ -1 & \frac{3}{2} \end{bmatrix}$$

Exchange the position of the elements in the main diagonal and multiply the other two elements with -1

Example 24

Given that matrix $C = \begin{bmatrix} 2 & -6 \\ 1 & -2 \end{bmatrix}$, calculate the inverse matrix of C.

Solution:

$$C^{-1} = \frac{1}{2(-2) - (-6)(1)}\begin{bmatrix} -2 & 6 \\ -1 & 2 \end{bmatrix}$$

$$= \frac{1}{2}\begin{bmatrix} -2 & 6 \\ -1 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} -1 & 3 \\ -\frac{1}{2} & 1 \end{bmatrix}$$

i-Technology

A scientific calculator can be used to find the determinant of a matrix. Scan the QR code or visit bit.do/VideoE204 to watch the related video.

Example 25

Given that matrix $D = \begin{bmatrix} m & -6 \\ 1 & -2 \end{bmatrix}$, calculate the value of m if

(a) matrix D does not have inverse matrix,

(b) $D^{-1} = \begin{bmatrix} -1 & 3 \\ -\frac{1}{2} & 1 \end{bmatrix}$.

Critical Mind

What is the inverse matrix of identity matrix?

Critical Mind

Why does A^{-1} not exist when $ad - bc = 0$?

Solution:

(a) Matrix D does not have inverse matrix, hence

$$\begin{aligned} ad - bc &= 0 \\ -2m - (-6)(1) &= 0 \\ -2m + 6 &= 0 \\ m &= 3 \end{aligned}$$

(b)

$$\begin{aligned} DD^{-1} &= I \\ \begin{bmatrix} m & -6 \\ 1 & -2 \end{bmatrix}\begin{bmatrix} -1 & 3 \\ -\frac{1}{2} & 1 \end{bmatrix} &= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \\ m(-1) + (-6)\left(-\frac{1}{2}\right) &= 1 \\ -m + 3 &= 1 \\ m &= 2 \end{aligned}$$

Checking Answer

$$D^{-1} = \frac{1}{-2m - (-6)(1)}\begin{bmatrix} -2 & 6 \\ -1 & m \end{bmatrix}$$

$$\begin{bmatrix} -1 & 3 \\ -\frac{1}{2} & 1 \end{bmatrix} = \frac{1}{-2m + 6}\begin{bmatrix} -2 & 6 \\ -1 & m \end{bmatrix}$$

Corresponding elements at row 1 and column 1:

$$\begin{aligned} -1 &= \frac{-2}{-2m + 6} \\ 2m - 6 &= -2 \\ m &= 2 \end{aligned}$$

Example 26

It is given that $\begin{bmatrix} 1 & 2 \\ 3 & 8 \end{bmatrix}A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ and matrix A has an order 2×2. Calculate matrix A.

Solution:

Since the product of $\begin{bmatrix} 1 & 2 \\ 3 & 8 \end{bmatrix}$ with A is an identity matrix, hence A is an inverse matrix of $\begin{bmatrix} 1 & 2 \\ 3 & 8 \end{bmatrix}$.

$$\begin{aligned} A &= \frac{1}{(1)(8) - (2)(3)}\begin{bmatrix} 8 & -2 \\ -3 & 1 \end{bmatrix} \\ &= \frac{1}{2}\begin{bmatrix} 8 & -2 \\ -3 & 1 \end{bmatrix} \\ &= \begin{bmatrix} 4 & -1 \\ -\frac{3}{2} & \frac{1}{2} \end{bmatrix} \end{aligned}$$

Self Practice 2.2f

1. For each of the following matrices, determine whether the inverse matrix exists. If yes, calculate the inverse matrix.

(a) $\begin{bmatrix} 6 & 0 \\ 0 & 1 \end{bmatrix}$ (b) $\begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix}$ (c) $\begin{bmatrix} -2 & 5 \\ 3 & -9 \end{bmatrix}$ (d) $\begin{bmatrix} 4 & 2 \\ 2 & 1 \end{bmatrix}$

2. Calculate the inverse matrices for the following matrices.

(a) $\begin{bmatrix} 5 & 6 \\ 2 & 3 \end{bmatrix}$ (b) $\begin{bmatrix} 2 & 3 \\ 3 & 5 \end{bmatrix}$ (c) $\begin{bmatrix} 4 & -2 \\ -3 & 2 \end{bmatrix}$ (d) $\begin{bmatrix} -2 & -5 \\ 2 & 7 \end{bmatrix}$

3. It is given that matrix $G = \begin{bmatrix} 2 & 1 \\ 3 & p \end{bmatrix}$. Calculate the value of p if

(a) matrix G does not have an inverse matrix,

(b) $G^{-1} = \begin{bmatrix} \frac{4}{5} & -\frac{1}{5} \\ -\frac{3}{5} & \frac{2}{5} \end{bmatrix}$.

4. It is given that $\begin{bmatrix} 4 & 10 \\ \frac{1}{2} & 1 \end{bmatrix} P = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ and matrix P has an order 2×2. Calculate matrix P.

How to use matrix method to solve simultaneous linear equations?

Learning Standard

Use the matrix method to solve simultaneous linear equations.

Simultaneous linear equations can be solved using the matrix method as shown in the steps below.

Simultaneous linear equations

$ax + by = p$
$cx + dy = q$

→

Matrix form $AX = B$

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} p \\ q \end{bmatrix}$$

where a, b, c, d, p and q are constants while x and y are unknowns

→

$$AX = B$$
$$A^{-1}AX = A^{-1}B$$
$$IX = A^{-1}B$$
$$X = A^{-1}B$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{ad - bc}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}\begin{bmatrix} p \\ q \end{bmatrix}$$

Example 27

Write the simultaneous linear equations below in the form of matrix.

$$3x + 4y = 12$$
$$5x - 6y = 7$$

Critical Mind

Is this multiplication feasible?
$AA^{-1}X = BA^{-1}$

Solution:

$$3x + 4y = 12$$
$$5x - 6y = 7$$

Coefficients of x form the elements for the first column of matrix A

Coefficients of y form the elements for the second column of matrix A

Matrix X

Constants form matrix B

$$\begin{bmatrix} 3 & 4 \\ 5 & -6 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 12 \\ 7 \end{bmatrix}$$

Critical Mind

If the equations in Example 27,

$5x - 6y = 7$
$3x + 4y = 12$

are written in the form of a matrix, does the arrangement of the equations affect the answer?

The simultaneous linear equations can be written as $\begin{bmatrix} 3 & 4 \\ 5 & -6 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 12 \\ 7 \end{bmatrix}$.

Example 28

Solve the simultaneous linear equations below using the matrix method.

$$x - 2y = 1$$
$$3x - 4y = 4$$

Solution:

$$\begin{bmatrix} 1 & -2 \\ 3 & -4 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 1 \\ 4 \end{bmatrix}$$ ← Write the simultaneous linear equations in the form of matrix

$$\begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{(1)(-4) - (-2)(3)}\begin{bmatrix} -4 & 2 \\ -3 & 1 \end{bmatrix}\begin{bmatrix} 1 \\ 4 \end{bmatrix}$$
$$= \frac{1}{2}\begin{bmatrix} 4 \\ 1 \end{bmatrix}$$
$$= \begin{bmatrix} 2 \\ \frac{1}{2} \end{bmatrix}$$

Hence, $x = 2$ and $y = \frac{1}{2}$ ← Final answer

Smart TIPS

Solving simultaneous linear equations means calculating the values of x and y. Therefore, the final answer states the values.

Checking Answer

Multiply $\begin{bmatrix} 1 & -2 \\ 3 & -4 \end{bmatrix}\begin{bmatrix} 2 \\ \frac{1}{2} \end{bmatrix}$.

$$\begin{bmatrix} 1 & -2 \\ 3 & -4 \end{bmatrix}\begin{bmatrix} 2 \\ \frac{1}{2} \end{bmatrix} = \begin{bmatrix} 2 - 1 \\ 6 - 2 \end{bmatrix}$$
$$= \begin{bmatrix} 1 \\ 4 \end{bmatrix}$$

Self Practice 2.2g

1. Write the simultaneous linear equations below in the form of matrix.
   (a) $x - y = 7$, $x + 3y = 5$
   (b) $3x + y = 0$, $5x + 2y = -14$
   (c) $7x + 2y = -11$, $2x - y = -10$
   (d) $3x + 2y - 14 = 0$, $4y = 5x - 5$
   (e) $2x + y + 4 = 0$, $y - 3x = 11$
   (f) $2x + y = -9$, $5x = -12$
   (g) $2x = 5y$, $\frac{x}{5} + 2y = 3$
   (h) $\frac{x}{y} = 4$, $0.8(x + 5) = 3y$

2. In a chess competition, the total number of participants is 100. The number of male participants, x, is 14 fewer than 2 times the number of female participants, y. Write the simultaneous linear equations that represent the above information in the form of matrix.

3. Solve the simultaneous linear equations below using the matrix method.

(a) $x - 2y = 5$, $2x - 3y = 10$

(b) $2x - 5y = 1$, $3x - y = -5$

(c) $2x - y = 8$, $x + y = 1$

(d) $3x + 2y = 4$, $9x + 4y = 14$

(e) $4x + 3y = 11$, $2y = 9 - 6x$

(f) $5x - 5y - 6 = 0$, $2x - 2.1 = 3y$

(g) $p + 3q = 4$, $3 + \frac{p}{2} = q$

(h) $m + n = 5$, $\frac{m}{2} - \frac{n}{4} = 1$

How to solve problems involving matrices?

Learning Standard

Solve problems involving matrices.

Write linear equations in the form of $ax + by = p$, $cx + dy = q$, where a, b, c, d, p and q are constants while x and y are unknowns → Write the simultaneous linear equations in the form of matrix $AX = B$ → Solve with multiplication with inverse matrix: $X = A^{-1}B$

Example 29

Based on the conversation above, how much is the price of a child ticket and an adult ticket?

Solution:

Understanding the problem

The price of 2 child tickets and 1 adult ticket is RM32. The price of 5 child tickets and 3 adult tickets is RM88.

x = the price of a child ticket

y = the price of an adult ticket

Devising a strategy

(a) Form two linear equations.

(b) Express the equations in the form of matrix and solve it.

Implementing the strategy

$$2x + y = 32$$
$$5x + 3y = 88$$

$$\begin{bmatrix} 2 & 1 \\ 5 & 3 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 32 \\ 88 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{(2)(3) - (1)(5)}\begin{bmatrix} 3 & -1 \\ -5 & 2 \end{bmatrix}\begin{bmatrix} 32 \\ 88 \end{bmatrix}$$

$$= \frac{1}{6 - 5}\begin{bmatrix} 96 - 88 \\ -160 + 176 \end{bmatrix}$$

$$= \begin{bmatrix} 8 \\ 16 \end{bmatrix}$$

Making a conclusion

The price of a child ticket is RM8 and an adult ticket is RM16.

Example 30

	Camera K	Camera L
Assembling	10 minutes	10 minutes
Packaging	5 minutes	9 minutes

Syarikat Komunikasi Era Baru produces two camera models, K and L. Each camera produced needs to go through two departments. They are the Assembling Department and the Packaging Department. The table above shows the time for assembling and packaging for each type of camera. It is given that the Assembling Department operates for 12 hours a day and the Packaging Department operates for 9 hours a day. Calculate the number of camera K and camera L produced in a day.

Solution:

Understanding the problem

The total time for assembling is 12 hours, which is equivalent to 720 minutes.

The total time for packaging is 9 hours, which is equivalent to 540 minutes.

x = number of camera K produced
y = number of camera L produced

Devising a strategy

(a) Form two linear equations.
(b) Express the equations in the form of matrix and solve it.

Implementing the strategy

$$10x + 10y = 720$$
$$5x + 9y = 540$$

$$\begin{bmatrix} 10 & 10 \\ 5 & 9 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 720 \\ 540 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{(10)(9) - (10)(5)}\begin{bmatrix} 9 & -10 \\ -5 & 10 \end{bmatrix}\begin{bmatrix} 720 \\ 540 \end{bmatrix}$$

$$= \frac{1}{40}\begin{bmatrix} 1080 \\ 1800 \end{bmatrix}$$

$$= \begin{bmatrix} 27 \\ 45 \end{bmatrix}$$

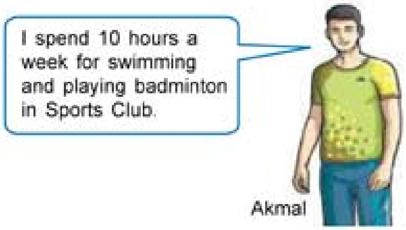
Making a conclusion

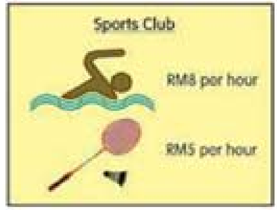
The number of camera K produced is 27 units and the number of camera L produced is 45 units.

Self Practice 2.2h

1. A research is conducted on the sales of two types of curry puffs with sardine and potato fillings. In the first hour, 24 curry puffs with sardine filling and 18 curry puffs with potato filling are sold, and the total amount of sales is RM28.80. In the next hour, 30 curry puffs with sardine filling and 14 curry puffs with potato filling are sold, and the total amount of sales is RM29.20. Calculate the prices for a curry puff with sardine filling and a curry puff with potato filling using the matrix method.

2. Akmal spends RM68 a week on the two sports as mentioned below. Calculate the time, in hours, Akmal spends in the Sports Club for swimming and playing badminton in a week using the matrix method.





3. Puan Komala and Puan Lily go to the market to buy papaya and banana. The table below shows the mass of the papaya and banana bought by them.

	Papaya	Banana
Puan Komala	4 kg	2 kg
Puan Lily	5 kg	3 kg

Puan Komala and Puan Lily pay RM26 and RM35 respectively for the two types of fruits. Calculate the prices for one kilogram of papaya and one kilogram of banana using the matrix method.

4. A building has several parking spaces for cars and motorcycles. One day, there were a total of 66 vehicles parked there and the total number of wheels was 190. Calculate the number of cars and the number of motorcycles parked that day using the matrix method. Assume that all motorcycles are two-wheeled.

5. Encik Jefri and Encik Tan invest in Unit Trust P and Unit Trust Q as shown in the table below.

	Unit Trust P	Unit Trust Q
Encik Jefri	RM5 000	RM3 000
Encik Tan	RM6 000	RM4 000

After a year, Encik Jefri gets a dividend of RM350 from the investment of both the unit trusts while Encik Tan gets a dividend of RM440. Calculate the dividend rates of Unit Trust P and Unit Trust Q using the matrix method.

Summary Arena

MATRICES

Matrices
- A set of numbers arranged in rows and columns to form a rectangular or a square array
- Written inside bracket [ ] or ( )

Order
Order $m \times n$ has m rows and n columns

Element
a_{ij} is the element of i^{th} row and j^{th} column

Equal matrices
$A = B$ if the order of both the matrices are the same and the corresponding elements are equal

Basic Operation on Matrices

Add and subtract matrices

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix} \pm \begin{bmatrix} e & f \\ g & h \end{bmatrix} = \begin{bmatrix} a \pm e & b \pm f \\ c \pm g & d \pm h \end{bmatrix}$$

Multiply a matrix by a number

$$n\begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} na & nb \\ nc & nd \end{bmatrix}$$

Multiply two matrices

$$\underset{m \times n}{A} \quad \underset{n \times p}{B} = \underset{m \times p}{AB}$$

$$\begin{bmatrix} a & b \end{bmatrix}\begin{bmatrix} c \\ d \end{bmatrix} = \begin{bmatrix} ac + bd \end{bmatrix}$$

$$\begin{bmatrix} c \\ d \end{bmatrix}\begin{bmatrix} a & b \end{bmatrix} = \begin{bmatrix} ca & cb \\ da & db \end{bmatrix}$$

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix}\begin{bmatrix} e & f \\ g & h \end{bmatrix} = \begin{bmatrix} ae + bg & af + bh \\ ce + dg & cf + dh \end{bmatrix}$$

Identity matrix, I, order $n \times n$ with element 1 along the main diagonal and the other elements are 0

$$\begin{bmatrix} 1 & 0 & 0 & \dots & 0 \\ 0 & 1 & 0 & \dots & 0 \\ 0 & 0 & 1 & \dots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \dots & 1 \end{bmatrix}$$

$$AI = IA = A$$

Inverse matrix
Inverse matrix of A is denoted by A^{-1}.

If $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$, then

$$A^{-1} = \frac{1}{ad - bc}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}.$$

$$AA^{-1} = A^{-1}A = I$$

Solving simultaneous linear equations

Simultaneous linear equations

$$ax + by = p$$
$$cx + dy = q$$

→ **Matrix form $AX = B$**

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} p \\ q \end{bmatrix}$$

where a, b, c, d, p and q are constants while x and y are unknowns

→

$$AX = B$$
$$A^{-1}AX = A^{-1}B$$
$$IX = A^{-1}B$$
$$X = A^{-1}B$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{ad - bc}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}\begin{bmatrix} p \\ q \end{bmatrix}$$

Reflection

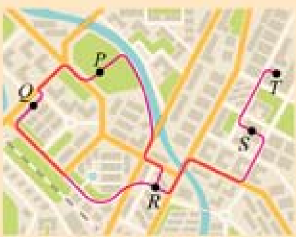
At the end of this chapter, I can

	🙂	😖
represent information from real situations in the form of matrices.		
determine the order of a matrix, hence identify certain elements in a matrix.		
determine whether two matrices are equal.		
add and subtract matrices.		
multiply a matrix by a number.		
multiply two matrices.		
explain the characteristics of identity matrix.		
explain the meaning of inverse matrix and hence determine the inverse matrix for a 2 × 2 matrix.		
use the matrix method to solve simultaneous linear equations.		
solve problems involving matrices.		

MINI PROJECT

Transportation companies use network to represent the routes of their transports. Network consists of vertices that are connected with edges.

In the diagram below, the vertices P, Q, R, S and T represent the cities while the edges represent the routes of bus between two cities. All the routes are roads that connect adjacent cities. This route system can be represented with the matrix as below.



$$\text{From}\begin{array}{c} \\ P \\ Q \\ R \\ S \\ T \end{array}\overset{\displaystyle\text{To}}{\begin{array}{c} \begin{array}{ccccc} P & Q & R & S & T \end{array} \\ \begin{bmatrix} 0 & 1 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 \\ 1 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 \end{bmatrix} \end{array}}$$

Prepare a report about the route system of buses (or any other types of transportation) in your area. Your report should consist of

(i) the introduction of the public transport system in your area,
(ii) the use of matrix to represent the route system of the public transport,
(iii) the meaning of the elements in the matrix.

Extensive Practice

Scan the QR code or visit bit.do/QuizE02 for interactive quiz

UNDERSTAND

1. State the number of rows and the number of columns of the matrix $\begin{bmatrix} 9 & -2 \\ 1 & 6 \\ 5 & 7 \end{bmatrix}$.

2. It is given that $A = \begin{bmatrix} 3 & 2 \\ 0 & -1 \\ 1 & 3 \end{bmatrix}$, $B = \begin{bmatrix} 1 \\ 4 \end{bmatrix}$ and $AB = C$. Determine the order of matrix C.

3. It is given that matrix $D = \begin{bmatrix} 4 & p \\ -2 & 3 \end{bmatrix}$. Calculate the value of p if the determinant of matrix D is 0.

4. Given that matrix $E = \begin{bmatrix} -1 & 2 \\ 5 & 4 \end{bmatrix}$, show that $E + E + E = 3E$.

5. Write the following simultaneous linear equations in the form of matrix.

$$m - 3 = 4n$$
$$3m + 2n - 2 = 0$$

MASTERY

6. It is given that $G = \begin{bmatrix} 4 & r \\ 1 & 2 \end{bmatrix}$ and $H = \begin{bmatrix} 1 & 3 \\ -1 & s \end{bmatrix}$. Calculate the values of r and s if $GH = HG$.

7. It is given that $A = \begin{bmatrix} 4 & -3 \\ 2 & 1 \end{bmatrix}$ and $AB = I$. Calculate matrix B.

8. Given that $P = \begin{bmatrix} 10 & -5 \\ -2 & 1 \\ 0 & 2z - 3 \end{bmatrix}$, $Q = \begin{bmatrix} 1 & y \\ 0.2 & -\frac{1}{3} \\ 8 & -1 \end{bmatrix}$, $R = \begin{bmatrix} 11 & -25 \\ y + 6x & -0.2 \\ 24 & 9 \end{bmatrix}$ and $0.8P + 3Q = R$, calculate the values of x, y and z.

9. It is given that matrix $F = \begin{bmatrix} -1 & 2 \\ 3 & -4 \end{bmatrix}$.
   (a) Calculate F^3.
   (b) Hence, calculate matrix G if $F^3 - 5G = 12F$.

10. Given that $\frac{1}{p}\begin{bmatrix} 2 & q \\ 1 & 4 \end{bmatrix}\begin{bmatrix} 4 & -6 \\ -1 & 2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, calculate the values of p and q.

11. (a) Write the simultaneous linear equations below in the form of matrix.
    (b) Calculate the values of x and y using the matrix method.

$$2y - x = 5$$
$$3x - 8y = -19$$

CHALLENGE

12. A marathon race has 128 participants. The number of male participants is 16 fewer than 2 times the number of female participants. Calculate the number of male participants and female participants in the marathon race using the matrix method.

13. It is given that the simultaneous linear equations $px + 4y = 10$ and $qx - 2y = 1$ have no solution. Express p in terms of q.

14. Faris took a course in a college. He registered for three subjects in the first semester. The overall mark for each subject was calculated based on the marks of coursework and exam according to the percentage of each section. Table 1 shows the marks obtained by Faris in the first semester. Table 2 shows the weightage given to coursework and exam.

	Coursework	Exam
Mathematics	80	70
English	60	75
Computer Science	74	84

Table 1

	First Semester
Coursework	60%
Exam	40%

Table 2

(a) Represent the information in Table 1 and Table 2 with matrices.
(b) Calculate the overall marks for Mathematics in the first semester using the matrix method.
(c) Determine the best performed subject in the first semester.

15. Syahirah is undergoing a diet plan involving two types of drink, P and Q. The table below shows the amount of protein and calories for a glass of the drinks.

	Drink P	Drink Q
Protein (g)	6	4
Calories (kcal)	95	110

The diet plan suggests that Syahirah consume a total of 16 g of protein and get 300 kcal per day from the two types of drinks.
(a) Form two linear equations with the above information.
(b) Calculate the number of glasses of drink P and drink Q Syahirah needs to consume daily according to the diet plan using the matrix method.

16. Mr Sanjay sold two brands of air conditioners, K and L. The prices of the air conditioners K and L are RM1 500 and RM2 000 respectively. The commissions for selling an air conditioner K and an air conditioner L are 3% and 4% respectively. In the month of May, Mr Sanjay sold 50 units air conditioners and received a total commission of RM2 880. Calculate the number of air conditioners K and L sold using the matrix method.

EXPLORING MATHEMATICS

Cryptography is a science of information safety. It involves techniques such as combining words into the form of images or writing words in secret codes so that the words cannot be read by a third party. During World War II, the German army used the Enigma machine to write their secret messages. Three Poland mathematicians managed to decrypt the messages from the Enigma machine and assisted the Allied Powers to end the war.

Use the code system below, send a message "GURU KELAS" to your friend.

	A	B	C	D	E	F	G	H	I	J	K	L	M	N
0	1	2	3	4	5	6	7	8	9	10	11	12	13	14
O	P	Q	R	S	T	U	V	W	X	Y	Z	!	?	.
15	16	17	18	19	20	21	22	23	24	25	26	27	28	29

Follow the following steps.

(a) Write the message in a few matrices with order 2×1.

(b) Use the lock $M = \begin{bmatrix} 2 & -1 \\ 1 & 0 \end{bmatrix}$ to encrypt the message, that is, matrix M is multiplied with every matrix formed in (a).

(c) The products obtained will be transformed into a secret message in alphabets based on the code system above. If the product is a negative number, add the product with 30. Send the secret message to your friend.

(d) When receiving the secret message, your friend needs to decrypt the message based on the following steps:

(i) write the secret message obtained in a few matrices with order 2×1.

(ii) multiply the key $K = \begin{bmatrix} 0 & 1 \\ -1 & 2 \end{bmatrix}$ with every matrix formed in (d)(i).

(iii) the products obtained are transformed into the original message by referring to the code system above. If the product is a negative number, add the product with 30.

Tips for the steps:

(a) For example, message "DI BAS", alphabets "D" = 4, "I" = 9, " " = 0, "B" = 2, "A" = 1 and "S" = 19. Hence the matrices formed are $\begin{bmatrix} 4 \\ 9 \end{bmatrix}$, $\begin{bmatrix} 0 \\ 2 \end{bmatrix}$ and $\begin{bmatrix} 1 \\ 19 \end{bmatrix}$.

(b) For example, $\begin{bmatrix} 2 & -1 \\ 1 & 0 \end{bmatrix}\begin{bmatrix} 4 \\ 9 \end{bmatrix}$, $\begin{bmatrix} 2 & -1 \\ 1 & 0 \end{bmatrix}\begin{bmatrix} 0 \\ 2 \end{bmatrix}$ and $\begin{bmatrix} 2 & -1 \\ 1 & 0 \end{bmatrix}\begin{bmatrix} 1 \\ 19 \end{bmatrix}$.

(c) For example, $\begin{bmatrix} 2 & -1 \\ 1 & 0 \end{bmatrix}\begin{bmatrix} 4 \\ 9 \end{bmatrix} = \begin{bmatrix} -1 \\ 4 \end{bmatrix}$.

Therefore, $\begin{bmatrix} -1 + 30 \\ 4 \end{bmatrix} = \begin{bmatrix} 29 \\ 4 \end{bmatrix}$.

By referring to the code system, $\begin{bmatrix} 29 \\ 4 \end{bmatrix}$ will be represented with $\begin{bmatrix} . \\ \text{D} \end{bmatrix}$.

A full secret message produced is ".D? MA".