CHAPTER 5

Congruency, Enlargement and Combined Transformations

What will you learn?

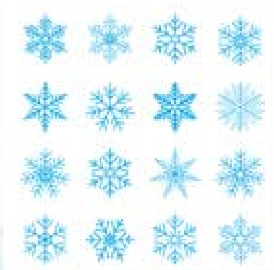
- Congruency
- Enlargement
- Combined Transformation
- Tessellation

Why study this chapter?

Astronomers use the telescope to observe the surface of a planet. The surface of the distant planet appears to be magnified through the telescope. However, the resulting image is inverted with its original object. Reflection is applied to get the actual picture.

Do you know?

Johannes Kepler (1571-1630) was a German mathematician and astronomer who had documented the study of tessellations in 1619. He used the concept of tessellation to explore and explain the structure of snowflakes.



For more information:

bit.do/DoYouKnowChap5

WORD BANK

scale factor	*faktor skala*
congruency	*kekongruenan*
similarity	*keserupaan*
reflection	*pantulan*
enlargement	*pembesaran*
rotation	*putaran*
tessellation	*teselasi*
transformation	*transformasi*
translation	*translasi*

Terengganu Drawbridge which is located in Kuala Terengganu is a new landmark for Terengganu in 2019. It is the longest drawbridge in the world with a length of 638 metres, connecting Kuala Terengganu and Kuala Nerus at the river mouth of Sungai Terengganu. The unique feature of the bridge is that it can be lifted in the centre to facilitate the passage of large ships beneath it. Observe the structure of the bridge. Which parts have congruent shapes?

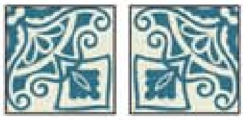
5.1 Congruency

How to differentiate between congruent and non-congruent shapes?

Learning Standard

Differentiate between congruent and non-congruent shapes based on sides and angles.

Observe each of the diagrams below. What is your observation regarding the size and shape of each pair of objects shown?


Two pieces of tiles


Petronas Twin Towers


The front and rear tyres of a car

The pairs of objects in the diagrams above have the same size and shape regardless of their position and arrangement. The pairs of objects are **congruent**.

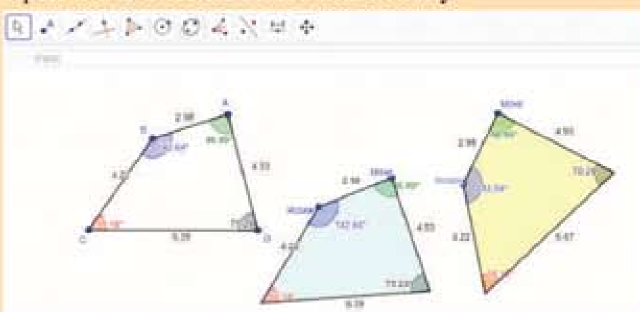
What are the characteristics used to identify that two objects are congruent?

MIND MOBILISATION 1 Pairs

Aim: To explore the congruent shapes based on sides and angles.

Steps:

1. Open the file GGB501 for this activity.

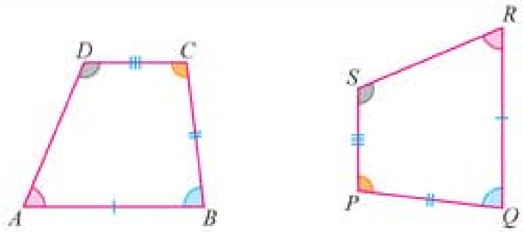


Scan the QR code or visit bit.do/GGB501BI to obtain the GeoGebra file for this activity.

2. Observe the measurements of the sides and angles for each of the quadrilaterals displayed.
3. Drag the points "Move" and "Rotate" to move the blue quadrilateral to overlap quadrilateral *ABCD*. Does the blue quadrilateral overlap quadrilateral *ABCD* completely? What is your observation on the measurements of the sides and angles?
4. Repeat step **3** for the yellow quadrilateral.

Discussion:
What is your conclusion about congruent shapes based on sides and angles?

The results of Mind Mobilisation 1 show that two **congruent** polygons have the same measurement for the corresponding sides and angles.



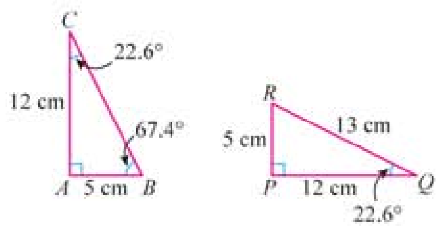
In the diagram above, quadrilaterals *ABCD* and *PQRS* are congruent with the conditions shown in the following table.

Lengths of corresponding sides	Sizes of corresponding angles
$AB = RQ$ $BC = QP$ $CD = PS$ $AD = RS$	$\angle BAD = \angle QRS$ $\angle ABC = \angle RQP$ $\angle BCD = \angle QPS$ $\angle ADC = \angle RSP$

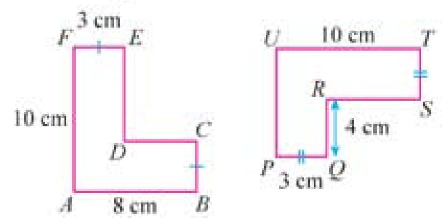
Example 1

Determine whether each pair of the following shapes are congruent.

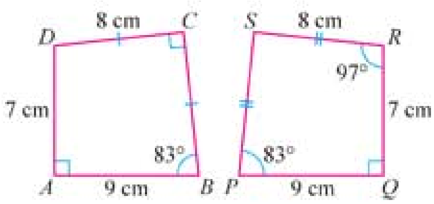
(a)



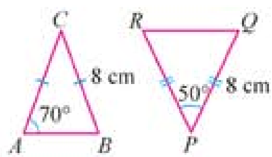
(b)



(c)



(d)



Solution:

(a) $BC = \sqrt{5^2 + 12^2}$ and $\angle PRQ = 180° - 90° - 22.6°$

$= 13$ cm $\qquad = 67.4°$

$= RQ \qquad = \angle ABC$

The measurements of all corresponding sides and angles are equal.
Hence, both shapes are congruent.

(b) $DC = 8 - 3$ or $PU = 4 + 3$
$= 5$ cm $= 7$ cm
$\neq RQ$ $\neq BA$

Hence, both shapes are not congruent.

(c) $\angle ADC = 360° - 90° - 83° - 90°$
$= 97°$
$= \angle QRS$

The measurements of all corresponding sides and angles are equal. Hence, both shapes are congruent.

(d) $\angle ACB = 180° - 70° - 70°$
$= 40°$
$\neq \angle QPR$

Hence, both shapes are not congruent.

Smart TIPS

If any corresponding side or angle of the first shape is not equal to the second shape, both shapes are not congruent.

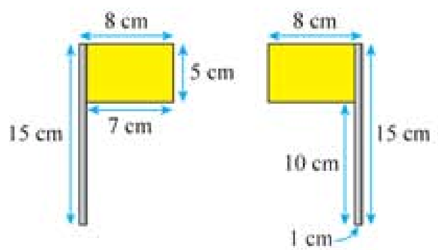
Info Bulletin

Triangles ABC and PQR which are congruent can be written as $\Delta ABC \cong \Delta PQR$.

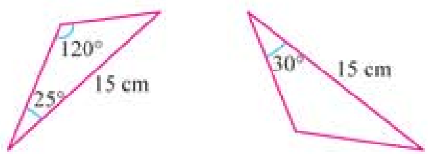
Self Practice 5.1a

1. Determine whether each pair of the following shapes are congruent.

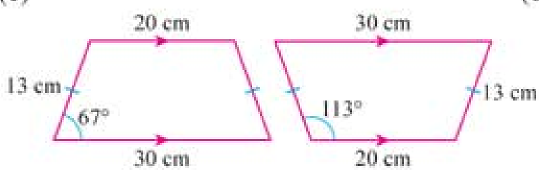
(a)



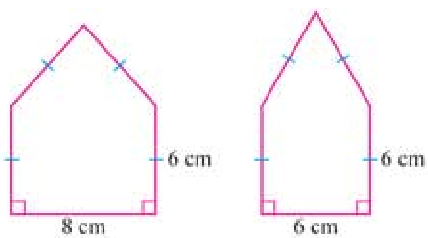
(b)



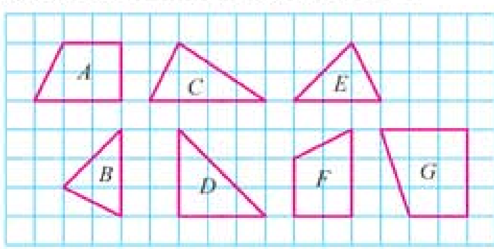
(c)



(d)

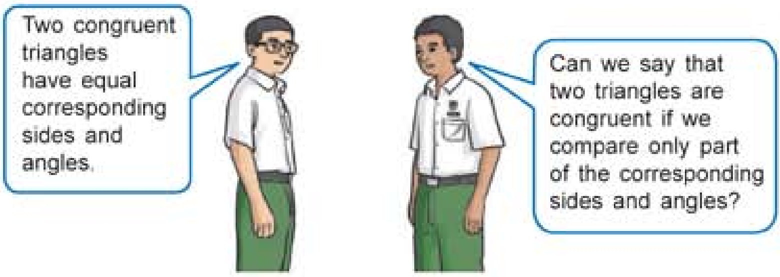


2. Determine pairs of congruent shapes in the diagram below.



CHAPTER 5

What are the characteristics of triangle congruency?



Learning Standard

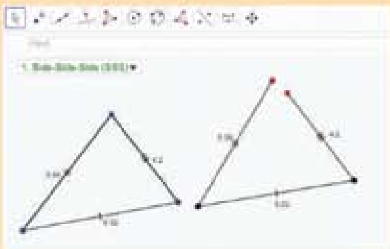
Make and verify the conjecture of triangle congruency based on sides and angles.

MIND MOBILISATION 2 Group

Aim: To make and verify the conjecture of triangle congruency.

Steps:

1. Open the file GGB502 for this activity.



Scan the QR code or visit bit.do/GGB502BI to obtain the GeoGebra file for this activity.

Scan the QR code or visit bit.do/WSChap5i to obtain the worksheet for this activity.

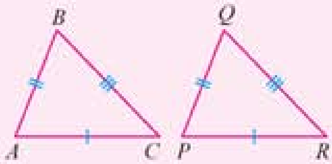
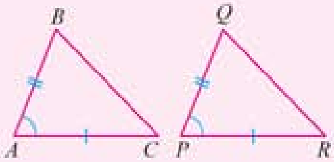
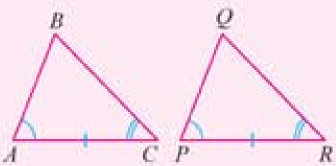
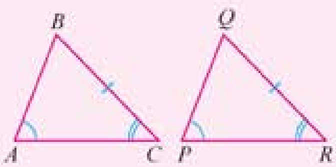
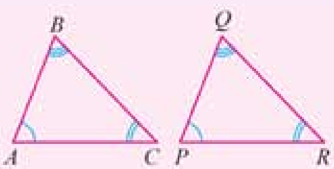
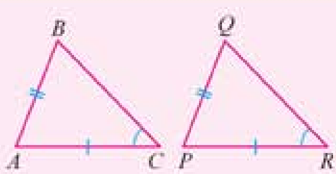
2. Select Side-Side-Side (SSS). Observe the measurements labelled on both diagrams.
3. Drag the red points to form a complete triangle. Drag the black points to change the position and orientation. Can you form congruent triangles? Can you find the non-congruent triangles?
4. Drag the blue points to change the shape of triangles and repeat step **3**.
5. Select Side-Angle-Side (SAS), Angle-Side-Angle (ASA), Angle-Angle-Side (AAS), Angle-Angle-Angle (AAA) and Side-Side-Angle (SSA) and repeat steps **2** to **4** for the exploration of other properties of congruent triangles.
6. Open the worksheet for this activity. Complete it based on your exploration.

	Can you form the congruent triangles?	Can you find the non-congruent triangles?	Conclusion
Side-Side-Side (SSS)			If ______, then the triangles are congruent.

Discussion:
What is your conclusion about triangle congruency based on the sides and angles?

CHAPTER 5

The results of Mind Mobilisation 2 show that congruent triangles have the following properties.

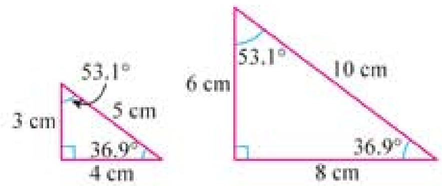
Properties of triangle congruency	
Side-Side-Side (SSS) • The length of corresponding sides are equal • $AC = PR$ $AB = PQ$ $BC = QR$	
Side-Angle-Side (SAS) • Two corresponding sides are equal in length and the size of the corresponding subtended angle between the two sides are equal • $AC = PR$ $\angle BAC = \angle QPR$ $AB = PQ$	
Angle-Side-Angle (ASA) • Two corresponding angles are equal and the length of the corresponding side between the two angles are equal • $\angle BAC = \angle QPR$ $AC = PR$ $\angle ACB = \angle PRQ$	
Angle-Angle-Side (AAS) • Two corresponding angles are equal and the length of one of the corresponding sides which does not lie between the two angles are equal • $\angle BAC = \angle QPR$ $\angle ACB = \angle PRQ$ $BC = QR$	
Angle-Angle-Angle (AAA) • All the three corresponding angles are equal • The area of the pair of triangles must be equal • $\angle BAC = \angle QPR$ $\angle ACB = \angle PRQ$ $\angle ABC = \angle PQR$	
Side-Side-Angle (SSA) • Two corresponding sides are equal in length and one of the corresponding angles which is not subtended between the two sides are equal • The area of the pair of triangles must be equal • $AC = PR$, $AB = PQ$, $\angle ACB = \angle PRQ$	

Example 2

Determine whether each pair of the following triangles satisfies the properties of triangle congruency. Justify your answer.

(a)

(b)



Solution:

(a) No. Although both triangles have two corresponding sides and one corresponding angle of the same measurement, the remaining side and angles are different in measurement. The areas are different.

(b) No. Although both triangles have three corresponding angles of the same measurement, all the corresponding sides are different in measurement. The areas are different.

Smart TIPS

Side-Side-Angle (SSA) and Angle-Angle-Angle (AAA) cannot be used to determine two triangles are congruent except the areas of the triangles are the same.

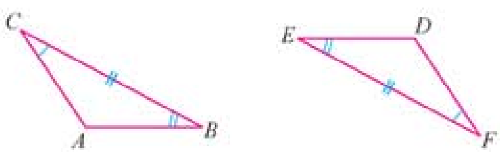
Example 3

The diagram below shows two congruent triangles.

It is given that the triangle congruence rule used to determine both triangles are congruent is Angle-Side-Angle (ASA). Complete the table below with the other characteristics involving Angle-Side-Angle (ASA).

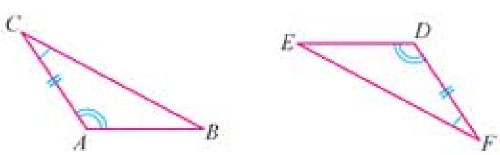
Angle	$\angle ACB = \angle DFE$
Side	
Angle	

Solution:



Angle	$\angle ACB = \angle DFE$
Side	$BC = EF$
Angle	$\angle ABC = \angle DEF$

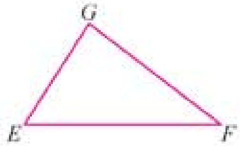
or



Angle	$\angle ACB = \angle DFE$
Side	$AC = DF$
Angle	$\angle BAC = \angle EDF$

Example 4

The diagram below shows a triangle *EFG*.



It is given that another triangle *KLM* has the same shape and size with triangle *EFG*. State the triangle congruence rule used to determine if both triangles are congruent.

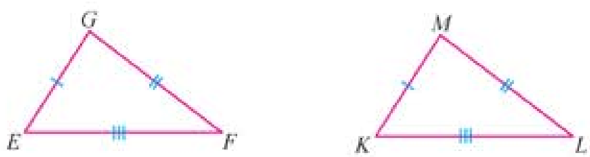
(a) $\angle EGF = \angle KML$, $\angle EFG = \angle KLM$, $EF = KL$
(b) $EG = KM$, $FG = LM$, $EF = KL$
(c) $EF = KL$, $\angle FEG = \angle LKM$, $EG = KM$

Solution:

(a)

Two corresponding angles and one corresponding side that does not lie between the two angles are given. Therefore, the triangle congruence rule is Angle-Angle-Side (AAS).

(b)



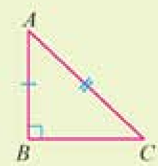
Three corresponding sides are given. Therefore, the triangle congruence rule is Side-Side-Side (SSS).

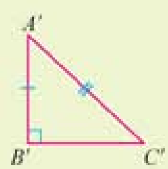
(c)

Two corresponding sides and one corresponding subtended angle between the two sides are given. Therefore, the triangle congruence rule is Side-Angle-Side (SAS).

Info Bulletin

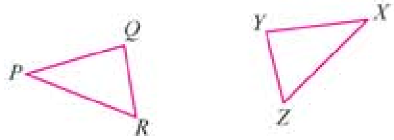
Right angle-Hypotenuse-Side (RHS) is also a special congruence property for right-angled triangle. If the hypotenuse and one of the corresponding non-hypotenuse sides of two right-angled triangles are equal in length, then the two triangles are congruent. This congruence property is based on Side-Side-Angle (SSA) rule.





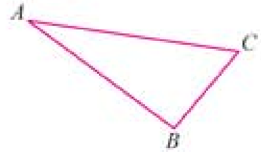
Self Practice 5.1b

1. The diagram below shows two triangles, *PQR* and *XYZ*.



It is given that both triangles *PQR* and *XYZ* are congruent. If $PQ = XY$, state the other characteristics involved if each of the following is used
(a) Side-Angle-Side
(b) Side-Side-Angle

2. The diagram on the right shows a triangle *ABC*. It is given that another triangle *PQR* is congruent with the triangle *ABC*. State the triangle congruence rule involved if
(a) $AB = PQ$, $BC = QR$ and $AC = PR$
(b) $AB = PQ$, $\angle ABC = \angle PQR$ and $\angle BAC = \angle QPR$

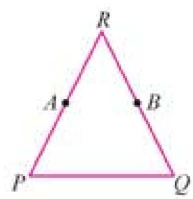


How to solve problems involving congruency?

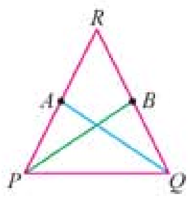
Learning Standard
Solve problems involving congruency.

Example 5

The diagram on the right shows an isosceles triangle *PQR* where $PR = QR$. *A* and *B* are the midpoints of the sides *PR* and *QR* respectively. Show that triangles *PBR* and *QAR* are congruent.



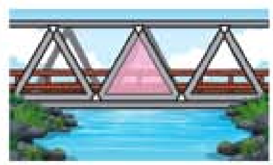
Solution:



Based on the diagram on the left,
- $PR = QR$
- $\angle PRB = \angle QRA$
- $BR = AR$

Triangles *PBR* and *QAR* satisfy the properties of Side-Angle-Side (SAS). Therefore, triangles *PBR* and *QAR* are congruent.

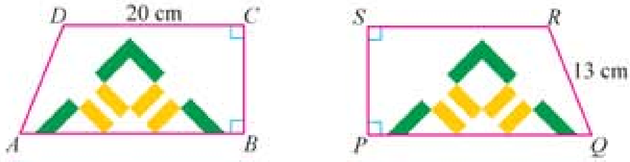
Application & Career



In the field of engineering, the concept of triangle congruence is used to build the bridge supports so that the bridge can be supported in a balanced state.

Example 6

Siva prepared two pieces of congruent trapezium shaped cards as shown in the diagram below. Each card is painted with half of a logo.



Siva put the two pieces of card together to form a pentagon with a complete logo. If the perimeter of the pentagon is 90 cm, calculate the area, in cm^2, of the pentagon.

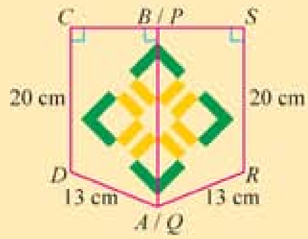
Solution:

Method 1

Understanding the problem

- Trapezium *ABCD* and trapezium *QPSR* are congruent.
- $AB = QP$, $BC = PS$, $CD = SR$, $AD = QR$.
- The perimeter of the pentagon formed is 90 cm.
- Calculate the area of the pentagon formed.

Devising a strategy



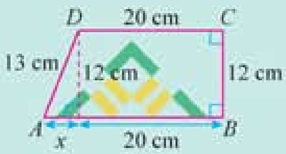
- $CS = 90\text{ cm} - AD - DC - QR - RS$
- Calculate the length of *AB*
- Area of pentagon
  $= 2 \times$ area of trapezium

Implementing the strategy

$CS = 90 - 13 - 20 - 13 - 20$
$\quad = 24\text{ cm}$

$CB = 12\text{ cm}$

$x = \sqrt{13^2 - 12^2}$
$\quad = 5\text{ cm}$



$AB = 20 + 5$
$\quad = 25\text{ cm}$

$\text{Area of pentagon} = 2 \times \frac{1}{2}(20 + 25)(12)$
$\quad = 540\text{ cm}^2$

Making a conclusion

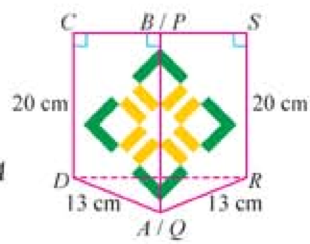
Hence, the area of the pentagon is 540 cm^2.

Method 2

Area of rectangle $CDRS = 24 \times 20$
$\quad = 480\text{ cm}^2$

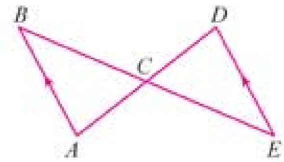
Area of triangle $DRA = \frac{1}{2}(24)(5)$
$\quad = 60\text{ cm}^2$

Area of pentagon = area of rectangle *CDRS* + area of triangle *DRA*
$\quad = 480 + 60$
$\quad = 540\text{ cm}^2$

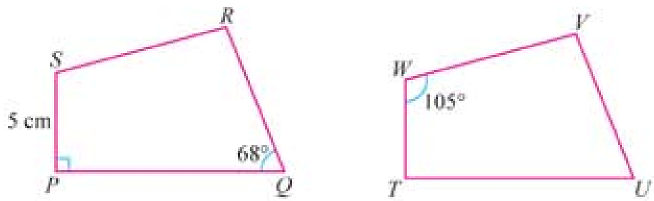


Self Practice 5.1c

1. In the diagram on the right, the lines *AB* and *DE* are parallel. *BE* and *AD* are straight lines. *C* is the midpoint of *AD*. Show that triangles *ABC* and *DEC* are congruent.



2. The diagram below shows two congruent quadrilaterals.



Given that the length of diagonal *WU* is 13 cm, calculate
(a) the length, in cm, of *PQ*,
(b) $\angle SRQ$.

3. Muizuddin divided his rectangular piece of land into two congruent sections as shown in the diagram on the right. Calculate
(a) the perimeter, in m, of each section of the land,
(b) the area, in m^2, of each section of the land.

5.2 Enlargement

What is the meaning of similarity of geometric objects?

Learning Standard
Explain the meaning of similarity of geometric objects.

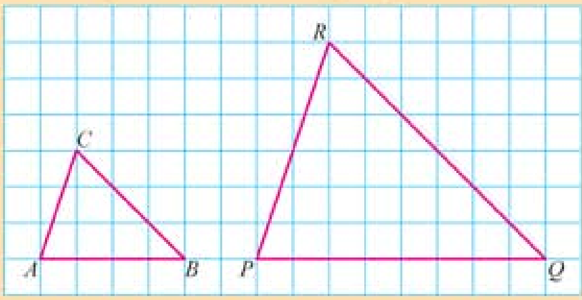
The photo below shows the house models which are **similar** to the real houses. This means that the house models have the **same shape** as the real houses despite the **different size**. What are the properties of two similar geometric objects?

MIND MOBILISATION 3 Pairs

Aim: To recognise the similarity of geometric objects.
Materials: Ruler and protractor

Steps:

1. By using ruler and protractor, measure the angles on each vertex and the length of each side of the two triangles in the following diagram.



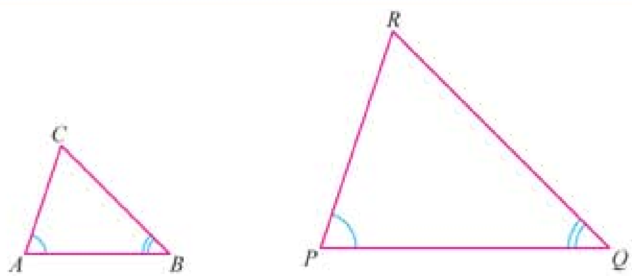
2. Complete the table below with your measurements.

Size of angle		Length of side		Ratio of the corresponding sides
Triangle ABC	Triangle PQR	Triangle ABC	Triangle PQR	
$\angle A =$	$\angle P =$	$AB =$	$PQ =$	$\frac{PQ}{AB} =$
$\angle B =$	$\angle Q =$	$BC =$	$QR =$	$\frac{QR}{BC} =$
$\angle C =$	$\angle R =$	$CA =$	$RP =$	$\frac{RP}{CA} =$

Discussion:
Both of the triangles are similar. State your conclusion about
(a) the corresponding angles of both triangles,
(b) the corresponding sides of both triangles.

The results of Mind Mobilisation 3 show that two geometric objects are **similar** when

- all the corresponding angles are equal, namely $\angle A = \angle P$, $\angle B = \angle Q$ and $\angle C = \angle R$
- all the ratios of corresponding sides are equal, namely $\frac{PQ}{AB} = \frac{QR}{BC} = \frac{RP}{CA}$



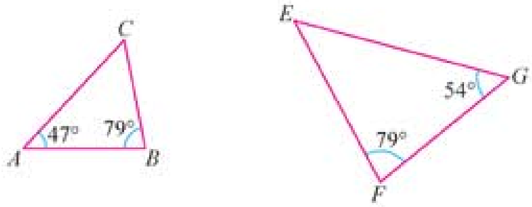
Smart TIPS

If all the corresponding sides of a pair of triangles are proportional, then all the corresponding angles are equal and vice versa.

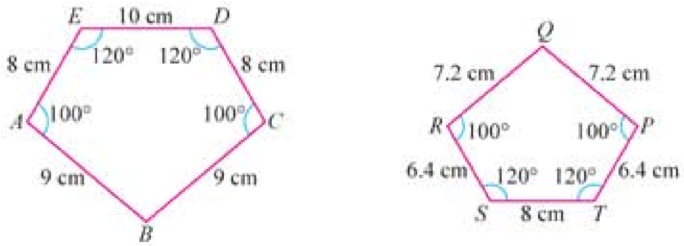
Example 7

Determine whether each pair of the following geometric objects are similar.

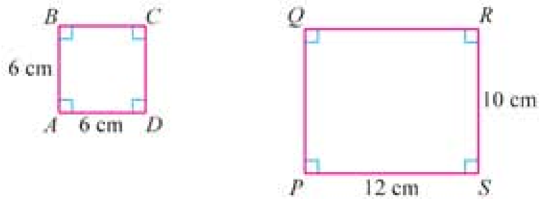
(a)



(b)



(c)



Solution:

(a) $\angle C = 180° - 47° - 79°$
$= 54°$
$= \angle G$
$\angle B = \angle F = 79°$
$\angle A = \angle E = 47°$

For a pair of triangles, all the corresponding sides are proportional when all the corresponding angles are equal. Hence, triangle *ABC* and triangle *EFG* are similar.

(b) All the corresponding angles are equal.

$$\frac{AB}{PQ} = \frac{BC}{QR} = \frac{9}{7.2} = \frac{5}{4}$$
$$\frac{AE}{PT} = \frac{CD}{RS} = \frac{8}{6.4} = \frac{5}{4}$$
$$\frac{ED}{TS} = \frac{10}{8} = \frac{5}{4}$$

All the ratios of the corresponding sides are equal. Hence, pentagon *ABCDE* and pentagon *PQRST* are similar.

(c) All the corresponding angles are equal.

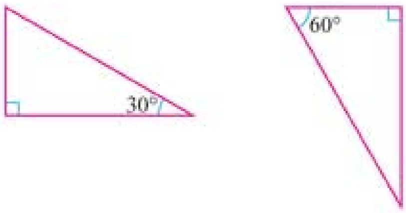
$$\frac{AB}{PQ} = \frac{DC}{SR} = \frac{6}{10} = \frac{3}{5}$$
$$\frac{AD}{PS} = \frac{BC}{QR} = \frac{6}{12} = \frac{1}{2}$$

The ratios of corresponding sides are not equal. Hence, quadrilateral *ABCD* and quadrilateral *PQRS* are not similar.

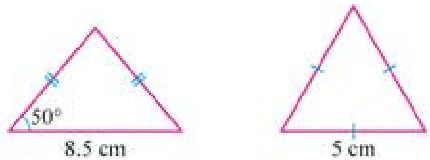
Self Practice 5.2a

1. Determine whether each pair of the following geometric objects are similar.

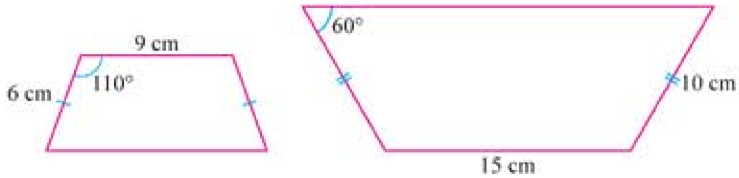
(a)



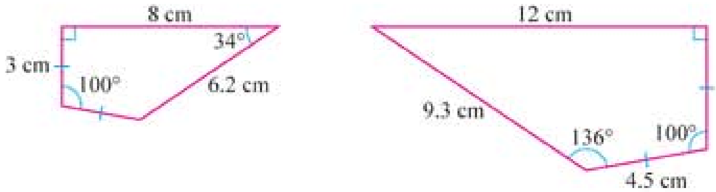
(b)



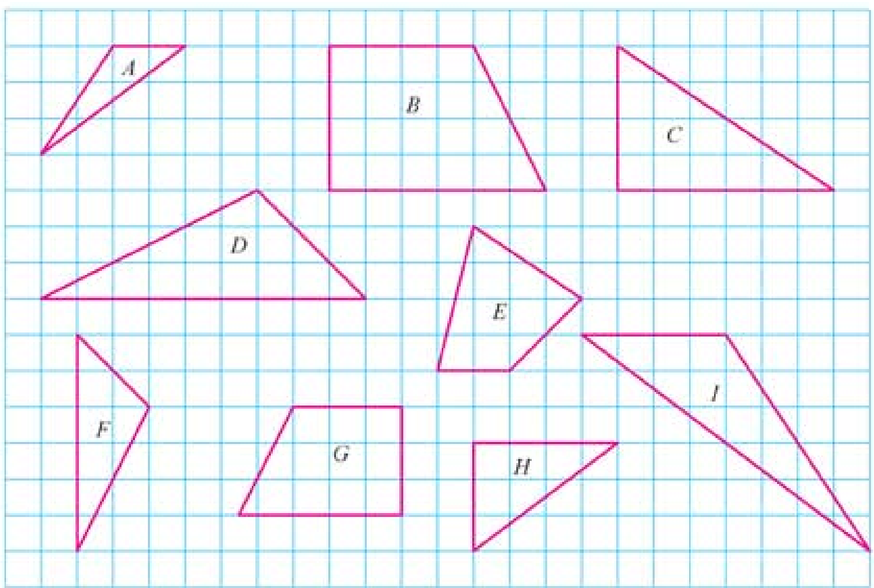
(c)



(d)



2. Determine pairs of similar objects in the diagram below.



What is the relationship between similarity and enlargement and how to describe enlargement?

Learning Standard

Make a connection between similarity and enlargement, hence describe enlargement using various representations.

A biologist uses a microscope to observe plant cells. The image produced through a microscope is thousands of times larger than its object. Do the object and the image produced satisfy the property of similarity?

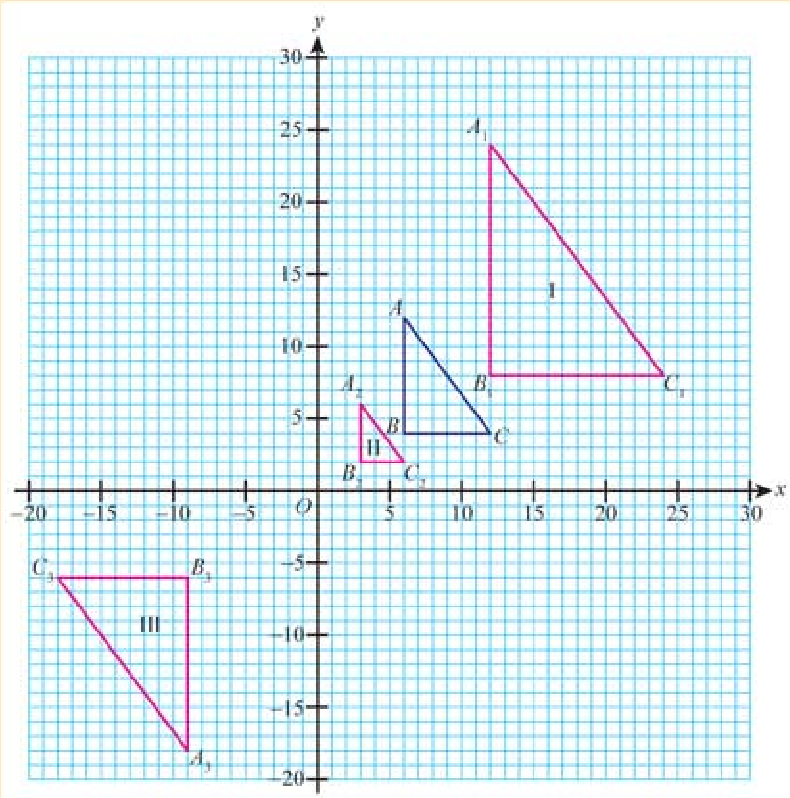
MIND MOBILISATION 4 Pairs

Aim: To make the connection between similarity and enlargement.

Materials: Ruler and protractor

Steps:

1. Observe the diagram below where $\Delta A_1B_1C_1$, $\Delta A_2B_2C_2$ and $\Delta A_3B_3C_3$ are the images of ΔABC under an enlargement.



2. Connect the corresponding points of all the images. Mark the intersection point between the three straight lines as point P.

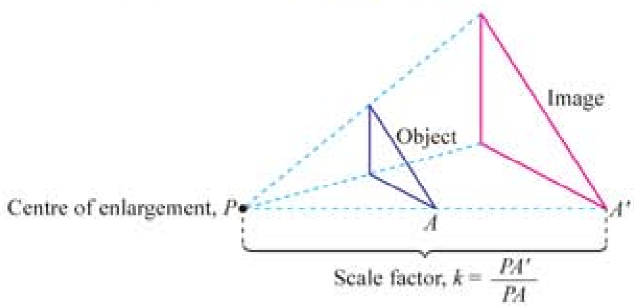
3. Measure the sides, in units, and angles for each triangle and complete the following table.

Triangle	Ratio of the corresponding sides	Ratio of the distance of vertex from point P	Equal corresponding angles? (Yes / No)	Similar with the triangle ABC? (Yes / No)
I	$\frac{A_1B_1}{AB} =$	$\frac{A_1P}{AP} =$		
	$\frac{B_1C_1}{BC} =$	$\frac{B_1P}{BP} =$		
	$\frac{A_1C_1}{AC} =$	$\frac{C_1P}{CP} =$		
II	$\frac{A_2B_2}{AB} =$	$\frac{A_2P}{AP} =$		
	$\frac{B_2C_2}{BC} =$	$\frac{B_2P}{BP} =$		
	$\frac{A_2C_2}{AC} =$	$\frac{C_2P}{CP} =$		
III	$\frac{A_3B_3}{AB} =$	$\frac{A_3P}{AP} =$		
	$\frac{B_3C_3}{BC} =$	$\frac{B_3P}{BP} =$		
	$\frac{A_3C_3}{AC} =$	$\frac{C_3P}{CP} =$		

Discussion:
1. What can you say about the ratio of the corresponding sides and the corresponding angles between the triangle ABC and its image?
2. Determine whether each image is similar to the triangle ABC.
3. Make a conclusion based on your findings.

The results of Mind Mobilisation 4 show that the object and image under an enlargement are similar.

Enlargement is a transformation in which all the points of an object move from a fixed point with a constant ratio. The fixed point is known as the **centre of enlargement** and the constant ratio is known as the **scale factor**.



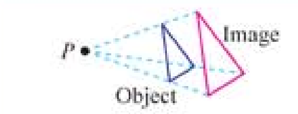
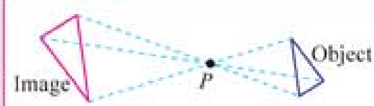
i-Technology

Scan the QR code or visit bit.do/GGB503BI to explore scale factor of enlargement.

In general, the scale factor k, of an enlargement can be determined as follows.

$$k = \frac{\text{distance of point of image from } P}{\text{distance of point of object from } P} \quad \text{or} \quad k = \frac{\text{length of side of image}}{\text{length of side of object}}$$

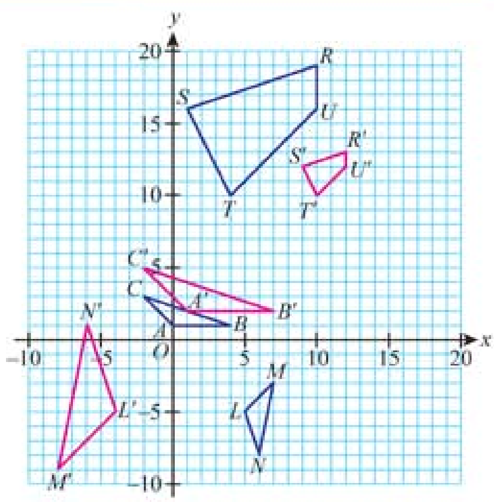
Different scale factors have different effects on enlargements:

Scale factor, k	Size of image	Position of image to the centre of enlargement P	
$k > 1$	**Larger** than the object	Be on the same side as the object	
$k = 1$	**Equal** in size to the object	Be on the same side as the object	
$0 < k < 1$	**Smaller** than the object	Be on the same side as the object	
$-1 < k < 0$	**Smaller** than the object	Be on the opposite side of the object	
$k = -1$	**Equal** in size to the object	Be on the opposite side of the object	
$k < -1$	**Larger** than the object	Be on the opposite side of the object	

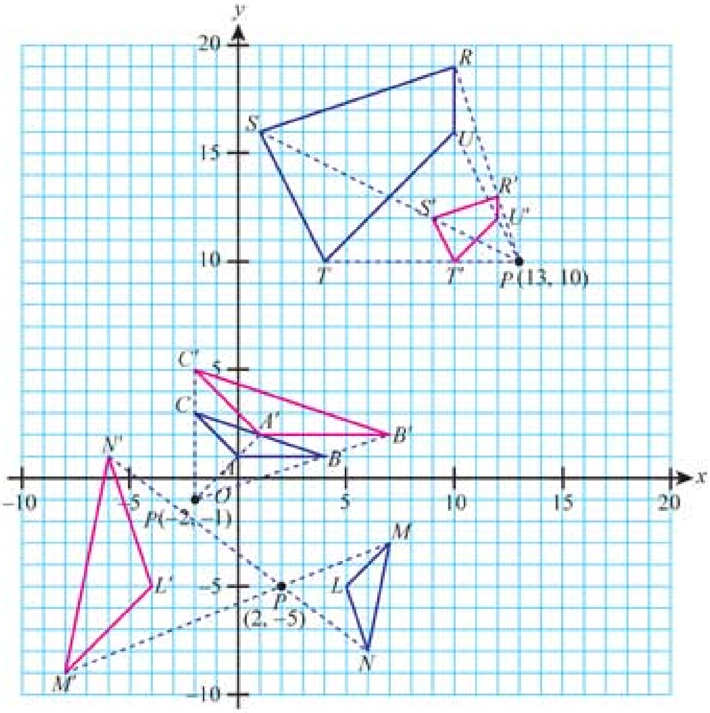
Example 8

The diagram on the right shows three objects and their images under the transformation of enlargement. Describe the enlargement by determining the scale factor and the centre of enlargement for the following:

(a) object ABC
(b) object $RSTU$
(c) object LMN



Solution:



(a) Scale factor $= \dfrac{A'B'}{AB}$

$= \dfrac{6}{4}$

$= \dfrac{3}{2}$ ← $k > 1$, image is larger than object

$A'B'C'$ is the image of ABC under an enlargement at centre (–2, –1) with a scale factor of $\dfrac{3}{2}$.

(b) Scale factor $= \dfrac{U'R'}{UR}$

$= \dfrac{1}{3}$ ← $0 < k < 1$, image is smaller than object

$R'S'T'U'$ is the image of $RSTU$ under an enlargement at centre (13, 10) with a scale factor of $\dfrac{1}{3}$.

(c) Scale factor $= \dfrac{\text{distance of } L' \text{ from centre of enlargement}}{\text{distance of } L \text{ from centre of enlargement}}$

$= -\dfrac{6}{3}$

$= -2$

The negative sign shows that the image is at the opposite side of the object. $k < -1$, image is larger than object.

$L'M'N'$ is the image of LMN under an enlargement at centre (2, –5) with a scale factor of –2.

Smart TIPS

The centre of enlargement can be determined from the intersection point between all the straight lines which are connecting each pair of the corresponding points.

Info Bulletin

In enlargement, each pair of the corresponding sides of the object and the image are parallel.

Smart TIPS

When describing an enlargement, we need to state the **centre of enlargement** and the **scale factor**.

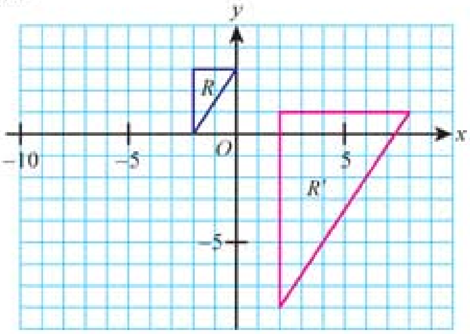
Smart TIPS

Negative scale factor will cause the image to appear on the other side of the centre of enlargement opposite to the object and the image will be inverted.

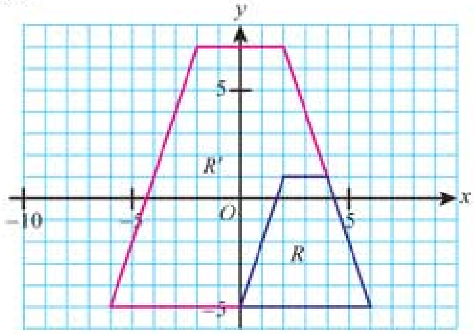
Self Practice 5.2b

1. It is given that R' is the image of object R. Describe the enlargement for each of the following.

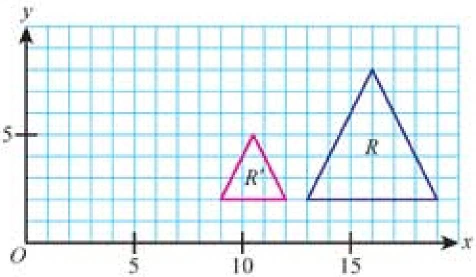
(a)



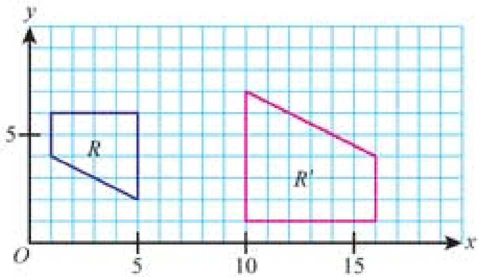
(b)



(c)

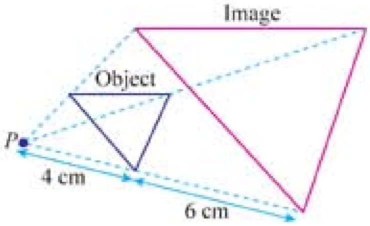


(d)

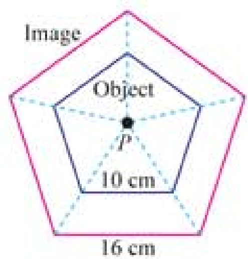


2. Describe the enlargement in each of the following diagrams.

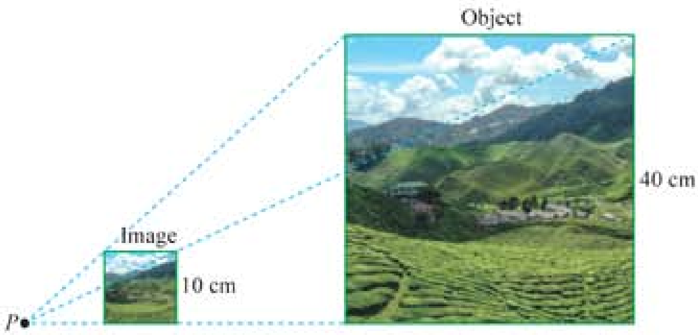
(a)



(b)



(c)



How to determine the image and object of an enlargement?

Learning Standard

Determine the image and object of an enlargement.

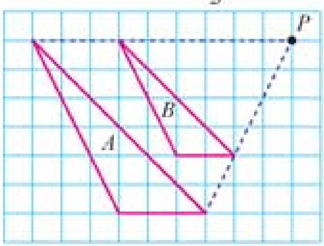
The flow chart below shows the steps to determine the image or object of an enlargement.

Identify the centre of enlargement → Identify the scale factor → Draw the projection line from the centre of enlargement to each vertex → Draw the image or object which has the similar shape according to the scale factor

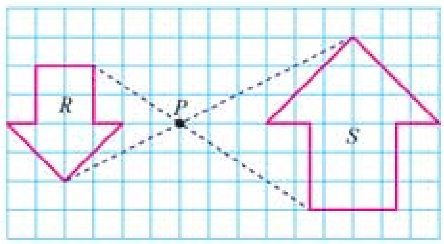
Example 9

Based on the given scale factor, determine the object and image for each of the following enlargements.

(a) Scale factor, $k = \frac{2}{3}$



(b) Scale factor, $k = -1.5$



Solution:

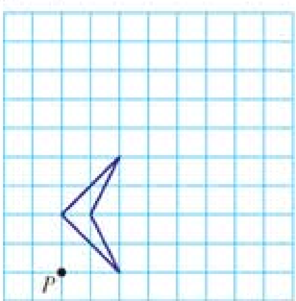
(a) When $k = \frac{2}{3}$, size of image is smaller than the object. A is object and B is image.

(b) When $k = -1.5$, size of image is larger than the object. R is object and S is image.

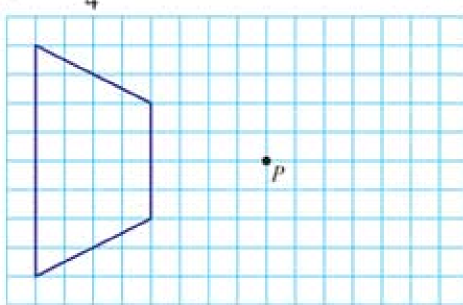
Example 10

Draw the image for each of the following objects using P as the centre of enlargement for the following scale factors.

(a) $k = 2$

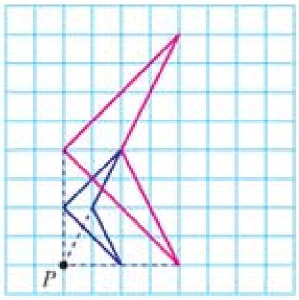


(b) $k = -\frac{3}{4}$

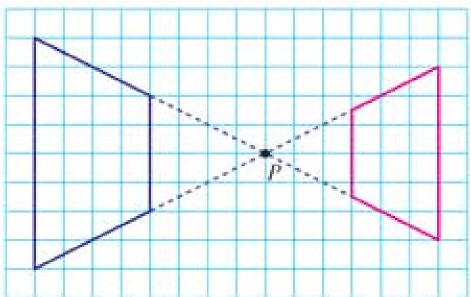


Solution:

(a) When $k = 2$, the distance of each vertex of the image from P is 2 times the distance of corresponding vertex of the object from P in the same direction.

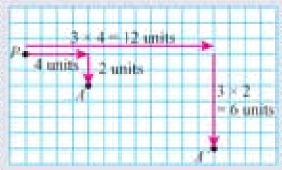


(b) When $k = -\frac{3}{4}$, the distance of each vertex of the image from P is $\frac{3}{4}$ times the distance of corresponding vertex of the object from P in the opposite direction.



Smart TIPS

We can use the horizontal distance and vertical distance to determine the ratio of the distance of a point from the centre of enlargement as the example below.



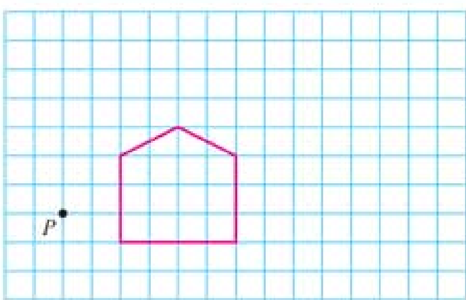
It is given that point A is object and point A' is its image, P is centre of enlargement and $k = 3$.

When $k = 3$, the vertical distance and horizontal distance of A' from P to the vertical distance and horizontal distance of A from P follow the ratio 3 : 1.

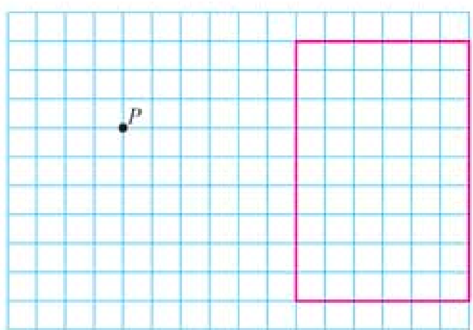
Example 11

Draw the object for each of the following images using P as the centre of enlargement for the following scale factors.

(a) $k = \frac{1}{2}$

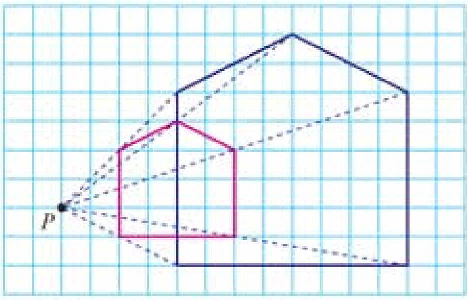


(b) $k = -3$

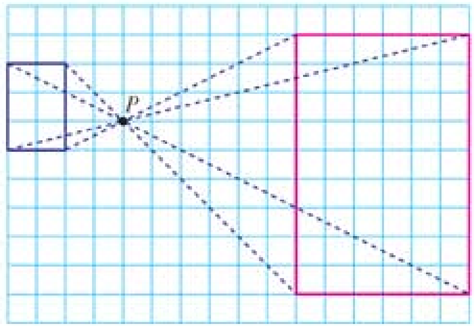


Solution:

(a) When $k = \frac{1}{2}$, the distance of each vertex of the object from P is 2 times the distance of corresponding vertex of the image from P in the same direction.



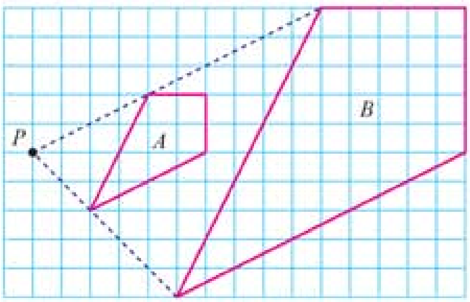
(b) When $k = -3$, the distance of each vertex of the object from P is $\frac{1}{3}$ times the distance of corresponding vertex of the image from P in the opposite direction.



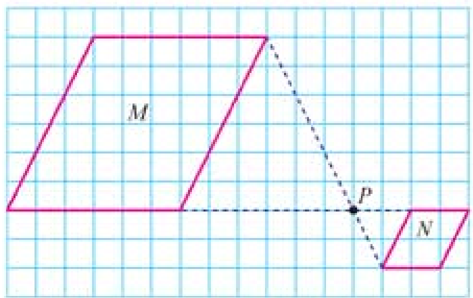
Self Practice 5.2c

1. Based on the given scale factor, determine the object and image for the following enlargements.

(a) Scale factor, $k = \frac{5}{2}$

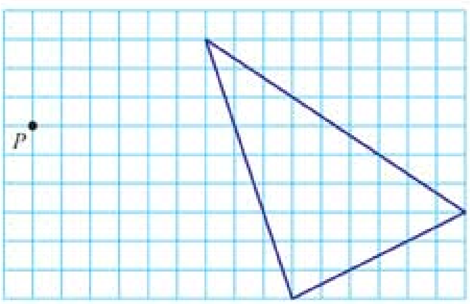


(b) Scale factor, $k = -3$

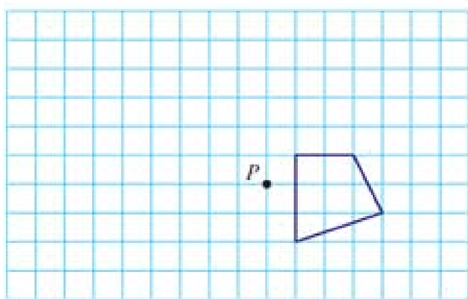


2. Draw the image for each of the following objects under the enlargement at centre P based on the scale factor given.

(a) $k = \frac{1}{3}$

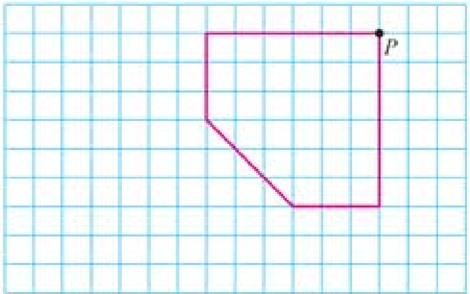


(b) $k = -2$

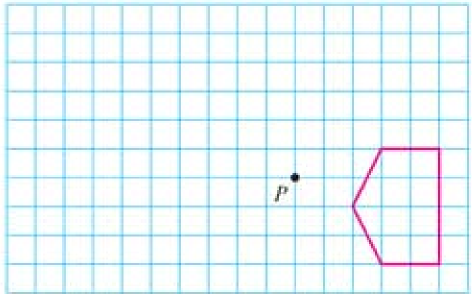


3. Draw the object for each of the following images under the enlargement at centre P based on the scale factor given.

(a) $k = \frac{3}{4}$



(b) $k = -\frac{1}{2}$



What is the relation between area of the image and area of the object of an enlargement?

Learning Standard

Make and verify conjecture on the relation between area of the image and area of the object of an enlargement.

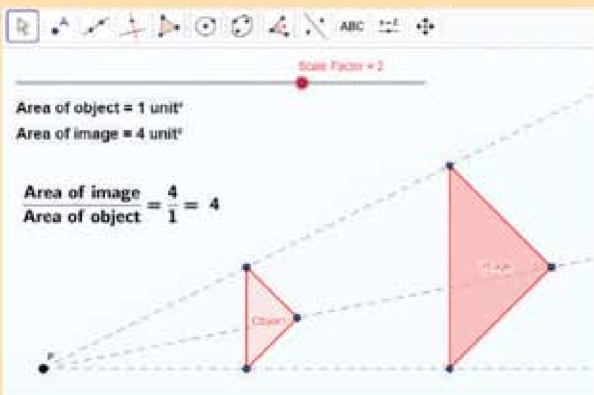
During enlargement, the lengths of corresponding sides are constantly proportional. What is the relation between area of the image and area of the object of an enlargement?

MIND MOBILISATION 5 Pairs

Aim: To explore the relation between area of the image and area of the object of an enlargement.

Steps:

1. Open the file GGB504 for this activity.



Scan the QR code or visit bit.do/GGB504BI to obtain the GeoGebra file for this activity.

2. Drag the slider 'Scale Factor' to some different values to observe the area of the object, area of the image and ratio of area of the image to area of the object.

3. Hence, complete the following table.

Scale factor, k	Area of the object (unit²)	Area of the image (unit²)	Ratio of area of the image to area of the object, $\frac{\text{area of the image}}{\text{area of the object}}$	k^2
$k = 2$	1	4	$\frac{4}{1} = 4$	$(2)^2 = 4$
$k = 3$				
$k = 4$				
$k = -1$				
$k = -2$				

Discussion:

1. What is the connection between scale factor and the ratio of area of the image to area of the object?
2. What is the relation between area of the image and area of the object of an enlargement?

The results of Mind Mobilisation 5 show that $\frac{\text{area of the image}}{\text{area of the object}} = k^2$ where k is the scale factor.

Hence, we can determine the area of the image of an enlargement with the formula:

$$\text{Area of the image} = k^2 \times \text{Area of the object}$$

Example 12

The table below shows the different values of area of the object, area of the image and scale factor under enlargement. Complete the table.

	Area of the object	Area of the image	Scale factor, k
(a)	5 cm²	45 cm²	
(b)	12 unit²		$\frac{7}{2}$
(c)		100 m²	−4

Solution:

(a)
$$k^2 = \frac{\text{area of the image}}{\text{area of the object}} = \frac{45}{5} = 9$$
$$k = \sqrt{9} = +3 \text{ or } -3$$

(b)
$$\left(\frac{7}{2}\right)^2 = \frac{\text{area of the image}}{12}$$
$$\text{area of the image} = \frac{49}{4} \times 12 = 147 \text{ unit}^2$$

(c)
$$(-4)^2 = \frac{100}{\text{area of the object}}$$
$$\text{area of the object} = \frac{100}{16} = 6.25 \text{ m}^2$$