CHAPTER 6

Ratios and Graphs of Trigonometric Functions

What will you learn?

- The Value of Sine, Cosine and Tangent for Angle θ, $0° \leqslant \theta \leqslant 360°$
- The Graphs of Sine, Cosine and Tangent Functions

Why study this chapter?

Engineers use graphs of trigonometric functions in the construction of concert halls to measure sound strength so that the sound is clearly heard. Geologists also use graphs of trigonometric functions to help them understand the formation of periodic pattern of earthquakes and waves.

Do you know?

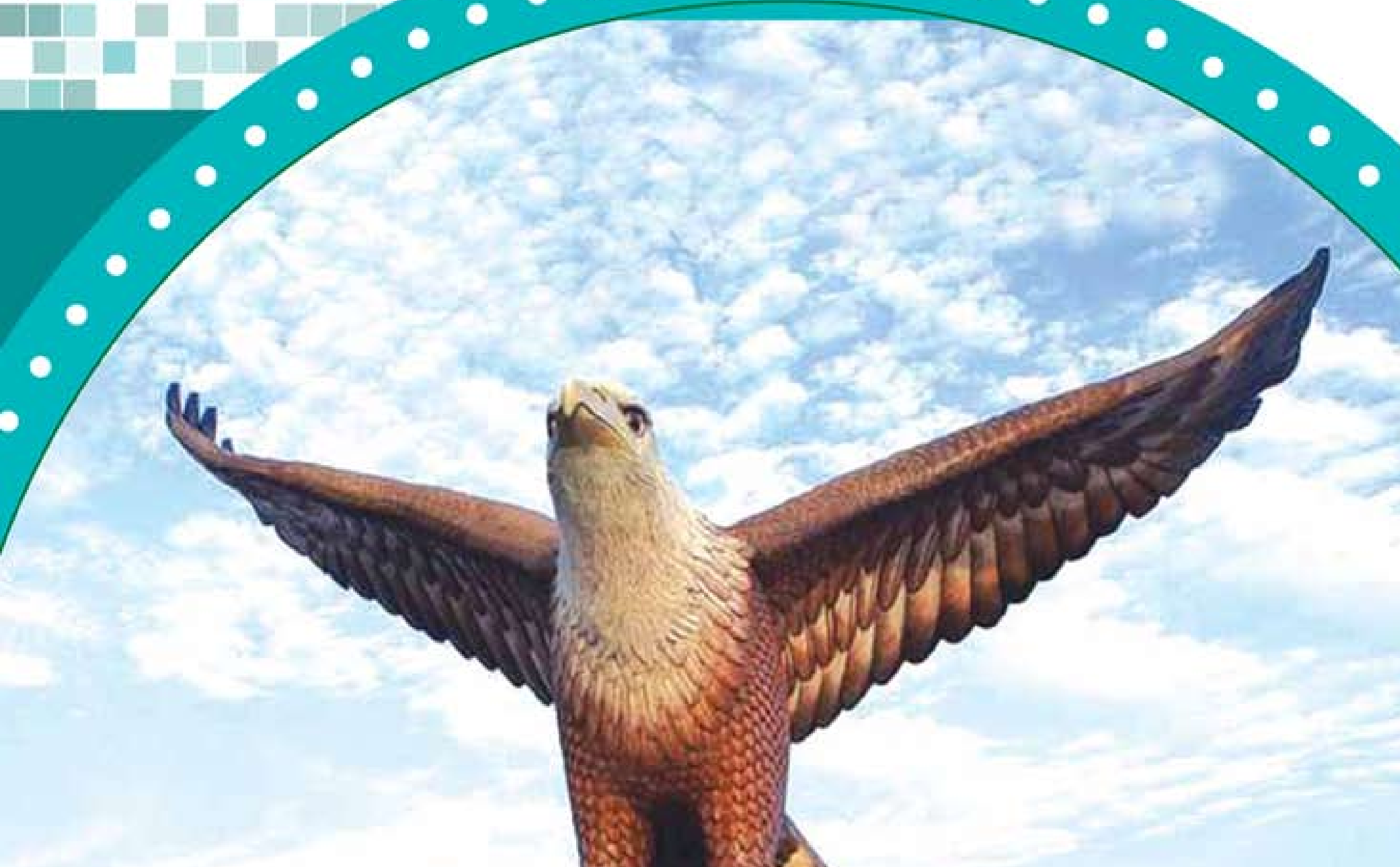
Hipparchus from Nicaea was an astronomer and mathematician known as the Father of Trigonometry for his contribution to trigonometry. He developed a trigonometric table in his attempt to understand the movement of stars and moons.

For more information:

bit.do/DoYouKnowChap6

WORD BANK

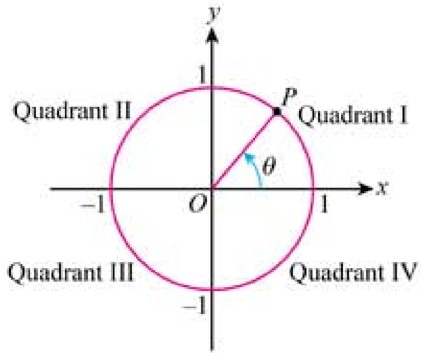
unit circle	*bulatan unit*
trigonometric function	*fungsi trigonometri*
cosine	*kosinus*
sine	*sinus*
corresponding reference angle	*sudut rujukan sepadan*
quadrant	*sukuan*
tangent	*tangen*



Langkawi Island, known as the Jewel of Kedah, is one of the tidal stations in Malaysia. A tidal station is built to collect tidal data around Langkawi Island. This data can be used to detect tsunami waves. As tides occur periodically, such tidal data can be modelled with graphs of trigonometric functions.

6.1 The Value of Sine, Cosine and Tangent for Angle θ, $0° \leqslant \theta \leqslant 360°$

The diagram on the right shows a unit circle. A **unit circle** is a circle that has a radius of 1 unit and is centred on the origin. The x-axis and y-axis divide the unit circle into 4 equal quadrants, namely quadrant I, quadrant II, quadrant III and quadrant IV.



It is given that P is a point that moves along the circumference of the unit circle and θ is the angle formed by the radius of the unit circle, OP, from the positive x-axis in an anticlockwise direction. It is found that

(a) point P is in quadrant I when $0° < \theta < 90°$,
(b) point P is in quadrant II when $90° < \theta < 180°$,
(c) point P is in quadrant III when $180° < \theta < 270°$,
(d) point P is in quadrant IV when $270° < \theta < 360°$.

What is the relationship between the function of sine, cosine and tangent for angles in quadrants II, III and IV with the corresponding reference angle?

Learning Standard

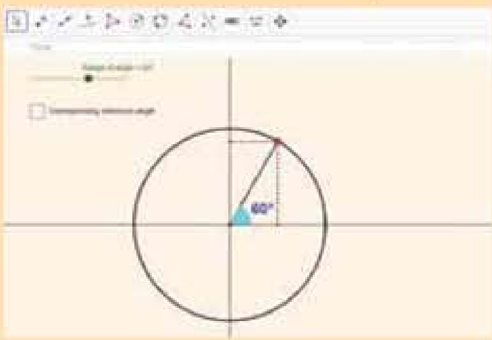
Make and verify conjecture about the value of sine, cosine and tangent for angles in quadrants II, III and IV with the corresponding reference angle.

MIND MOBILISATION 1 Pairs

Aim: To explore the relationship between the function of sine, cosine and tangent for angles in quadrants II, III and IV with the corresponding reference angle.

Steps:

1. Open the file GGB601 for this activity.



Scan the QR code or visit bit.do/GGB601BI to obtain the GeoGebra file for this activity.

2. Drag the red point to quadrants II, III and IV. Observe the blue angle. [The slider 'Range of angle' can be dragged to change the displayed angle.]
3. Click 'Corresponding reference angle'.
4. Drag the red point and observe the corresponding reference angles in quadrants II, III and IV.

5. Write the relationship between the corresponding reference angle, α, with the blue angle, θ, in each quadrant.

Quadrant I	Quadrant II	Quadrant III	Quadrant IV
$\alpha =$	$\alpha =$	$\alpha =$	$\alpha =$

6. Drag the red point and choose any blue angles in quadrants II, III and IV to complete the table below. Use a scientific calculator to do the calculations.

Quadrant ☐ :

Blue angle, $\theta =$ ☐ Corresponding reference angle, $\alpha =$ ☐

$\sin\theta$	$\sin\alpha$	$\cos\theta$	$\cos\alpha$	$\tan\theta$	$\tan\alpha$

7. Based on the table in step **6**, compare the values of sine, cosine and tangent of blue angle, θ, with the corresponding reference angle, α. Complete each of the following with positive or negative sign.

Quadrant II

$\sin\theta =$ ☐ $\sin\alpha$

$\cos\theta =$ ☐ $\cos\alpha$

$\tan\theta =$ ☐ $\tan\alpha$

Quadrant III

$\sin\theta =$ ☐ $\sin\alpha$

$\cos\theta =$ ☐ $\cos\alpha$

$\tan\theta =$ ☐ $\tan\alpha$

Quadrant IV

$\sin\theta =$ ☐ $\sin\alpha$

$\cos\theta =$ ☐ $\cos\alpha$

$\tan\theta =$ ☐ $\tan\alpha$

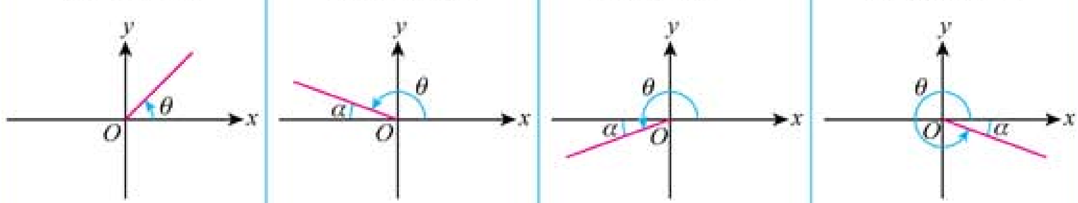
Discussion:

1. What are the characteristics of the corresponding reference angles and their relationship with angles in quadrants II, III and IV?
2. What is your conclusion about the relationship of the function of sine, cosine and tangent for angles in quadrants II, III and IV with the corresponding reference angle?

The results of Mind Mobilisation 1 show that;

(a) The **corresponding reference angle**, α, is always less than $90°$. Angles in quadrants II, III and IV have corresponding reference angles, α. The angle in quadrant I itself is the corresponding reference angle, $\alpha = \theta$.

Quadrant I	Quadrant II	Quadrant III	Quadrant IV



$\alpha = \theta$	$\alpha = 180° - \theta$	$\alpha = \theta - 180°$	$\alpha = 360° - \theta$

The reference angles in quadrants II, III and IV are the corresponding angles in quadrant I.

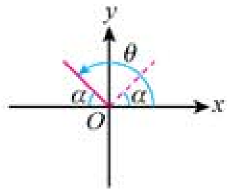
(b) The relationship between the function of sine, cosine and tangent for angles in quadrants II, III and IV with the corresponding reference angle can be summarised as follows:

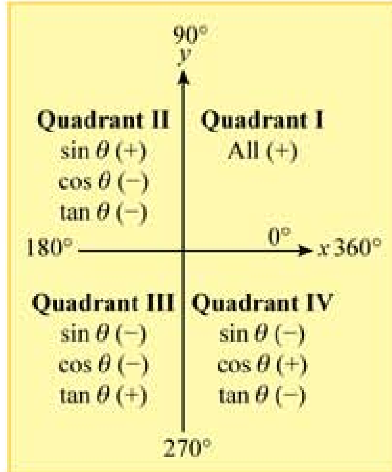
Quadrant II

$\sin\theta = +\sin\alpha = +\sin(180° - \theta)$

$\cos\theta = -\cos\alpha = -\cos(180° - \theta)$

$\tan\theta = -\tan\alpha = -\tan(180° - \theta)$



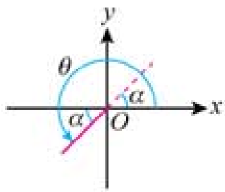


Quadrant III

$\sin\theta = -\sin\alpha = -\sin(\theta - 180°)$

$\cos\theta = -\cos\alpha = -\cos(\theta - 180°)$

$\tan\theta = +\tan\alpha = +\tan(\theta - 180°)$

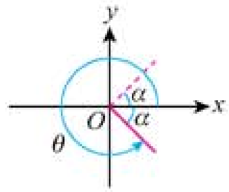


Quadrant IV

$\sin\theta = -\sin\alpha = -\sin(360° - \theta)$

$\cos\theta = +\cos\alpha = +\cos(360° - \theta)$

$\tan\theta = -\tan\alpha = -\tan(360° - \theta)$



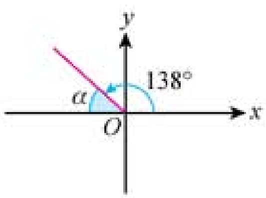
Example 1

Determine the quadrant and the corresponding reference angle for each of the following.

(a) 138° (b) 239° (c) 312°

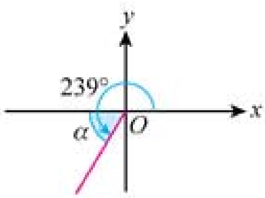
Solution:

(a)



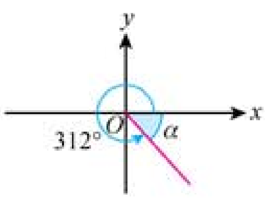
138° is located in quadrant II.
Corresponding reference angle, α
$= 180° - 138°$
$= 42°$

(b)



239° is located in quadrant III.
Corresponding reference angle, α
$= 239° - 180°$
$= 59°$

(c)



312° is located in quadrant IV.
Corresponding reference angle, α
$= 360° - 312°$
$= 48°$

Smart TIPS

Corresponding reference angle is an acute angle.

Critical Mind

To obtain the corresponding reference angle, α, why is the formula $\theta - 180°$ used in Example 1(b)?

Critical Mind

Calculate the corresponding reference angle if the given angle is 498°.

Example 2

State the relationship between each of the following trigonometric functions with its corresponding reference angle.

(a) $\sin 167°$ (b) $\cos 258°$ (c) $\tan 349°$

Solution:

only $\sin \theta$ is positive

(a) Angle 167° is located in quadrant II.

$\sin \theta = \sin (180° - \theta)$

$\sin 167° = \sin (180° - 167°)$

$\sin 167° = \sin 13°$

only $\tan \theta$ is positive

(b) Angle 258° is located in quadrant III.

$\cos \theta = -\cos (\theta - 180°)$

$\cos 258° = -\cos (258° - 180°)$

$\cos 258° = -\cos 78°$

only $\cos \theta$ is positive

(c) Angle 349° is located in quadrant IV.

$\tan \theta = -\tan (360° - \theta)$

$\tan 349° = -\tan (360° - 349°)$

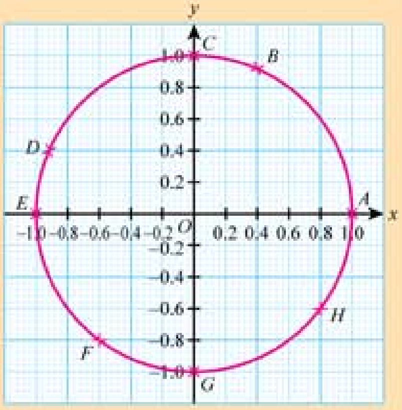
$\tan 349° = -\tan 11°$

MIND MOBILISATION 2 Pairs

Aim: To explore the relationship between the values of sine, cosine and tangent with the values of *x*-coordinate and *y*-coordinate for angles in quadrants II, III and IV in a unit circle.

Steps:

1. Draw the *x*-axis and the *y*-axis with the origin *O* on a graph paper and a circle centred at *O* with a radius of 1 unit as shown in the diagram on the right.
2. Plot the points as the diagram on the right.
3. Copy and complete the table as follows.



Point	*x*-coordinate	*y*-coordinate	$\frac{y\text{-coordinate}}{x\text{-coordinate}}$
A			
B			

4. It is given that θ is the angle formed by the radius of a unit circle from the positive *x*-axis in an anticlockwise direction. Measure the angle, θ from the drawn graph and complete the table as follows.

Radius	Angle θ	$\sin \theta$	$\cos \theta$	$\tan \theta$
OA				
OB				

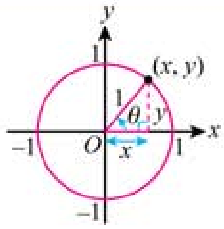
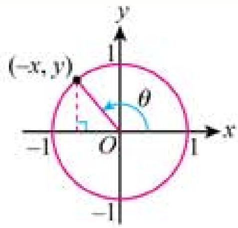
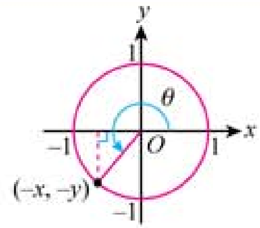
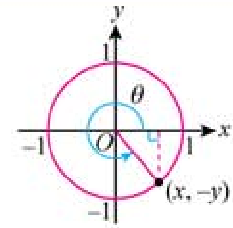
Discussion:

1. What is the relationship between the values of *x*-coordinate, *y*-coordinate and the ratio of *y*-coordinate to *x*-coordinate with the values of $\sin \theta$, $\cos \theta$ and $\tan \theta$?

The results of Mind Mobilisation 2 show that;

$\sin \theta = y\text{-coordinate}$ $\cos \theta = x\text{-coordinate}$ $\tan \theta = \frac{y\text{-coordinate}}{x\text{-coordinate}}$

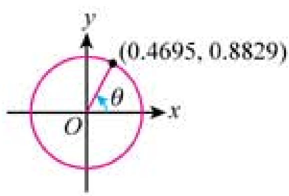
In general,

Quadrant I	Quadrant II	Quadrant III	Quadrant IV
			
$\sin \theta = +y$ $\cos \theta = +x$ $\tan \theta = +\frac{y}{x}$	$\sin \theta = +y$ $\cos \theta = -x$ $\tan \theta = -\frac{y}{x}$	$\sin \theta = -y$ $\cos \theta = -x$ $\tan \theta = +\frac{y}{x}$	$\sin \theta = -y$ $\cos \theta = +x$ $\tan \theta = -\frac{y}{x}$

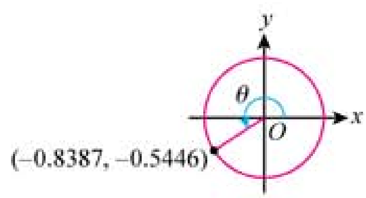
Example 3

Each of the following diagrams shows a unit circle and angle θ. Determine the values of $\sin \theta$, $\cos \theta$ and $\tan \theta$.

(a)



(b)



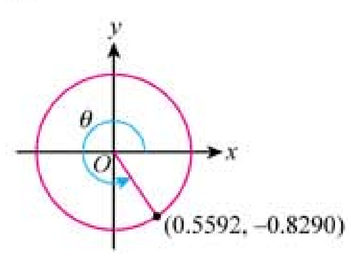
Solution:

(a) $\sin \theta = 0.8829$
$\cos \theta = 0.4695$
$\tan \theta = \frac{0.8829}{0.4695}$
$= 1.8805$

(b) $\sin \theta = -0.5446$
$\cos \theta = -0.8387$
$\tan \theta = \frac{-0.5446}{-0.8387}$
$= 0.6493$

Self Practice 6.1a

1. Determine the value of corresponding reference angle for each of the following.
   (a) 97° (b) 189° (c) 278° (d) 164.2°
   (e) 253.6° (f) 305.7° (g) 128°53′ (h) 215°42′
2. State the relationship between each of the following trigonometric functions with its corresponding reference angle.
   (a) sin 101° (b) cos 194° (c) tan 246°
   (d) tan 294.5° (e) sin 339.8° (f) cos 112.3°
   (g) cos 287°45′ (h) tan 96°31′ (i) sin 203°26′
3. The diagram on the right shows a unit circle and angle θ. Determine the values of $\sin \theta$, $\cos \theta$ and $\tan \theta$.



How to determine the value of sine, cosine and tangent for angles in quadrants II, III and IV?

Determine the corresponding reference angle. → Determine positive or negative sign for sine, cosine and tangent for angles in quadrants II, III and IV.

Learning Standard

Determine the value of sine, cosine and tangent for angles in quadrants II, III and IV based on the corresponding reference angle.

Example 4

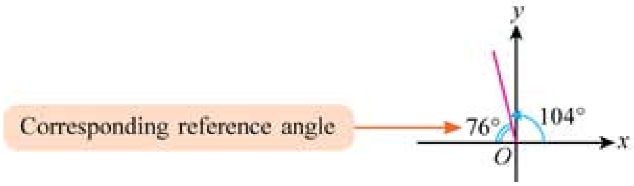
Determine the value for each of the following based on the corresponding reference angle.

(a) $\sin 104°$ (b) $\cos 208.2°$ (c) $\tan 318°17'$

Solution:

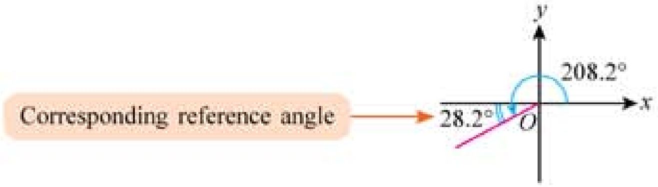
(a) $\sin 104° = +\sin (180° - 104°)$
$= +\sin 76°$
$= 0.9703$

sine values are positive in quadrant II



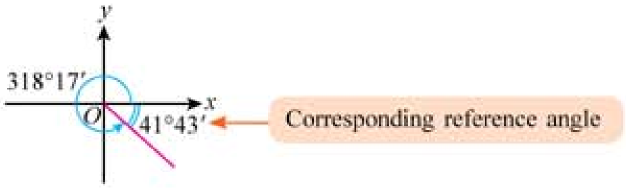
(b) $\cos 208.2° = -\cos (208.2° - 180°)$
$= -\cos 28.2°$
$= -0.8813$

cosine values are negative in quadrant III



(c) $\tan 318°17' = -\tan (360° - 318°17')$
$= -\tan 41°43'$
$= -0.8915$

tangent values are negative in quadrant IV



Smart TiPS

Quadrant I: **A**ll positive
Quadrant II: **s**in θ positive
Quadrant III: **t**an θ positive
Quadrant IV: **c**os θ positive

Tips to remember:
Use appropriate acronym.

i-Technology

To determine the value of tan 41°43′ from the scientific calculator, press

[tan] [4] [1] [°′″]
[4] [3] [°′″] [=]

Make sure your scientific calculator is in "Deg" mode.

CHAPTER 6

Self Practice 6.1b

1. Determine the value of each of the following based on the corresponding reference angle.

(a) sin 128°	(b) sin 236°	(c) sin 337°
(d) cos 196°	(e) cos 289°	(f) cos 127°
(g) tan 221°	(h) tan 134°	(i) tan 316°
(j) tan 321.4°	(k) sin 341.7°	(l) cos 99.3°
(m) cos 307°39′	(n) tan 102°38′	(o) sin 197°42′

How to determine the value of sine, cosine and tangent for angles in quadrants II, III and IV corresponding to angles 30°, 45° and 60°?

Example 5

Without using a scientific calculator, determine the value for each of the following based on the corresponding reference angle.

(a) cos 150° (b) tan 225° (c) sin 300°

Solution:

(a) $\cos 150° = -\cos (180° - 150°)$
$= -\cos 30°$
$= -\dfrac{\sqrt{3}}{2}$

cosine values are negative in quadrant II

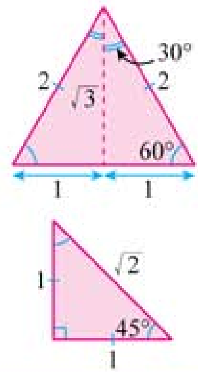
(b) $\tan 225° = +\tan (225° - 180°)$
$= +\tan 45°$
$= 1$

tangent values are positive in quadrant III

(c) $\sin 300° = -\sin (360° - 300°)$
$= -\sin 60°$
$= -\dfrac{\sqrt{3}}{2}$

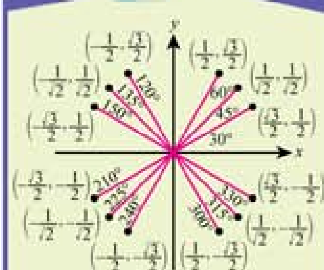
sine values are negative in quadrant IV

MEMORY BOX



Ratio \ Angle	30°	60°	45°
sin θ	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$
cos θ	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$
tan θ	$\frac{1}{\sqrt{3}}$	$\sqrt{3}$	1

Info Bulletin



Self Practice 6.1c

1. Determine the values of sine, cosine and tangent for each of the following angles without using a scientific calculator.

(a) 120°	(b) 135°	(c) 210°
(d) 240°	(e) 315°	(f) 330°

2. Given that $\sin \theta = \dfrac{\sqrt{3}}{2}$, calculate $\cos \theta$ and $\tan \theta$ without using a scientific calculator.

How do you determine the angle when the value of sine, cosine and tangent are given?

Learning Standard

Determine the angle when the value of sine, cosine and tangent are given.

Identify the quadrant based on the positive or negative sign on the value of trigonometric function.	→	Determine the corresponding reference angle.	→	Calculate angle θ based on the quadrant identified.

Example 6

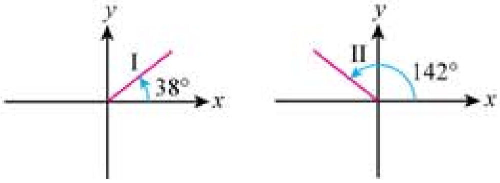
(a) Given that $\sin \theta = 0.6157$ and $0° \leqslant \theta \leqslant 360°$, calculate angle θ.
(b) Given that $\cos \theta = -0.4226$ and $0° \leqslant \theta \leqslant 360°$, calculate angle θ.
(c) Given that $\tan \theta = -1.4826$ and $0° \leqslant \theta \leqslant 360°$, calculate angle θ.

Solution:

(a) $\sin \theta = 0.6157$ ← Positive sign. θ is in quadrant I or II

Corresponding reference angle
$= \sin^{-1} 0.6157$
$= 38°$

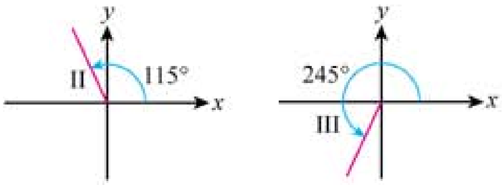
$\theta = 38°$ or $(180° - 38°)$
$= 38°$ or $142°$



(b) $\cos \theta = -0.4226$ ← Negative sign. θ is in quadrant II or III

Corresponding reference angle
$= \cos^{-1} 0.4226$
$= 65°$

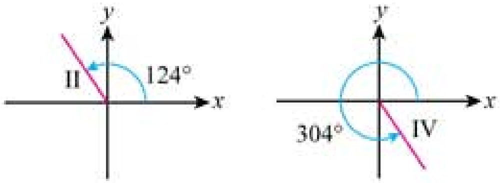
$\theta = (180° - 65°)$ or $(180° + 65°)$
$= 115°$ or $245°$



(c) $\tan \theta = -1.4826$ ← Negative sign. θ is in quadrant II or IV

Corresponding reference angle
$= \tan^{-1} 1.4826$
$= 56°$

$\theta = (180° - 56°)$ or $(360° - 56°)$
$= 124°$ or $304°$

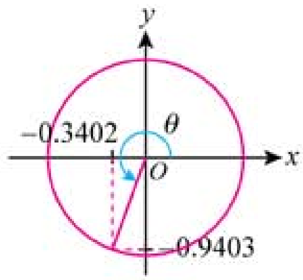


CHAPTER 6

Self Practice 6.1d

1. Given that $0° \leqslant \theta \leqslant 360°$, calculate angle θ for each of the following. Round off the answer to 1 decimal place.
   (a) $\sin \theta = 0.9397$ (b) $\cos \theta = 0.9336$ (c) $\tan \theta = 0.8391$
   (d) $\tan \theta = -1.198$ (e) $\cos \theta = -0.6018$ (f) $\sin \theta = -0.7314$
   (g) $\cos \theta = -0.5829$ (h) $\sin \theta = -0.8395$ (i) $\tan \theta = 0.7391$

2. The diagram on the right shows a unit circle with $0° \leqslant \theta \leqslant 360°$. Calculate angle θ. Round off the answer to 1 decimal place.



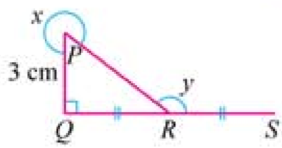
How to solve problems involving sine, cosine and tangent?

Learning Standard

Solve problems involving sine, cosine and tangent.

Example 7

In the diagram on the right, QRS is a straight line. Given that $QS = 8$ cm and $QR = RS$, calculate
(a) $\cos x$
(b) $\tan y$



Solution:

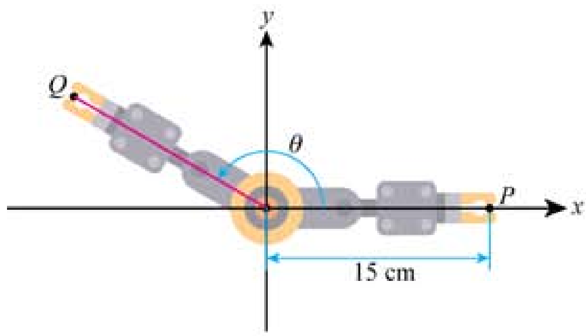
$QR = 8 \text{ cm} \div 2 = 4 \text{ cm}$ and $PR = \sqrt{3^2 + 4^2} = 5 \text{ cm}$

(a) $270° < x < 360°$,
hence, angle x is in quadrant IV.
$\cos x = +\cos \angle QPR$ ← $\angle QPR$ is the corresponding reference angle of angle x
$= +\frac{3}{5}$

(b) $90° < y < 180°$,
hence, angle y is in quadrant II.
$\tan y = -\tan \angle PRQ$ ← $\angle PRQ$ is the corresponding reference angle of angle y
$= -\frac{3}{4}$

Example 8

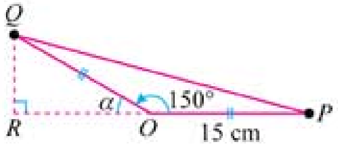
A group of students produces a robotic arm with the length of 15 cm as shown in the diagram on the right. The robotic arm is programmed to move an object from point P to point Q. Given that $\cos \theta = -0.866$ and $0° \leqslant \theta \leqslant 180°$, calculate the angle θ and the distance, in cm, between point P and point Q.



Solution:

$\cos \theta = -0.866$

Negative sign. θ is in quadrant II ($0° \leqslant \theta \leqslant 180°$)



Corresponding reference angle
$= \cos^{-1} 0.866$
$= 30°$

$\theta = 180° - 30°$
$= 150°$

$\alpha = 180° - 150°$
$= 30°$

$\sin 30° = \dfrac{QR}{15}$

$QR = 15 \sin 30°$
$= 7.5$ cm

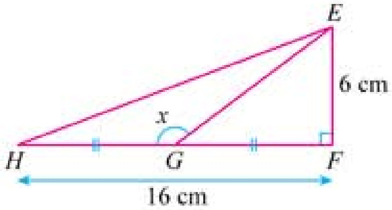
$\cos 30° = \dfrac{OR}{15}$

$OR = 15 \cos 30°$
$= 12.99$ cm

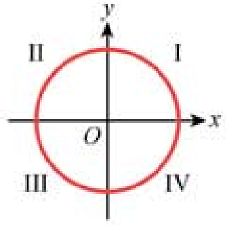
$PQ = \sqrt{QR^2 + PR^2}$
$= \sqrt{7.5^2 + 27.99^2}$
$= 28.98$ cm

Self Practice 6.1e

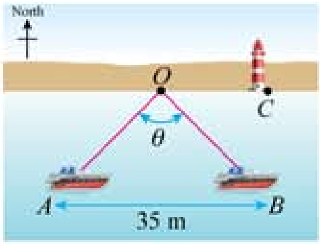
1. In the diagram on the right, HGF is a straight line. Given that $HG = GF$, calculate
   (a) $\cos x$,
   (b) $\sin x$.



2. Yong Ying plays a game of darts. The dartboard is round in shape and has four quadrants as shown in the diagram on the right. She finds that the darts she throws will usually hit the quadrants with the value of $\sin \theta = 0.809$. State
   (a) the quadrants,
   (b) the angles θ.



3. A ship sails from position A to position B such that $OA = OB$. The route of the ship is parallel to the sea bank. From position B, the ship will sail north towards the lighthouse, C. If $OA = 20$ m, calculate
   (a) the angle θ,
   (b) the distance, in m, between the ship in position B and the lighthouse,
   (c) the value of $\tan \angle AOC$.



4. The diagram on the right shows an alarm clock.
   (a) Calculate the angle θ moved through by the minute hand if the minute hand is being rotated anticlockwise from number 3 to number 10.
   (b) Hence, calculate the value of $\cos \theta$.

6.2 The Graphs of Sine, Cosine and Tangent Functions

After doing a heart check-up at the hospital, the doctor will explain the condition of our heart based on the result of the electrocardiogram. Electrocardiogram graphs have the characteristics of trigonometric function graphs. What are the characteristics?

What are the characteristics of the graphs of trigonometric functions, $y = \sin x$, $y = \cos x$ and $y = \tan x$ for $0° \leqslant x \leqslant 360°$?

Learning Standard

Draw graphs of trigonometric functions, $y = \sin x$, $y = \cos x$ and $y = \tan x$ for $0° \leqslant x \leqslant 360°$, hence compare and contrast the characteristics of the graphs.

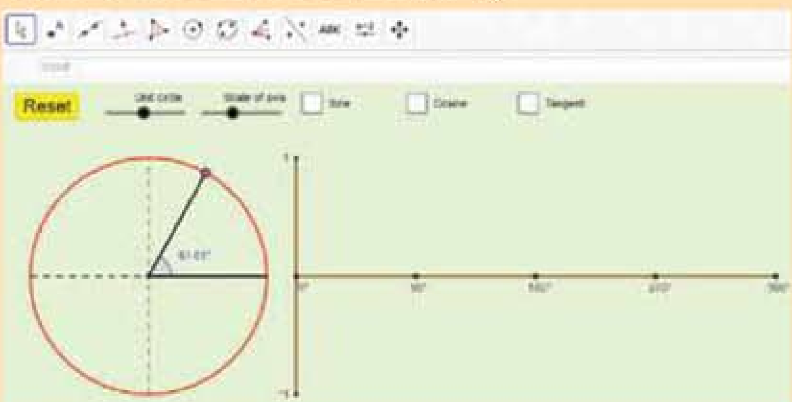
MIND MOBILISATION 3 Pairs

Aim: To draw graphs of trigonometric functions, $y = \sin x$, $y = \cos x$ and $y = \tan x$ for $0° \leqslant x \leqslant 360°$, hence compare and contrast the characteristics of the graphs.

Steps:

Activity I

1. Open the file GGB602 for this activity.



2. Click 'Sine' and then drag the red point.
3. Observe the shape, the value on y-axis when the angle is 0°, maximum value, minimum value and x-intercept of the graph formed.
4. Repeat Steps **2** to **3** with 'Cosine' and 'Tangent'.
[Click 'Reset' if you want to erase the graph formed.]

Scan the QR code or visit bit.do/GGB602BI to obtain the GeoGebra file for this activity.

CHAPTER 6

Activity II

1. Open the worksheet by scanning the QR code for this activity. Complete the table given.

x	0°	15°	30°	45°	60°	75°	90°
$\sin x$							
$\cos x$							
$\tan x$							

Scan the QR code or visit bit.do/WSChap6 to obtain the worksheet for this activity.

2. By using the scale of 2 cm to 30° on the x-axis and 2 cm to 1 unit on the y-axis, draw the graphs of the functions $y = \sin x$, $y = \cos x$ and $y = \tan x$ separately on graph papers.
3. Observe the shape, maximum value, minimum value, x-intercept and y-intercept of each of the graphs.
4. Complete the following table.

	Maximum value	Minimum value	x-intercept	y-intercept
$y = \sin x$				
$y = \cos x$				
$y = \tan x$				

Discussion:

For the graphs of sine, cosine and tangent,

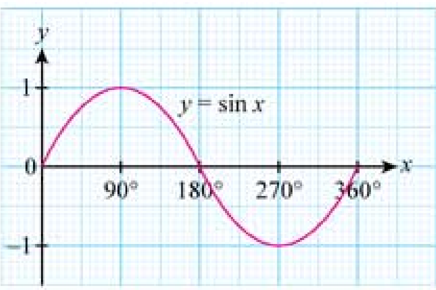
(a) what is the shape of each graph?

(b) what are the maximum value and minimum value of each graph?

(c) what are the x-intercept and y-intercept of each graph?

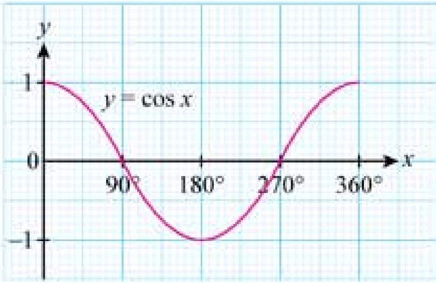
The results of Mind Mobilisation 3 show that the characteristics of the graphs of sine, cosine and tangent are as follows.

Sine graph, $y = \sin x$



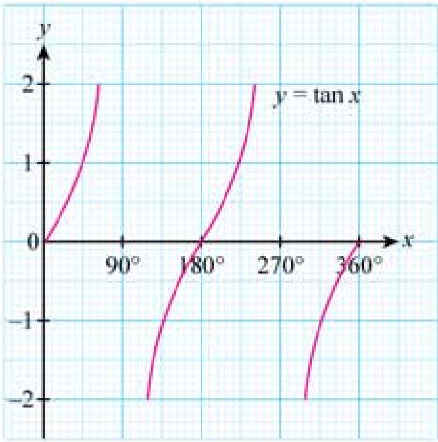
- has a maximum value of 1 when $x = 90°$ and a minimum value of –1 when $x = 270°$
- intercepts x-axis at $x = 0°$, 180° and 360° (x-intercept)
- intercepts y-axis at $y = 0$ (y-intercept)

Cosine graph, $y = \cos x$



- has a maximum value of 1 when $x = 0°$ and $360°$ and a minimum value of -1 when $x = 180°$
- intercepts x-axis at $x = 90°$ and $270°$ (x-intercept)
- intercepts y-axis at $y = 1$ (y-intercept)

Tangent graph, $y = \tan x$



- maximum value is ∞ and minimum value is $-\infty$
- intercepts x-axis at $x = 0°$, $180°$ and $360°$ (x-intercept)
- intercepts y-axis at $y = 0$ (y-intercept)
- the values of tan 90° and tan 270° are undefined

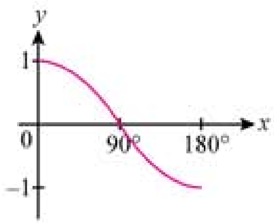
Critical Mind

- What is meant by an undefined value?
- Why are tan 90° and tan 270° undefined?

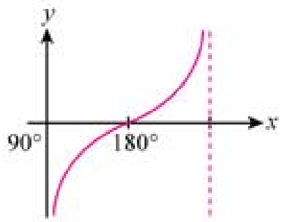
Self Practice 6.2a

1. Determine whether each of the following trigonometric graphs is a sine, cosine or tangent graph.

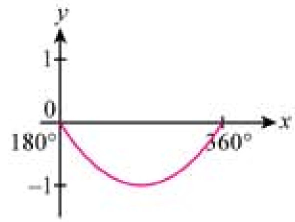
(a)



(b)

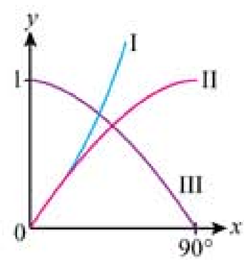


(c)



2. (a) Sketch the graph for $y = \sin x$ where $90° \leqslant x \leqslant 270°$.
   (b) Sketch the graph for $y = \cos x$ where $45° \leqslant x \leqslant 225°$.

3. The diagram on the right shows three trigonometric graphs for angles between 0° and 90°.
   (a) Identify the equations for graphs I, II and III.
   (b) State the maximum value and minimum value of graphs I, II and III, if any.

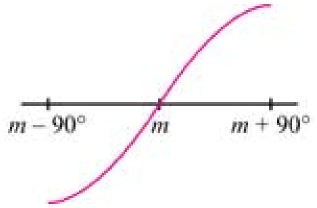


CHAPTER 6

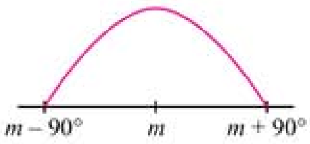
4. For each of the following trigonometric functions, state the value of x when the value of y is maximum and state the maximum value.
(a) $y = \sin x,\ 0° \leqslant x \leqslant 360°$
(b) $y = \cos x,\ 0° \leqslant x \leqslant 360°$
(c) $y = \tan x,\ 0° \leqslant x \leqslant 360°$

5. Each diagram below shows part of a graph of trigonometric function such that $0° \leqslant x \leqslant 360°$. State the function and the value of m.

(a)



(b)

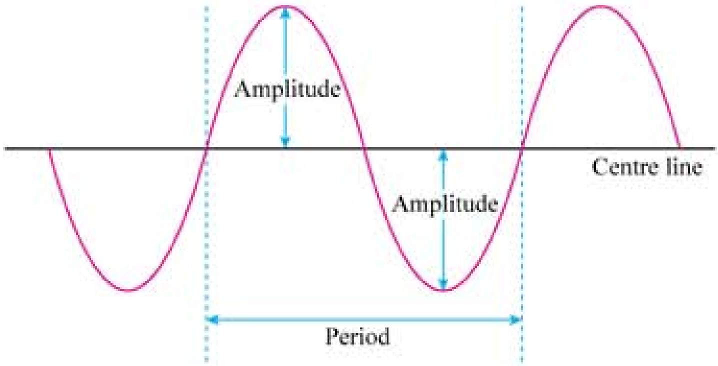


What are the effects of changes in constants a, b and c on the graphs of trigonometric functions $y = a \sin bx + c$, $y = a \cos bx + c$ and $y = a \tan bx + c$?

Learning Standard

Investigate and make generalisations about the effects of changes in constants a, b and c on the graphs of trigonometric functions:
(i) $y = a \sin bx + c$
(ii) $y = a \cos bx + c$
(iii) $y = a \tan bx + c$
for $a > 0$, $b > 0$.

The diagram below shows the characteristics of a periodic function, namely period and amplitude.



Smart TiPS

Amplitude can also be determined by the formula:
$$\left(\frac{\text{maximum value} - \text{minimum value}}{2}\right)$$

Period is the interval of one complete cycle. The trigonometric function is a periodic function. The graph of a trigonometric function repeats itself on a specific interval. For example, the graph of sine function repeats itself on an interval of 360°. We can say that the function $y = \sin x$ is a periodic function with a period of 360°. Amplitude is the maximum distance measured from the centre line.

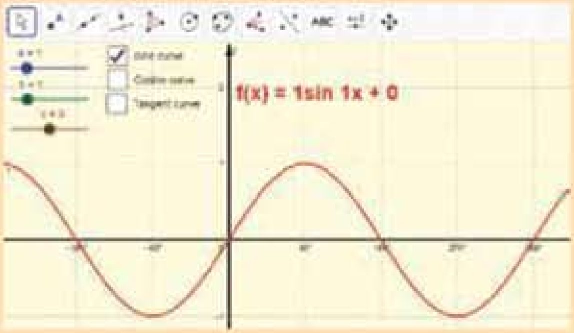
If given that the trigonometric functions $y = a \sin bx + c$, $y = a \cos bx + c$ and $y = a \tan bx + c$, what would happen to the shape and position of the graphs of the trigonometric functions if the values of constants a, b and c change?

MIND MOBILISATION 4 Group

Aim: To investigate and make generalisations about the effects of changes in constants a, b and c on the graphs of trigonometric functions $y = a \sin bx + c$, $y = a \cos bx + c$ and $y = a \tan bx + c$.

Steps:

1. Open the file GGB603 for this activity.



Scan the QR code or visit bit.do/GGB603BI to obtain the GeoGebra file for this activity.

2. Drag the slider of the value of a and observe the change on the graph displayed.
3. Drag the slider of the value of b and observe the change on the graph displayed.
4. Drag the slider of the value of c and observe the change on the graph displayed.
5. Repeat steps **2** to **4** for 'Cosine curve' and 'Tangent curve'.

Discussion:

What are your conclusions about the effects of changes in constants a, b and c on the graphs of trigonometric functions $y = a \sin bx + c$, $y = a \cos bx + c$ and $y = a \tan bx + c$ for $a > 0$, $b > 0$?

The results of Mind Mobilisation 4 show that
(a) when a changes, the maximum and minimum values change,
(b) when b changes, the graph will be compressed or expanded,
(c) when c changes, the graph will move vertically up or down.
The value of a affects the amplitude of the function, the value of b affects the period of the function and the value of c affects the position of the graph of the function.

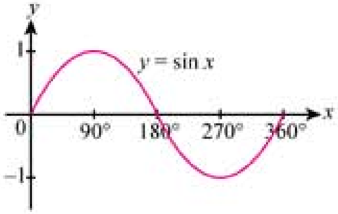
In general,

	$y = a \sin bx + c$, $a > 0, b > 0$	$y = a \cos bx + c$, $a > 0, b > 0$	$y = a \tan bx + c$, $a > 0, b > 0$
The value of a changes	• Maximum and minimum values change • Amplitude of function = a		• Graph curvature changes • No amplitude
The value of b changes	• The period of the function changes • When the value of b increases, the graph appears to compress horizontally, the period of the function decreases • The period of sine and cosine functions = $\frac{360°}{b}$; The period of tangent function = $\frac{180°}{b}$		
The value of c changes	• The position of the graph changes • When $c > 0$, the graph moves c units vertically up the x-axis • When $c < 0$, the graph moves c units vertically down the x-axis		

Example 9

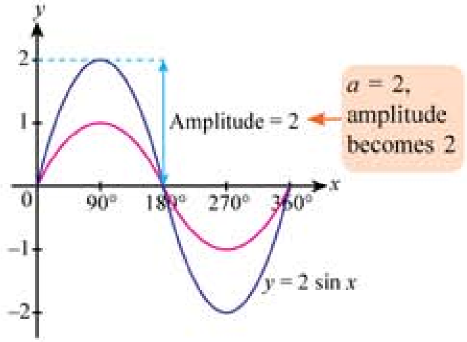
The diagram on the right shows a graph of the function $y = \sin x$ for $0° \leqslant x \leqslant 360°$. Sketch each of the following trigonometric functions on the same axes.

(a) $y = 2 \sin x$ (b) $y = \sin 2x$
(c) $y = \sin x + 1$ (d) $y = 2 \sin 2x$

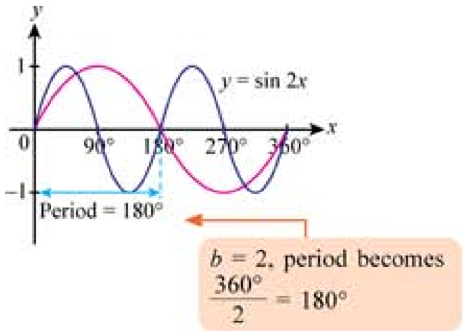


Solution:

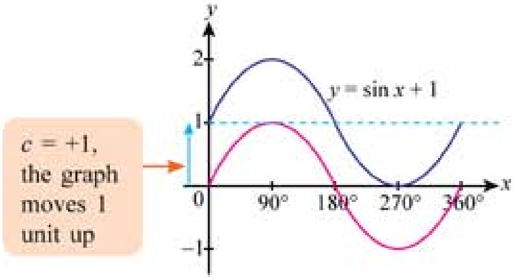
(a)



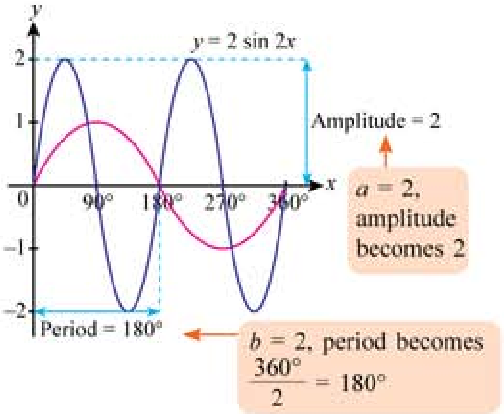
(b)



(c)



(d)



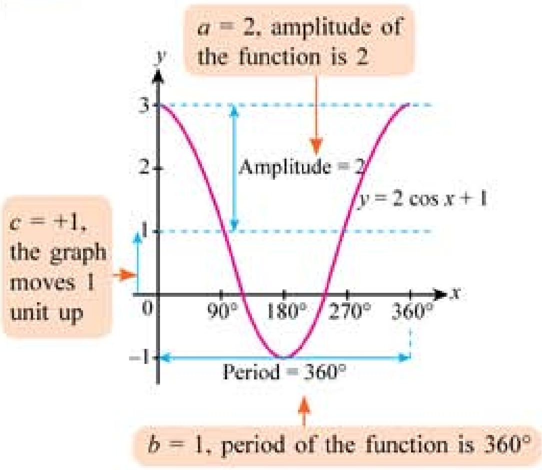
Example 10

Sketch each of the following trigonometric functions for $0° \leqslant x \leqslant 360°$.

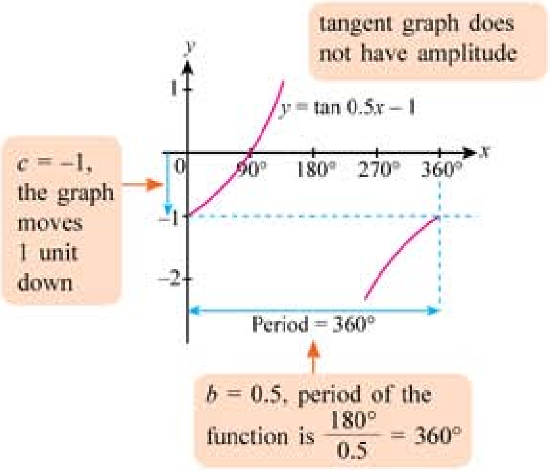
(a) $y = 2 \cos x + 1$ (b) $y = \tan 0.5x - 1$

Solution:

(a)



(b)



Self Practice 6.2b

1. Determine the amplitude and the period of each of the following trigonometric functions.

(a) $y = 4 \sin x$ (b) $y = 3 \sin 2x$ (c) $y = 2 \sin 3x - 4$

(d) $y = \cos 4x$ (e) $y = 4 \cos 2x$ (f) $y = 3 \cos 3x + 1$

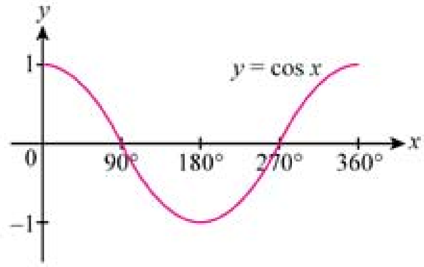
(g) $y = \frac{1}{3} \tan 3x$ (h) $y = 3 \tan \frac{1}{3}x$ (i) $y = 3 \tan 2x + 2$

2. The diagram on the right shows a graph of the function $y = \cos x$ for $0° \leqslant x \leqslant 360°$. Sketch each of the following trigonometric functions on the same axes.

(a) $y = \frac{1}{2} \cos x$

(b) $y = \cos \frac{x}{2}$

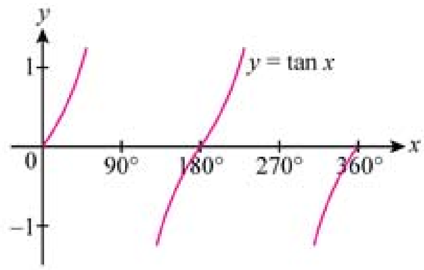
(c) $y = \cos x - 2$



3. The diagram on the right shows a graph of the function $y = \tan x$ for $0° \leqslant x \leqslant 360°$. Sketch each of the following trigonometric functions on the same axes.

(a) $y = \tan 2x$

(b) $y = \tan x + 2$



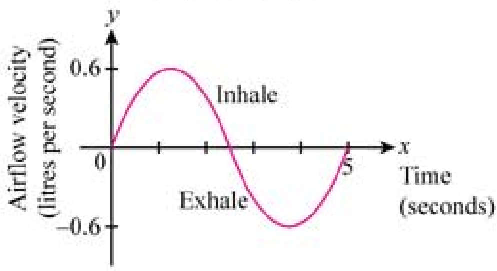
How to solve problems involving graphs of sine, cosine and tangent functions?

Learning Standard

Solve problems involving graphs of sine, cosine and tangent functions.

Example 11

Airflow velocity in normal respiratory cycle



The graph above shows a complete respiratory cycle. This cycle consists of the process of inhalation and exhalation. It occurs every 5 seconds. Airflow velocity is positive when we inhale and negative when we exhale. This velocity is measured in litre per second.

(a) If y represents the velocity of airflow after x seconds, state a function in the form of $y = a \sin bx + c$ which models the airflow in normal respiratory cycle shown as the graph above.

(b) What is the velocity of airflow, in litres per second, when the time is 7 seconds?

Solution:

(a) From the graph, amplitude = 0.6, therefore $a = 0.6$

period = 5 seconds, therefore $\frac{360°}{b} = 5$

$b = 72$

there is no movement of going up or down the x-axis, therefore $c = 0$

Hence, a function that models the airflow in normal respiratory cycle shown as the graph is $y = 0.6 \sin 72x$.

(b) $y = 0.6 \sin 72x$

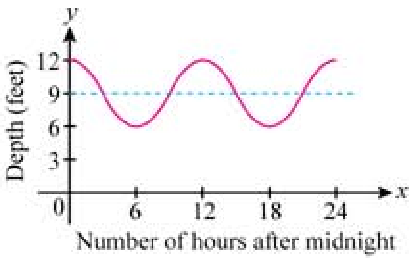
When $x = 7$, $y = 0.6 \sin (72 \times 7)$

$y = 0.35$

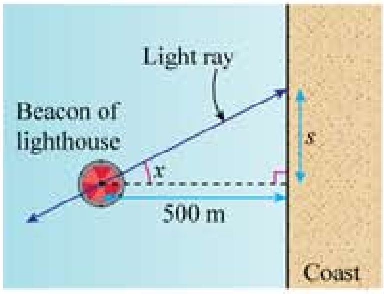
Hence, the airflow velocity is 0.35 litre per second when the time is 7 seconds.

Self Practice 6.2c

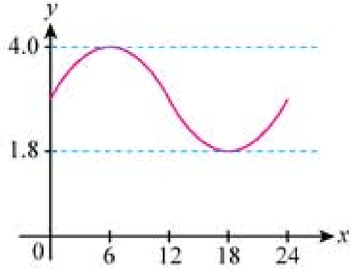
1. The graph on the right shows the depth of water recorded in a dockyard.
   (a) If y represents the depth of water, in feet, and x represents the number of hours after midnight, use the function in the form of $y = a \cos bx + c$ to model the depth of water shown as the graph.
   (b) At what time will the water be the deepest?



2. The diagram on the right shows the view from the top of a lighthouse and the coast. The beacon of the lighthouse sends out a light ray as shown in the diagram. When the beacon rotates, the light ray moves along the coast at an angle, x.
   (a) Write a function for the distance s.
   (b) State the amplitude and the period of the function.



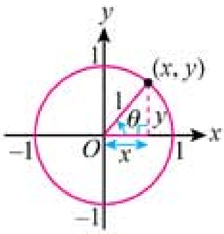
3. The graph on the right depicts the water level recorded at a port. It is given that y represents the water level, in m, and x represents time, in hours. State the trigonometric function of the graph in the form of $y = a \sin bx + c$.



Summary Arena

Sine, Cosine and Tangent

Unit circle



$\sin \theta = y\text{-coordinate}$

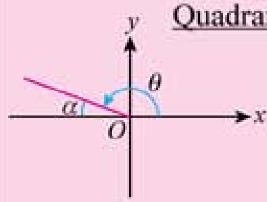
$\cos \theta = x\text{-coordinate}$

$\tan \theta = \dfrac{y\text{-coordinate}}{x\text{-coordinate}}$

Corresponding reference angles

Let α be a corresponding reference angle

Quadrant II



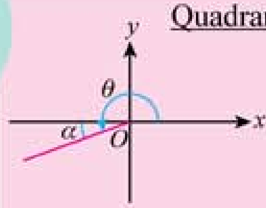
$\alpha = 180° - \theta$

$\sin \theta = +\sin \alpha$

$\cos \theta = -\cos \alpha$

$\tan \theta = -\tan \alpha$

Quadrant III



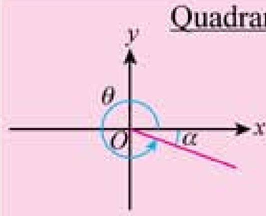
$\alpha = \theta - 180°$

$\sin \theta = -\sin \alpha$

$\cos \theta = -\cos \alpha$

$\tan \theta = +\tan \alpha$

Quadrant IV



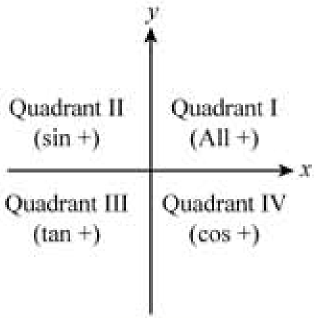
$\alpha = 360° - \theta$

$\sin \theta = -\sin \alpha$

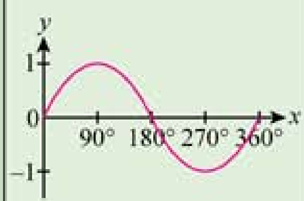
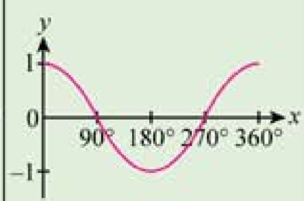
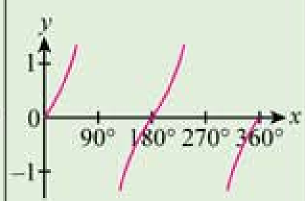
$\cos \theta = +\cos \alpha$

$\tan \theta = -\tan \alpha$

Signs of values of sin θ, cos θ and tan θ



Graphs of sin x, cos x and tan x for $0° \leqslant x \leqslant 360°$

	$y = \sin x$	$y = \cos x$	$y = \tan x$
Shape of graph			
Maximum value	1	1	∞
Minimum value	–1	–1	–∞
x-intercept	0°, 180°, 360°	90°, 270°	0°, 180°, 360°
y-intercept	0	1	0

Reflection

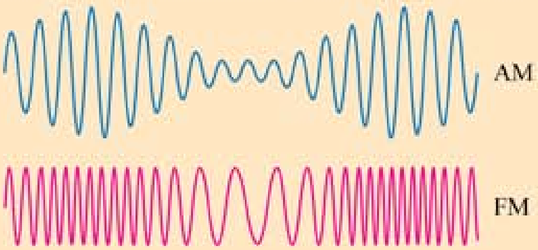
At the end of this chapter, I can

	🙂	😖
make and verify conjecture about the value of sine, cosine and tangent for angles in quadrants II, III and IV with the corresponding reference angle.		
determine the value of sine, cosine and tangent for angles in quadrants II, III and IV based on the corresponding reference angle.		
determine the angle when the value of sine, cosine and tangent are given.		
solve problems involving sine, cosine and tangent.		
draw graphs of trigonometric functions $y = \sin x$, $y = \cos x$ and $y = \tan x$ for $0° \leqslant x \leqslant 360°$, hence compare and contrast the characteristics of the graphs.		
investigate and make generalisations about the effects of changes in constants a, b and c on the graphs of trigonometric functions: (i) $y = a \sin bx + c$ (ii) $y = a \cos bx + c$ (iii) $y = a \tan bx + c$ for $a > 0$, $b > 0$.		
solve problems involving graphs of sine, cosine and tangent functions.		

MINI PROJECT

Radio broadcasting is an example of electronic communication that can be found today. Sounds like music and voices that we hear from the radio are broadcasted in the form of waves. In the case of AM radio, sound is transmitted through amplitude modulation whereas for FM radio, sound is transmitted through frequency modulation.

In groups of four, do a brief report that explains the difference between amplitude modulation and frequency modulation based on the characteristics of the graphs of the trigonometric functions that you have learnt. You are encouraged to use appropriate graphs, tables, and mind maps to present your group report.



Extensive Practice

Scan the QR code or visit bit.do/QuizE06 for interactive quiz

UNDERSTAND

1. State the relationship between each of the following trigonometric functions with its corresponding reference angle.
(a) $\tan 154°$ (b) $\sin 234°$ (c) $\cos 314°$

2. Determine the value of each of the following based on its corresponding reference angle.
(a) $\cos 116°$ (b) $\tan 211°38'$ (c) $\sin 305.6°$

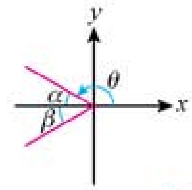
3. Given that $\tan \theta = -0.7265$ and $0° \leqslant \theta \leqslant 360°$, calculate the angle θ.

4. Sketch the graph of $y = \cos x$ for $90° \leqslant x \leqslant 270°$.

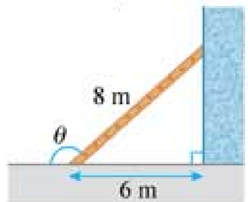
MASTERY

5. State the maximum and minimum values of the graph of function $y = 3 \sin 2x - 1$ for $0° \leqslant x \leqslant 360°$.

6. In the diagram on the right, $\theta = 150°$ and $\alpha = \beta$. Determine the values of
(a) $\cos \alpha$
(b) $\tan \beta$

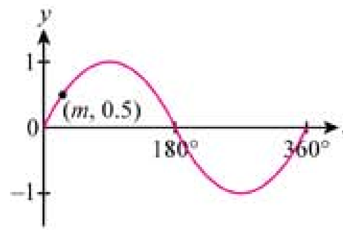


7. The diagram on the right shows a piece of wood with a length of 8 m leaning against a vertical wall. The horizontal distance from the wall to the wood is 6 m. Calculate the value of $\sin \theta$.

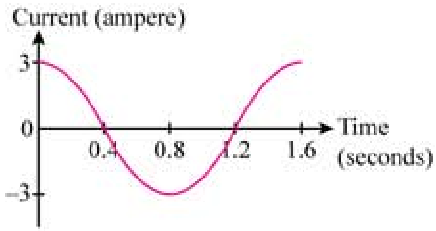


8. Sketch the graph of the function $y = 3 \sin 2x + 1$ for $0° \leqslant x \leqslant 360°$.

9. The diagram on the right shows a graph of a trigonometric function for $0° \leqslant x \leqslant 360°$.
(a) Write the trigonometric function.
(b) State the value of m.

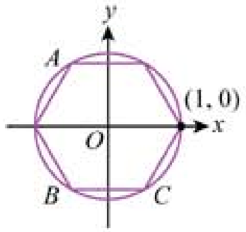


10. The diagram on the right shows a graph obtained on the screen of an oscilloscope when a supply of alternating current is connected to it.
(a) Which type of trigonometric functions is represented by the graph?
(b) State the amplitude of the current.
(c) State the period of the current.

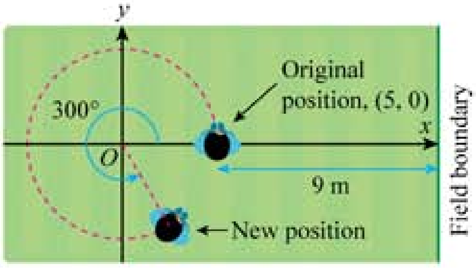


CHALLENGE

11. A regular hexagon is drawn in a unit circle as shown in the diagram on the right. If one of the vertices of the hexagon is at (1, 0), determine the coordinates of vertices *A*, *B* and *C*.



12. Your school band performs on the field. Band members make a circular formation with a diameter of 10 m. Assume that you are one of the members and your original position is 9 m from the field boundary. After you have moved, your new position is as shown in the diagram on the right. If you are required to move to the field boundary from your new position, what is the shortest distance, in m, would you walk through?

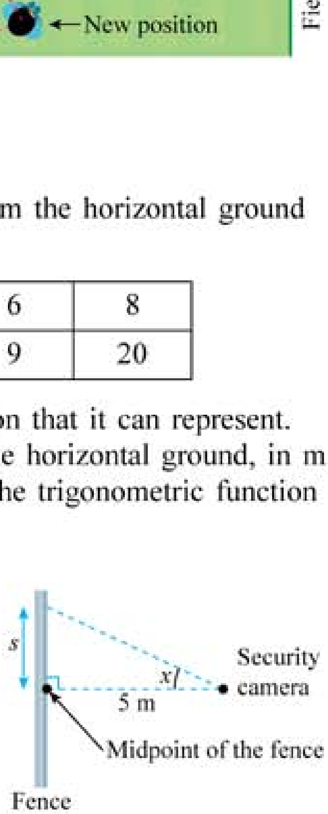


13. The table below shows the height of Ming Seng's position from the horizontal ground as he rides on the Ferris wheel.

Time (minutes)	0	2	4	6	8
Height (m)	20	31	20	9	20

(a) Based on the table, state the type of trigonometric function that it can represent.
(b) Hence, if *y* is the height of Ming Seng's position from the horizontal ground, in m, and *x* is the time in minutes, sketch the graph and state the trigonometric function representing the information above.

14. The diagram on the right shows a security camera in front of a fence of an apartment. The camera is mounted on a pole located 5 m from the midpoint of the fence. Write a trigonometric function that expresses the distance, *s*, in m, along the fence from its midpoint in terms of *x*.



EXPLORING MATHEMATICS

Do you know that trigonometric functions can be applied when programming a character which is jumping in a computer game?

Try to relate the movement patterns of computer game characters with trigonometric functions. Get the relevant information from legitimate sources such as interview a programmer, refer to a reading material or others. Write a short journal on

- the characteristics of trigonometric functions,
- the use of trigonometric functions in computer games,
- two other examples of real-life applications involving trigonometric functions.