CHAPTER 3 GRAVITATION

What are Newton's Universal Law of Gravitation and Kepler's Laws?

Why is it important to know the values of the gravitational acceleration of planets in the Solar System?

How do man-made satellites help to improve human life?

Let's Study

3.1 Newton's Universal Law of Gravitation
3.2 Kepler's Laws
3.3 Man-made Satellites

Information Page

There are various types of man-made satellites revolving in their respective orbits in outer space. Satellites are invented for communication purposes, weather forecasts and Earth observations. Why are these satellites able to revolve in their respective orbits?

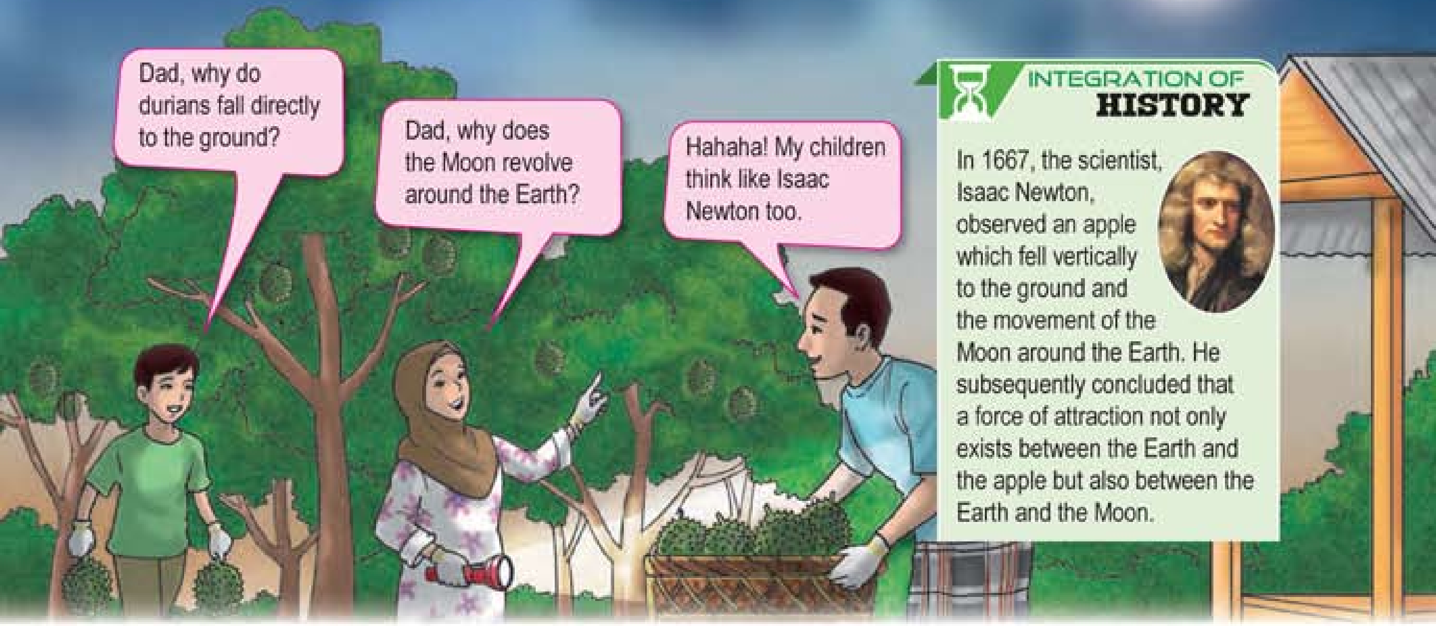
Several hundred years ago, Isaac Newton, a scientist, conceptualised a universal law connecting all heavenly bodies and also man-made satellites. Curiosity about the universe has encouraged scientists to launch spaceships and satellites which can overcome the Earth's gravity. At present, there are spaceships which are moving far away from Earth. Such spaceships enable photographs of planets to be taken to benefit scientific inventions and technology.

Animated launching of satellite

http://bt.sasbadi.com/p4077a

Learning Standards and List of Formulae

3.1 Newton's Universal Law of Gravitation



INTEGRATION OF HISTORY

In 1667, the scientist, Isaac Newton, observed an apple which fell vertically to the ground and the movement of the Moon around the Earth. He subsequently concluded that a force of attraction not only exists between the Earth and the apple but also between the Earth and the Moon.

Figure 3.1 *Situation at a durian orchard*

Activity 3.1

Pattern Recognition | ICS | ISS

Aim: To discuss gravitational force between two bodies in the universe

Instructions:

1. Carry out a Hot Seat activity in groups.
2. Study Figure 3.2 and discuss the three situations below.

A person who jumps up will return to the ground. What force causes the person to return to the ground?

Air molecules remain in the atmosphere without escaping to outer space. What force acts between the molecules in the atmosphere and the Earth?

The Moon revolves around the Earth without drifting away from its orbit. The Earth exerts a pulling force on the Moon. Does the Moon also exert a force on the Earth?

Figure 3.2 *Situations involving gravitational force between two bodies*

3. Surf the web to gather relevant information.
4. Choose a group representative to answer questions from other groups.

Gravitational force is known as **universal force** because it acts between any two bodies in the universe. Figure 3.3 shows the gravitational force between the Sun, the Earth and the Moon. How can the gravitational force between two bodies be explained?

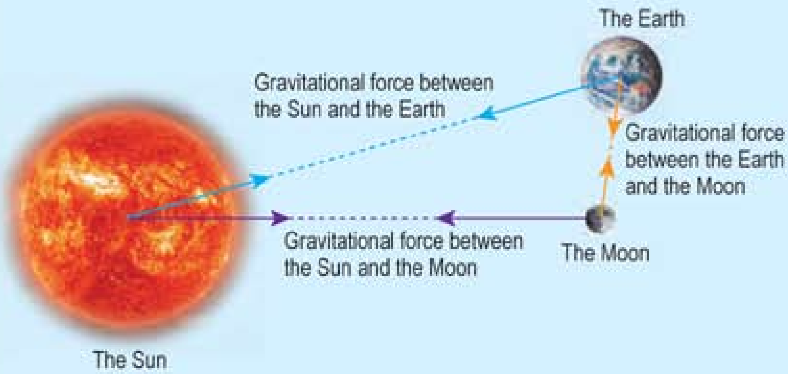


Figure 3.3 *Gravitational force as a universal force*

Gravitational force

http://bt.sasbadi.com/p4079

In the year 1687, Isaac Newton presented two relationships that involve gravitational force between two bodies:

- gravitational force is directly proportional to the product of the masses of the two bodies, that is $F \propto m_1 m_2$
- gravitational force is inversely proportional to the square of the distance between the centres of the two bodies, that is $F \propto \frac{1}{r^2}$

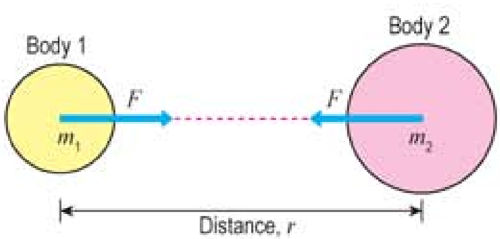


Figure 3.4 *Gravitational force between two bodies*

The two relationships above are formulated in Figure 3.5 to obtain **Newton's Universal Law of Gravitation**.

$$F \propto m_1 m_2 \quad \text{and} \quad F \propto \frac{1}{r^2} \;\rightarrow\; F \propto \frac{m_1 m_2}{r^2}$$

Figure 3.5 *Formulation of Newton's Universal Law of Gravitation*

Info File

1. Gravitational force exists between two bodies.
2. Both bodies experience gravitational force of the same magnitude.

Info File

Why does a fallen leaf move towards the ground?

Both the leaf and the Earth experience the same gravitational force. This causes the leaf and the Earth to move towards one another. As the mass of the Earth is very much larger than the mass of the leaf, gravitational force does not have an apparent effect on the Earth's movement. As such, we only observe the leaf falling to the ground.

3.1.1

Newton's Universal Law of Gravitation states that the gravitational force between two bodies is directly proportional to the product of the masses of both bodies and inversely proportional to the square of the distance between the centres of the two bodies.

Info File

The value of gravitational constant, G can be determined through experiment.

Newton's Universal Law of Gravitation can be expressed as follows:

$$F = \frac{Gm_1m_2}{r^2}$$

F = gravitational force between two bodies
m_1 = mass of first body
m_2 = mass of second body
r = distance between the centre of the first body and the centre of the second body
G = gravitational constant ($G = 6.67 \times 10^{-11}$ N m² kg⁻²)

SMART INFO

$$F = \frac{Gm_1m_2}{r^2}$$

Two bodies of masses m_1 and m_2 that are separated at a distance of r, experience a gravitational force of F respectively.

If you know the masses of two bodies and the distance between the centres of the two bodies, you can calculate the magnitude of the gravitational force between the two bodies. Study the examples given.

Example 1

Calculate the gravitational force between a durian and the Earth.
Mass of durian = 2.0 kg
Mass of the Earth = 5.97×10^{24} kg
Distance between the centre of the durian and the centre of the Earth
= 6.37×10^6 m

Figure 3.6

Solution:

Step 1
List the given information in symbols.

$m_1 = 2.0$ kg
$m_2 = 5.97 \times 10^{24}$ kg
$r = 6.37 \times 10^6$ m
$G = 6.67 \times 10^{-11}$ N m^2 kg^{-2}

Step 2
Identify and write down the formula used.

Gravitational force,
$$F = \frac{Gm_1m_2}{r^2}$$

Step 3
Substitute numerical values into the formula and perform the calculations.

$$F = \frac{(6.67 \times 10^{-11}) \times 2.0 \times (5.97 \times 10^{24})}{(6.37 \times 10^6)^2}$$
$$= 19.63 \text{ N}$$

Example 2

A rocket at a launching pad experiences a gravitational force of 4.98×10^5 N. What is the mass of the rocket?
[Mass of the Earth = 5.97×10^{24} kg, distance between the centre of the Earth and the centre of the rocket = 6.37×10^6 m]

Solution:

Gravitational force, $F = 4.98 \times 10^5$ N
Mass of the Earth, $m_1 = 5.97 \times 10^{24}$ kg
Mass of rocket = m_2
Distance between the centre of the Earth and the centre of the rocket,
$r = 6.37 \times 10^6$ m
$G = 6.67 \times 10^{-11}$ N m^2 kg^{-2}

Gravitational force,

$$F = \frac{Gm_1m_2}{r^2}$$

$$4.98 \times 10^5 = \frac{(6.67 \times 10^{-11}) \times (5.97 \times 10^{24}) \times m_2}{(6.37 \times 10^6)^2}$$

$$m_2 = \frac{(4.98 \times 10^5) \times (6.37 \times 10^6)^2}{(6.67 \times 10^{-11}) \times (5.97 \times 10^{24})}$$

$$= 5.07 \times 10^4 \text{ kg}$$

Solving Problems Involving Newton's Universal Law of Gravitation

Gravitational force acts between any two bodies, including those on the Earth, planets, the Moon or the Sun. What are the effects of mass and distance between two bodies on gravitational force?

Activity 3.2

Logical Reasoning | CPS

Aim: To solve problems involving Newton's Universal Law of Gravitation for two bodies at rest on the Earth

Instructions:
1. Work in pairs.
2. Imagine you and your partner are bodies at rest on the Earth.
3. Record your mass, m_1 and the mass of your partner, m_2 in Table 3.1.

Table 3.1

Pair	Mass, *m* / kg		*r* / m	*F* / N
	m_1	m_2		
1			2.0	
			4.0	
2			2.0	
			4.0	

4. Calculate the gravitational force, F using both your masses and the distances given in the table.
5. Change partners and repeat steps 3 and 4.

Discussion:
1. How do the masses of two bodies influence the gravitational force between them?
2. What is the effect of distance between two bodies on gravitational force between them?
3. Why is the magnitude of gravitational force between you and your partner small?

The effects of mass and distance between two objects on gravitational force are shown in Figure 3.7.

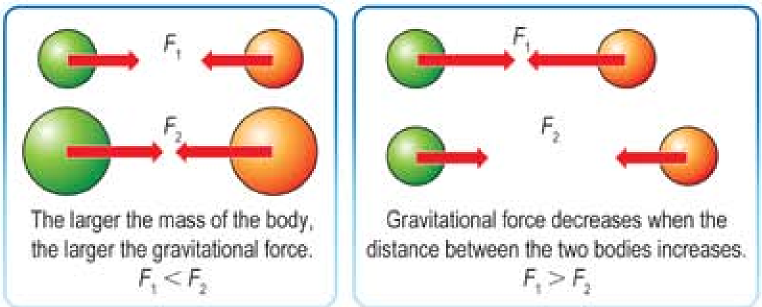


***Figure 3.7** Effects of mass and distance between two bodies on gravitational force*

Gravitational force between two bodies depends on the mass of the bodies, as well as the distance between them.

Info File

Even though gravitational force is a universal force, two persons on the Earth's surface will not feel the effect of the gravitational force. This is because the gravitational force between two bodies of small mass has a very small magnitude. For example, two persons of masses 50 kg and 70 kg respectively, only experience a gravitational force of 2.3×10^{-7} N if they stand at a distance of 1 m from each other.

Activity 3.3

Abstraction CPS

Aim: To solve problems involving Newton's Universal Law of Gravitation for
(i) objects on the Earth's surface
(ii) the Earth and satellite
(iii) the Earth and the Sun

Instructions:

1. Work in pairs.
2. Study Figure 3.8 and answer the questions.

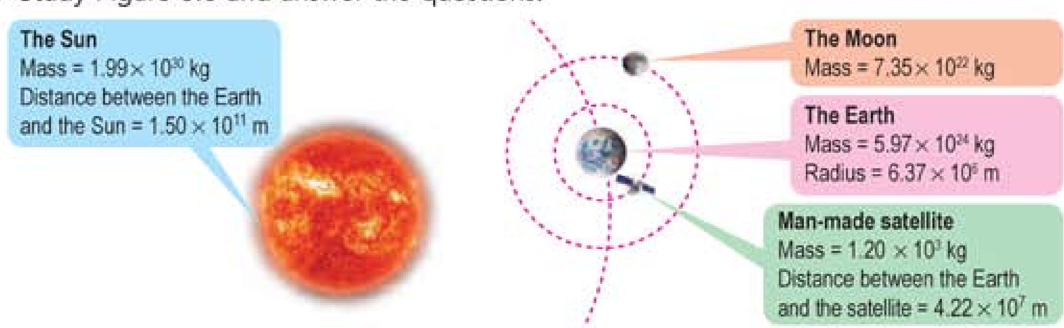


***Figure 3.8** The Sun, the Earth, the Moon and a man-made satellite*

Discussion:

1. What is the gravitational force on the man-made satellite before it is launched?
2. Compare:
   (a) the mass of the Earth, the mass of the man-made satellite and the mass of the Sun
   (b) between the Earth-satellite distance and the Sun-Earth distance.
3. Predict the difference in the magnitude of the gravitational force between the Earth and the man-made satellite and the gravitational force between the Sun and the Earth.
4. Calculate:
   (a) the gravitational force between the Earth and the man-made satellite
   (b) the gravitational force between the Earth and the Sun.
   Does your answer match your prediction in question 3?
5. The gravitational force between the Earth and the Moon is 2.00×10^{20} N. What is the distance between the centre of the Earth and the centre of the Moon?

Relating Gravitational Acceleration, g on the Surface of the Earth with Universal Gravitational Constant, G

According to Newton's Second Law of Motion, gravitational force can be expressed as $F = mg$. From Newton's Universal Law of Gravitation, gravitational force is expressed as $F = \frac{Gm_1m_2}{r^2}$.

What is the relationship between g and G?

Info File

From Newton's Second Law of Motion, $F = ma$.

When involving gravitational acceleration, g,

$F = mg$

Activity 3.4

Algorithms CPS

Aim: To derive the formula for gravitational acceleration, g using the formulae $F = mg$ and $F = \frac{GMm}{r^2}$

Instructions:

1. Work in pairs.
2. Scan the QR code and download Figure 3.9 from the website given.
3. Discuss and complete Figure 3.9 to derive the relationship between g and G.

Download Figure 3.9

http://bt.sasbadi.com/p4083

M = mass of the Earth
m = mass of the object
r = distance between the centre of the Earth and the centre of the object

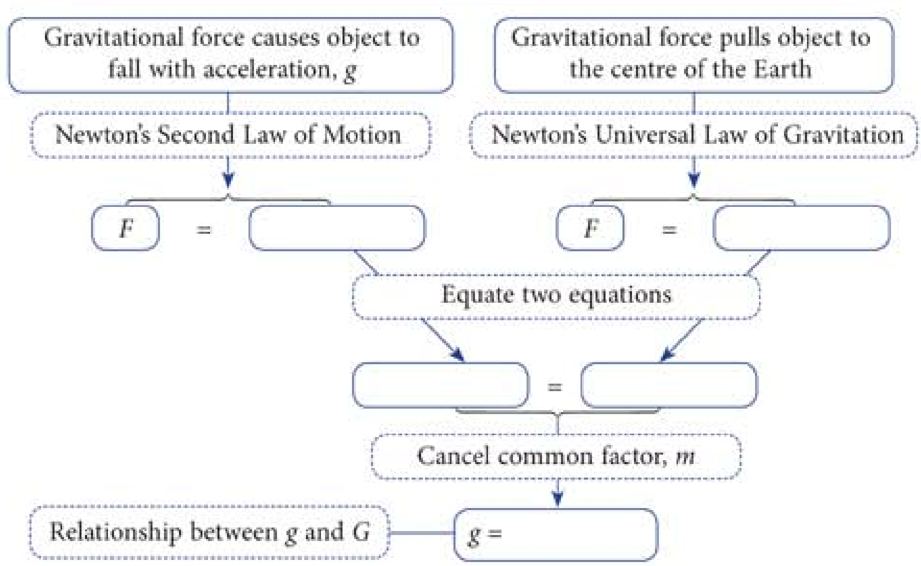


Figure 3.9 *Relationship between g and G*

Discussion:

1. What is the relationship between gravitational acceleration, g and gravitational constant, G?
2. What are the factors that influence the value of gravitational acceleration?

Activity 3.5

Logical Reasoning | CPS

Aim: To discuss the variation in the values of g with r

Instructions:

1. Work in pairs.
2. Calculate the value of gravitational acceleration for the five distances given in Figure 3.10.

> **Gravitational acceleration below the surface of the Earth**
>
> *http://bt.sasbadi.com/p4084*

- Mass of the Earth, $M = 5.97 \times 10^{24}$ kg
- Radius of the Earth, $R = 6.37 \times 10^{6}$ m
- Gravitational constant, $G = 6.67 \times 10^{-11}$ N m^2 kg^{-2}

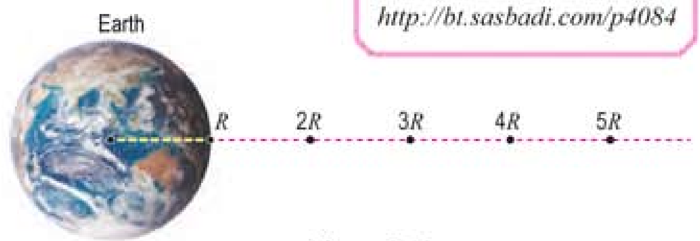


Figure 3.10

3. Complete Table 3.2.

Table 3.2

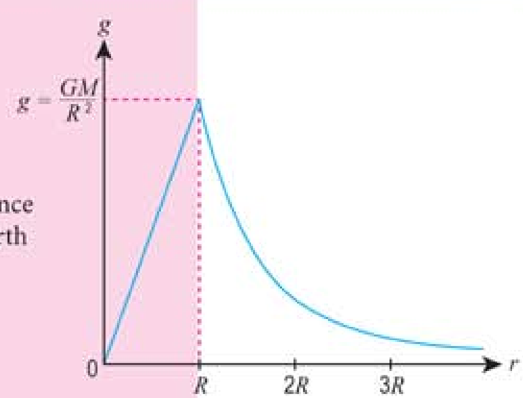
Distance from centre of the Earth, r	R	$2R$	$3R$	$4R$	$5R$
Gravitational acceleration, g / m s^{-2}					

Discussion:

1. What is the value of gravitational acceleration on the Earth's surface?
2. Plot a graph of g against r.
3. How does the value of gravitational acceleration change when the distance from the centre of the Earth increases?
4. Discuss the condition where the value of gravitational acceleration is almost zero.

Figure 3.11 shows a sketch of a graph with various values of gravitational acceleration, g and distance, r from the centre of the Earth.

Value g is **directly proportional** to the distance from the centre of the Earth for $r < R$.



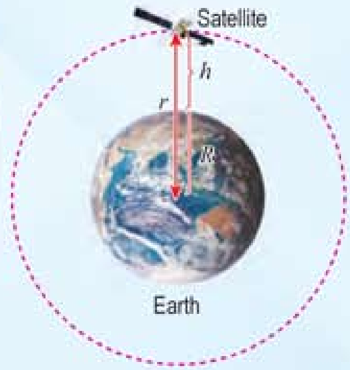
Value g is **inversely proportional** to the square of the distance from centre of the Earth for $r \geq R$.

Figure 3.11 *Variation of g with r*

Figure 3.12 shows a satellite at height, h from the surface of the Earth. R is the radius of the Earth and r is the distance of the satellite from the centre of the Earth, which is the radius of the orbit.

At height, h from the surface of the Earth, distance from the centre of the Earth is $r = (R + h)$.

With this, gravitational acceleration, $g = \frac{GM}{(R + h)^2}$



On the surface of the Earth, height, $h = 0$. Therefore, r = radius of the Earth, R.

Gravitational acceleration on the surface of the Earth, $g = \frac{GM}{R^2}$, where M is the mass of the Earth.

***Figure 3.12** A satellite at height, h from the surface of the Earth*

Example 1

Mass of the Earth is 5.97×10^{24} kg and radius of the Earth is 6.37×10^6 m. Calculate gravitational acceleration on the surface of the Earth. [$G = 6.67 \times 10^{-11}$ N m² kg⁻²]

Solution:

Step ❶
List the given information with symbols.

$M = 5.97 \times 10^{24}$ kg
$G = 6.67 \times 10^{-11}$ N m^2 kg^{-2}
$r = 6.37 \times 10^6$ m

Step ❷
Identify and write down the formula used.

$$g = \frac{GM}{r^2}$$

Step ❸
Make numerical substitution into the formula and perform the calculations.

$$g = \frac{(6.67 \times 10^{-11}) \times (5.97 \times 10^{24})}{(6.37 \times 10^6)^2}$$
$$= 9.81 \text{ m s}^{-2}$$

Example 2

A radar imaging satellite orbits around the Earth at a height of 480 km. What is the value of gravitational acceleration at the position of the satellite?
[$G = 6.67 \times 10^{-11}$ N m^2 kg^{-2}, $M = 5.97 \times 10^{24}$ kg, $R = 6.37 \times 10^6$ m]

Solution:

Height of orbit, h = 480 km
= 480 000 m

$$g = \frac{GM}{(R + h)^2}$$
$$= \frac{(6.67 \times 10^{-11}) \times (5.97 \times 10^{24})}{(6.37 \times 10^6 + 480\,000)^2}$$
$$= 8.49 \text{ m s}^{-2}$$

3.1.3

Importance of Knowing the Value of Gravitational Acceleration

Gravitational force is a universal force. Therefore, the formula $g = \frac{GM}{R^2}$ can be used to calculate gravitational acceleration on the surface of other bodies such as planets, the Moon and the Sun. Which planet has the largest gravitational acceleration? Which planet has the smallest gravitational acceleration?

Space debris

http://bt.sasbadi.com/p4086a

Activity 3.6

Logical Reasoning | CPS | ISS

Aim: To compare different gravitational accelerations for the Moon, the Sun and the planets in the Solar System

Instructions:

1. Carry out the Think-Pair-Share activity.
2. Gather information on the mass, M and radius, R of the Sun, the Moon as well as the other remaining planets in the Solar System.
3. Present the gathered information in a table.
4. Calculate the gravitational acceleration, g for each of the bodies.

Discussion:

1. Which planet has the largest gravitational acceleration?
2. Which planet has gravitational acceleration closest to the gravitational acceleration of the Earth?
3. What factors determine the value of the gravitational acceleration of a planet?

When the value of the gravitational acceleration on the surface of a planet is known, the **magnitude of the gravitational force** acting on an object on the surface of the planet can be calculated. Knowledge on the value of gravitational acceleration plays an important role in space exploration and continuity of life.

Activity 3.7

Logical Reasoning | ICS | ISS

Aim: To discuss the importance of knowledge on gravitational acceleration of planets in space exploration and continuity of life

Instructions:

1. Work in groups.
2. Gather information on the importance of knowledge on gravitational acceleration of planets in space exploration and continuity of life.
3. Present the results of your discussion in the form of i-Think map.

Gravitational acceleration

http://bt.sasbadi.com/p4086b

We live on Earth where gravitational acceleration is 9.81 m s^{-2}. While exploring space whether far from Earth or near other planets, the body of astronauts can be exposed to low or high gravity conditions. What are the effects of gravity on the growth of humans?

Activity 3.8

Logical Reasoning ISS ICS

Aim: To gather information on the effects of gravity on the growth of humans

Instructions:

1. Carry out the Round Table activity.

Table 3.3

Factor	Effect of low gravity	Effect of high gravity
Difference in density		
Bone fragility		
Size of lungs		
Blood circulatory system		
Blood pressure		

2. Based on Table 3.3, gather information on the effects of gravity on the growth of humans by visiting websites or from other suitable reading materials.
3. Complete Table 3.3.
4. Present a multimedia presentation entitled Effects of Gravity on the Growth of Humans.

Effects of gravity

http://bt.sasbadi.com/p4087

3.1.4

Centripetal Force in the Motion of Satellites and Planets

Figure 3.13 shows three positions of a satellite which orbits around the Earth at a uniform speed. Observe the direction of the velocity of the satellite at each position.

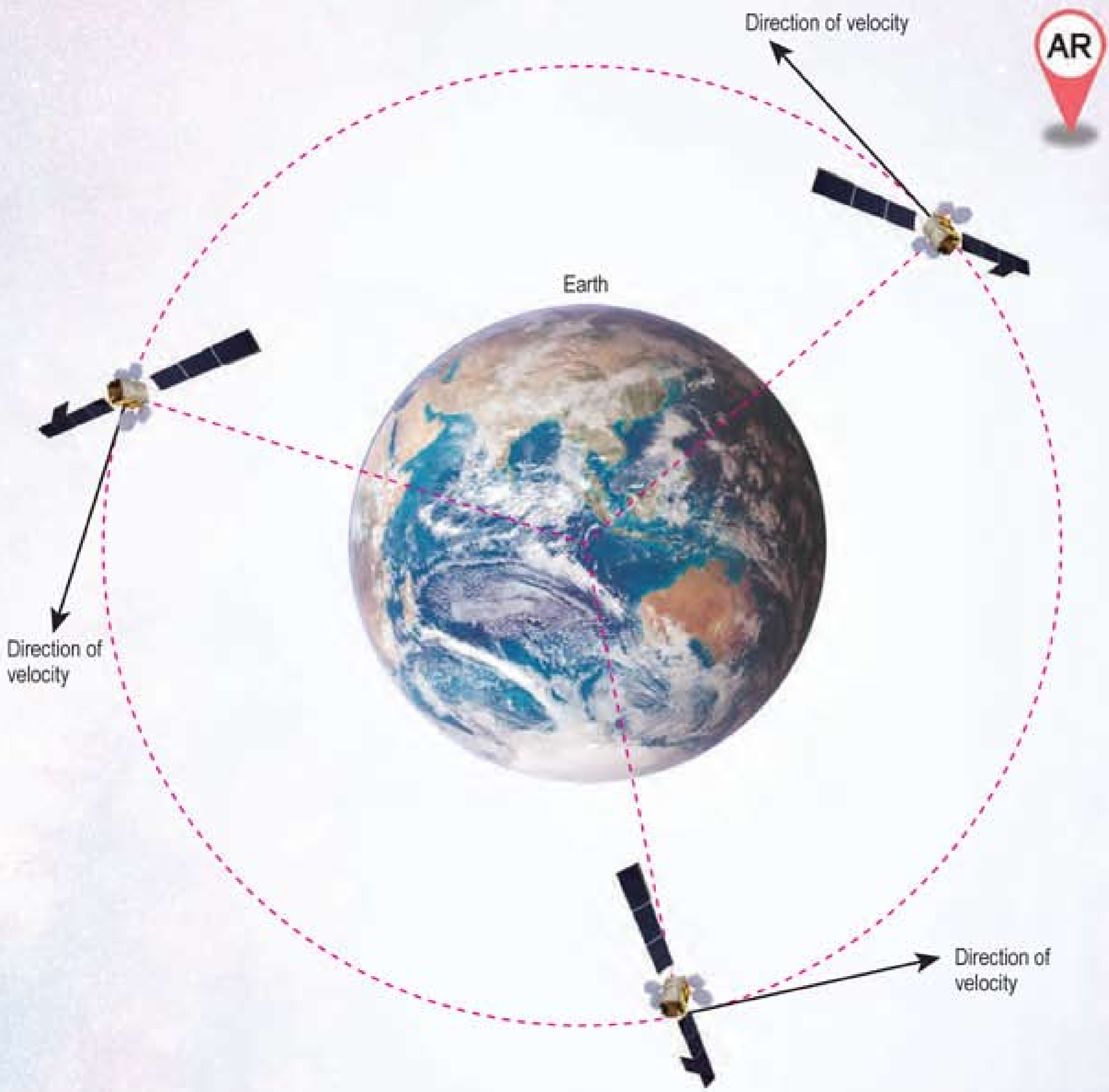


Figure 3.13 *Satellite in circular motion*

An object in circular motion always experiences changes in the direction of its motion even though its speed is fixed. In Chapter 2, you learnt that a force is required to change the direction of motion of a body. What force acts on a body which is in a circular motion?

SMART INFO

When a body moves in a circle at uniform speed, the body is said to be in uniform circular motion.

Activity 3.9

Aim: To understand centripetal force using Centripetal Force Kit

Apparatus: Centripetal Force Kit (a plastic tube, rubber stopper, slotted weight holder, three 50 g slotted weights, crocodile clip, thick string) and ruler

Instructions:

1. Set up the apparatus as shown in Figure 3.14 for circular motion of radius, r = 50 cm. The total mass of the slotted weights and its holder is 100 g.

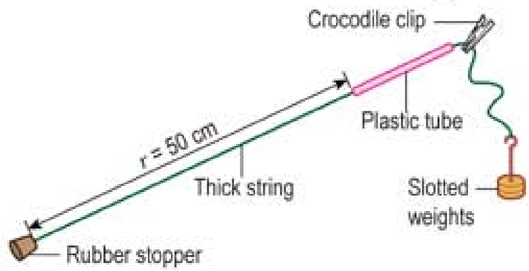


Figure 3.14

SMART INFO

The slotted weights cause a tension in the string that acts as centripetal force during the circular motion of the rubber stopper.

2. Hold the plastic tube in your right hand and the slotted weights in your left hand. Rotate the rubber stopper at a constant speed in a horizontal circle above your head as shown in Figure 3.15. Make sure that the crocodile clip stays at a distance of about 1 cm from the lower end of the plastic tube so that the radius is fixed.

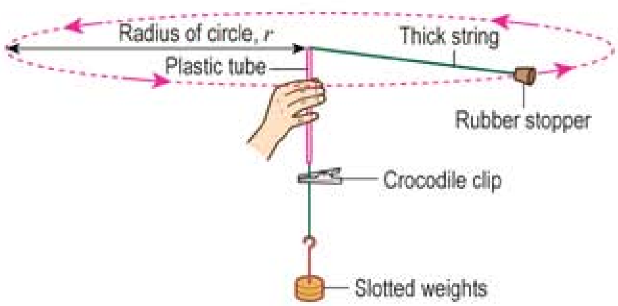


Figure 3.15

Video demonstrating the use of Centripetal Force Kit

http://bt.sasbadi.com/p4089

3. Release the slotted weights and continue rotating the rubber stopper. Observe the speed of motion of the rubber stopper.
4. Repeat steps 1 to 3 with a total mass of 200 g of slotted weights. Compare the speed of motion of the rubber stopper with the speed of motion in step 3.
5. Repeat step 4. When the rubber stopper is rotating, pull the lower end of the string downwards so that the rubber stopper rotates with a decreasing radius. Feel how the tension in the string acting on your left hand changes.

Discussion:

1. When the rubber stopper makes a circular motion, the stretched string exerts a force on the rubber stopper. What is the direction of the force?
2. What is the relationship between the speed of the rubber stopper and the centripetal force?
3. How does the centripetal force change when the rubber stopper makes a circular motion with a smaller radius?

3.1.5

For a body in circular motion, a force acts on the body in a direction towards the centre of the circle. This force is called **centripetal force**.

Figure 3.16 shows the tension in the rope that acts as the centripetal force for the motion of the rubber stopper. The magnitude of the centripetal force depends on the mass of the body, the linear speed and the radius of the circle. Centripetal force can be calculated using the formula:

$F = \frac{mv^2}{r}$, where F = centripetal force
m = mass
v = linear speed
r = radius of circle

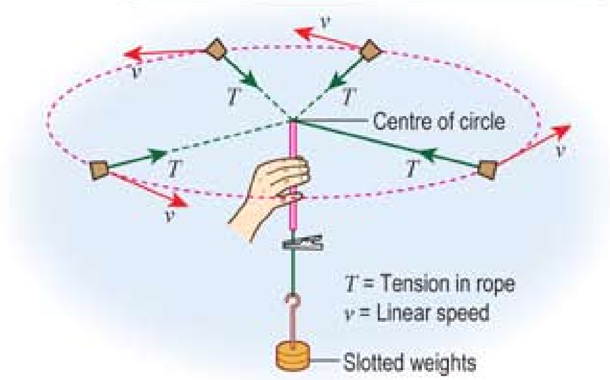


Figure 3.16 Tension in the string acting as centripetal force

SMART INFO

When a body is rotated at a certain uniform speed with the string almost horizontal, the effect of gravitational force on the circular motion of the body can be ignored. Though the speed is uniform, the direction of motion of the body keeps changing.

SMART INFO

Linear speed shows how fast a body moves in a circular motion.

Example 1

Figure 3.17 shows a hammer throw athlete swinging an iron ball in a horizontal circle before releasing it. What is the centripetal force that acts on the iron ball when the iron ball is moving at a speed of 20 m s^{-1}?

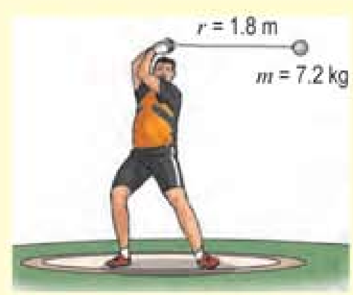


Figure 3.17

Solution:

Step 1
List the given information with symbols.

m = 7.2 kg
v = 20 m s^{-1}
r = 1.8 m

Step 2
Identify and write down the formula used.

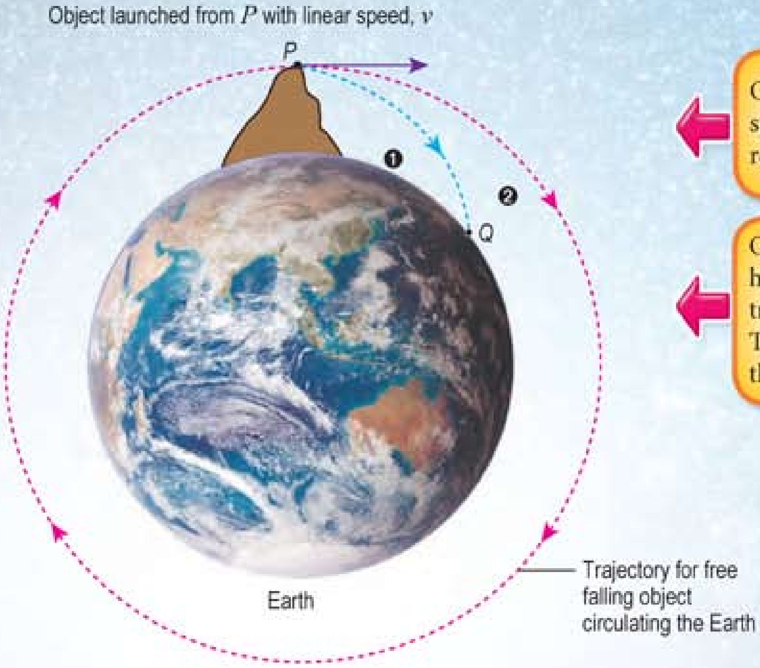
$F = \frac{mv^2}{r}$

Step 3
Make numerical substitution into the formula and perform the calculations.

Centripetal force, $F = \frac{7.2 \times 20^2}{1.8}$
$= 1\ 600$ N

Can a satellite orbit the Earth without being driven by a rocket engine? The possibility of such movement was predicted by Isaac Newton in the 17^{th} century as shown in Figure 3.18.

AR



Object launched with low linear speed will follow trajectory ❶ and reaches the Earth at Q.

Object launched with sufficiently high linear speed will follow trajectory ❷ circulating the Earth. The object will not return to the Earth.

Simulation of prediction of Isaac Newton

http://bt.sasbadi.com/p4091b

Figure 3.18 *Prediction of Isaac Newton*

The prediction of Newton is now a reality with so many man-made satellites orbiting around the Earth without being driven by any thrust. Satellites always experience gravitational force acting towards the centre of the Earth. The gravitational force on satellites acts as centripetal force.

By comparing the formula for force, $F = ma$

and formula for centripetal force, $F = \frac{mv^2}{r}$, we obtain:

Centripetal acceleration, $a = \frac{v^2}{r}$,

where v = linear speed of satellite

r = radius of the orbit of satellite

INTEGRATION OF HISTORY

Although Isaac Newton did not have the facilities to carry out simulation or experiment, he was able to visualise the experiment on the movement of bodies around the Earth. His original sketch is shown below.

3.1.5

Example 1

Figure 3.19 shows a weather satellite orbiting the Earth at a height, h = 480 km. Linear speed of the satellite is 7.62×10^3 m s^{-1}. The radius of the Earth, R is $R = 6.37 \times 10^6$ m. What is the centripetal acceleration of the satellite?

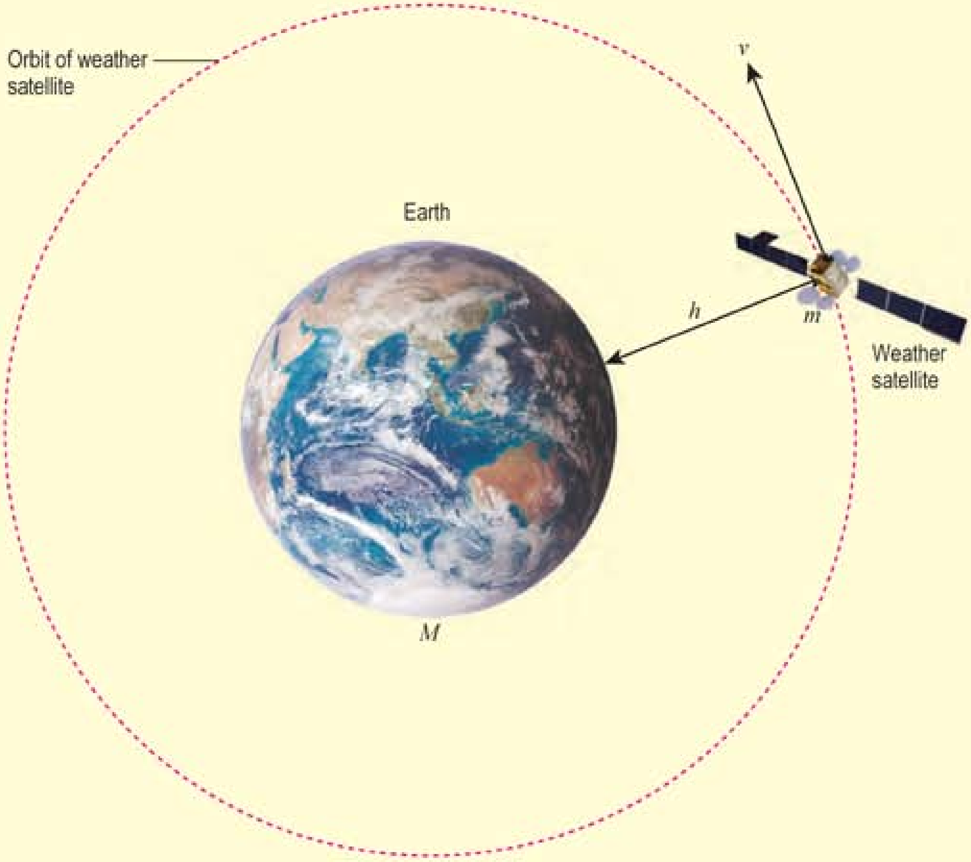


Figure 3.19

Solution:

Step ❶
List the given information in symbols.

Height of satellite, h = 480 km
= 480 000 m
Linear speed of satellite, $v = 7.62 \times 10^3$ m s^{-1}
Radius of the Earth, $R = 6.37 \times 10^6$ m

Step ❷
Identify and write down the formula used.

$$a = \frac{v^2}{r}$$

Step ❸
Substitute numerical values into the formula and perform the calculations.

$$a = \frac{v^2}{(R + h)}$$
$$= \frac{(7.62 \times 10^3)^2}{(6.37 \times 10^6 + 480\ 000)}$$
$$= 8.48 \text{ m s}^{-2}$$

Mass of the Earth and the Sun

Formula for the mass of the Earth and the Sun can be derived by using the formula of Newton's Universal Law of Gravitation and the formula for centripetal force.

Activity 3.10

Algorithms CPS

Aim: To determine the mass of the Earth and the Sun

Instructions:

1. Observe Figure 3.20.
2. Figure 3.20 shows the orbit of the Moon around the Earth.

SMART INFO

Circumference of a circle with radius r, is $2\pi r$.

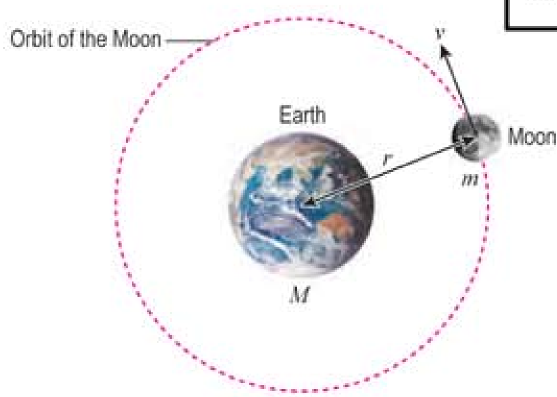


Figure 3.20

M = mass of the Earth
m = mass of the Moon
r = radius of the Moon's orbit
T = period of revolution of the Moon around the Earth
v = linear speed of the Moon

3. Discuss and complete the boxes below.

Distance travelled by the Moon when making one complete orbit around the Earth	=	
Linear speed of the Moon, v	=	$\frac{\text{Distance}}{\text{Time}}$
v	=	

4. Scan the QR code, download and print Figure 3.21 from the website given. Complete it to determine the formula for the mass of the Earth.

Download Figure 3.21

http://bt.sasbadi.com/p4094

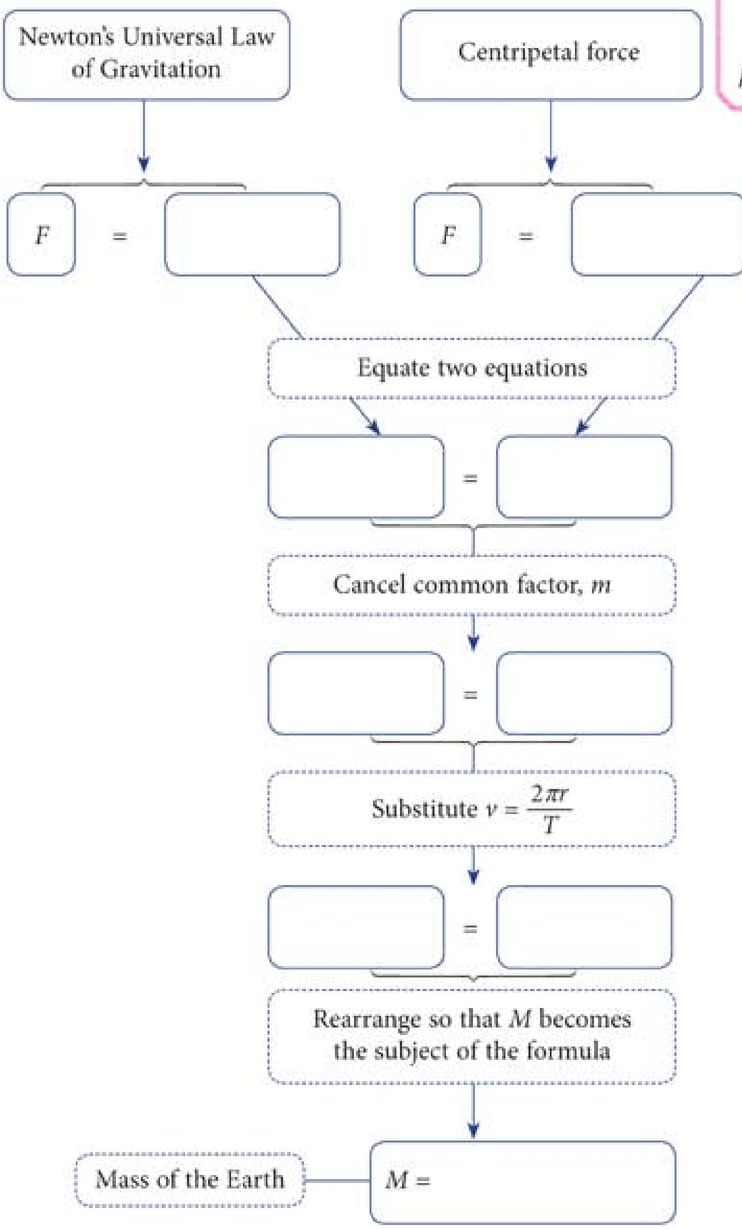


***Figure 3.21** Determining formula for the mass of the Earth*

Discussion:

1. What is the formula to determine the mass of the Earth?
2. Period of revolution of the Moon around the Earth, T is 2.36×10^6 s and radius of the Moon's orbit, r is 3.83×10^8 m. Calculate the mass of the Earth, M.
3. The Earth revolves around the Sun in a period of one year and the radius of the orbit is 1.50×10^{11} m. Calculate the mass of the Sun.

Formula used to determine the mass of the Earth or the Sun	$M = \frac{4\pi^2 r^3}{GT^2}$
Data needed to calculate the mass of the Earth	• radius of the orbit of any satellite or the Moon • period of revolution
Data needed to calculate the mass of the Sun	• radius of the orbit of any planet • period of revolution of planet concerned

Figure 3.22 *Formula and data used to calculate the mass of the Earth and the Sun*

Formative Practice 3.1

1. State Newton's Universal Law of Gravitation.
2. State two factors which influence the magnitude of the gravitational force between two bodies.
3. A piece of space junk of mass 24 kg is at a distance of 7.00×10^6 m from the centre of the Earth. What is the gravitational force between the space junk and the Earth?
[$G = 6.67 \times 10^{-11}$ N m^2 kg^{-2}, mass of the Earth = 5.97×10^{24} kg]
4. A weather satellite orbits the Earth at a height of 560 km. What is the value of gravitational acceleration at the position of the satellite?
[$G = 6.67 \times 10^{-11}$ N m^2 kg^{-2}, mass of the Earth = 5.97×10^{24} kg, radius of the Earth = 6.37×10^6 m]
5. A man-made satellite of mass 400 kg orbits the Earth with a radius of 8.2×10^6 m. Linear speed of the satellite is 6.96×10^3 m s^{-1}. What is the centripetal force acting on the satellite?
6. Figure 3.23 shows Mercury orbiting the Sun with a radius of 5.79×10^{10} m and a period of revolution of 7.57×10^6 s. Calculate the mass of the Sun.

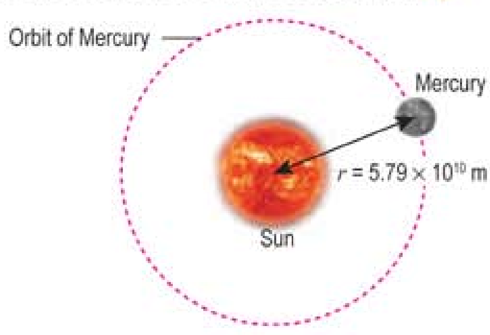


Figure 3.23

3.2 Kepler's Laws

Kepler's First, Second and Third Laws

When you were in Form 3, you knew about Kepler, a German astronomist, mathematician and astrologist who modified the heliocentric model according to Kepler's Law. Do you know that there are three Kepler's Laws? Let us get to know these three laws.

INTEGRATION OF HISTORY

Johannes Kepler worked as an assistant to astronomist Tycho Brahe. His strong determination motivated him to study Brahe's astronomical data. Finally, Kepler succeeded in formulating three laws that describe the movement of planets around the Sun.

Kepler's First Law	All planets move in elliptical orbits with the Sun at one focus (Law of Orbits)

Carry out Activity 3.11 to get a clear picture regarding Kepler's first law.

Activity 3.11

Aim: To sketch the shape of an ellipse based on the concept of dual foci of ellipse
Materials: Pencil, 20 cm thread, two thumbtacks, A4 paper, softboard and cellophane tape

Instructions:

1. Scan the QR code and print the template from the website given on a piece of A4 paper. Place it on a softboard.
2. Stick the thumbtacks at points F_1 and F_2 on the softboard.
3. Tie two ends of the thread to the two thumbtacks respectively.
4. Tighten the thread with the tip of a pencil as shown in Figure 3.24.
5. Move the pencil from the major axis to the left of F_1 to the major axis to the right of F_2 to sketch half an ellipse.
6. Repeat step 5 below major axis to obtain the shape of a complete ellipse.
7. Remove the thumbtacks and thread.
8. Draw a small circle to represent the Sun at F_1.
   Draw a small circle to represent the Earth on the circumference of the ellipse.

Template Activity 3.11

http://bt.sasbadi.com/p4096

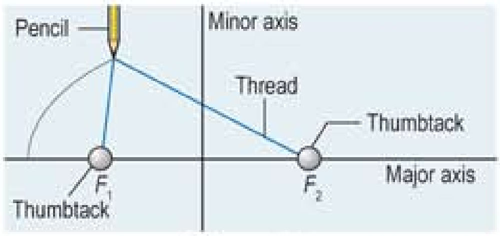


Figure 3.24

Discussion:

1. Describe how the distance between the Earth and the Sun changes when the Earth makes a complete orbit around the Sun.
2. Discuss how the shape of the Earth's orbit would be if the major axis is almost as long as the minor axis.

The planets in the Solar System have **elliptical** shaped orbits. Figure 3.25 shows the Sun always stays on a focus of the ellipse. The **major axis** is longer than the **minor axis**. Most orbits of the planets in the Solar System have major axis and minor axis of almost the same length. As such, the shape of the elliptical orbit of the planets in the Solar System is almost round. Planets can be assumed to make circular motion around the Sun. The **radius of orbit** is the average value of the distance between the planet and the Sun.

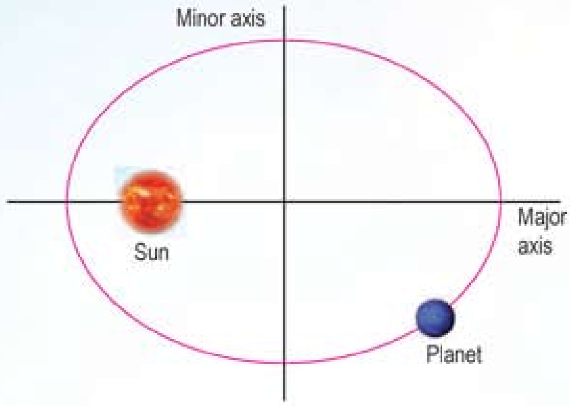


***Figure 3.25** Orbit of planet around the Sun*

Kepler's Second Law	A line that connects a planet to the Sun sweeps out equal areas in equal times (Law of Areas).

Observe Figure 3.26. If a planet takes the same amount of time to move from *A* to *B* and from *C* to *D*, the area *AFB* is the same as the area *CFD*. Distance *AB* is longer than distance *CD*. This means the planet is moving at a higher linear speed from *A* to *B* than from *C* to *D*.

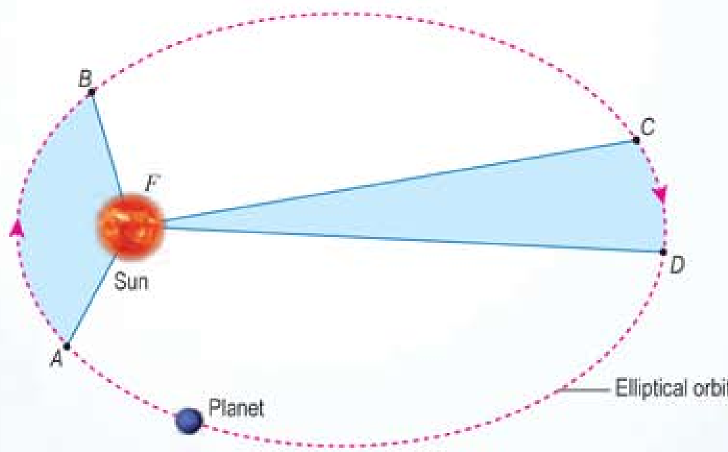


***Figure 3.26** Motion of planet in its orbit*

Kepler's Third Law	The square of the orbital period of any planet is directly proportional to the cube of the radius of its orbit (Law of Periods).

Mathematically,
$T^2 \propto r^3$
T = orbital period of a planet
r = radius of orbit

A planet which orbits with a larger radius has a longer orbital period. As such, planets which are further from the Sun take a longer time to complete one orbit around the Sun.

For example, the Earth takes 1 year to make one complete orbit while Saturn takes 29.5 years. Figure 3.27 shows the orbits and orbital periods of planets.

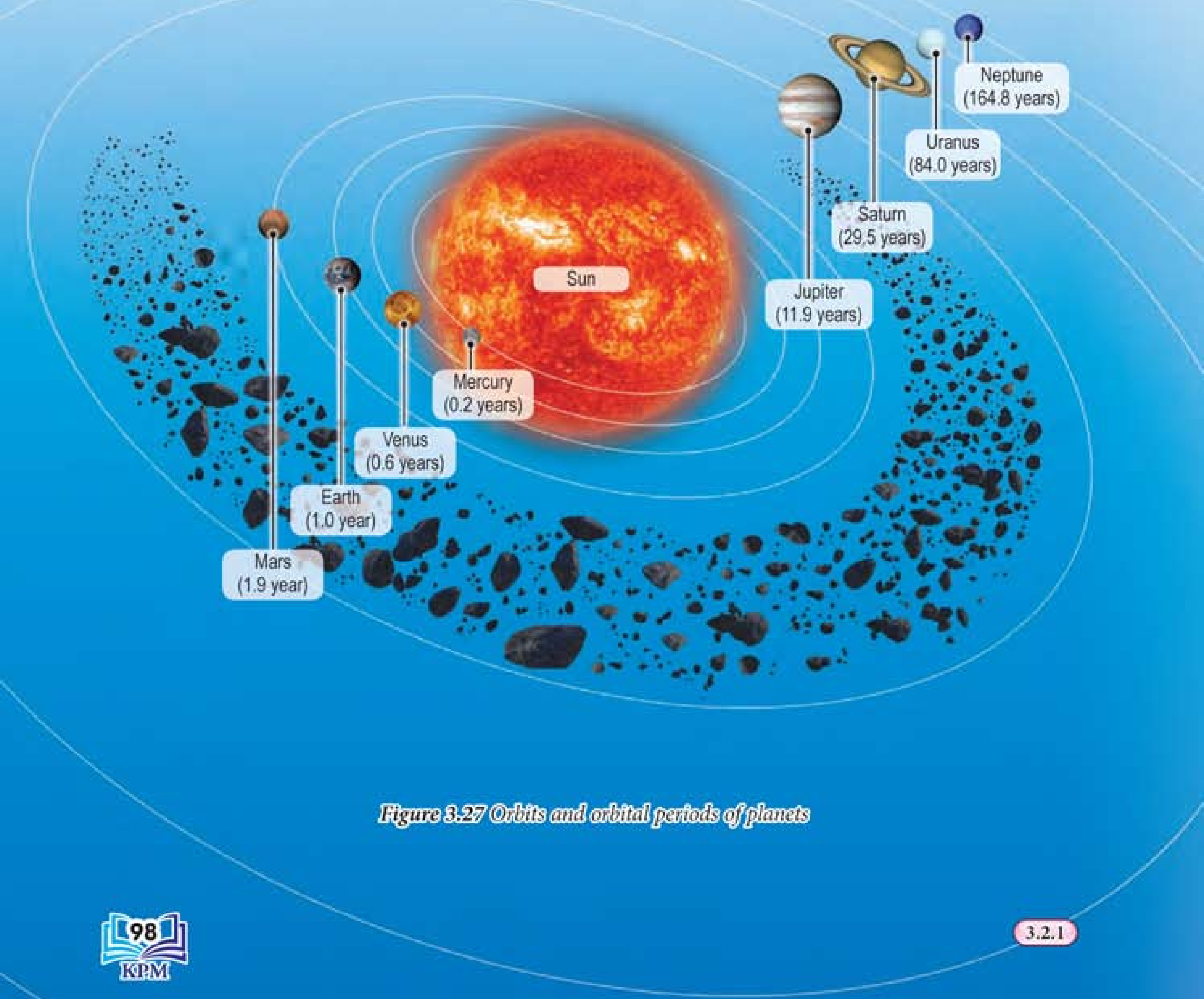


Figure 3.27 Orbits and orbital periods of planets

Kepler's third law can be formulated using **Newton's Universal Law of Gravitation** and concept of **circular motion**. Planets make circular motions around the Sun. The **centripetal force** is the same as the gravitational force between the Sun and the planet. Observe Figure 3.28 which shows the orbit of a planet around the Sun.

Assuming that the orbit of the planet around the Sun is circular, we can derive the relationship between the orbital period of the planet and the radius of the orbit as in Kepler's third law.

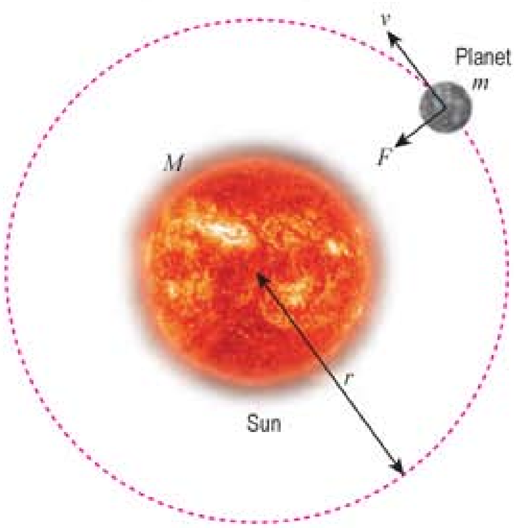


Figure 3.28 Orbit of a planet

Mass of the Sun = M
Mass of the planet = m
Radius of orbit = r
Gravitational force = F
Linear speed of planet = v
Orbital period = T

Gravitational force acting on the planet, $F = \frac{GMm}{r^2}$

The gravitational force acts as a centripetal force for the planet to make circular motion around the Sun.

Centripetal force, $F = \frac{mv^2}{r}$

Therefore,
Centripetal force = Gravitational force

$$\frac{mv^2}{r} = \frac{GMm}{r^2}$$

$$v^2 = \frac{GM}{r} \text{ .... [1]}$$

Linear speed of planet, $v = \frac{\text{Distance travelled in one complete orbit}}{\text{Orbital period}}$

$$= \frac{2\pi r}{T} \text{ .... [2]}$$

Substitute [2] into [1]:

$$\left(\frac{2\pi r}{T}\right)^2 = \frac{GM}{r}$$

$$T^2 = \left(\frac{4\pi^2}{GM}\right)r^3$$

SMART INFO

Perimeter of orbit = $2\pi r$

As GM is constant, $T^2 \propto r^3$

$T^2 \propto r^3$ is Kepler's third law.

3.2.2

Figure 3.29 shows the formulation of Kepler's third law. When Kepler's third law is applied in the system of planets and the Sun, M is referred to as the mass of the Sun. Kepler's third law can also be applied to the system of satellites and the Earth, with M referring to the mass of the Earth.

Kepler's third law, that is $T^2 \propto r^3$ — can be formulated by equating:

- centripetal force, $F = \dfrac{mv^2}{r}$
- gravitational force, $F = \dfrac{GMm}{r^2}$

$v = \dfrac{2\pi r}{T}$ — As such — the relationship between orbital period of planet, T and radius of its orbit, r is

$$T^2 = \left(\frac{4\pi^2}{GM}\right) r^3$$

$$T^2 = kr^3$$

where the constant, $k = \dfrac{4\pi^2}{GM}$

***Figure 3.29** Formulating Kepler's Third Law*

Solving Problems Using Kepler's Third Law Formula

From Kepler's third law, relationship between orbital period, T and radius of orbit, r is

$$T^2 = \left(\frac{4\pi^2}{GM}\right) r^3$$

Compare two planets.

For planet 1, $T_1^2 = \left(\dfrac{4\pi^2}{GM}\right) r_1^3$ .... (1)

For planet 2, $T_2^2 = \left(\dfrac{4\pi^2}{GM}\right) r_2^3$ .... (2)

(1) ÷ (2) gives $\dfrac{T_1^2}{T_2^2} = \dfrac{r_1^3}{r_2^3}$

The equation $\dfrac{T_1^2}{T_2^2} = \dfrac{r_1^3}{r_2^3}$ can be used to calculate the orbital period, T or radius of orbit, r.

Example 1

Figure 3.30 shows the planets, Earth and Mars, orbiting the Sun.

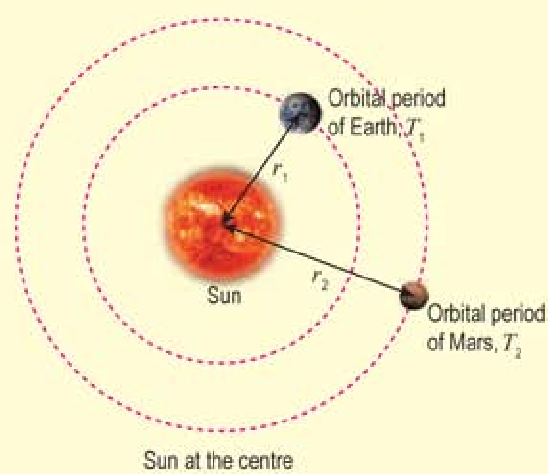


Figure 3.30

3.2.2 3.2.3

(a) The radius of the orbit of planet Mars can be determined by comparing the orbit of Mars with the orbit of the Earth. What information is needed to determine the radius of the orbit of Mars?

(b) The radius of the orbit of the Earth is 1.50×10^{11} m, orbital period of the Earth and Mars is 1.00 year and 1.88 years respectively. Calculate the radius of the orbit of Mars.

Solution:

(a) Radius of the orbit of the Earth, orbital period of the Earth and orbital period of Mars.

(b)

Step ❶
List the given information in symbols.

Radius of orbit of the Earth, $r_1 = 1.50 \times 10^{11}$ m
Radius of orbit of Mars $= r_2$
Orbital period of the Earth, $T_1 = 1.00$ years
Orbital period of Mars, $T_2 = 1.88$ years

Step ❷
Identify and write down the formula used.

$$\frac{T_1^2}{T_2^2} = \frac{r_1^3}{r_2^3}$$

Step ❸
Substitute numerical values into the formula and perform the calculations.

$$\frac{1.00^2}{1.88^2} = \frac{(1.50 \times 10^{11})^3}{r_2^3}$$

$$r_2^3 = \frac{(1.50 \times 10^{11})^3 \times 1.88^2}{1.00^2}$$

$$r_2 = \sqrt[3]{\frac{(1.50 \times 10^{11})^3 \times 1.88^2}{1.00^2}}$$

$$= 2.28 \times 10^{11} \text{ m}$$

SMART INFO

The equation $\frac{T_1^2}{T_2^2} = \frac{r_1^3}{r_2^3}$ involves the orbital period of a planet divided by the orbital period of another planet. The same unit needs to be used for both periods.

Example 2

Figure 3.31 shows that a research satellite needs to orbit at a height of 380 km to capture clear images of the surface of the Earth. What is the orbital period of the satellite?

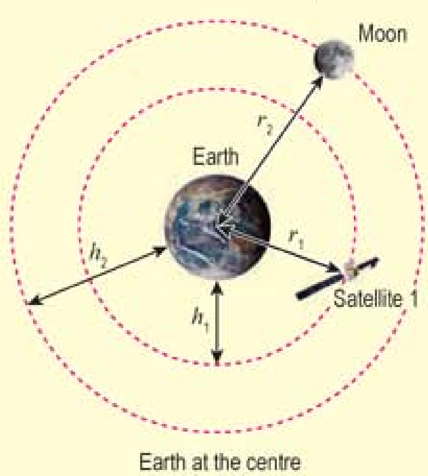


Figure 3.31

[Radius of the orbit of the Moon = 3.83×10^8 m, orbital period of the Moon = 655.2 hours]

Solution:

Radius of orbit of the satellite, $r_1 = (6.37 \times 10^6) + (380 \times 10^3)$
$= 6.75 \times 10^6$ m

Radius of orbit of the Moon, $r_2 = 3.83 \times 10^8$ m

Orbital period of the satellite $= T_1$

Orbital period of the Moon, $T_2 = 655.2$ hours

$$\frac{T_1^2}{T_2^2} = \frac{r_1^3}{r_2^3}$$

$$\frac{T_1^2}{655.2^2} = \frac{(6.75 \times 10^6)^3}{(3.83 \times 10^8)^3}$$

$$T_1^2 = \frac{(6.75 \times 10^6)^3 \times 655.2^2}{(3.83 \times 10^8)^3}$$

$$T_1 = \sqrt{\frac{(6.75 \times 10^6)^3 \times 655.2^2}{(3.83 \times 10^8)^3}}$$

$= 1.53$ hours

Formative Practice 3.2

1. State Kepler's first law.

2. (a) State Kepler's second law.
   (b) Figure 3.32 shows the orbit of a planet around the Sun. Compare the linear speed of the planet at positions X, Y and Z.

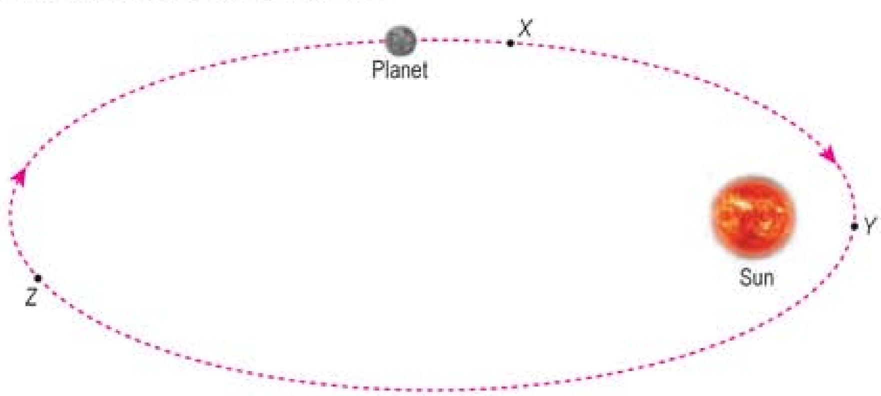


Figure 3.32

3. (a) State Kepler's third law.
   (b) At what height should a satellite be if the satellite is required to orbit the Earth in a period of 24 hours?
   [Orbital period of the Moon = 27.3 days, radius of orbit of the Moon = 3.83×10^8 m]