CHAPTER 6

# Angles and Tangents of Circles


## What will you learn?

- **6.1** Angle at the Circumference and Central Angle Subtended by an Arc
- **6.2** Cyclic Quadrilaterals
- **6.3** Tangents to Circles
- **6.4** Angles and Tangents of Circles

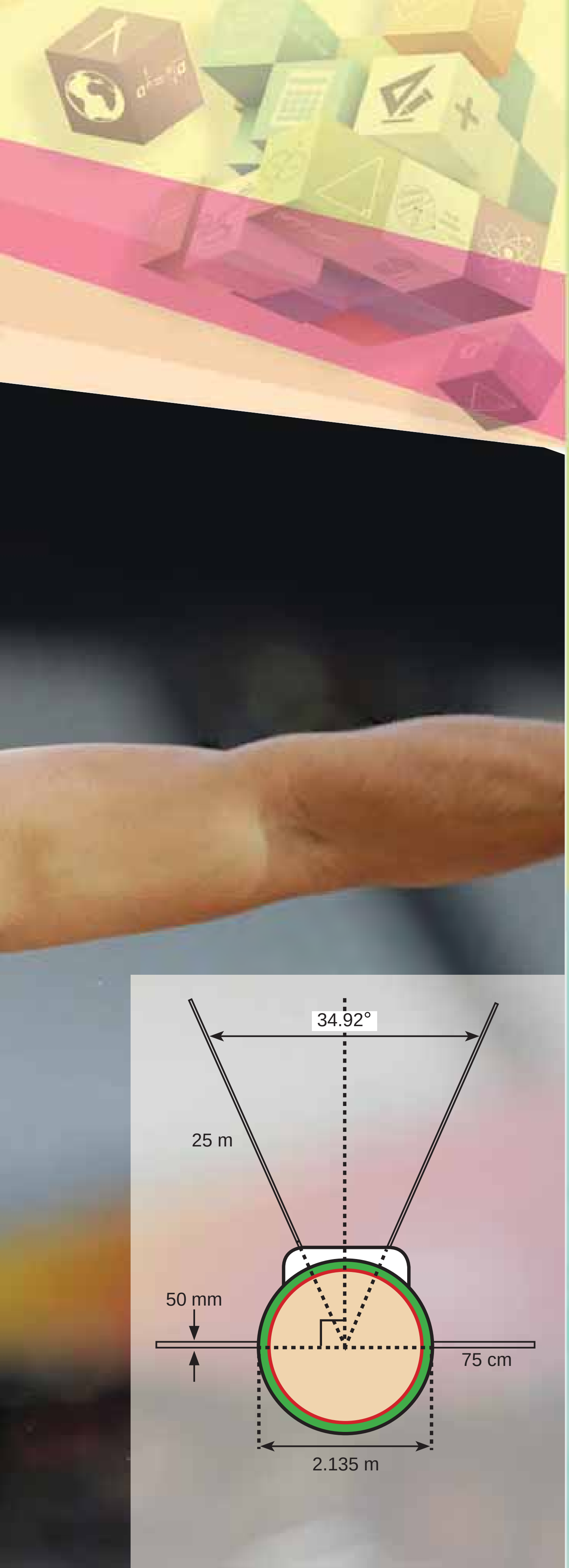
## Why do you learn this chapter?

- The circle is a unique shape and it has special properties. Its uniqueness allows circles to be used in various fields.
- The concept of angles and tangents of circles are used in industry, road construction, painting, astronomy, sports and so on.

Shot-put is an athletics event. The shot-put area is circular with a diameter of 2.135 m. The circle is divided into two parts or two semicircles with a white line of 50 mm thickness. Two straight lines are drawn from the centre of the circle at an angle of 34.92° between each other to determine the shot-put area.

Muhammad Ziyad Zolkefli is a national paralympic athlete. He won the gold medal in the T20 shot-put event at the 10th Fazza International Athletics Championship, Grand Prix (GP) World Athletics in Dubai, United Arab Emirates.

Have you ever participated in a shot-put event?




## Exploring Era

Thales and Pythagoras are famous Greek mathematicians. Thales' theorem states that when the three vertices of a triangle touch the circumference of the circle and one of the sides of the triangle is the diameter, then the angle subtended by the diameter is 90°. This theory was based on the influence of Ancient Egypt, India and Mesopotamia. Ancient mathematicians studied the circle as it was considered a perfect shape.

http://bukutekskssm.my/Mathematics/F3/ExploringEraChapter6.pdf

## WORD BANK

| | |
|---|---|
| diameter | *diameter* |
| arc | *lengkok* |
| circumference | *lilitan* |
| axis of symmetry | *paksi simetri* |
| chord | *perentas* |
| semicircle | *semi bulatan* |
| symmetry | *simetri* |
| tangent | *tangen* |
| alternate segments | *tembereng selang-seli* |
| point of tangency | *titik ketangenan* |

# 6.1 Angle at the Circumference and Central Angle Subtended by an Arc

## What are the angles at the circumference of a circle?

A circle is a unique two-dimensional shape. This is because the number of sides of the circle is infinite. The uniqueness of its shape allow round-shaped objects such as wheels to move easily. Have you ever seen a vehicle wheel in other shapes?

Angles formed in circles also have their own properties.

**LEARNING STANDARD**

Make and verify conjectures about the relationships between angles at the circumference and central angle subtended by particular arcs, and hence use the relationships to determine the values of angles in circles.

**DISCUSSION CORNER**

Compare the bicycle wheels below, which wheel allows you to reach your destination faster?

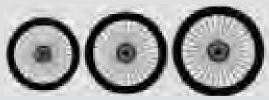


Diagram 1 shows two chords, $PQ$ and $QR$ which meet at point $Q$ at the circumference of the circle.

$\angle PQR$ is the **angle at the circumference** of the circle subtended by the arc $PR$.

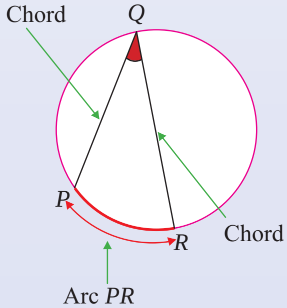



Diagram 1

In Diagram 2,

(a) $\angle PQS$ and $\angle PRS$ are angles at the circumference of the circle subtended by **major arc** $PS$.

(b) $\angle QPR$ and $\angle QSR$ are angles at the circumference of the circle subtended by **minor arc** $QR$.

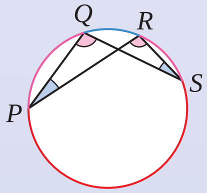
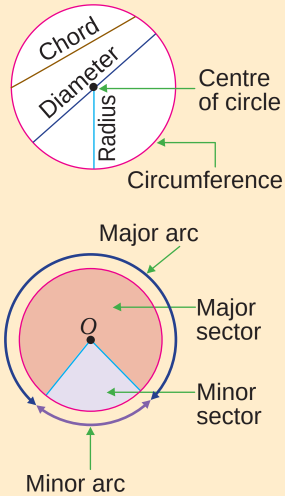



Diagram 2

**FLASHBACK**



## Are angles at the circumference of a circle subtended by the same arc equal?

### Brainstorming 1

In groups

**Aim:** To verify that angles at the circumference subtended by the same arc are equal.

**Materials:** A4 paper, compasses, protractor, ruler and pencil.

**Steps:**

1. Draw a circle of radius 5 cm. Draw a chord $PQ$ (Diagram 1).
2. Draw a chord $QR$ that forms 30° at point $Q$ (Diagram 2). Other groups are encouraged to form acute angles between 20° and 40°.
3. Mark the point $S$ on the circumference and draw chords $PS$ and $RS$ (Diagram 3).
4. Measure $\angle PSR$ and record it in the table below.
5. Repeat step 3 with point $T$ and chords $PT$ and $RT$ (Diagram 4).
6. Measure $\angle PTR$ and record it in the table.

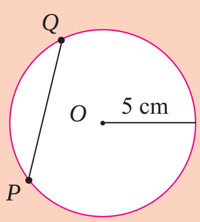



Diagram 1

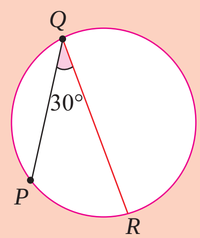



Diagram 2

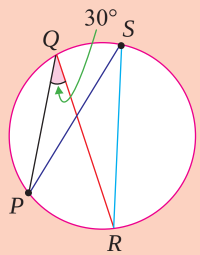



Diagram 3

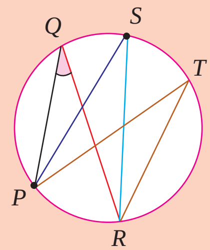



Diagram 4

| $\angle PQR$ | $\angle PSR$ | $\angle PTR$ | | |
|---|---|---|---|---|
| 30° | | | | |

7. You may repeat step 3 with other points on major arc $PR$. Measure the angle formed and record in the table.
8. Display your group's findings in the Mathematics corner. Give feedback on the findings of other groups.

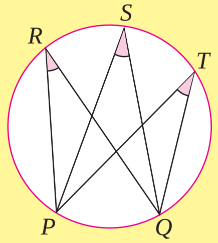
**Discussion:**

What can you say about the angles at the circumference of the circle subtended by arc $PR$?

From Brainstorming 1, it is found that:

The angles subtended by arc $PR$, $\angle PQR$, $\angle PSR$ and $\angle PTR$, are equal.

In general,




**Angles** at the **circumference subtended** by the **same arc** are **equal**.
$\angle PRQ = \angle PSQ = \angle PTQ$

You can also use dynamic software to verify the properties of angles at the circumference subtended by the same arc.

## Brainstorming 2

In pairs

**Aim:** To verify that angles at the circumference subtended by the same arc are equal.

**Materials:** Dynamic software

**Steps:**

1. Start with *New Sketch* and click on the *Compass Tool* to draw a circle (Diagram 1).
2. Click on *Point Tool* and mark three points (Diagram 2).
3. Click on *Text Tool* and label the three points marked in step 2 (Diagram 3).

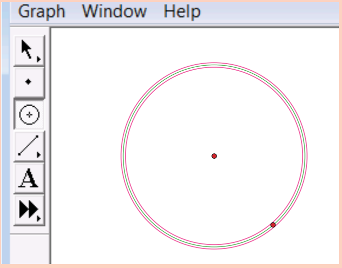



Diagram 1

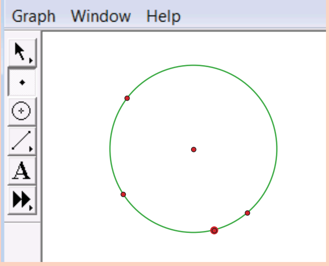



Diagram 2

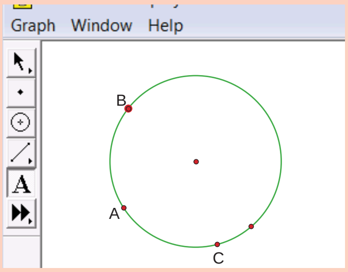



Diagram 3

4. Click on *Straightedge Tool* and draw two straight lines connecting point *A* and point *B* as well as point *B* and point *C* (Diagram 4).
5. Click on *Selection Arrow Tool* and click on points *A*, *B* and *C* (Diagram 5).
6. Click *Measure* and select *Angle*. The value of $\angle ABC$ will be displayed (Diagram 6).

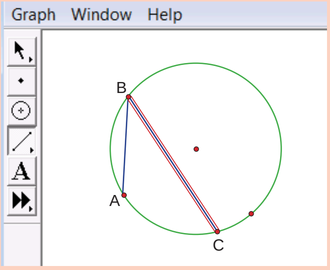



Diagram 4

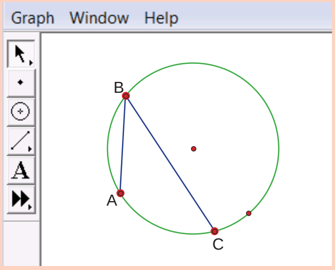



Diagram 5

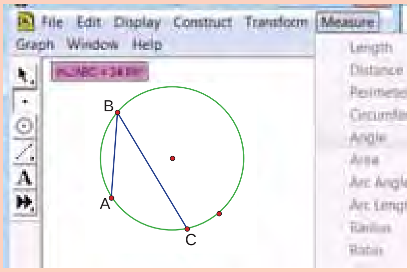



Diagram 6

**7.** Repeat steps 2 to 4 for point *D* and step 5 to select points *A*, *D* and *C* (Diagram 7).

**8.** Repeat step 6. The value of $\angle ADC$ will be displayed (Diagram 8). Notice that the values of $\angle ABC$ and $\angle ADC$ are the same.

**9.** You can try this with another point on the major arc *AC* to determine the value of the angle at the circumference.

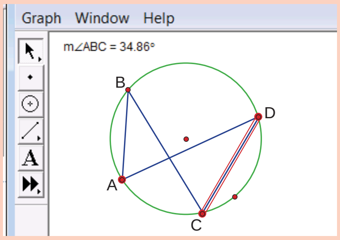



Diagram 7

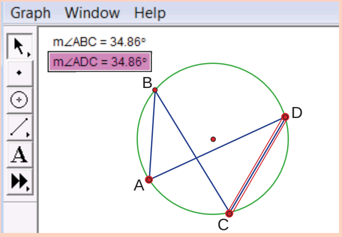



Diagram 8

**Discussion:**

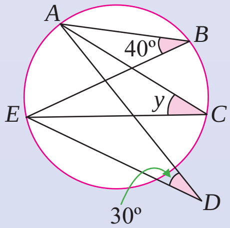
What can be concluded from your observations in the above activities?

From Brainstorming 2, it is found that:

The angles at the circumference subtended by the same arc are equal.

**Example 1**

Based on the diagram on the right, calculate the value of *y*.




***Solution:***

$y = \angle ABE = 40°$

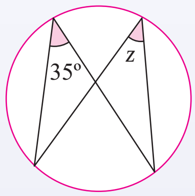
**BULLETIN**

$y = \angle ABE = 40°$.
$\angle ADE \neq 40°$ because $\angle ADE$ is not an angle at the circumference of the circle subtended by arc *AE*.

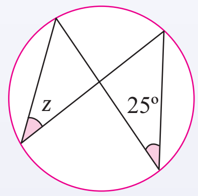
## MIND TEST 6.1a

**1.** Calculate the value of z.

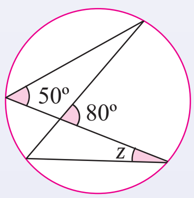
(a)



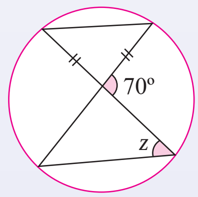

(b)




(c)

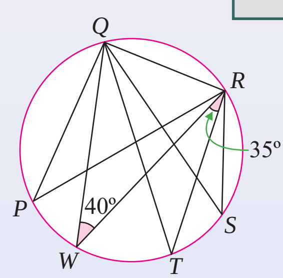



(d)



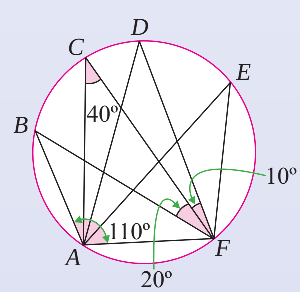

**2.** In the diagram on the right, chords $QW = RW$. Given that $\angle QWR = 40°$ and $\angle WRT = 35°$, determine the value of

(a) $\angle QSR$ (b) $\angle WQT$
(c) $\angle WRQ$ (d) $\angle QRT$



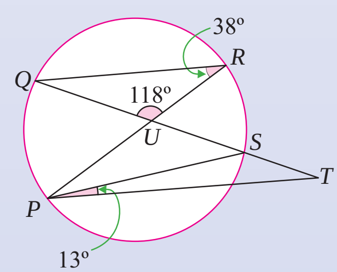

**3.** In the diagram on the right, $\angle BAF = 110°$, $\angle ACF = 40°$, $\angle CFD = 10°$ and $\angle BFC = 20°$. Determine the value of

(a) $\angle ABF$ (b) $\angle BFA$
(c) $\angle CAD$ (d) $\angle DAF$



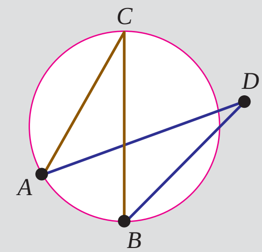

**4.** In the diagram on the right, $\angle QRP = 38°$, $\angle QUR = 118°$ and $\angle SPT = 13°$. Determine the value of

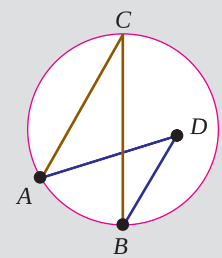
(a) $\angle RPS$ (b) $\angle PTQ$




### DISCUSSION CORNER

Is $\angle ADB = \angle ACB$? Discuss.



**Are angles at the circumference of a circle subtended by arcs of the same length equal and are angles at the circumference proportional to the arc length?**

## Brainstorming 3

In groups

**Aim:** **1.** To verify that angles at the circumference subtended by arcs of the same length are equal.

**2.** To verify that angles at the circumference is proportional to the length of the arc.

**Materials:** Compasses, protractor, pencil, ruler and A4 paper.

**Steps:**

1. Draw a circle of radius 5 cm. Without adjusting the gap of the compasses, divide the circumference of circle into six parts (Diagram 1 - Diagram 3).
2. Draw two angles at the circumference that are subtended by two different parts of the same length and label them (Diagram 4).

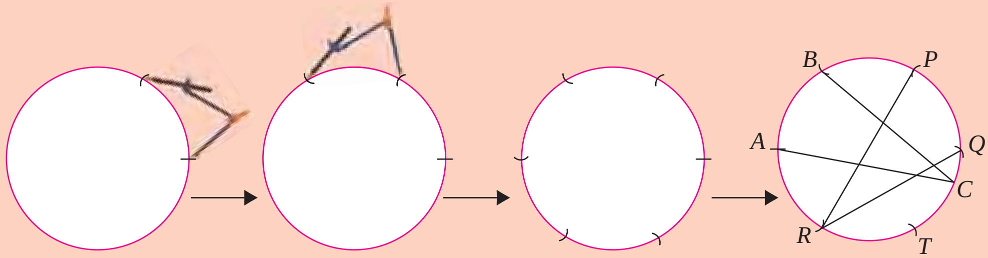



Diagram 1 Diagram 2 Diagram 3 Diagram 4

3. Measure ∠*BCA* and ∠*PRQ*. Record them in Table 1.
4. Repeat step 1. Draw chords with different arc lengths (Diagram 5). Measure ∠*RPT* and ∠*BQR*. Record them in Table 2.

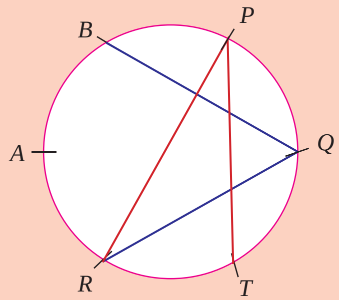



Diagram 5

| Arcs | |
|---|---|
| *BA* | *PQ* |
| ∠*BCA* | ∠*PRQ* |
| | |

Table 1

| Arcs | |
|---|---|
| *RT* | *BR* = 2*RT* |
| ∠*RPT* | ∠*BQR* |
| | |

Table 2

**Discussion:**

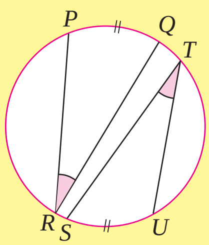
1. What can you conclude about angles at the circumference subtended by arcs of the same length?
2. What is your conclusion on the effects of changing the arc length to the angles subtended at the circumference?

From Brainstorming 3, it is found that:

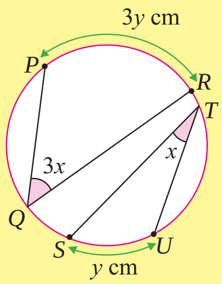
(a) $\angle BCA = \angle PRQ$ [Arc length $AB$ = Arc length $PQ$].

(b) $\angle BQR = 2 \times \angle RPT$ [Arc length $BR = 2 \times$ Arc length $RT$].

In general,



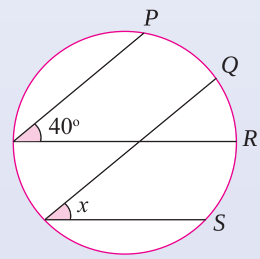

**Angles** at the **circumference** subtended by **arcs of the same length** are equal. If arc length $PQ$ = arc length $SU$ then $\angle PRQ = \angle STU$.




**The size of an angle** at the **circumference** subtended by an arc is **proportional** to **the arc length**.

## Example 2

The diagram on the right shows a circle with length of arcs $PR = QS$. Determine the value of $x$. Give reasons for your answer.

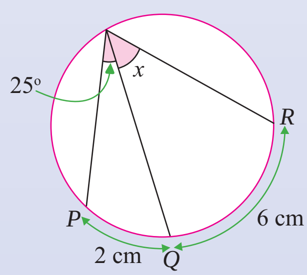



**Solution:**

$x = 40°$ because $\angle x$ and $\angle 40°$ are at the circumference and length of arcs $PR = QS$.

## Example 3

Based on the diagram on the right, determine the value of $x$.




**Solution:**

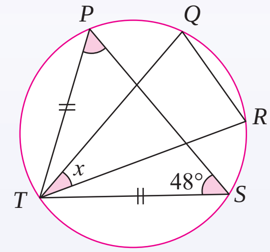
$$\frac{x}{25°} = \frac{6 \text{ cm}}{2 \text{ cm}}$$

$$x = 3(25°)$$

$$x = 75°$$

## Example 4

Given the length of minor arc $PS$ is two times the length of arc $QR$, determine the value of $x$.




**Solution:**

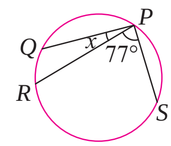
$\angle PTS = 180° - 2(48°)$ ← Sum of angles in a triangle is 180°.

$\angle PTS = 84°$

Thus, $x = \dfrac{84°}{2}$ ← $QR = \dfrac{PS}{2}$

$x = 42°$

**QUIZ**

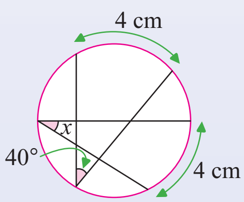



If arc length of $RS = \dfrac{7}{2}QR$, determine the value of $x$.

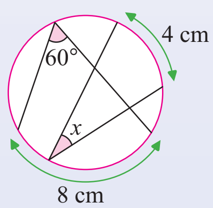
## MIND TEST 6.1b

**1.** Based on the diagrams below, calculate the value of $x$.

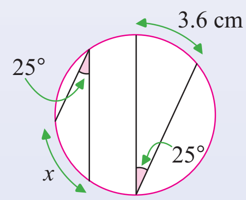
(a)



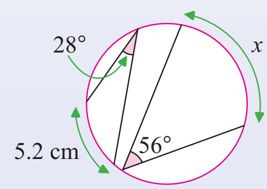

(b)




(c)



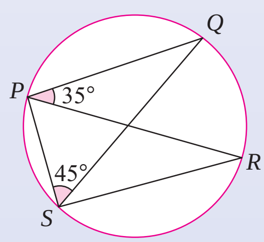

(d)




**2.** The diagram on the right shows a circle. Given that the length of arcs $RS = 2QR$, $\angle QPR = 35°$ and $\angle PSQ = 45°$, determine the value of

(a) $\angle SPR$

(b) $\angle SRP$

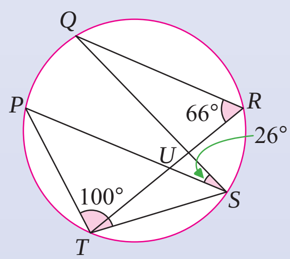



**3.** In the diagram on the right, the length of arcs $QPT = 3RS$. Given that $\angle QRT = 66°$, $\angle QST = 26°$ and $\angle PTS = 100°$, determine the value of

(a) $\angle RQS$

(b) $\angle TUS$

(c) $\angle TPS$




CHAPTER 6

**What is the relationship between angles at the centre of a circle and angles at the circumference that are subtended by the same arc?**

## Brainstorming 4

In pairs

**Aim:** To verify the relationship between angles at the centre of a circle and angles at the circumference subtended by the same arc.

**Materials:** Dynamic software

**Steps:**

1. Start with *New Sketch* and click on *Compass Tool* to draw a circle.
2. Use *Point Tool* to place three points around its circumference (Diagram 1).
3. Use *Text Tool* to label all points at the circle with $A$, $B$, $C$ and centre as $D$ (Diagram 2).
4. Use *Straightedge Tool* to construct lines from one point to another (Diagram 3).

Diagram 1

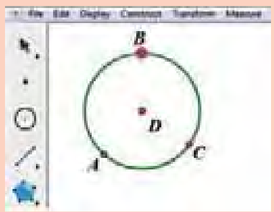



Diagram 2

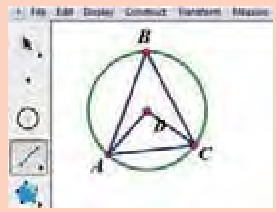



Diagram 3

5. Use *Selection Arrow Tool* to select points $A$, $B$ and $C$.
6. Click on the menu *Measure* and select *Angle*. The value of $\angle ABC$ will be displayed.
7. Repeat steps 5 and 6 to get $\angle ADC$. The value of $\angle ADC$ will be displayed (Diagram 4).

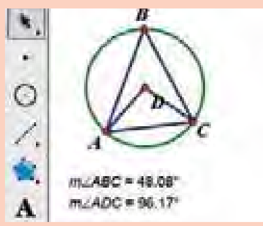



Diagram 4

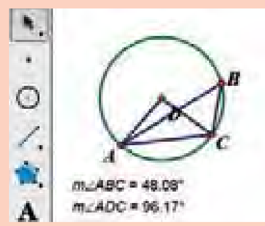



Diagram 5

8. What is the relationship between $\angle ABC$ and $\angle ADC$?
9. Click on point $B$ and move it along the circumference of the circle as shown in Diagram 5. Is the value of $\angle ABC$ still the same as the value obtained in step 6?

**Discussion:**

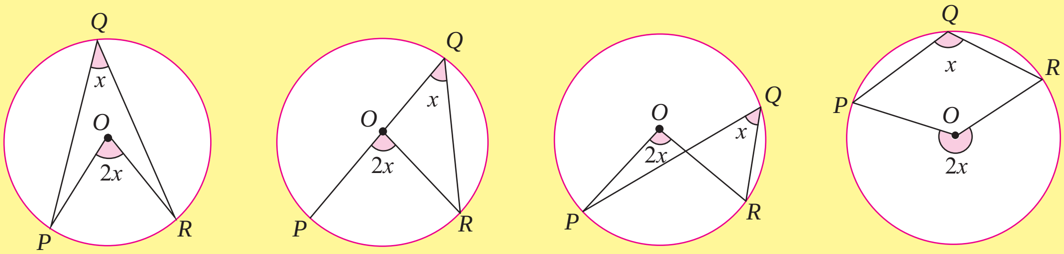
What can you conclude about the relationship between angles at the centre of a circle and angles at the circumference of a circle subtended by the same arc?

From Brainstorming 4, it is found that:

(a) $\angle ADC = 2 \times \angle ABC$

(b) The value of $\angle ABC$ is constant even though point $B$ is moved along the circumference of the circle.

In general,

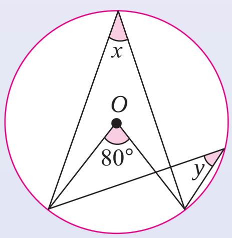



The size of the **angle** at the **centre of a circle** (central angle) subtended by the same arc is **twice** the size of angle at the **circumference**.

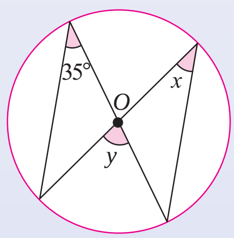
## Example 5

Determine the value of $x$ and $y$ for each of the following.

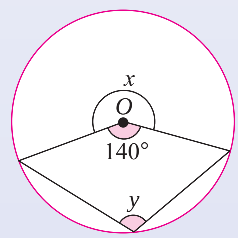
(a)




(b)




(c)




**Solution:**

(a) $x = \frac{1}{2}(80°)$
$x = 40°$
$y = 40°$

(b) $x = 35°$
$y = 2(35°)$
$y = 70°$

(c) $x = 360° - 140°$
$x = 220°$
$y = \frac{1}{2}(220°)$
$y = 110°$

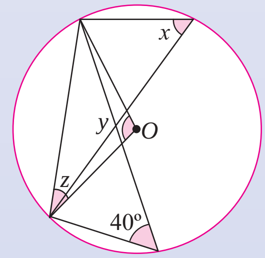
## MIND TEST 6.1c

**1.** The diagram on the right shows a circle with centre $O$. Determine the value of

(a) $x$

(b) $y$

(c) $z$

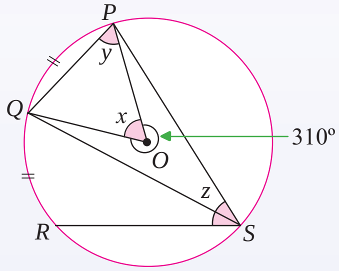


**2.** The diagram on the right shows a circle with centre $O$. Given that the length of arcs $PQ = QR$ and major angle $POQ = 310°$, calculate the value of

(a) $x$

(b) $y$

(c) $z$

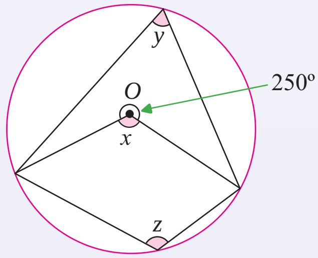



**3.** The diagram on the right shows a circle with centre $O$. Calculate the value of

(a) $x$

(b) $y$

(c) $z$

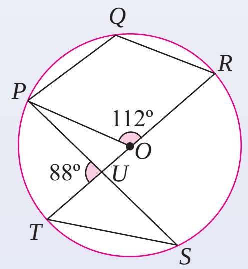



**4.** The diagram on the right shows a circle with centre $O$. Given that $\angle POR = 112°$ and $\angle PUT = 88°$, determine the value of

(a) $\angle PQR$

(b) $\angle UST$

(c) $\angle RTS$





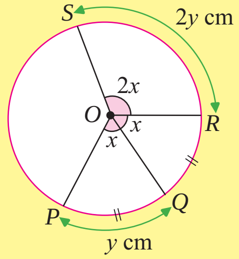

### Are the central angles of a circle proportional to the arc length?

You have learned that:

**1.** Angles at the circumference of a circle subtended by the same arc are equal.

**2.** Angles at the circumference of a circle subtended by an arc are proportional to its arc length.

Both of the concepts above can also be applied to the central angle. In general,




For central angles of a circle subtended by an arc:

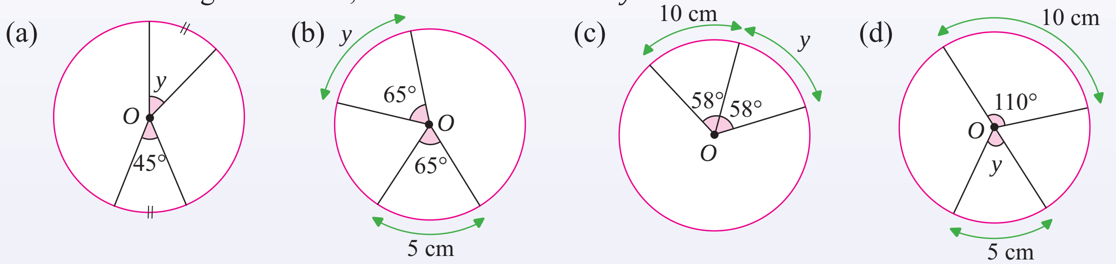
(a) the sizes of the angles are equal if their arc lengths are equal.

(b) the change in size of an angle is proportional to the change in the arc length.

## MIND TEST 6.1d

**1.** Based on the diagrams below, calculate the value of *y*.

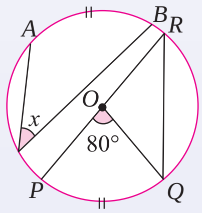
(a) (b) (c) (d)




**2.** The diagram on the right shows a circle with centre *O* where the length of arcs *AB* = *PQ*. Determine

(a) the value of *x*

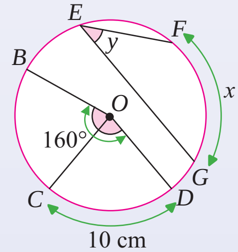
(b) the angle that has the same value as *x*




**3.** The diagram on the right shows a circle with centre *O*. It is given that length of arc *CD* = 10 cm and ∠*BOD* = 160°. If the length of arcs *BCD* = 2*CD* and ∠*FEG* = $\frac{1}{4}$ ∠*BOD*, determine

(a) the value of *y*, in cm

(b) the length of *x*, in cm




**What is the value of angles at the circumference subtended by the diameter?**

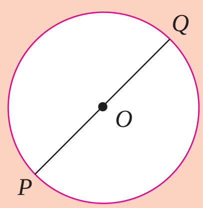
## Brainstorming 5

In pairs

**Aim:** To determine the angles subtended by the diameter.

**Materials:** Compasses, protractor, pencil, ruler and drawing paper.

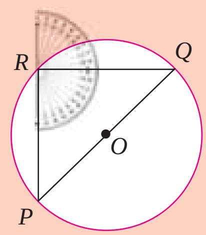
**Steps:**




**1.** Draw a circle with centre *O* and diameter *PQ* as in the diagram.

**2.** Draw two chords, *PR* and *QR* as in the diagram. Measure the value of ∠*PRQ*.

**3.** Change the position of point *R* at the circumference of the circle. Measure the new value of ∠*PRQ*.




**Discussion:**

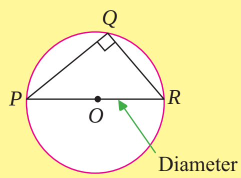
**1.** What can you conclude about the value of ∠*PRQ* when the position of point *R* is changed at the circumference?

**2.** What is the value of the angle at the circumference of a circle subtended by the diameter?

From Brainstorming 5, it is found that:

For all positions of point $R$ at the circumference of the circle subtended by diameter $PQ$, the value of $\angle PRQ$ is 90°.

In general,



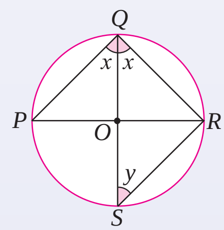

The angle at the circumference of circle subtended by the diameter is 90°. If $PQR$ is a semicircle, then $\angle PQR = 90°$.

**DISCUSSION CORNER**

Is a diameter a chord? Discuss.

**Example 6**

The diagram on the right shows a circle with centre $O$ where points $P$, $Q$, $R$ and $S$ lie on the circumference of the circle. Given that $PR$ and $QS$ are diameters, calculate the value of $y$.




***Solution:***

$PR = QS$

Thus, $2x = 90°$

$x = 45°$

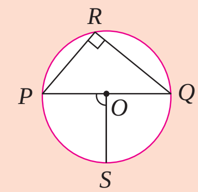
$y + x + \angle QRS = 180°$

$y + 45° + 90° = 180°$

$y = 180° - 45° - 90°$

$y = 45°$

**BULLETIN**

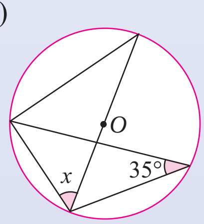



If arc lengths $PRQ = 2PS$ then, $\angle PRQ = \angle POS$

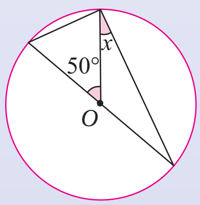
**MIND TEST 6.1e**

1. The diagrams below show circles with centre $O$. Calculate the value of $x$.

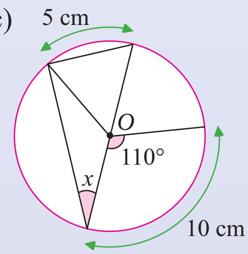
(a)



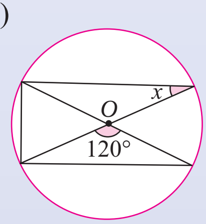

(b)



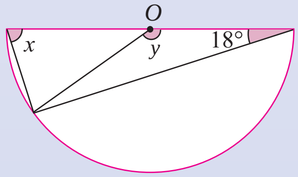

(c)



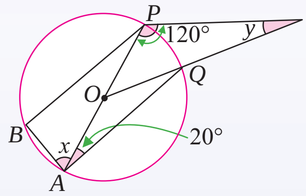

(d)




2. The diagram on the right shows a semicircle with centre $O$. Determine the value of $x + y$.



**3.** The diagram on the right shows a circle with centre $O$. If length of arcs $AB = PQ$, calculate the value of $x + y$.




## How do you solve problems involving angles in circles?

**LEARNING STANDARD**

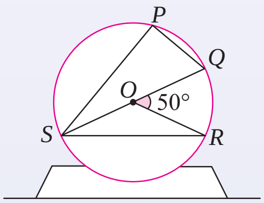
Solve problems involving angles in circles.

**Example 7**

A sculpture is constructed in the shape of a circle with centre at $O$ as in the diagram. The points on the circumference form arc $PQ$ which is of the same length as arc $QR$. Line $SQ$ passes through $O$. Determine the value of

(a) $\angle QSR$

(b) $\angle PQS$




**Solution:**

(a) $\angle QSR = \frac{1}{2} \angle QOR$

$= \frac{1}{2}(50°)$

$= 25°$

(b) $\angle PSQ = \angle QSR = 25°$

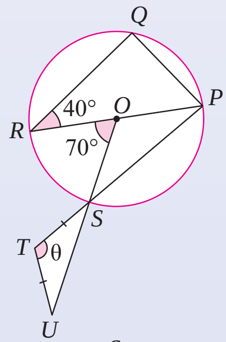
$\angle PQS + 90° + 25° = 180°$

$\angle PQS = 180° - 90° - 25°$

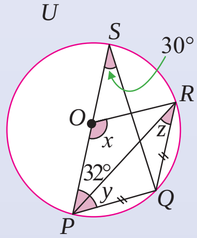
$= 65°$

**MIND TEST 6.1f**

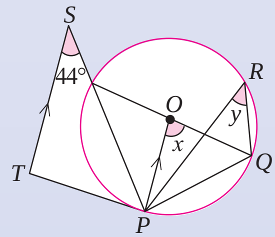
**1.** The diagram on the right shows a circle with centre $O$. $OSU$ and $PST$ are straight lines. Given that the diameter of the circle is 16 cm, $\angle ROS = 70°$, $\angle QRP = 40°$ and $ST = TU$,
(a) calculate the value of $\theta$
(b) calculate the length of $PQ$, in cm, correct to three significant figures




**2.** The diagram on the right shows a circle with centre $O$. Given that $PQ = QR$, $\angle PSQ = 30°$ and $\angle SPR = 32°$, calculate the value of $x + y + z$.




**3.** The diagram on the right shows a circle with centre $O$. Given that $TS$ is parallel to $PO$ and $\angle TSP = 44°$, calculate the value of $x + y$.




CHAPTER 6

## 6.2 Cyclic Quadrilaterals

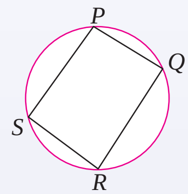
### What do you know about cyclic quadrilaterals?

**LEARNING STANDARD**

Recognise and describe cyclic quadrilaterals.

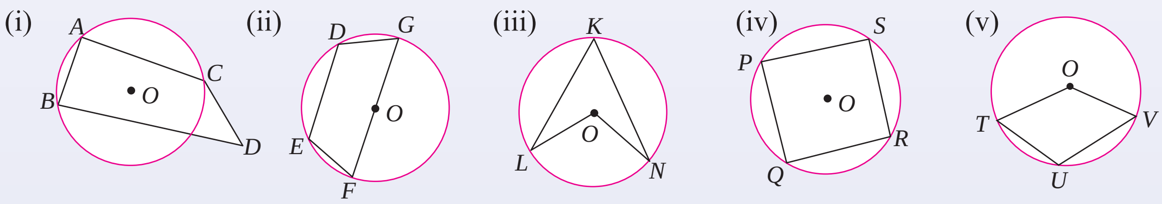
A **cyclic quadrilateral** is a quadrilateral in a circle where **all four vertices** of the quadrilateral lie on the **circumference of the circle**.

*PQRS* in the diagram on the right is a cyclic quadrilateral. ∠*P* and ∠*R* as well as ∠*S* and ∠*Q* are known as **opposite angles** in the cyclic quadrilateral.




**Example 8**

For each of the following circles, *O* is centre of the circle.




(a) Identify the cyclic quadrilateral found in each of the above circles and explain your answer.

(b) State the opposite angles in each cyclic quadrilateral that you have identified.

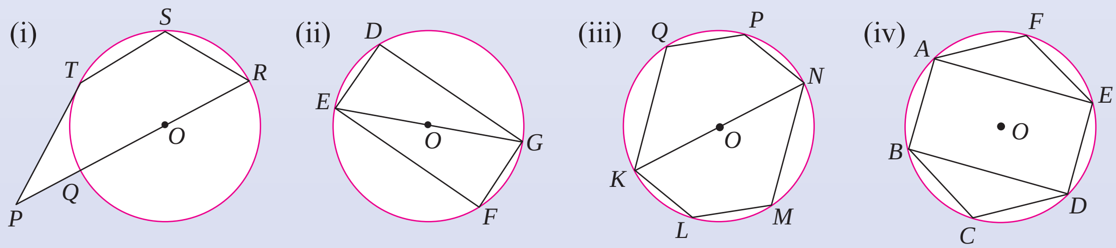
**Solution:**

(a) (i) Vertex *D* does not lie on the circumference, hence *ABCD* is not a cyclic quadrilateral.
(ii) All vertices are on the circumference, hence *DEFG* is a cyclic quadrilateral.
(iii) Vertex *O* does not lie on the circumference, hence *KLON* is not a cyclic quadrilateral.
(iv) All vertices are on the circumference, hence *PQRS* is a cyclic quadrilateral.
(v) Vertex *O* does not lie on the circumference, hence *OTUV* is not a cyclic quadrilateral.

(b) (i) None (ii) ∠*D* and ∠*F*, ∠*E* and ∠*G* (iii) None
(iv) ∠*P* and ∠*R*, ∠*Q* and ∠*S* (v) None

**MIND TEST 6.2a**

**1.** For each of the following circles, *O* is the centre of the circle.




(a) Identify the cyclic quadrilateral found in each circle above and explain your answer.

(b) State the opposite angles in each cyclic quadrilateral that you have identified.

## What are the relationships between angles of a cyclic quadrilateral?

### Brainstorming 6

In pairs

**Aim:** To determine the relationship between opposite interior angles of a cyclic quadrilateral.

**Materials:** Dynamic software

**Steps:**

1. Start with *New Sketch* and click on *Compass Tool* to draw a circle.
2. Click on *Straightedge Tool* to construct four lines from one point to another point on its circumference (Diagram 1).
3. Use *Text Tool* to label all points connecting the line with $A$, $B$, $C$ and $D$.
4. Use *Selection Arrow Tool* to select $D$, $A$, and $B$.
5. Click on the menu *Measure* and select *Angle*. The value of $\angle DAB$ will be displayed.
6. Repeat steps 4 and 5 to get $\angle ABC$, $\angle BCD$ and $\angle CDA$ (Diagram 2).

**Discussion:**

1. What are the relationships between $\angle DAB$, $\angle ABC$, $\angle BCD$ and $\angle ADC$?
2. What can you conclude about the relationships between the angles of a cyclic quadrilateral?

**LEARNING STANDARD**

Make and verify conjectures about the relationships between angles of cyclic quadrilaterals, and hence use the relationships to determine the values of angles of cyclic quadrilaterals.

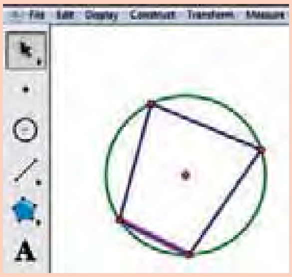


Diagram 1

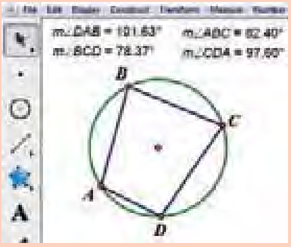


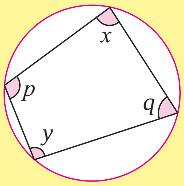

Diagram 2

From Brainstorming 6, it is found that:

(a) $\angle DAB + \angle BCD = 180°$ and $\angle ABC + \angle ADC = 180°$

(b) **The sum of the opposite** interior angles in a cyclic quadrilateral is 180°.

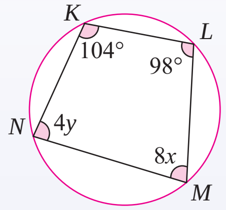
In general,




The sum of opposite interior angles in a cyclic quadrilateral is 180°.
$\angle x + \angle y = 180°$ and $\angle p + \angle q = 180°$

**Example 9**

The diagram on the right shows a cyclic quadrilateral *KLMN*. Calculate the value of
(a) $x$ (b) $y$




***Solution:***

(a) The interior angles $\angle LKN$ and $\angle LMN$ are opposite in the cyclic quadrilateral.

Thus, $\angle LKN + \angle LMN = 180°$

$104° + 8x = 180°$

$8x = 180° - 104°$

$8x = 76°$

$x = \dfrac{76°}{8}$

$x = 9.5°$

(b) The interior angles $\angle KNM$ and $\angle KLM$ are opposite in the cyclic quadrilateral.

Thus, $\angle KNM + \angle KLM = 180°$

$4y + 98° = 180°$

$4y = 180° - 98°$

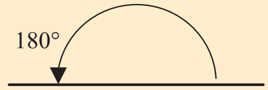
$4y = 82°$

$y = \dfrac{82°}{4}$

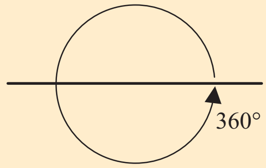
$y = 20.5°$

**FLASHBACK**

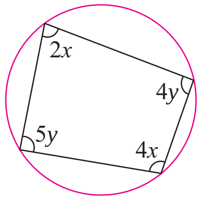
Angle on a straight line is 180°.




Angle of a full rotation is 360°.




**QUIZ**

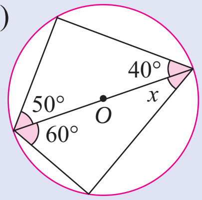



Calculate the value of $x + y$.

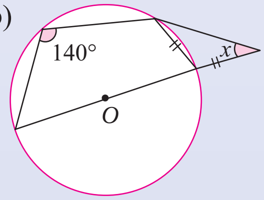
## MIND TEST 6.2b

**1.** The diagrams below show circles with centre $O$. Calculate the value of $x$.

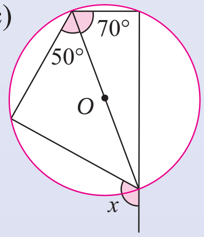
(a)



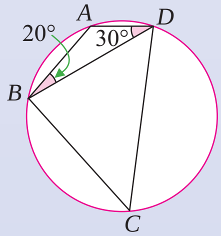

(b)



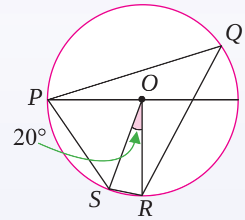

(c)




**2.** The diagram on the right shows a cyclic quadrilateral *ABCD*. Given that $\angle ADB = 30°$ and $\angle ABD = 20°$, calculate the value of $\angle BCD$.



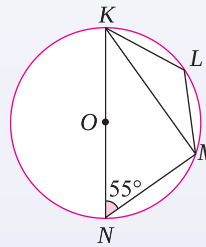
**3.** The diagram on the right shows a circle with centre $O$. If $\Delta POS$ is an equilateral triangle and $\angle SOR = 20°$, calculate the value of $\angle PQR$.



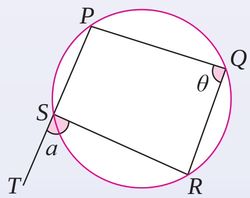

**4.** The diagram on the right shows a circle with centre $O$. Given that $\angle KNM = 55°$ and $KL = LM$, determine the value of

(a) $\angle KLM$

(b) $\angle LMN$



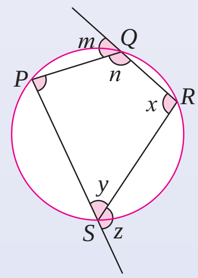

### What is the relationship between the exterior angle with the corresponding opposite interior angle?




The diagram shows a cyclic quadrilateral $PQRS$. The chord $PS$ is extended to $T$. $\angle TSR$, $a$, is the **exterior angle** of the cyclic quadrilateral $PSRQ$. $\angle PQR$, $\theta$, is known as the **opposite interior angle** corresponding to $a$.

## Example 10

In the diagram on the right, $PQRS$ is a cyclic quadrilateral. Given that $m$ and $z$ are exterior angles, state the opposite interior angles corresponding to $m$ and $z$.

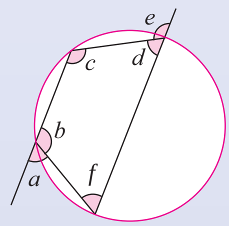



***Solution:***

$y$ is the opposite interior angle corresponding to $m$.
$n$ is the opposite interior angle corresponding to $z$.

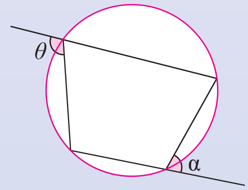
## MIND TEST 6.2c

**1.** Copy and complete the table below based on the diagram on the right.




| Exterior angle | Corresponding opposite interior angle |
|---|---|
| | |
| | |

**2.** Draw a circle as shown in the diagram. Label the corresponding opposite interior angles for the exterior angle $\theta$ and $\alpha$ with symbols $p$ and $q$ respectively.

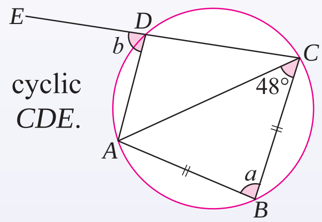


## How do you solve problems involving cyclic quadrilaterals?

**LEARNING STANDARD**

Solve problems involving cyclic quadrilaterals.

**Example 11**




The diagram on the right shows a cyclic quadrilateral $ABCD$ and a straight line $CDE$. Calculate the value of

(a) $a$
(b) $b$

**Solution:**

(a) $\angle ACB = \angle CAB = 48°$

$\angle ACB + \angle CAB + a = 180°$

$48° + 48° + a = 180°$

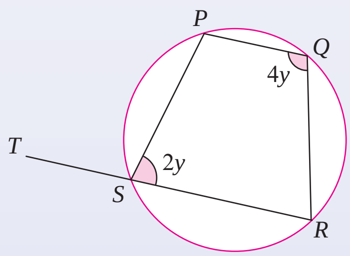
$a = 180° - 48° - 48°$

$a = 84°$

(b) $b = a$

Thus, $b = 84°$

**Example 12**




The diagram on the right shows a cyclic quadrilateral $PQRS$ and a straight line $RST$. Calculate the value of $\angle PST$.

**Solution:**

$\angle PQR + \angle PSR = 180°$

$4y + 2y = 180°$

$6y = 180°$

$y = 30°$

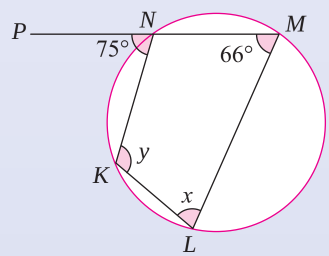
$\angle PST = \angle PQR$

$= 4y$

$= 4(30°)$

$\angle PST = 120°$

**Example 13**




The diagram on the right shows a cyclic quadrilateral $KLMN$ and a straight line $MNP$. Calculate the value of

(a) $x$
(b) $y$

**Solution:**

(a) $\angle PNK$ is an exterior angle. The opposite interior angle corresponding to it is angle $x$.

Thus,

$x = 75°$

(b) $y$ and $\angle NML$ are opposite interior angles of cyclic quadrilateral $KLMN$.

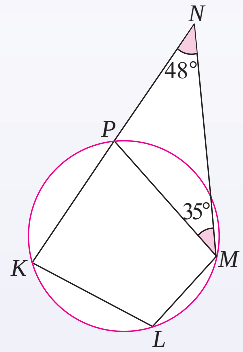
Thus, $y = 180° - \angle NML$

$y = 180° - 66°$

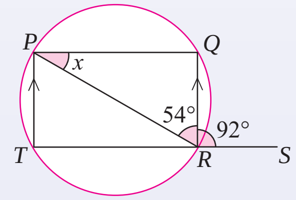
$y = 114°$

## MIND TEST 6.2d

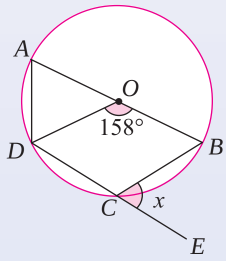
**1.** The diagram on the right shows a cyclic quadrilateral *KLMP* and a straight line *KPN*. Given that $\angle KNM = 48°$ and $\angle NMP = 35°$, calculate the value of $\angle MLK$.



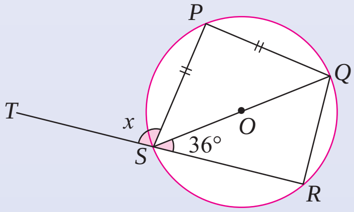

**2.** The diagram on the right shows a cyclic quadrilateral *PQRT* and a straight line *TRS*. The sides *PT* and *QR* are parallel. Given that $\angle PRQ = 54°$ and $\angle QRS = 92°$, calculate the value of $x$.



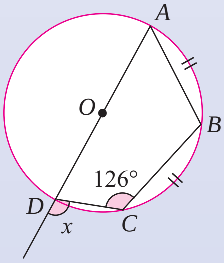

**3.** In the diagram on the right, the cyclic quadrilateral *ABCD* lies in a circle with centre *O*. Calculate the value of $x$ if *DCE* is a straight line and $\angle DOB = 158°$.




**4.** The diagram on the right shows a circle with centre *O*. *PQRS* is a cyclic quadrilateral. It is given that $\angle QSR = 36°$. If the length of $PS = PQ$ and *RST* is a straight line, calculate the value of $x$.




**5.** The diagram on the right shows a circle with centre *O*. Given that $\angle BCD = 126°$, length of arcs $AB = BC$ and *AOD* is a straight line, calculate the value of $x$.



## 6.3 Tangents to Circles

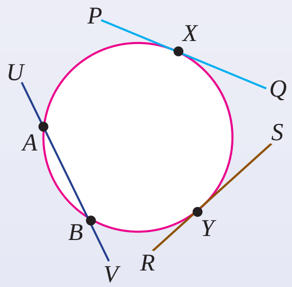
### What do you understand about the tangents to circles?

**LEARNING STANDARD**

Recognise and describe the tangents to circles.

You have learnt that the circle is a unique shape and has many special properties.


In the diagram on the left, point $T$ on the wheel will only touch the road once, when it makes a complete circle. The road serves as a tangent to the wheel which is round and the point $T$ is the point of tangency when it touches the road.



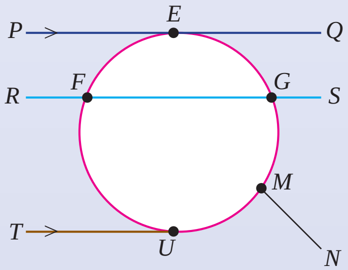

In the diagram on the left, straight lines $PQ$ and $RS$ each touches the circle at point $X$ and point $Y$ while straight line $UV$ passes through point $A$ and point $B$ on the circle. Thus,

(a) $PQ$ and $RS$ – Tangents to the circle.
(b) $X$ and $Y$ – Points of tangency of $PQ$ and $RS$, respectively.
(c) $UV$ – Not a tangent.
(d) $A$ and $B$ – Not points of tangency of $UV$.

**Tangent** to a circle is a straight line that touches the circle at only one point. The point of contact between tangent and the circle is the **point of tangency**.

**Example 14**

Are all straight lines and points shown in the diagram on the right tangents to the circle and points of tangency? State the reasons for your answer.




***Solution:***

$PQ$ and $TU$ are tangents to the circle because they touch the circle at only one point. Point $E$ and point $U$ are points of tangency of $PQ$ and $TU$ respectively.

$RS$ is not a tangent to the circle because it passes through two points on the circle. Hence, point $F$ and point $G$ are not points of tangency of $RS$. $MN$ is not a tangent to the circle because it will touch two points on the circle if extended. Thus, point $M$ is not a point of tangency.

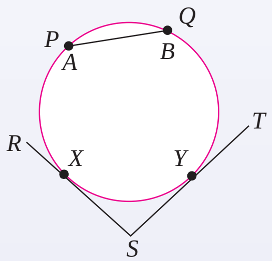
## MIND TEST 6.3a

**1.** In the diagrams below, identify points and lines which are

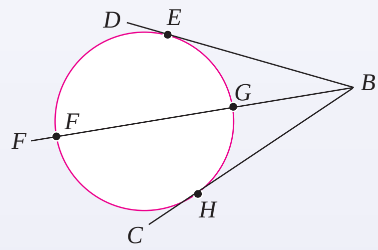
(i) tangents (ii) points of tangency (iii) not a tangent (iv) not a point of tangency

State the reasons for your answer.

(a)




(b)




**What do you know about the value of the angle between tangent and radius at the point of tangency?**

## Brainstorming 7

In pairs

**LEARNING STANDARD**

Make and verify conjectures about the angle between tangent and radius of a circle at the point of tangency.

**Aim:** To measure the angle between tangent and radius of a circle at the point of tangency.

**Materials:** Dynamic software

**Steps:**

**1.** Start with *New Sketch* and click on the *Compass Tool* to draw a circle (Diagram 1).

**2.** Click on *Straightedge Tool* to draw a straight line from the centre of the circle to a point on the circumference (Diagram 2).

**3.** Click on *Arrow Tool* to select point on the circumference and straight line.

**4.** Click *Construct* and select *Perpendicular Line* (Diagram 3).

**5.** Use *Point Tool* to mark the points and label them with the *Text tool* as *A*, *B* and *C* (Diagram 4).

**6.** Use *Selection Arrow Tool* to select *A*, *B* and *C*.

Diagram 1

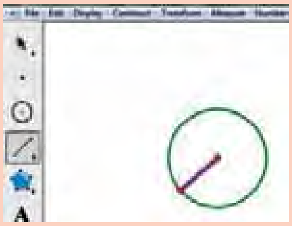


Diagram 2

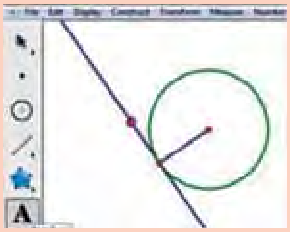


Diagram 3

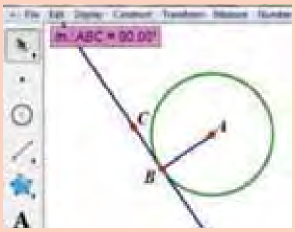


Diagram 4

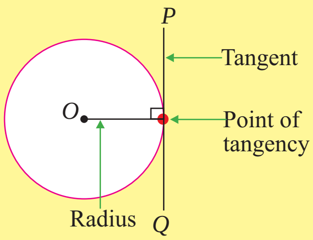
7. Click on the menu *Measure* and select *Angle*. The value of *ABC* will be displayed.
8. Repeat step 2 to step 7 to draw tangent lines on the other side of the circle and determine the angle between tangent and radius at the point of tangency.

**Discussion:**
What conclusions can you draw about the value of the angle between tangent and radius at the point of tangency?

From Brainstorming 7, it is found that:

When tangent and radius intersect at the point of tangency, a right angle is formed. Thus $\angle ABC = 90°$.

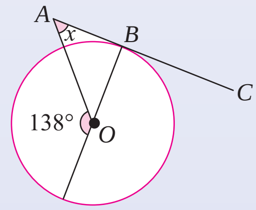
In general,




The radius of a circle that intersects with tangent to the circle at the point of tangency will form a 90° angle with the tangent.

**Example 15**

The diagram on the right shows a circle with centre *O* which meets the straight line *ABC* at point *B* only. Calculate the value of *x*.




***Solution:***

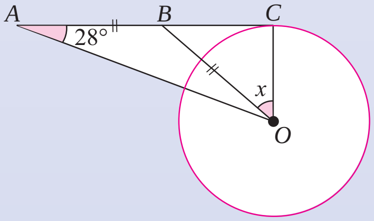
Line *ABC* is a tangent to the circle and it touches the circle at point *B*. Thus, the angle $\angle OBA = 90°$.

$$\angle AOB + 138° = 180°$$
$$\angle AOB = 180° - 138°$$
$$= 42°$$

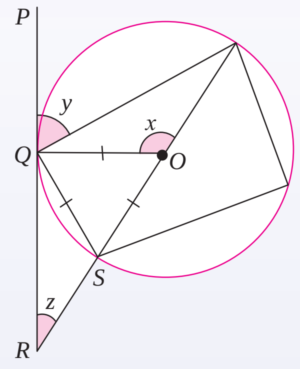
$$x + \angle AOB = 90°$$
$$x = 90° - \angle AOB$$
$$x = 90° - 42°$$
$$x = 48°$$

**MIND TEST 6.3b**

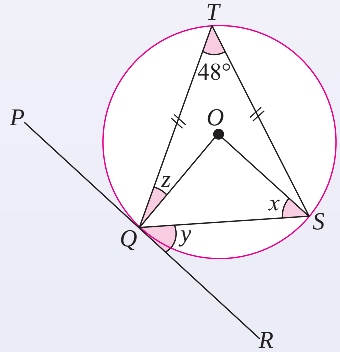
1. In the diagram on the right, *ABC* is a straight line and *O* is the centre of the circle. Given that *AB* = *OB* and $\angle BAO = 28°$, calculate the value of *x*.



2. The diagram on the right shows a circle with centre $O$. Given that $OQS$ is an equilateral triangle and $PQR$ is a tangent to the circle, calculate the value of
   (a) $x$
   (b) $y$
   (c) $z$




3. In the diagram on the right, $O$ is the centre of the circle and $PQR$ is a tangent to the circle. Given that $QT = ST$ and $\angle QTS = 48°$, calculate the value of $x + y + z$.




## What are the properties related to two tangents to a circle?

### Brainstorming 8

In pairs

**LEARNING STANDARD**

Make and verify conjectures about the properties related to two tangents to a circle.

**Aim:** To determine the properties related to two tangents to a circle.

**Materials:** Drawing paper, compasses, protractor, ruler and pencil.

**Steps:**

1. Draw a circle of radius 3 cm with centre $O$. Draw a straight line 8 cm from the centre $O$ and label as $OA$ (Diagram 1).
2. Draw another circle of radius 7 cm with point $A$ as centre of the circle. Mark the intersection points of both circles as $B$ and $C$ (Diagram 2).
3. Draw straight lines $OB$, $OC$, $AB$ and $AC$ (Diagram 3).

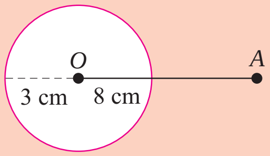


Diagram 1

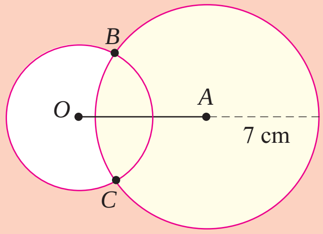


Diagram 2

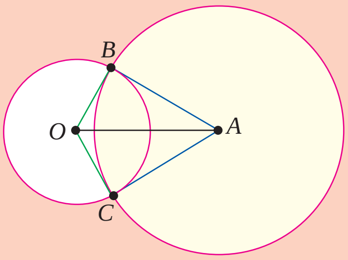


Diagram 3

4. Measure the following and complete the table below.

| ∠AOB | ∠AOC | ∠OBA | ∠OCA | ∠OAB | ∠OAC | Length | | | |
|---|---|---|---|---|---|---|---|---|---|
| | | | | | | OB | OC | AB | AC |
| | | | | | | | | | |

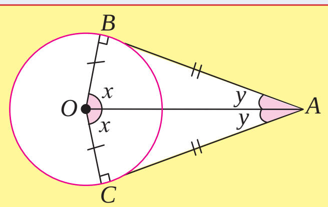
5. Display your group's findings in the Mathematics corner. Compare your group's answers with other groups.

**Discussion:**
What are your conclusions regarding the pairs of ∠*AOB* and ∠*AOC*, ∠*OBA* and ∠*OCA*, ∠*OAB* and ∠*OAC* and also the length of lines *OB*, *OC*, *AB* and *AC*?

From Brainstorming 8, it is found that:

(a) ∠*AOB* = ∠*AOC*, ∠*OBA* = ∠*OCA* and ∠*OAB* = ∠*OAC*
(b) Length of *OB* = length of *OC* and length of *AB* = length of *AC*

In general,



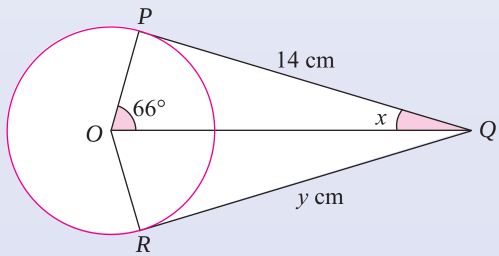

If two tangents to a circle with centre *O* and points of tangency *B* and *C* meet at point *A*, then

- *BA* = *CA*
- ∠*BOA* = ∠*COA*
- ∠*OAB* = ∠*OAC*



**Example 16**

The diagram on the right shows a circle centred at *O*. Tangents *PQ* and *RQ* meet at point *Q*. Calculate
(a) the value of *x*
(b) the value of *y*
(c) the radius of the circle




***Solution:***

(a) Right-angled triangle Δ*OPQ* and ∠*OPQ* = 90°.

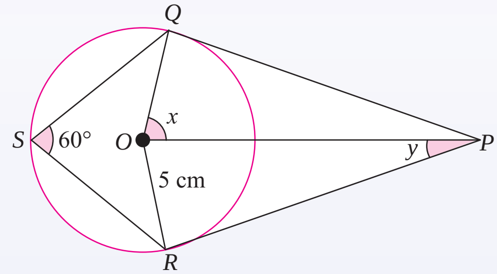
Thus, $x + 66° = 90°$
$x = 90° - 66°$
$x = 24°$

(b) Length of *PQ* = *QR* = *y*
Thus, *y* = 14 cm

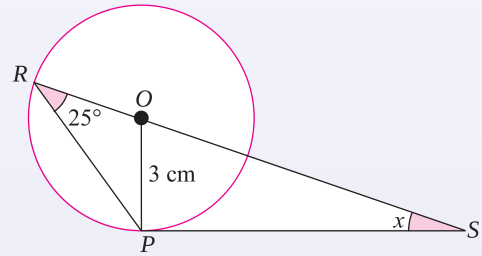
(c) $\tan 24° = \frac{OP}{14}$
$OP = 14 \times \tan 24°$
Radius, *OP* = 6.233 cm

## MIND TEST 6.3c

**1.** The diagram on the right shows a circle of radius 5 cm centred at $O$. Given that $PQ$ and $PR$ are tangents to the circle and $\angle QSR = 60°$, calculate
(a) the value of $x$ (b) the value of $y$
(c) the length of $PQ$ (d) the length of $OP$




**2.** In the diagram on the right, $O$ is the centre of circle with radius 3 cm and $ROS$ is a straight line. Given that $\angle ORP = 25°$ and $PS$ is a tangent to the circle, calculate
(a) the value of $x$ (b) the length of $PS$
(c) the length of $RS$




### What is the relationship of the angle between tangent and chord with the angle in the alternate segment which is subtended by the chord?

**LEARNING STANDARD**

Make and verify conjectures about the relationship of angle between tangent and chord with the angle in the alternate segment which is subtended by the chord.

In Diagram 1 (a), $PQR$ is a tangent to the circle.
$\angle x$ is the angle between the chord $QS$ and tangent $PQR$ on a minor segment.

$\angle y$ is the angle of the major segment or alternate segment which is subtended by the chord $QS$.

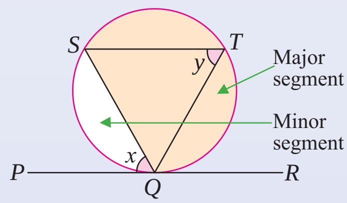


Diagram 1(a)

In Diagram 1 (b), $O$ is the centre of the circle. $OQ$ and $OS$ are radii of the circle and $PQR$ is a tangent to the circle. Thus,

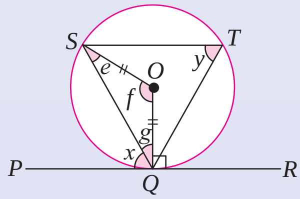


Diagram 1(b)

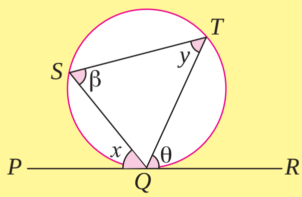
(a) $x + g = 90°$
$g = 90° - x$
$e = g$

Substitute

(b) $f = 180° - 2g$
$f = 180° - 2(90° - x)$
$f = 180° - 180° + 2x$
$f = 2x$ ← (1)

(c) $y = \dfrac{f}{2}$ ← (2)
Substitute (1) in (2)
$y = \dfrac{2x}{2}$
$y = x$

Based on the statements of Diagram 1 (a) and Diagram 1 (b), we can conclude that:

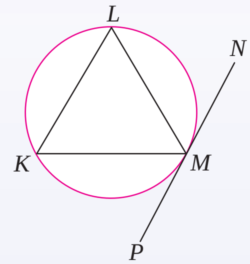



$\angle x = \angle y$ and $\angle \theta = \angle \beta$ because the angles between the chords and the tangents are equal to the angles at the alternate segments subtended by the chords.

## Example 17

The diagram on the right shows triangle $KLM$ and $PMN$ is a tangent to the circle. Determine the angles in the alternate segment for

(a) $\angle PMK$ (b) $\angle NML$



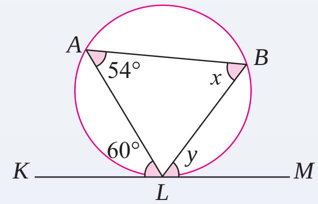

**Solution:**

(a) $\angle KLM$ (b) $\angle LKM$

## Example 18

The diagram on the right shows the triangle $ABL$ inside a circle. Given that $KLM$ is a tangent to the circle, determine the value of

(a) $x$ (b) $y$




**Solution:**

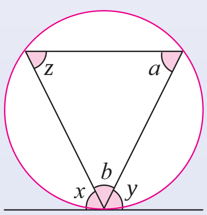
(a) $x = 60°$ because $x$ is an angle in the alternate segment of $\angle KLA$ which is subtended by chord $AL$.

(b) $y = 54°$ because $\angle LAB$ is an angle in the alternate segment of $y$ which is subtended by chord $BL$.

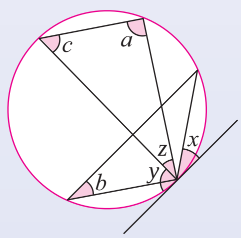
## MIND TEST 6.3d

**1.** State the pair of angles with the same value in the following circles.

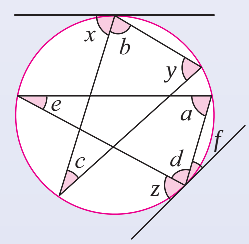
(a)



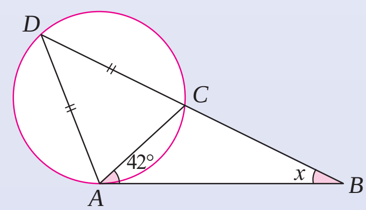

(b)



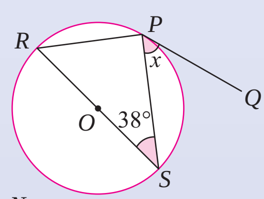

(c)



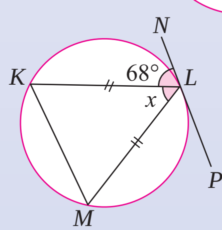

**2.** The diagram on the right shows a circle where $AB$ is a tangent to the circle. Given $\angle BAC = 42°$, calculate the value of $x$.




**3.** The diagram on the right shows a circle with centre $O$. $PQ$ is a tangent to the circle. Given $\angle PSR = 38°$, calculate the value of $x$.




**4.** The diagram on the right shows a circle where $PLN$ is a tangent to the circle. $\Delta KLM$ is an isosceles triangle. Given $\angle KLN = 68°$, calculate the value of $x$.



## How do you solve problems involving tangents to circles?

### What do you know about common tangents?

**LEARNING STANDARD**

Solve problems involving tangents to circles.

A common tangent to two circles is a straight line that is a tangent to both the circles.

Notice the following pairs of circles and their common tangents.

**1.**

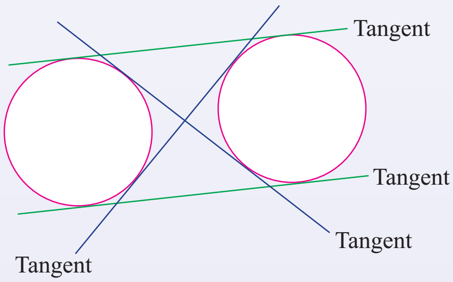



Diagram 1(a)

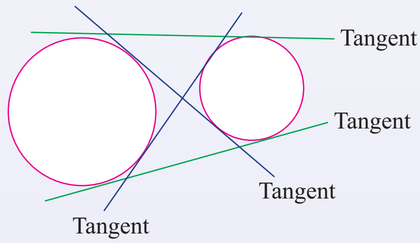



Diagram 1(b)

**2.**

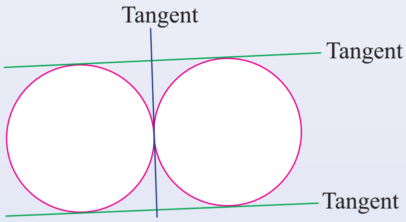



Diagram 2(a)

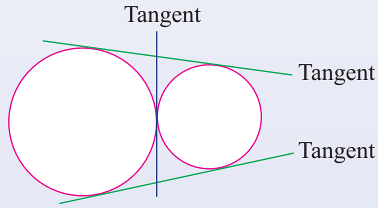



Diagram 2(b)

**3.**

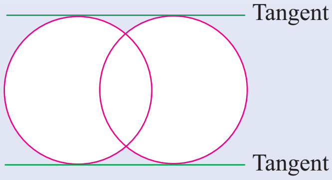



Diagram 3(a)

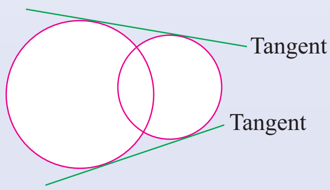



Diagram 3(b)

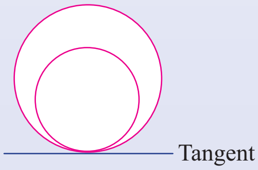



Diagram 3(c)

From the above diagrams it is found that if two circles of equal sizes or different sizes that are

(a) not touching each other, as shown in Diagram l(a) and Diagram l(b), will produce four common tangents

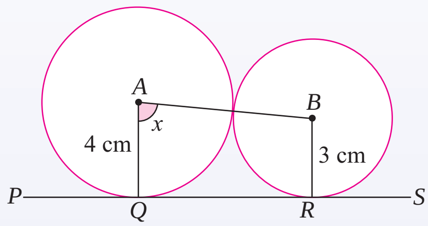
(b) touching at one point, as shown in Diagram 2(a) and Diagram 2(b), will produce three common tangents

(c) intersecting, as shown in Diagram 3(a) and Diagram 3(b), will produce two common tangents

(d) overlapping, as shown in Diagram 3(c), will produce only one common tangent

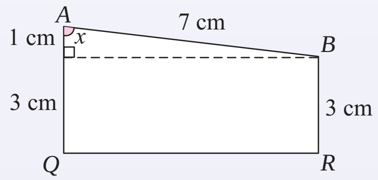
## Example 19

The diagram on the right shows two circles centred at $A$ and $B$ with radius 4 cm and 3 cm respectively. Given that $PQRS$ is a common tangent to both circles, calculate the value of $x$.




**Solution:**

$$\cos x = \frac{1}{7}$$
$$x = \cos^{-1}\left(\frac{1}{7}\right)$$
$$x = 81.79°$$

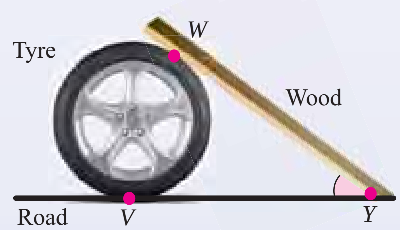



## Example 20

A piece of wood is mounted on a tyre as shown in the diagram. It is given that $V$ is the point of contact between the tyre and the road, $W$ is the point of contact between the wood and the tyre while $Y$ is the point of contact between the wood and the road. The diameter of the tyre is 50 cm and the distance of $WY$ is 1.2 metres. Assuming that the road is a straight line, calculate

(a) the distance of $VY$

(b) the distance between the centre of the tyre and the point $Y$ in metres. State your answer correct to two decimal places






### Understanding the problem

(a) $VY$ and $WY$ are tangents to the circle. The diameter of the tyre is 50 cm and the distance $WY$ is 1.2 metres.

(b) The distance between the centre of the tyre to point $Y$.

### Planning a strategy

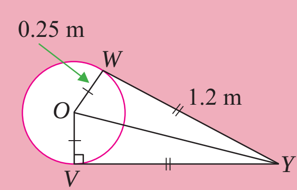
Draw a diagram and label it with the given values.

Diameter = 50 cm = 0.5 metre
Radius = 25 cm = 0.25 metre
$WY$ = 1.2 metres

### Implementing the strategy

(a) $VY = WY = 1.2$ m.

(b) $OY = \sqrt{1.2^2 + 0.25^2}$
$OY = \sqrt{1.5025}$
$OY = 1.23$ m (2 d.p.)



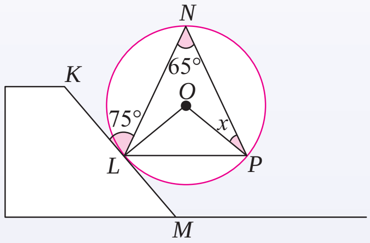

### Making a conclusion

(a) $\Delta OWY$ and $\Delta OVY$ are congruent.
Thus, $VY = WY = 1.2$ metre.

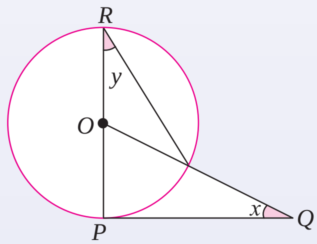
(b) The distance between the centre of the tyre and point $Y$, $OY = 1.23$ m.

## MIND TEST 6.3e

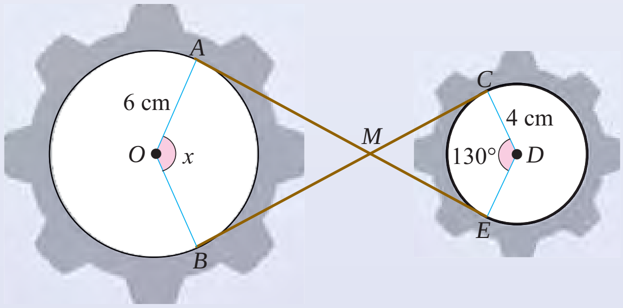
1. The diagram on the right shows the cross section of a barrel and a wall viewed from the top. The barrel is centred at $O$. The wall $KLM$ touches the barrel at point $L$. Given that $\angle KLN = 75°$ and $\angle LNP = 65°$, calculate the value of $x$.




2. The diagram on the right shows a circle with centre $O$. $PQ$ is a tangent to the circle. Given that $PQ = 2OP$, determine the value of $\angle x$ and $\angle y$. Give your answer in minutes and degrees.

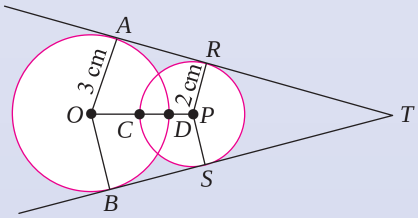



3. The diagram below shows part of the gear system on a machine. Straight chains $AE$ and $BC$ meet both gears at points $A$, $B$, $C$, and $E$. The gears are circular with centres $O$ and $D$ respectively. Given that $OA = 6$ cm, $DC = 4$ cm and $\angle CDE = 130°$, calculate




(a) the value of $x$
(b) the length in cm, correct to four significant figures, of
(i) $AM$ (ii) $CM$ (iii) $OD$

4. The diagram on the right shows two circles with radius 3 cm and 2 cm centred at $O$ and $P$ respectively. Given the length of $CD = DP$, calculate the length, in cm, correct to two decimal places, of
(a) $OP$ (b) $BS$ (c) $BST$



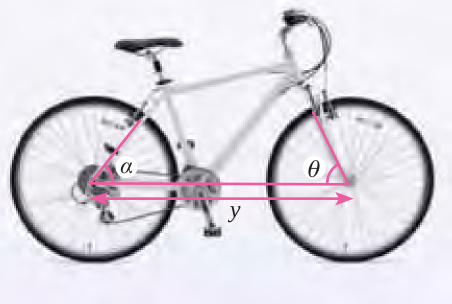
## 6.4 Angles and Tangents of Circles

### How do you solve problems involving angles and tangents to the circle?

**LEARNING STANDARD**

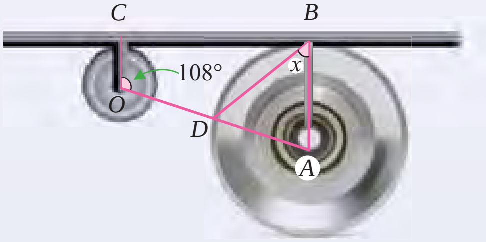
Solve problems involving angles and tangents of circles.

A circle is a familiar shape that we come across in our daily routine. One example is the wheel of the bicycle. Can you calculate the length of $y$, $\angle\alpha$ and $\angle\theta$?




**Example 21**

The diagram on the right shows two pulleys centred at $O$ and $A$ respectively, which are suspended from the ceiling $BC$. The rope $ADO$ connects both pulleys. Calculate the value of $x$.




**Solution:**

**Understanding the problem**

$BC$ is a tangent to the circles at points $C$ and $B$.

$\angle OCB = \angle ABC = 90°$

$\angle AOC = 108°$

Identify $\angle ABD$, $x$

**Planning a strategy**

$\angle OCB + \angle ABC + \angle AOC + \angle OAB = 360°$

$\angle ABD = \angle ADB = x$

**Implementing the strategy**

$\angle OAB + 90° + 90° + 108° = 360°$

$\angle OAB = 360° - 90° - 90° - 108°$
$= 72°$

$AB$ and $AD$ are radii. Thus,

$\angle ABD = \angle ADB = x$

$x = \dfrac{180° - 72°}{2}$

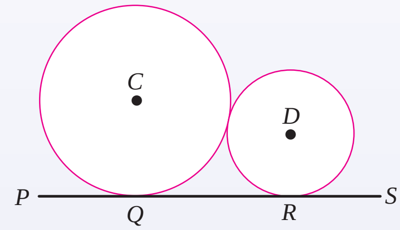
$x = 54°$

**Making a conclusion**

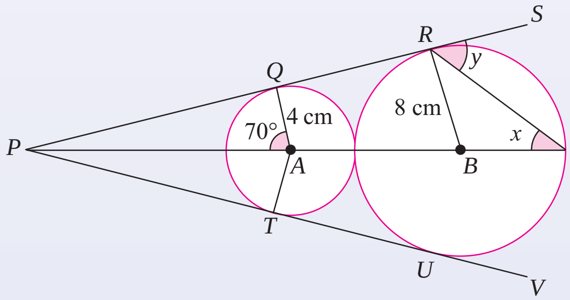
The value of $x = 54°$

## MIND TEST 6.4a

1. The diagram on the right shows two circles with centres $C$ and $D$. Given radii of the two circles are 6 cm and 3 cm respectively, and $PQRS$ is a common tangent to both circles, calculate
   (a) the length of $QR$, in cm. State the answer correct to three significant figures.
   (b) the area of quadrilateral $CDRQ$, in cm$^2$. State the answer correct to four significant figures.




2. The diagram below shows two circles centred at $A$ and $B$ with radius 4 cm and 8 cm respectively. $PQRS$ and $TUV$ are common tangents to both circles and $\angle PAQ = 70°$.

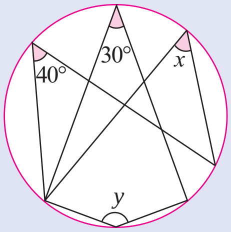



   Calculate
   (a) the value of $x$
   (b) the value of $y$
   (c) the length of $QR$, in cm, correct to four significant figures

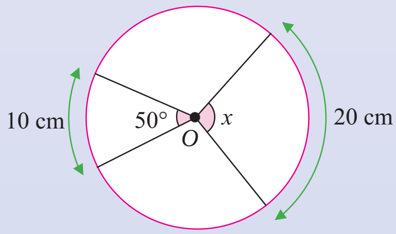
## Dynamic Challenge

### Test Yourself

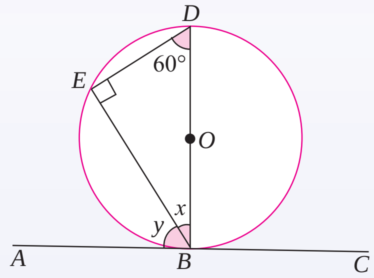
1. The diagram on the right shows a circle. Calculate the value of $x$ and $y$.



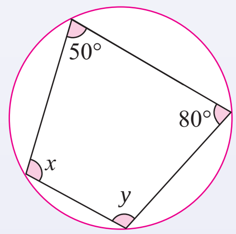

2. The diagram on the right shows a circle with centre $O$. Calculate the value of $x$.



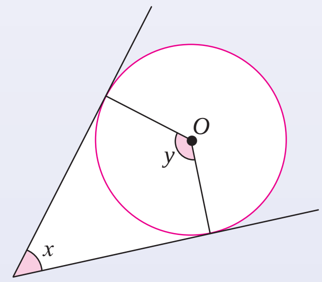
3. The diagram on the right shows a circle with centre $O$. $ABC$ is a tangent to the circle. Given that the $\angle BDE = 60°$, calculate the value of
   (a) $x$
   (b) $y$



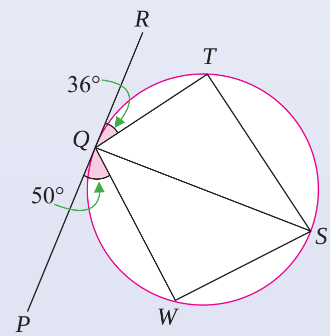

4. The diagram on the right shows a cyclic quadrilateral. Calculate the value of $x + y$.




5. A circle with centre $O$ has two tangents to the circle as shown in the diagram on the right. What is the relationship between angle $x$ and angle $y$?

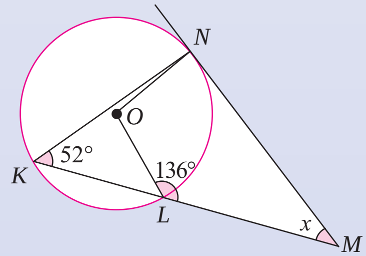



6. The diagram on the right shows a circle. Given that $PQR$ is a tangent to the circle, $\angle RQT = 36°$ and $\angle PQW = 50°$, calculate the value of angle $TSW$.

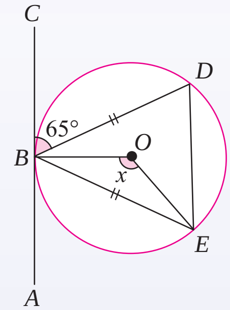



**Skills Enhancement**

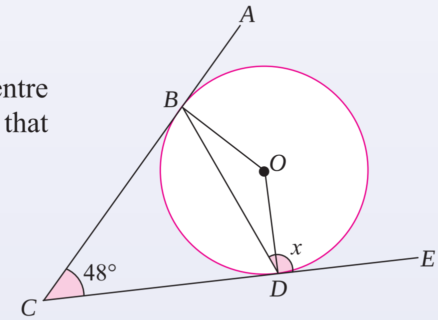
1. In the diagram on the right, $O$ is centre of the circle and $MN$ is a tangent to the circle. Given that $\angle LKN = 52°$ and $\angle MLO = 136°$, calculate the value of $x$.



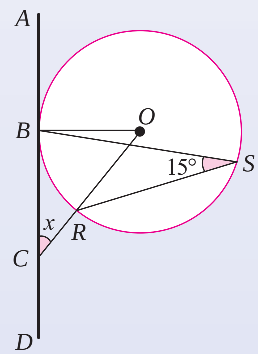
2. The diagram on the right shows a circle with centre *O*. *ABC* is a tangent to the circle. Given that *BD* = *BE* and ∠*CBD* = 65°, calculate the value of *x*.




3. The diagram on the right shows a circle with centre *O*. *ABC* and *CDE* are tangents to the circle. Given that ∠*BCD* = 48°, calculate the value of *x*.

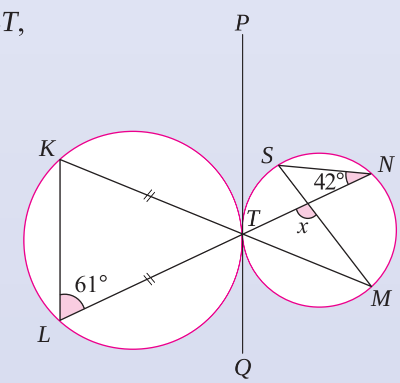



4. The diagram on the right shows a circle with centre *O*. *AD* is a tangent to the circle. Given that ∠*BSR* = 15°, calculate the value of *x*.

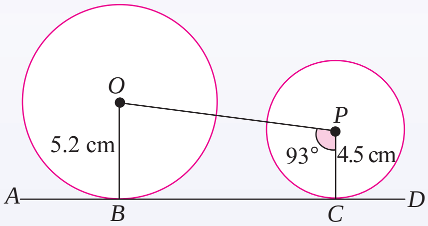



## Self Mastery

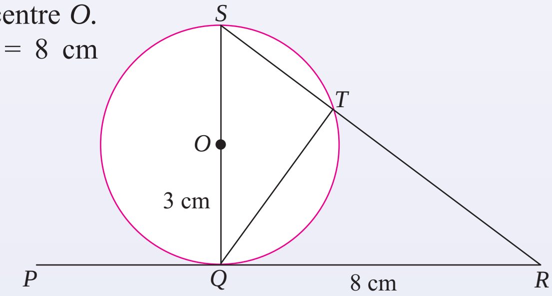
1. The diagram on the right shows two circles. *PTQ* is a common tangent to both circles. Given the length of *KT* = *LT*, ∠*KLT* = 61° and ∠*SNT* = 42°, calculate
   (a) ∠*LTQ*
   (b) the value of *x*



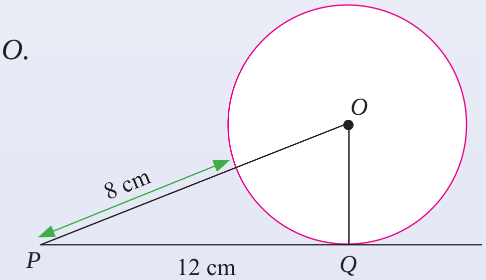
2. The diagram on the right shows two circles centred at $O$ and $P$ respectively. $ABCD$ is a common tangent to both circles. Calculate the area of trapezium $OBCP$, in $cm^2$, correct to three significant figures.




3. The diagram on the right shows a circle with centre $O$. Given that radius of the circle is 3 cm, $QR$ = 8 cm and $PQR$ is a tangent to the circle, determine
   (a) $\angle TRQ$
   (b) the length $ST$, in cm

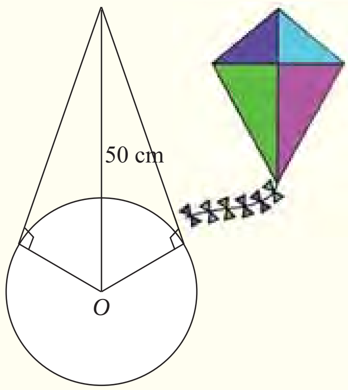



4. The diagram on the right shows a circle with centre $O$. $PQ$ is a tangent to the circle. Calculate the
   (a) radius of the circle, in cm
   (b) length of $OP$, in cm
   (c) area of $\Delta OPQ$, in $cm^2$






## PROJECT

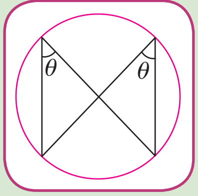


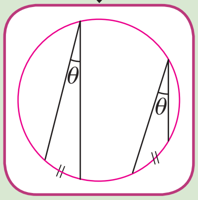

Kite-flying is a traditional game in our country. Kites can be constructed by using the concept of tangents to circles. With the knowledge of congruence and tangency that you have learnt, make a kite which has a length of 50 cm. Look for guidance from the diagram provided on the left.

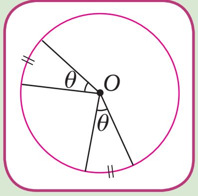
## CONCEPT MAP

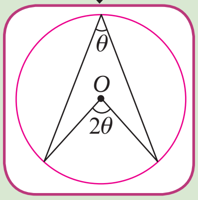
**Circles**

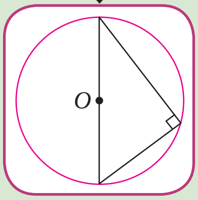
**Properties of angles in a circle**



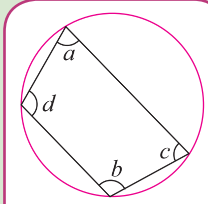



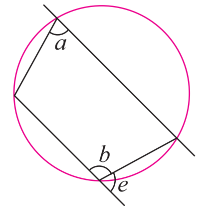




**Cyclic quadrilaterals**

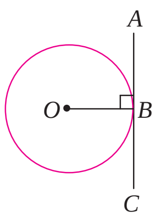



$\angle a + \angle b = 180°$
$\angle c + \angle d = 180°$

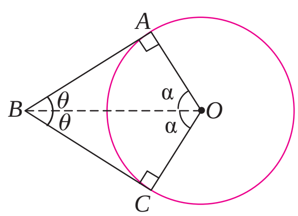



$\angle a = \angle e$

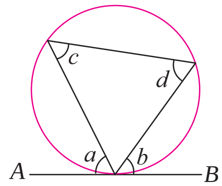
**Tangents to circles**




$OB$ = radius
$ABC$ = tangent to circle




$AB$ and $BC$ are tangents.
$\angle ABC + \angle AOC = 180°$
$BA = BC$




$\angle a = \angle d$
$\angle b = \angle c$

CHAPTER 6

## SELF-REFLECT

**At the end of this chapter, I can:**

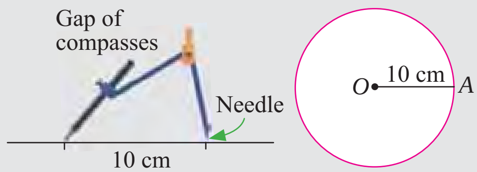
| | | 😊 | ☹️ |
|---|---|---|---|
| **1.** | Make and verify conjectures about the relationships between angles at the circumference, angle at the circumference and central angle subtended by particular arcs, and hence use the relationships to determine the values of angles in circle. | | |
| **2.** | Solve problems involving angles in circles. | | |
| **3.** | Recognise and describe cyclic quadrilaterals. | | |
| **4.** | Make and verify conjectures about the relationships between angles of cyclic quadrilateral, and hence use the relationships to determine the values of angles of cyclic quadrilateral. | | |
| **5.** | Solve problems involving cyclic quadrilaterals. | | |
| **6.** | Recognise and describe the tangents to circles. | | |
| **7.** | Make and verify conjectures about the angle between a tangent and radius of a circle at the point of tangency. | | |
| **8.** | Make and verify conjectures about the properties related to two tangents to a circle. | | |
| **9.** | Make and verify conjectures about the relationship of angles between tangents and chords with the angle in the alternate segment which is subtended by the chord. | | |
| **10.** | Solve problems involving tangents to circles. | | |
| **11.** | Solve problems involving angles and tangents of circles. | | |

## EXPLORING MATHEMATICS

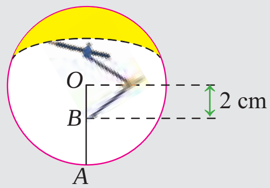
The appearance of the moon changes according to phases. Students can draw shapes of the moon at different phases to be used as decoration.

**Materials**: Drawing paper, compasses, pencil, ruler and scissors.

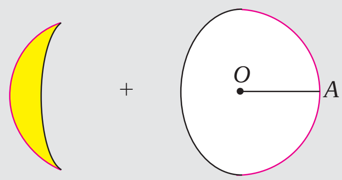
**1.** Draw a circle with radius 10 cm.



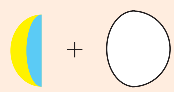
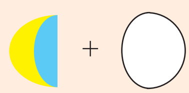

**2.** Then set the gap of compasses at 10 cm and place its needle at point *B*, which is 2 cm from point *O*. Draw a circle as in the diagram on the right.



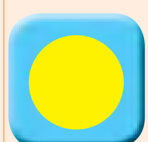
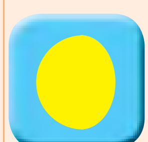
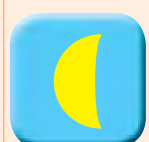
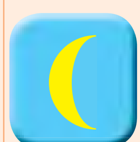

**3.** The shaded part can be cut out and used as a crescent. The unshaded part can be used as a gibbous.




**4.** Step 1 and 2 can be repeated by changing the distance of *OB*. The distance of *OB* can be extended to 2 cm to obtain two different shapes of moon. The distance can be further extended to from different shapes. The examples are as follows:

| Distance of *OB* | Resulting shape |
|---|---|
| 1 cm |  |
| 2 cm |  |

These shapes can be used as a tool to teach the changing phases of the moon or as decoration.

| Day | 4 | 8 | 12 | 15 | 19 | 23 | 27 |
|---|---|---|---|---|---|---|---|
| Phase of the moon |  Waxing Crescent |  First Quarter |  Waxing Gibbous |  Full Moon |  Waning Gibbous |  Last Quarter |  Waning Crescent |