**Self Practice 5.2d**

1. The table below shows the different values of area of the object, area of the image and scale factor under enlargement. Complete the table.

| Area of the object | Area of the image | Scale factor, $k$ |
|---|---|---|
| 18 unit$^2$ | 72 unit$^2$ | |
| 54 m$^2$ | | $\frac{1}{3}$ |
| | 31.25 cm$^2$ | $\frac{5}{4}$ |

## How to solve problems involving enlargement?

**Learning Standard**

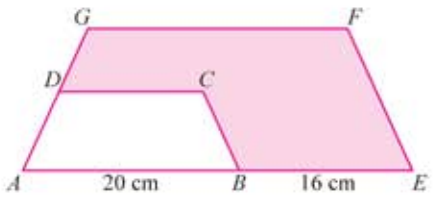
Solve problems involving enlargement.

**Example 13**

In the diagram on the right, trapezium $AEFG$ is the image of trapezium $ABCD$ under an enlargement at centre $A$. Given that the area of the coloured region is 168 cm$^2$, calculate

(a) the scale factor of the enlargement,

(b) the area, in cm$^2$, of trapezium $AEFG$.




**Solution:**

(a) Scale factor, $k = \frac{AE}{AB}$

$= \frac{20 + 16}{20}$

$= \frac{9}{5}$

(b) Let the area of trapezium $ABCD = L_1$
and the area of trapezium $AEFG = L_2$

$L_2 - L_1 = 168$

$L_2 - \frac{L_2}{\left(\frac{9}{5}\right)^2} = 168$

$L_2 - \frac{25}{81}L_2 = 168$

$\frac{56}{81}L_2 = 168$

$L_2 = 243$

Area of image $= k^2 \times$ Area of object

$L_2 = \left(\frac{9}{5}\right)^2 \times L_1$

$L_1 = \frac{L_2}{\left(\frac{9}{5}\right)^2}$

Hence, the area of trapezium $AEFG = 243$ cm$^2$.

**Alternative Method**

$L_2 = x$

$L_1 = \frac{x}{\left(\frac{9}{5}\right)^2}$

$= \frac{25}{81}x$

$L_2 - L_1 = 168$

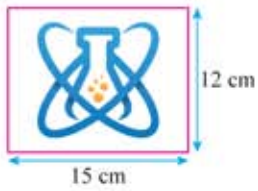
$x - \frac{25}{81}x = 168$

$\frac{56}{81}x = 168$

$x = 243$ cm$^2$

**Example 14**

The diagram below shows a rectangular logo drawn by a member of STEM Association.




On the school entrepreneur day, the STEM Association intends to put up a banner at their stall. The banner is designed based on the enlargement of the logo. If the area of the banner is 0.8 $m^2$, what are the measurements of the length and width, in cm, of the banner?

**Solution:**

Area of the logo = $15 \times 12 = 180\ cm^2$ and area of the banner = $0.8\ m^2 = 8\ 000\ cm^2$.

$$k^2 = \frac{\text{area of the image}}{\text{area of the object}}$$

$$= \frac{8\ 000\ cm^2}{180\ cm^2}$$

$$= \frac{400}{9}$$

$$k = \frac{20}{3}$$

$$k = \frac{\text{length of side of image}}{\text{length of side of object}}$$

$$\frac{20}{3} = \frac{\text{Length}}{15}$$

$$\text{Length} = 15 \times \frac{20}{3}$$

$$= 100\ cm$$

$$\text{Width} = \frac{\text{Area}}{\text{Length}}$$

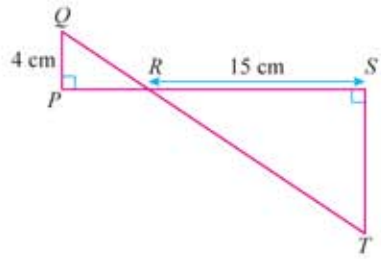
$$= \frac{8\ 000}{100}$$

$$= 80\ cm$$

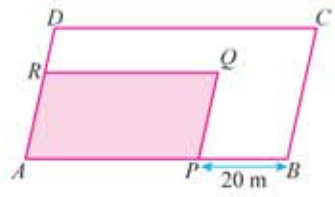
The measurement of the banner is 100 cm in length and 80 cm in width.

**Self Practice 5.2e**

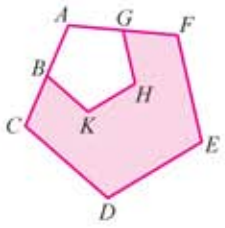
1. The diagram on the right shows two right-angled triangles where $\Delta PQR$ is the image of $\Delta STR$ under an enlargement. It is given that $5PR = 2RS$. Calculate
   (a) the scale factor of the enlargement,
   (b) the area, in $cm^2$, of the whole diagram.




2. The diagram on the right shows the plan of a garden in the shape of a parallelogram. It is given that $ABCD$ is the image of $APQR$ with the enlargement at centre $A$ and the area of the garden is 1 350 $m^2$. If the area of the coloured region is 600 $m^2$, calculate the length, in m, of $AP$.



**3.** In the diagram on the right, the pentagon $ACDEF$ is the image of pentagon $ABKHG$ under an enlargement where the vertex $A$ is the centre of enlargement. It is given that $B$ is the midpoint of side $AC$ and the area of the pentagon $ABKHG$ is 17 cm$^2$. Calculate the area, in cm$^2$, of the coloured region.




**4.** The diagram on the right shows a circular badge used in an environmental protection programme which is launched by the Environmental Association. This association intends to paint a mural on the wall of a building based on the enlargement of the badge. It is given that the area of the mural is $4\pi$ m$^2$. What is the scale factor of the enlargement?

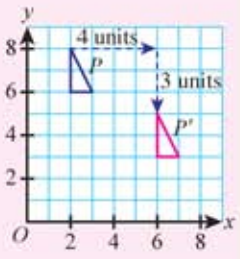
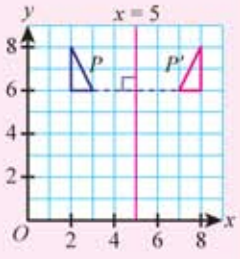
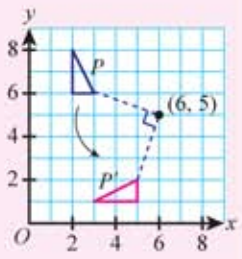
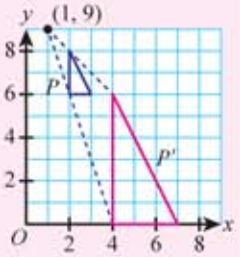

## 5.3 Combined Transformation

### How to determine the image and object of a combined transformation?

**Learning Standard**

Determine the image and object of a combined transformation.

We have learnt 4 types of transformations as shown in the examples below where $P'$ is the image of $P$ under the transformation stated.

| Translation | Reflection | Rotation | Enlargement |
|---|---|---|---|
| Translation $\begin{pmatrix} 4 \\ -3 \end{pmatrix}$ | Reflection on line $x = 5$ | Rotation of 90° anticlockwise at centre (6, 5) | Enlargement at centre (1, 9) with scale factor 3 |
|   |   |   |   |

To determine the object when an image is given, the transformation of translation needs to be performed in the opposite direction. For example, translation $\begin{pmatrix} 2 \\ -4 \end{pmatrix}$ becomes $\begin{pmatrix} -2 \\ 4 \end{pmatrix}$. For the transformation of rotation, rotation in clockwise direction will become rotation in anticlockwise direction. For the transformation of enlargement, enlargement with a scale factor $k = 2$ will become reciprocal, that is enlargement with scale factor $k = \frac{1}{2}$.

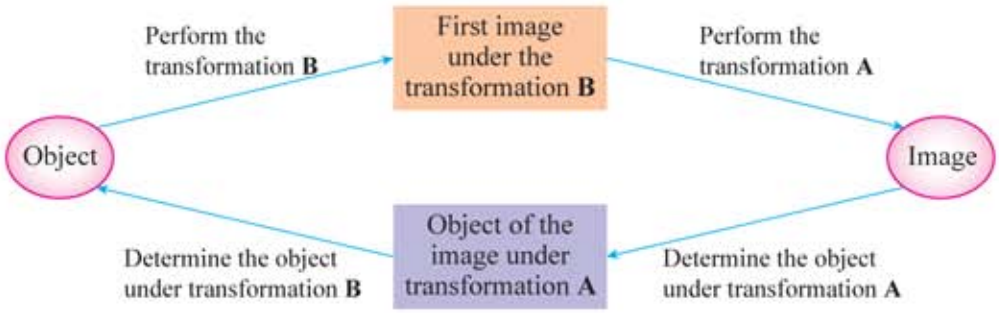
**Critical Mind**

Why the transformation of reflection does not change like the other three transformations when finding the object if the image is given?

An object can perform more than one transformation and will produce an image based on the transformations involved. In general, the combination between the transformation **A** and transformation **B** can be written as transformation **AB** or transformation **BA** in the order of the desired transformation.

Combined transformation **AB** means transformation **B** followed by transformation **A**.

The diagram below shows the steps to determine the image or object of a combined transformation **AB**.




**Example 15**

The diagram on the right shows several triangles drawn on a Cartesian plane. It is given that transformation

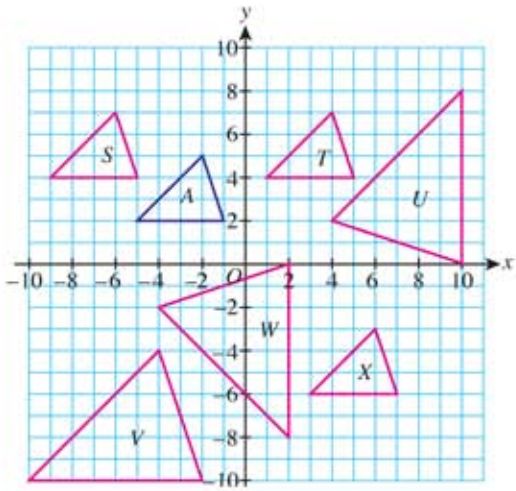
**P** = translation $\begin{pmatrix} 3 \\ 1 \end{pmatrix}$

**Q** = rotation of 90° anticlockwise at centre (3, 4)

**R** = enlargement at centre (8, 0) with scale factor 2

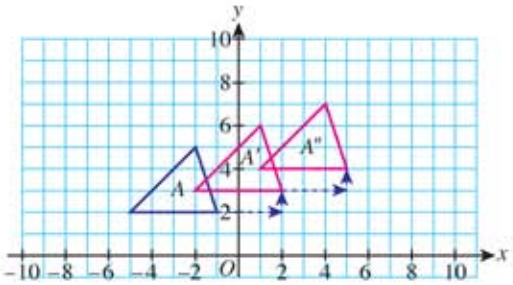
Determine the image of triangle $A$ under the combined transformation

(a) $\mathbf{P}^2$ (b) **RQ**



**Solution:**

(a) Combined transformation $\mathbf{P}^2$ means transformation **P** performed 2 times in a row.

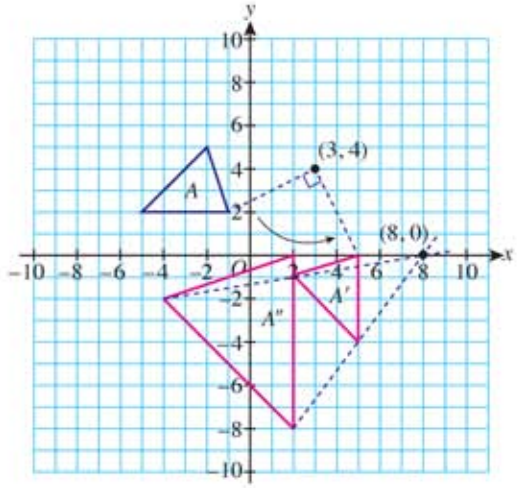



$A'$ is the image of $A$ under transformation **P**.
$A''$ is the image of $A'$ under transformation **P**.

$$A \xrightarrow{\mathbf{P}} A' \xrightarrow{\mathbf{P}} A''$$

Image of triangle $A$ under the combined transformation $\mathbf{P}^2$ is triangle $T$.

(b) Combined transformation **RQ** means transformation **Q** followed by transformation **R.**




$A'$ is the image of $A$ under transformation **Q**.
$A''$ is the image of $A'$ under transformation **R**.

$$A \xrightarrow{\mathbf{Q}} A' \xrightarrow{\mathbf{R}} A''$$

Image of triangle $A$ under the combined transformation **RQ** is triangle $W$.



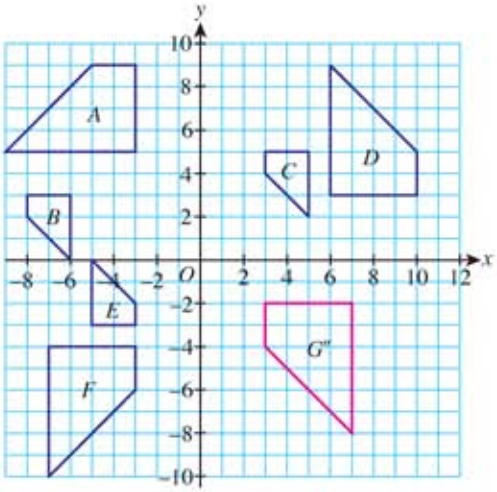
**Example 16**

In the diagram on the right, trapezium $G''$ is the image of an object under a combined transformation. It is given that transformation

**T** = enlargement at centre (1, 4) with scale factor –2
**V** = rotation of 180° at the origin
**W** = reflection on line $y = 1$

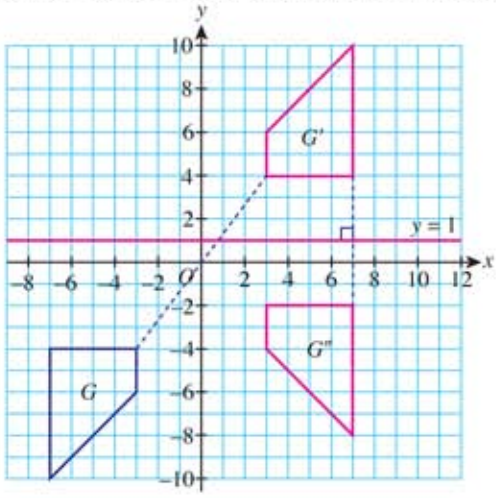
Determine the object of trapezium $G''$ under the combined transformation

(a) **WV** (b) **VT**



**Solution:**

(a) To find the object, transformation **W** is followed by transformation **V**.

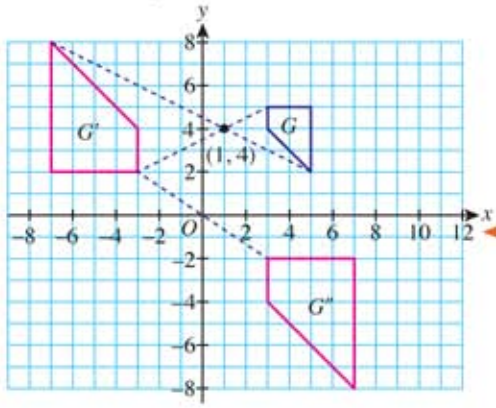



$G'$ is the object of $G''$ under transformation **W**.
$G$ is the object of $G'$ under transformation **V**.

$$G'' \xrightarrow{\mathbf{W}} G' \xrightarrow{\mathbf{V}} G$$

Object of trapezium $G''$ under the combined transformation **WV** is trapezium $F$.

(b) To find the object, transformation **V** is followed by transformation **T**.




$G'$ is the object of $G''$ under transformation **V**.
$G$ is the object of $G'$ under transformation **T**.

$$G'' \xrightarrow{\mathbf{V}} G' \xrightarrow{\mathbf{T}} G$$

To determine object $G$ under transformation **T**, perform enlargement with scale factor $k = -\frac{1}{2}$ on image $G'$.

Object of trapezium $G''$ under the combined transformation **VT** is trapezium $C$.

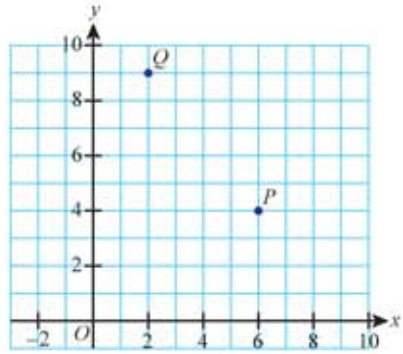
## Example 17

The diagram on the right shows the point $P$ and point $Q$ on a Cartesian plane. It is given that transformation

**A** = translation $\begin{pmatrix} -5 \\ 2 \end{pmatrix}$

**B** = reflection on line $y = x$

**C** = rotation of 90° clockwise at centre (5, 8)

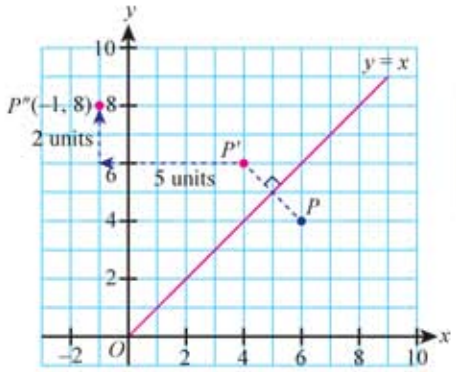



(a) Determine the image of point $P$ under the combined transformation
(i) **AB** (ii) **BC**

(b) It is given that point $Q$ is the image of point $K$ under a transformation **CA**. Determine the point $K$.

**Solution:**

(a) (i) Combined transformation **AB** means transformation **B** followed by transformation **A**.

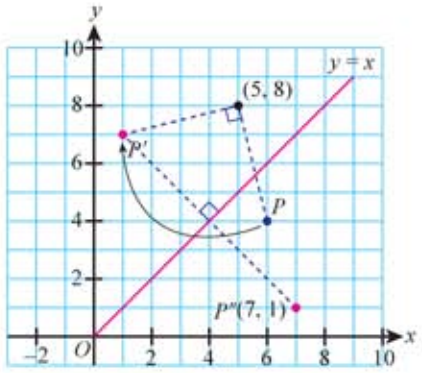



$P'$ is the image of $P$ under transformation **B**.
$P''$ is the image of $P'$ under transformation **A**.

$$P \xrightarrow{\mathbf{B}} P' \xrightarrow{\mathbf{A}} P''$$

Hence, the image of point $P$ under the combined tranformation **AB** is $P''(-1, 8)$.

(ii) Combined transformation **BC** means transformation **C** followed by transformation **B**.

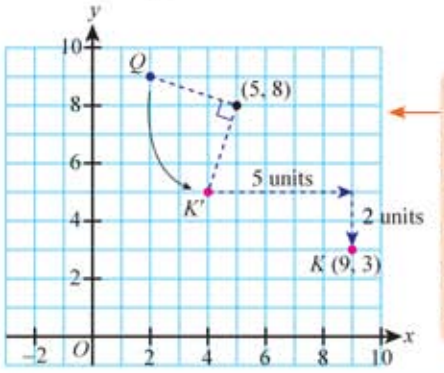



$P'$ is the image of $P$ under transformation **C**.
$P''$ is the image of $P'$ under transformation **B**.

$$P \xrightarrow{\mathbf{C}} P' \xrightarrow{\mathbf{B}} P''$$

Hence, the image of point $P$ under the combined tranformation **BC** is $P''(7, 1)$.

(b) To find the object, transformation **C** is followed by transformation **A**.




To determine object $K'$ under transformation **C**, perform anticlockwise rotation on image $Q$.
To determine object $K$ under transformation **A**, perform translation $\begin{pmatrix} 5 \\ -2 \end{pmatrix}$ on image $K'$.

**Smart TiPS**

When determining the object, the order of transformations is reversed to determine the image under the same combined transformation.

$K'$ is the object of $Q$ under transformation **C**.
$K$ is the object of $K'$ under transformation **A**.

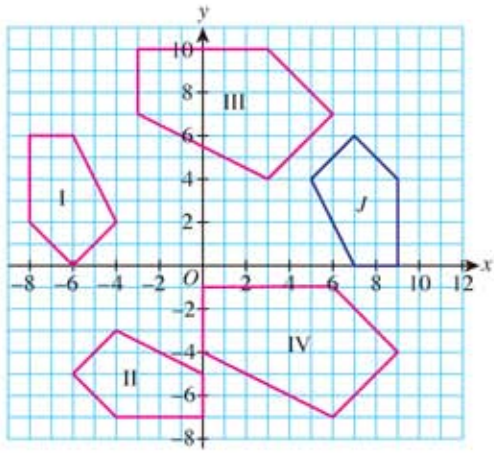
$$Q \xrightarrow{\mathbf{C}} K' \xrightarrow{\mathbf{A}} K$$

Hence, $K(9, 3)$

## Self Practice 5.3a

**1.** The diagram on the right shows several pentagons drawn on a Cartesian plane. It is given that transformation

**A** = reflection on line $y = x$
**B** = rotation of 180° at centre (1, 0)
**C** = enlargement at centre (6, 7) with scale factor $\frac{3}{2}$

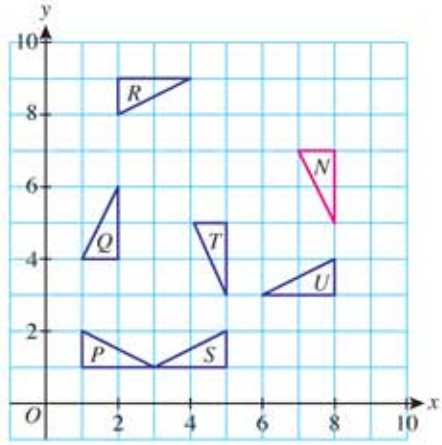



Determine the image of pentagon $J$ under the combined transformation
(a) **AB**
(b) **CA**

**2.** In the diagram on the right, the right-angled triangle $N$ is the image of an object under a combined transformation. It is given that transformation

**A** = translation $\begin{pmatrix} 3 \\ 2 \end{pmatrix}$

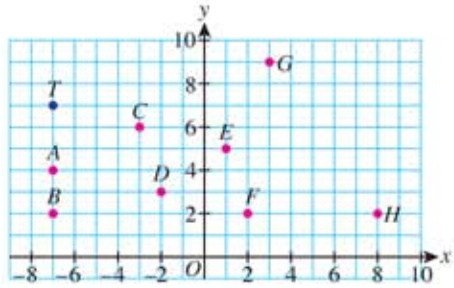
**B** = rotation of 270° clockwise at centre (6, 5)
**C** = reflection on line $x + y = 6$




Determine the object of triangle $N$ under the combined transformation
(a) **AC**
(b) **BA**

**3.** It is given that transformation

**U** = translation $\begin{pmatrix} 4 \\ -1 \end{pmatrix}$
**V** = reflection on line $x = 1$
**W** = rotation of 90° anticlockwise at centre (5, 5)




The diagram on the right shows several points drawn on a Cartesian plane.
(a) Determine the image of point $T$ under the combined transformation
  (i) $\mathbf{U}^2$
  (ii) **WV**
(b) It is given that $H$ is the image of a point under transformation **UV**. Determine the point.

## What is the effect of commutative law on combined transformation?

**Learning Standard**

Make and verify the conjecture about commutative law in combined transformation.

It is given that **P** and **Q** are two different transformations. Is the image under the combined transformation **PQ** the same as the image under the combined transformation **QP**?

**MIND MOBILISATION 6** Pairs

**Aim**: To make and verify the conjecture about commutative law in combined transformation.

**Steps**:

1. Open the file GGB505 for this activity.

Scan the QR code or visit bit.do/GGB505BI to obtain the GeoGebra file for this activity.

Scan the QR code or visit bit.do/WSChap5ii to obtain the worksheet for this activity.

2. On the menu, select the types of transformation for Transformation P and Transformation Q.
3. Drag the slider 'P followed by Q' and 'Q followed by P' in turns to observe the images produced. Is the image produced by 'P followed by Q' the same as the image produced by 'Q followed by P'?
4. Open the worksheet for this activity. Complete it based on your exploration.

| Transformation P | Description of transformation P | Transformation Q | Description of transformation Q | Are images produced by 'P followed by Q' and 'Q followed by P' the same? |
|---|---|---|---|---|
| Translation | | Translation | | |
| Translation | | Reflection | | |

**Discussion**:

1. State whether each combination of two transformations satisfies the commutative law.
2. What is your conclusion about the commutative law in combined transformation?

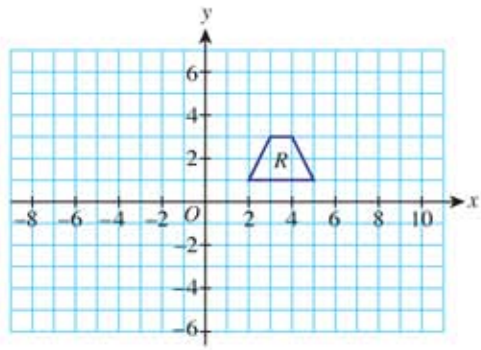
The results of Mind Mobilisation 6 show that most of the images produced by combined transformation **PQ** are not the same as the images produced by combined transformation **QP**.

In general,

- A combined transformation **AB** satisfies the commutative law if the images under the combined transformation **AB** and combined transformation **BA** are the same.
- A combined transformation **AB** does not satisfy the commutative law if the images under the combined transformation **AB** and combined transformation **BA** are not the same.

## Example 18

The diagram on the right shows an object on a Cartesian plane. It is given that transformation

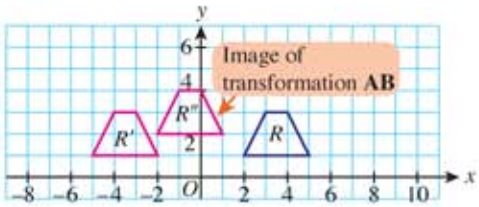


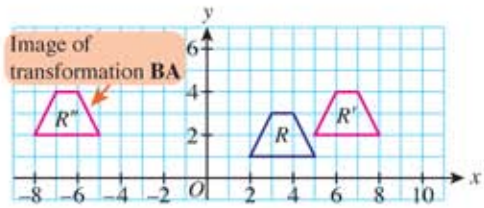

$\mathbf{A}$ = translation $\begin{pmatrix} 3 \\ 1 \end{pmatrix}$
$\mathbf{B}$ = reflection on the $y$-axis
$\mathbf{C}$ = rotation of 180° at the origin

Determine whether each of the following combined transformations satisfies the commutative law.
(a) Combined transformation **AB**
(b) Combined transformation **BC**

**Solution:**

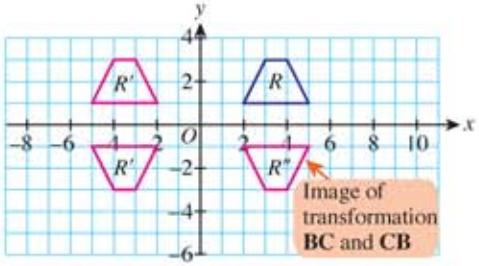
(a)




Images under the combined transformations **AB** and **BA** are not the same, so the combined transformation **AB** does not satisfy the commutative law.

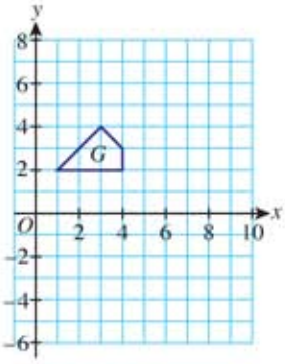
(b)




Images under the combined transformations **BC** and **CB** are the same, so the combined transformation **BC** satisfies the commutative law.

## Self Practice 5.3b

**1.** The diagram on the right shows an object on a Cartesian plane. It is given that transformation




$\mathbf{P}$ = enlargement at the origin with scale factor 2
$\mathbf{Q}$ = rotation of 90° clockwise at the origin
$\mathbf{R}$ = translation $\begin{pmatrix} -1 \\ 4 \end{pmatrix}$

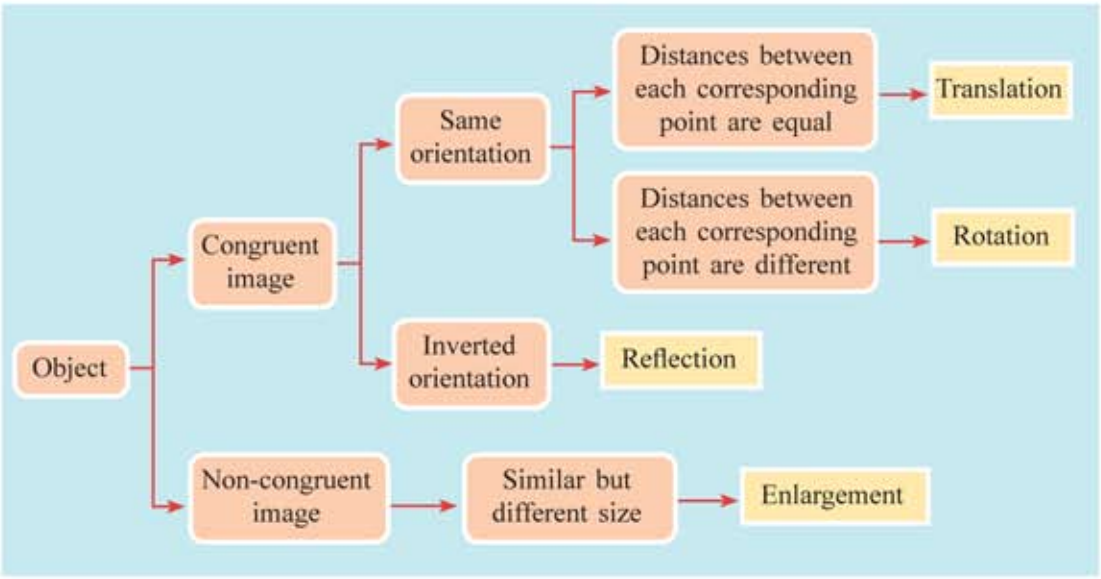
Determine whether each of the following combined transformations satisfies the commutative law.
(a) Combined transformation **PQ**
(b) Combined transformation **PR**

## How to describe the combined transformation?

**Learning Standard**

Describe combined transformation.

We can describe a transformation based on the properties of the image. The diagram below shows the key points to describe a transformation.

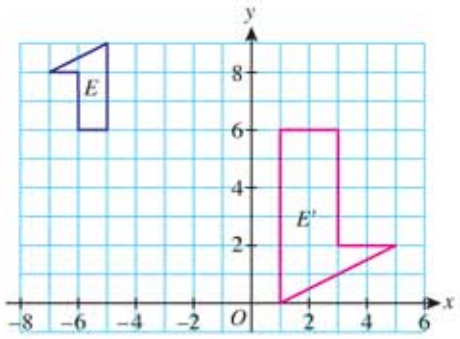



When describing a combined transformation **AB**, we need to describe transformation **B** first, followed by transformation **A**.



**Example 19**

In the diagram below, $E'$ is the image of object $E$ under the combined transformation **ST**.

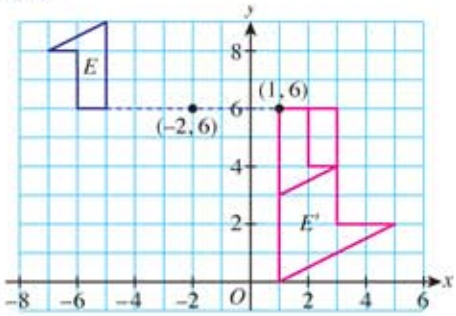



(a) Describe the transformation
  (i) **T**
  (ii) **S**

(b) Hence, describe a single transformation which is equivalent to the combined transformation **ST**.

**Solution:**

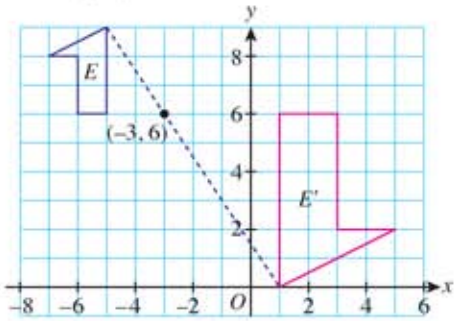
(a)




(i) Transformation **T** is a rotation of 180° at centre (–2, 6).

(ii) Transformation **S** is an enlargement at centre (1, 6) with scale factor 2.

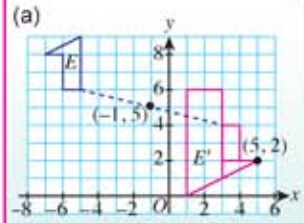
(b)




The single transformation which is equivalent to the combined transformation **ST** is an enlargement at centre (–3, 6) with scale factor –2.

**Alternative Method**

(a)




(i) Transformation **T** is a rotation of 180° at centre (–1, 5).

(ii) Transformation **S** is an enlargement at centre (5, 2) with scale factor 2.

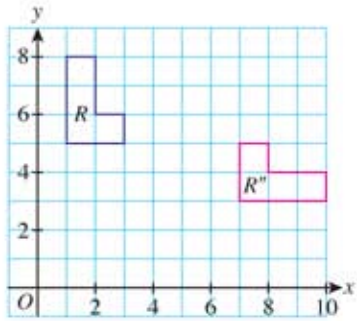
**Info Bulletin**

Not all the combined transformations can be described in a single transformation.

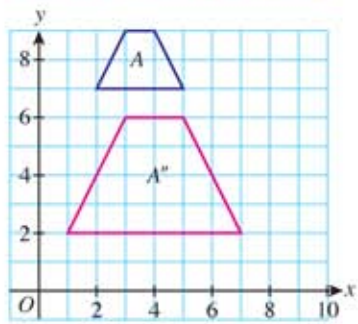


## Self Practice 5.3c

**1.** In the diagram on the right, $R''$ is the image of object $R$ under the combined transformation **AB**. Describe transformation **A** and transformation **B**.



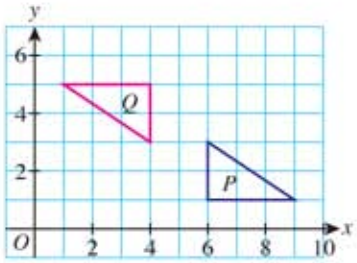

**2.** In the diagram on the right, $A''$ is the image of object $A$ under the combined transformation **PQ**. Describe transformation **P** and transformation **Q**.



**3.** It is given that transformation **A** is a reflection on line $x = 5$ and $Q$ is the image of $P$ under the combined transformation **AB**. Describe

(a) transformation **B**,

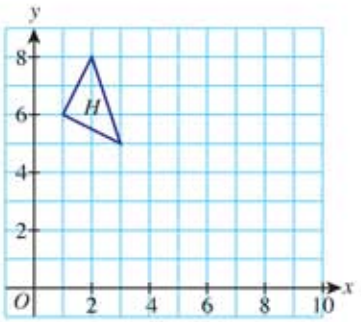
(b) a single transformation which is equivalent to the combined transformation **AB**.




**4.** It is given that transformation **M** is a rotation of 90° clockwise at centre (4, 4) and transformation **N** is a reflection on line $x = 4$.

(a) Draw the image of object $H$ under the combined transformation **MN**.

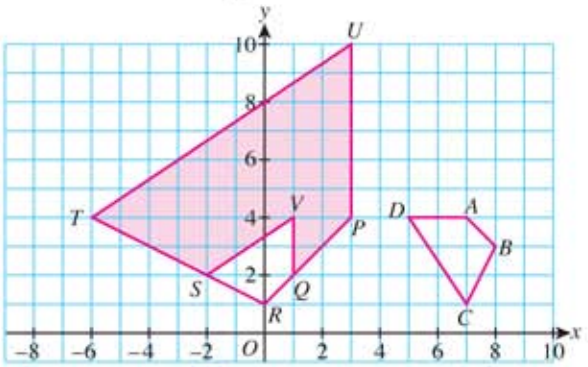
(b) Hence, describe a single transformation which is equivalent to the combined transformation **MN**.




## How to solve problems involving combined transformation?

**Example 20**

The diagram below shows the map of a tropical forest. The coloured region is the habitat of a group of wildlife. Quadrilaterals $ABCD$ and $QRSV$ are the wildlife observatories.




The concept of transformation is used to construct the observatory such that $ABCD$ is the image of $PRTU$ under the combined transformation **UV**.

(a) Describe in full, the transformation

(i) **V** (ii) **U**

(b) Given that the area of observatory $ABCD$ is 4.5 $m^2$, calculate the area, in $m^2$, of the coloured region.

**Learning Standard**

Solve problems involving combined transformation.

**My Malaysia**

According to Wildlife Conservation Act 2010, wildlife means any species of wild animal or wild bird, whether totally protected or protected, vertebrate or invertebrate, live or dead, mature or immature and whether or not may be tamed or bred in captivity.

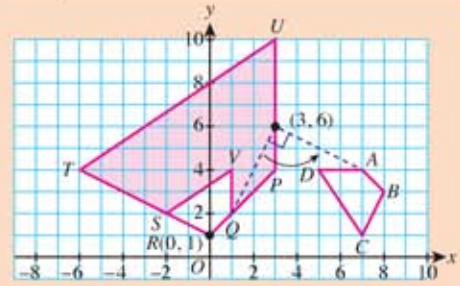
(Source: bit.do/WLConservationAct) (Accessed on 14 September 2020)

**Solution:**

(a) (i) **V** is an enlargement at centre (0, 1) with scale factor $\frac{1}{3}$.

(ii) **U** is a rotation of 90° anticlockwise at centre (3, 6).

Combined transformation **UV** is an enlargement followed by a rotation. Hence, **V** is the enlargement and **U** is the rotation.




(b) Area of $ABCD = k^2 \times$ Area of $PRTU$

$$4.5 = \left(\frac{1}{3}\right)^2 \times \text{Area of } PRTU$$

Area of $PRTU = 4.5 \times 9$

$= 40.5 \text{ m}^2$

Area of coloured region $= 40.5 - 4.5$

$= 36 \text{ m}^2$

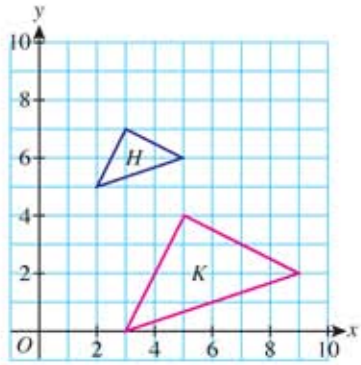
## Self Practice 5.3d

**1.** In the diagram on the right, triangle $K$ is the image of triangle $H$ under the combined transformation **PQ**.

(a) Describe transformation **P** and transformation **Q**.

(b) Describe a single transformation which is equivalent to the transformation **PQ**.

(c) Given that the area of triangle $K$ is 10 unit$^2$, calculate the area, in unit$^2$, of triangle $H$.

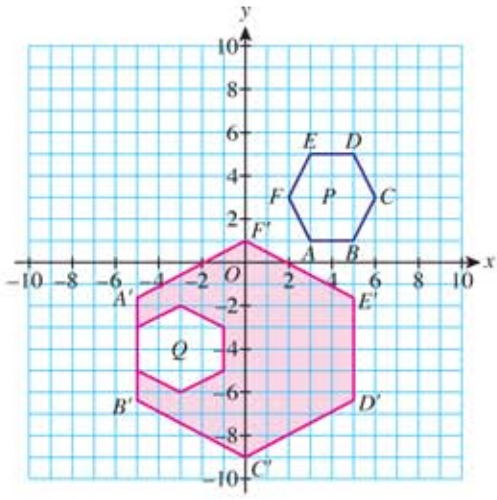



**2.** The diagram on the right shows a plan of a water theme park. The coloured region is for water tunnels. The hexagonal regions $P$ and $Q$ are food courts. During the construction of the water theme park, the architects used the concept of transformation to design the plan such that $Q$ is the image of $P$ under a reflection and the hexagon $A'B'C'D'E'F'$ is the image of $P$ under the combined transformation **UV**.

(a) Determine the axis of reflection.

(b) Describe transformation **U**.

(c) Given that the area of the food court $P$ is 60 m$^2$, calculate the area, in m$^2$, of the coloured region.



# 5.4 Tessellation

## What is the meaning of tessellation?

**Learning Standard**

Explain the meaning of tessellation.

Beehives have a unique inner structure. This structure can provide the maximum space to keep honey. What are the characteristics of the structure of a beehive and how is this structure constructed?

The structure of a beehive is a **tessellation**.

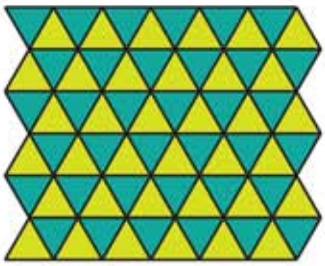
A **tessellation** is a pattern of recurring shapes that fills a plane without leaving empty spaces or overlapping.

**Mathematics is fun!**

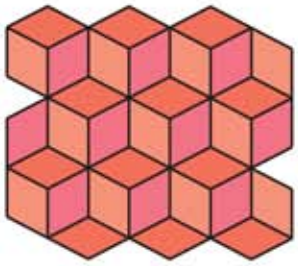
**Test Your Understanding About Tessellation**
Scan the QR code or visit bit.do/GGB506BI. Drag the points on the screen until a tessellation is created. Print your work and show to your teacher.

The diagram below shows some examples of tessellation.

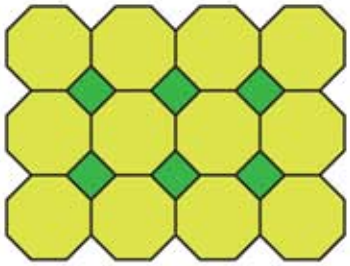
(i) Tessellation consisting of equilateral triangles



(ii) Tessellation consisting of rhombuses



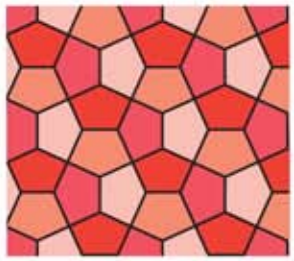
(iii) Tessellation consisting of a combination of squares and regular octagons



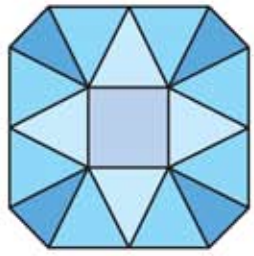
**Example 21**

Determine whether each of the following patterns is a tessellation.

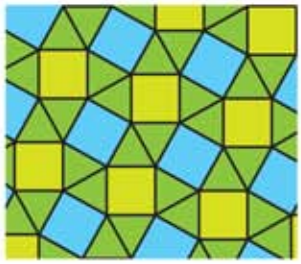
(a)



(b)



(c)



(d)

**Solution:**

(a) A tessellation consisting of pentagons only.
(b) Not a tessellation because it has a shape which does not recur, which is a square.
(c) A tessellation consisting of combination of equilateral triangles and squares.
(d) A tessellation consisting of the recurring patterns without overlapping.

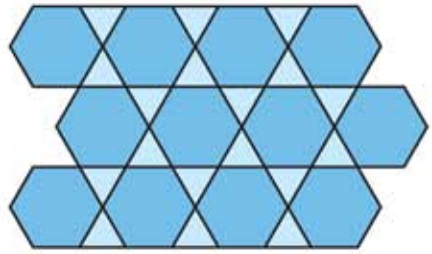


**Self Practice 5.4a**

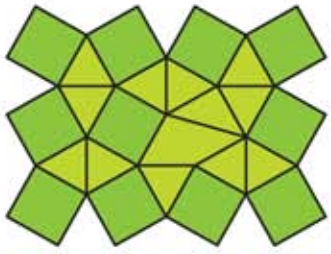
**1.** Determine whether each of the following patterns is a tessellation.

(a)

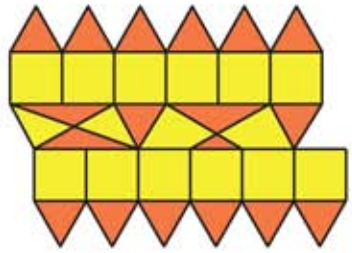
(b)



(c)



(d)



## How to design the tessellation?

**Learning Standard**

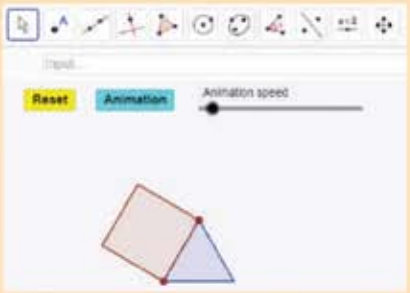
Design tessellation involving isometric transformation.

**MIND MOBILISATION 7** Pairs

**Aim**: To design tessellation by using isometric transformation

**Steps:**

1. Open the file GGB507 for this activity.




Scan the QR code or visit bit.do/GGB507BI to obtain the GeoGebra file for this activity.

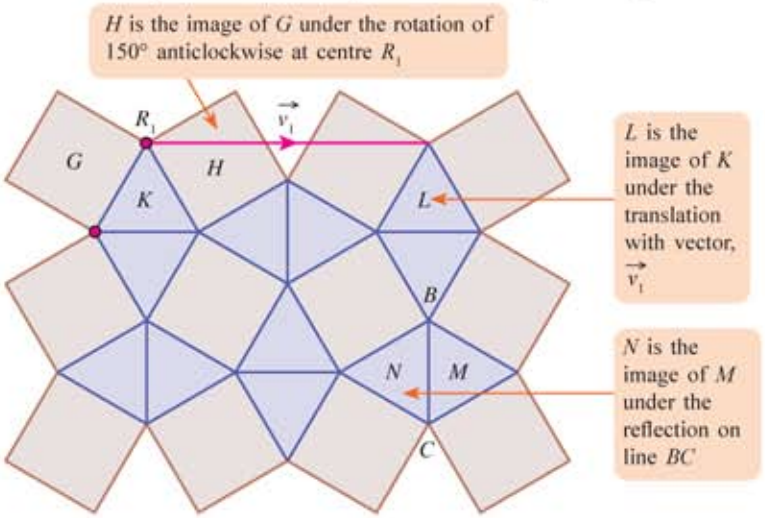
2. Click 'Animation'.
3. Observe the transformation involved in producing a tessellation.
4. Drag slider 'Animation speed' at an appropriate speed.
5. Click 'Reset' if you want to make another observation.

**Discussion:**

1. How many types of transformations are used to design a tessellation in this activity? State all the transformations.
2. Discuss how the combined transformation is used to design a tessellation.

The results of Mind Mobilisation 7 show that we can design a tessellation by using isometric transformations such as translation, reflection and rotation.

The following shows some steps involved in producing the tessellation in the activity above.




**MEMORY BOX**

Isometric transformation is the transformation that produces an image that is congruent with an object.

**Example 22**

The diagram below shows a tessellation consisting of equilateral triangles which are produced by isometric transformation. State the transformation involved to produce

(a) shape $Q$ from shape $P$

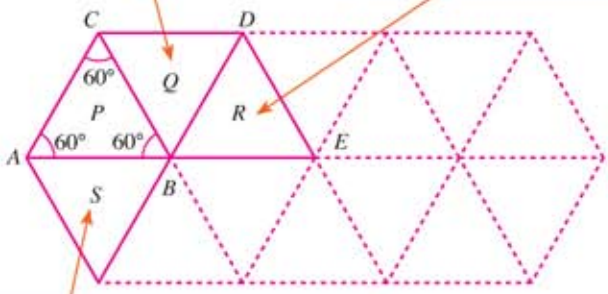
(b) shape $R$ from shape $Q$

(c) shape $S$ from shape $P$

**Solution:**

$Q$ is the image of $P$ under
- the reflection on line $BC$ or
- the rotation of 60° clockwise at centre $B$

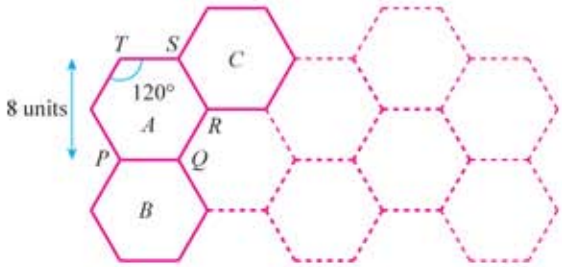
$R$ is the image of $Q$ under
- the reflection on line $BD$ or
- the rotation of 60° anticlockwise at centre $D$




$S$ is the image of $P$ under
- the reflection on line $AB$ or
- the rotation of 60° clockwise at centre $A$

**Self Practice 5.4b**

**1.** The diagram below shows a tessellation consisting of regular hexagons which are produced by isometric transformation.



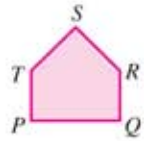

State the transformation involved to produce

(a) shape $B$ from shape $A$

(b) shape $C$ from shape $A$

**Interactive Platform**

The diagram below shows a pentagon $PQRST$.




By using the pentagon, discuss in groups on how to design several different tessellations by using combined transformations.

**Info Bulletin**

$R$ is also the image of $P$ under the translation $\begin{pmatrix} d \\ 0 \end{pmatrix}$, such that $d$ = distance of $AB$

**Info Bulletin**

**Escher Tessellation**

Example of Escher tessellation

Maurits Cornelis Escher (1898-1972) was a Dutch graphic artist. He designed a lot of artworks based on the concept of mathematics. He introduced a unique and attractive way to produce the pattern of tessellation which is known as Escher tessellation. An Escher tessellation is a tessellation in which its shapes are easily identified in real life.

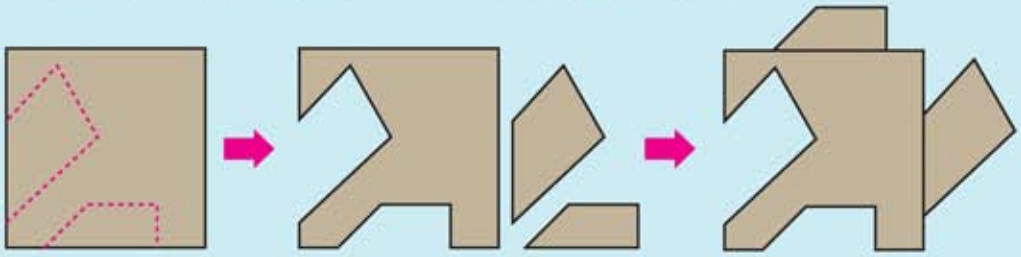
## PROJECT

**Aim:** To design an Escher tessellation

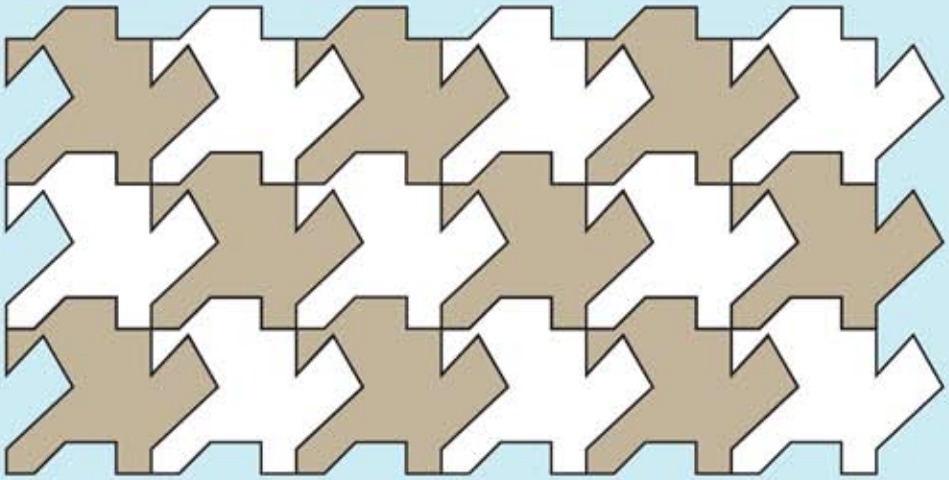
**Materials:** Cardboard, coloured papers, scissors, glue

**Steps:**

1. Carry out this project in groups.
2. Cut the provided coloured papers into 18 squares of equal size.
3. Choose two sides on a square, draw and cut out two different patterns. Then, paste the remaining part on the cardboard and the two cut patterns are attached on the other two sides of the remaining part as shown in the diagram below.



4. Repeat step **3** to produce another 17 identical patterns.
5. Arrange and paste all the shapes produced on the cardboard to produce the Escher tessellation as shown in the diagram below. What is the characteristic that differentiates the usual tessellation with the Escher tessellation?



6. Repeat this project to design several different Escher tessellations.
7. Display the design of your group in the class.
8. Present
   (a) the transformation used in the design of your group.
   (b) your conclusion about Escher tessellation.

# Summary Arena

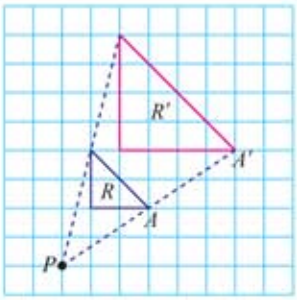
**Congruent shapes**

- The corresponding sides and angles are of the same sizes.

**Triangle congruency**

| | | |
|---|---|---|
| • Side-Side-Side (SSS)  | • Side-Angle-Side (SAS)  | • Angle-Side-Angle (ASA)  |
| • Angle-Angle-Side (AAS)  | • Angle-Angle-Angle (AAA)  Area must be equal | • Side-Side-Angle (SSA)  Area must be equal |

**Enlargement**




- Object and image are similar
- Scale factor, $k = \frac{PA'}{PA}$
- Area of image $= k^2 \times$ Area of object

**Combined transformation**

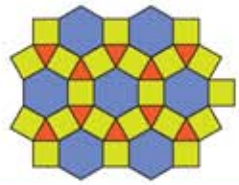
- Combined transformation **AB** means transformation **B** followed by transformation **A**

To determine image from object under transformation **AB**, the order is transformation **B** followed by transformation **A**.

To determine object from image, the order of transformations is reversed to determine the image under the same combined transformation.

**Tessellation**

- Tessellation is a pattern of recurring shapes that fills a plane without leaving empty spaces or overlapping
- Tessellation can be designed from an isometric transformation



## Reflection

At the end of this chapter, I can

| |  |  |
|---|---|---|
| differentiate between congruent and non-congruent shapes based on sides and angles. | | |
| make and verify the conjecture of triangle congruency based on sides and angles. | | |
| solve problems involving congruency. | | |
| explain the meaning of similarity of geometric objects. | | |
| make a connection between similarity and enlargement, hence describe enlargement using various representations. | | |
| determine the image and object of an enlargement. | | |
| make and verify conjecture on the relation between area of the image and area of the object of an enlargement. | | |
| solve problems involving enlargement. | | |
| determine the image and object of a combined transformation. | | |
| make and verify the conjecture about commutative law in combined transformation. | | |
| describe combined transformation. | | |
| solve problems involving combined transformation. | | |
| explain the meaning of tessellation. | | |
| design tessellation involving isometric transformation. | | |

## MINI PROJECT

Batik is a part of Malaysian cultural heritage. The design of *batik Malaysia* is world-renowned. Use your creativity to design a colourful batik pattern by using the Escher tessellation. You may use the dynamic geometry software or concrete materials to produce the batik pattern. Then, display your work in the class.

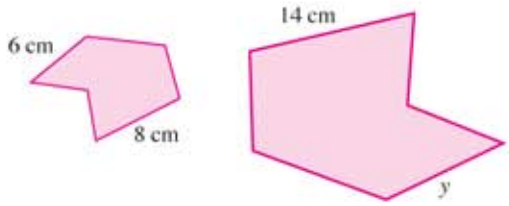
## Extensive Practice

Scan the QR code or visit bit.do/QuizE05 for interactive quiz

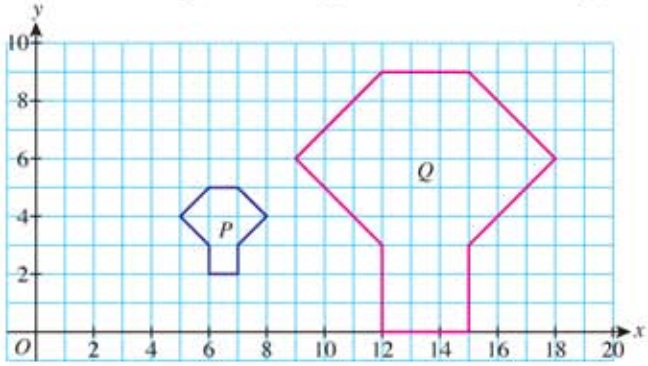
**UNDERSTAND**

**1.** The diagram on the right shows two congruent trapeziums with a pair of sides of equal length. Calculate the value of $x$.


**2.** The diagram on the right shows two similar geometric objects. Calculate the value of $y$.



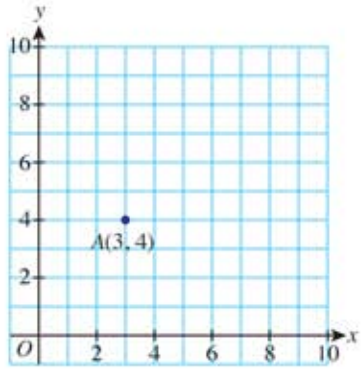

**3.** The diagram below shows that $Q$ is the image of $P$ under an enlargement.



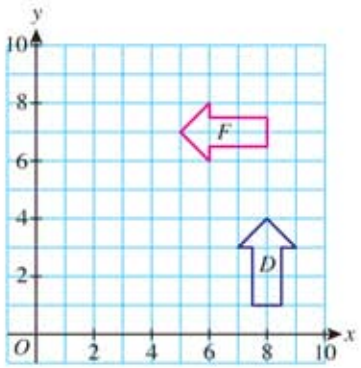

Describe the enlargement.

**4.** It is given that the areas of shape $A$ and shape $B$ are 18 cm$^2$ and 32 cm$^2$ respectively. If shape $B$ is the image of shape $A$ under an enlargement, determine the scale factor of the enlargement.

**5.** The diagram on the right shows that point $A$ lies on a Cartesian plane. It is given that transformation **R** is translation $\begin{pmatrix} 5 \\ -2 \end{pmatrix}$ and transformation **S** is a reflection on line $y = 7$. State the coordinates of the image of point $A$ under the combined transformation **RS.**



**6.** The diagram on the right shows that $F$ is the image of $D$ under the combined transformation **GH**. It is given that transformation **H** is a reflection on line $x = 6$. Describe transformation **G**.

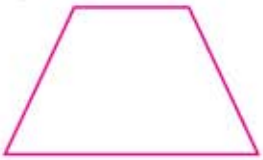



MASTERY

**7.** Based on each of the following polygons, design a tessellation using isometric transformation.

(a) isosceles triangle

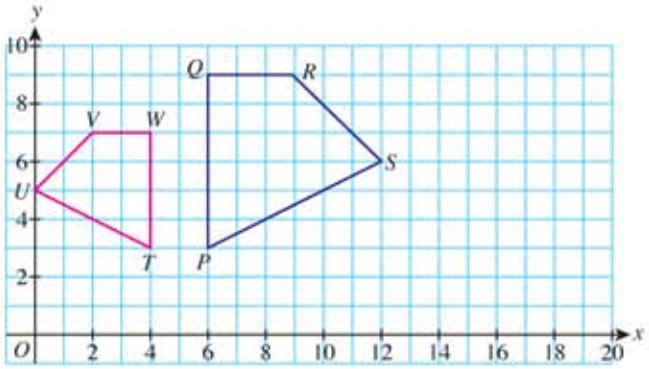
(b) trapezium



(c) pentagon

(d) hexagon

**8.** The diagram below shows two quadrilaterals $PQRS$ and $TUVW$ drawn on a Cartesian plane.



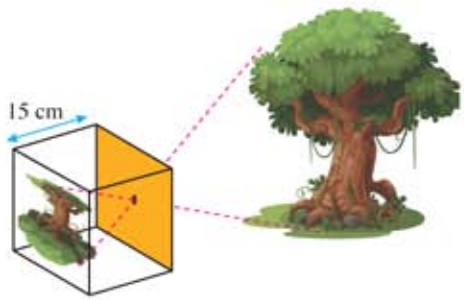

(a) Quadrilateral $TUVW$ is the image of quadrilateral $PQRS$ under the combined transformation **MN**. Describe
   (i) transformation **N**,
   (ii) transformation **M**.

(b) Given that the area of quadrilateral $TUVW$ is 10 unit$^2$, calculate the area, in unit$^2$, of the quadrilateral $PQRS$.

CHALLENGE

**9.** The diagram on the right shows two circles with the same centre $O$. The large circle is the image of the small circle under an enlargement at centre $O$. It is given that the radius of the small circle is 3 cm and the area of the coloured region is $47.25\pi$ cm$^2$. Calculate the scale factor of the enlargement.


**10.** Zurina built a pinhole camera to observe the image of a tree as shown in the diagram below.

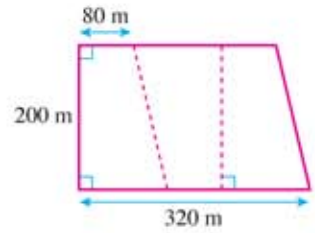



It is given that the scale factor of the image produced is $-\frac{1}{80}$. What is the horizontal distance, in cm, between the tree and its image?

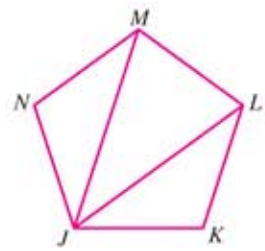
**11.** Madam Noriah has a trapezium-shaped piece of land. She divided her land into three congruent sections as shown in the diagram on the right. Calculate

(a) the perimeter, in m, of the land,

(b) the area, in m$^2$, of each section of the land.




**12.** The diagram on the right shows a regular pentagon $JKLMN$. It is given that the triangle $JNM$ is the image of triangle $JKL$ under a combined transformation **XY**. Describe transformation **X** and transformation **Y**.



EXPLORING MATHEMATICS

Diagram (a) below shows several shapes drawn on a square grid.

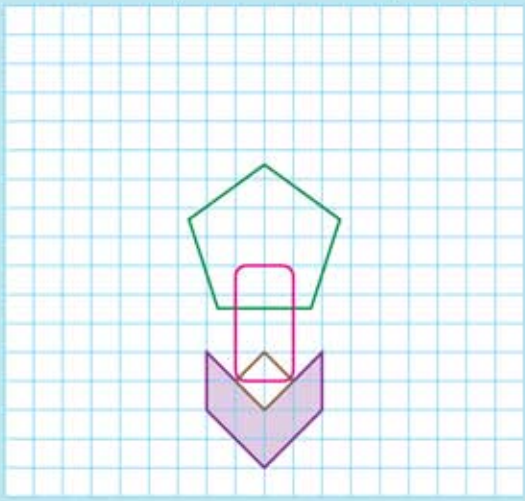
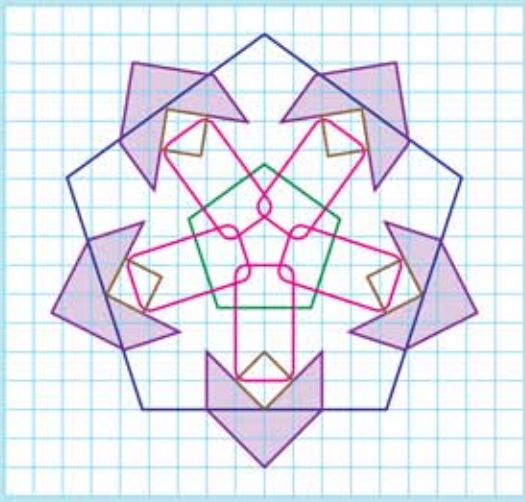


Diagram (b) below shows a logo designed by applying the transformation to the shapes in Diagram (a) and it is drawn on the square grid.



Based on your observation and discussion in groups, describe the transformations that have been used to design the logo.

Then, by choosing a combination of polygons, design a logo for an association or club in your school by applying transformations to the combination of polygons. Present the logo designed based on the following aspects:

(i) The meaning represented by each part or colour of the logo.
(ii) The appropriate motto.
(iii) The combined transformations used.