CHAPTER 8

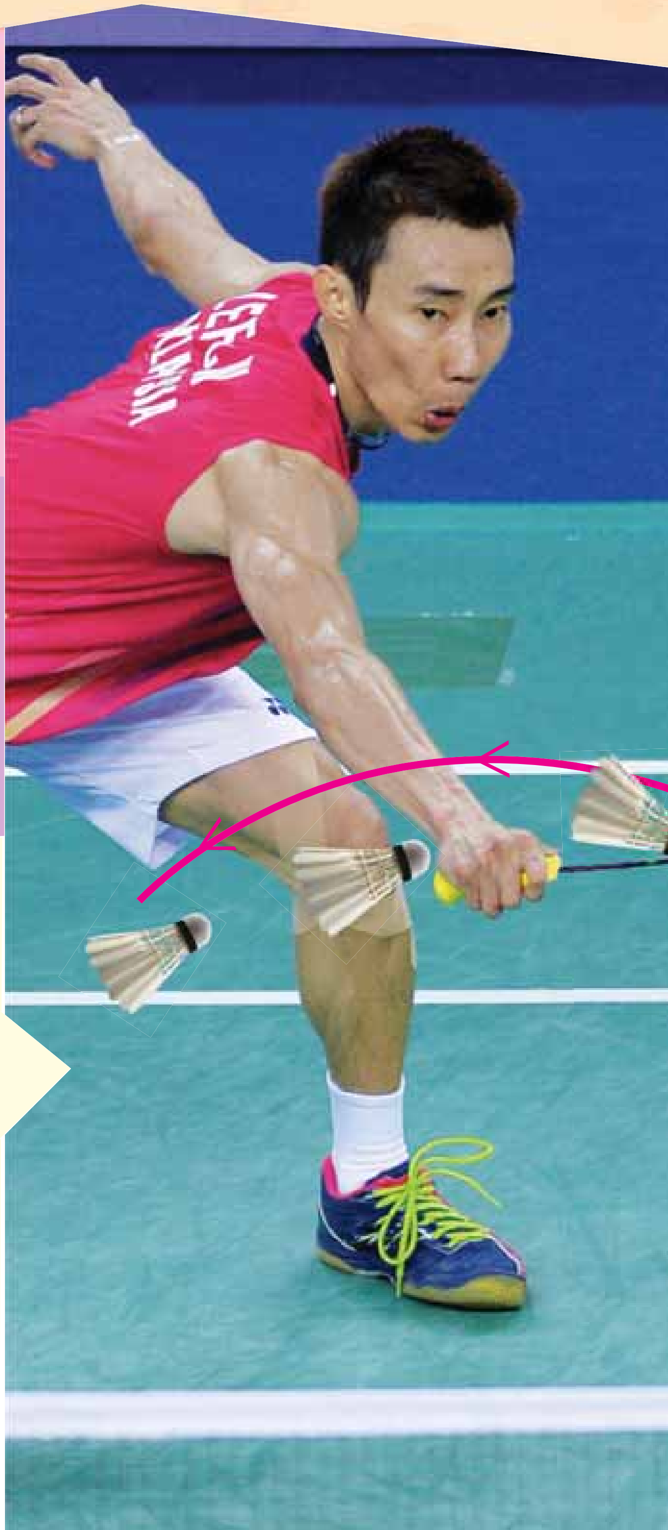
Loci in Two Dimensions

What will you learn?

- 8.1 Loci
- 8.2 Loci in Two Dimensions

Why do you learn this chapter?

- Knowledge about loci allows one to estimate or predict the path of the moving points based on certain conditions.
- The concept of loci is used in construction, engineering drawings, aviation, satellite movements and so on.



National badminton champion Datuk Lee Chong Wei currently holds the record for the fastest smash since September 2015 when he did the shot with a speed of 408 kilometres per hour (km/h). He won the 2015 Hong Kong Open Badminton Championships which was held at the Hung Hom Coliseum. According to Badminton World Federation (BWF), the speed of the shot made by Chong Wei was recorded and measured using Hawk Eye technology that has been adopted in several major tournaments since September 2015. Do you know that the movement of a shuttlecock follows certain conditions?

Exploring Era

Apollonius (260 – 190 BC) was an ancient Greek mathematician who was very interested in studying problems of loci. He had conducted research on various forms of loci such as the straight lines and certain curves. However, the most outstanding Greek mathematician in studying loci was Pappus (290 AD – 350 AD). Pappus's loci materials are still being researched by mathematicians today.

http://bukutekskssm.my/Mathematics/F3/ExploringEraChapter8.pdf

WORD BANK

- equidistant • *berjarak sama*
- circle • *bulatan*
- arc • *lengkok*
- curve • *lengkung*
- locus • *lokus*
- loci • *lokus-lokus*
- perpendicular bisector • *pembahagi dua sama serenjang*
- angle bisector • *pembahagi dua sudut*

8.1 Locus

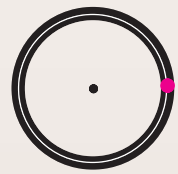
What do you understand about loci?

LEARNING STANDARD

Recognise loci in real life situations and hence explain the meaning of locus.

In the picture on the right, a piece of coloured sticker is pasted on the tyre of a bicycle.

What is the shape generated by the sticker when the bicycle is pedalled?



The shape formed by the sticker is a circle as shown in the diagram on the right. Does this shape comply with certain conditions?

The picture below shows a ball being kicked by a football player. The movement of a point on the ball yields a curve.

BULLETIN

Malaysia's football fans will always remember 'Super Mokh', the late Datuk Mokhtar Dahari, who fired a solid 40-metre shot against England Squad 3 in 1978.

The picture on the right shows a rocket being launched. The movement of a point on the rocket will produce a straight line.

A locus is a trace or trajectory formed by a set of points in a plane or three-dimensional space that satisfies certain conditions.

Brainstorming 1

In groups

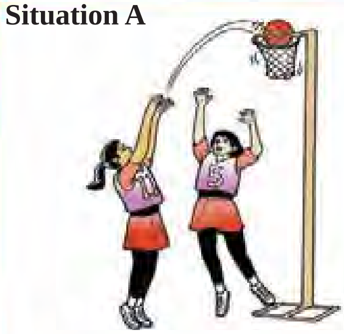
Aim: To identify two-dimensional loci in daily life situations.

Materials: Situation cards.

Steps:

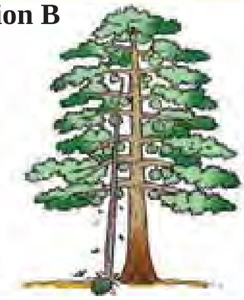
1. Each group is given several situation cards that show activities involving movements in daily activities as shown below.

Situation A



Throwing a ball into the net.

Situation B



A durian falling from a tree.

Situation C

An airplane landing.

Situation D

The moving tip of the wiper on the windshield.

2. Discuss in the group and sketch the locus of a point on the object involved in the given situations.
3. Present the loci sketch and compare your answers with other groups.

Discussion:
Discuss five other movements in daily activities that can be categorised as loci.

From Brainstorming 1, it is found that:

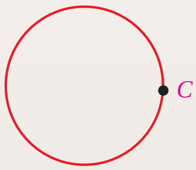
The shapes of two-dimensional loci can be seen in the form of straight lines, arcs and curves.

Example 1

Point C is drawn on a blade of a revolving fan as shown in the diagram. Elaborate and sketch the locus of point C.

Solution:

This locus is a circle.



MIND TEST 8.1a

1. Explain and sketch the locus of point C on each object in the following diagrams.

(a) A ball centred at C rolling along an inclined plane.

(b) Point C on a swinging pendulum.

(c) Point C on a spinning yo-yo.

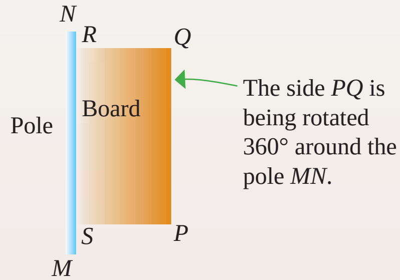
(d) Point C on the shoe of a child who is playing on a slide at the playground.

2. State and sketch the locus of a point on

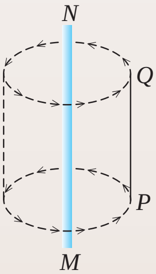
(a) a coconut falling from a tree
(b) a moving car on a straight road
(c) a leaping frog

Example 2

The diagram on the right shows a pole MN. A rectangular board $PQRS$ is attached to the pole where $PQRS$ is movable. If the side PQ is rotated 360° around MN, what is the three-dimensional shape formed?



Solution:



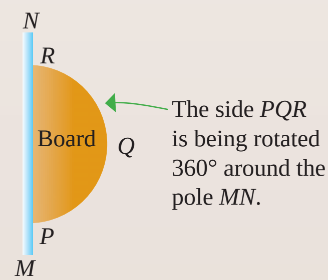
The shape formed when the side PQ is rotated 360° around pole MN is a right **cylinder**.

FLASHBACK

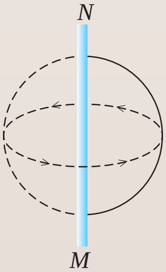
Cylinders, spheres, cones, prisms and pyramids are examples of **three-dimensional shapes**.

Example 3

The diagram on the right shows a pole MN. A semicircular board PQR is attached to the pole where PQR is movable. If PQR is rotated 360° around MN, what is the three-dimensional shape formed?



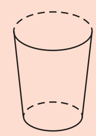
Solution:



The shape formed when the semicircular board is rotated 360° around pole MN is a **sphere**.

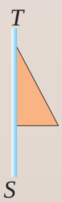
BULLETIN

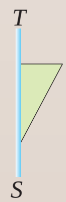
This three-dimensional shape is known as a frustum.

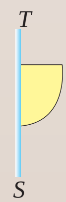


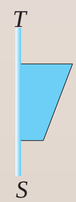
MIND TEST 8.1b

1. Sketch the three-dimensional loci when the two-dimensional shaded shapes are rotated 360° around pole ST.

(a)


(b)


(c)


(d)


8.2 Loci in Two Dimensions

What is the locus of points that are of constant distance from a fixed point?

Brainstorming 2 In pairs

LEARNING STANDARD

Describe the locus of points that are of constant distance from a fixed point.

Aim: To determine the locus of points that are of constant distance from a fixed point.

Materials: Blank paper, a pencil and a ruler.

Steps:

1. Mark a fixed point O on a sheet of paper (Diagram 1).
2. Measure 5 cm from the point O and mark ×.
3. Repeat step 2 as many times as possible (Diagram 2).

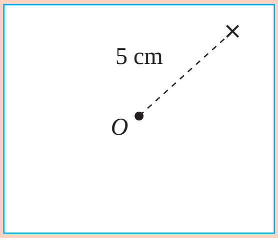


Diagram 1

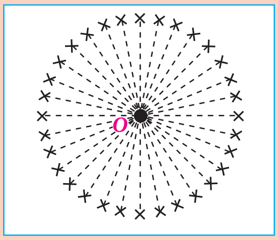


Diagram 2

4. Note the location of the points marked with × (Diagram 2).
5. Repeat steps 1 to 3 with different distances from the fixed point O.
6. Are the resulting geometric shapes the same as the shape obtained in step 4? Explain.

Discussion:

What is the geometric shape generated by the location of the dots ×? Explain.

From Brainstorming 2, it is found that:

Points marked at the same distance from a fixed point O forms a circle.

In general,

The locus of a point that is equidistant from a **fixed point** is a **circle centred at that fixed point**.

Brainstorming 3

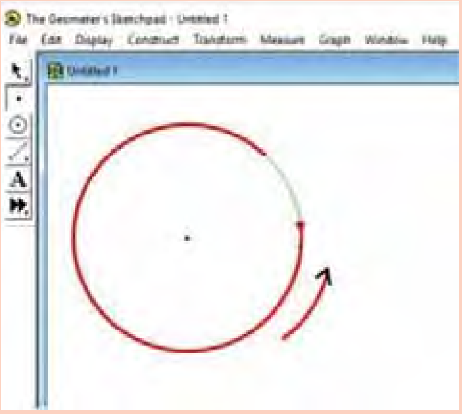
In pairs

Aim: To construct locus of points that are of constant distance from a fixed point.

Materials: Dynamic software

Steps:

1. Start with *New Sketch*.
2. Select *Compass Tool* and draw a circle.
3. Select *Point Tool* and mark.
4. Open *Display* menu and select *Trace Point* followed by *Animate Point*.
5. Observe the animation of the movement generated.



Discussion:

What is the geometric shape generated from the movement of the marked point?

From Brainstorming 3, it is found that:

A point that always moves at the same distance from a fixed point forms a circle.

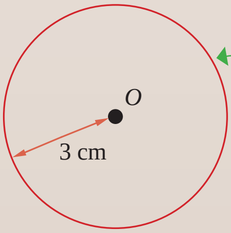
How do you construct a locus of points that are of constant distance from a fixed point?

Example 4

Construct a locus of point P which is always 3 cm from a fixed point O.

Solution:

1. Mark point O.
2. Set the gap of the compasses at 3 cm.
3. Construct a circle of radius 3 cm centred at the point O.



What is the locus of points that are equidistant from two fixed points?

Brainstorming 4

In pairs

LEARNING STANDARD

Describe the locus of points that are of equidistant from two fixed points.

Aim: To determine the locus of points that are equidistant from two fixed points.

Materials: Plain paper, a compasses, a ruler and a pencil.

Steps:

1. Mark two fixed points *P* and *Q* which are 8 cm apart (Diagram 1).
2. Using the compasses, mark the intersection, 4.5 cm from point *P* and point *Q* (Diagram 2).
3. Repeat step 2 with distances more than 4.5 cm from point *P* and point *Q* (Diagram 3).

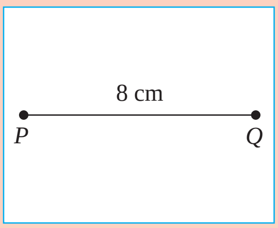


Diagram 1

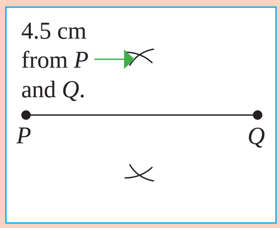


Diagram 2

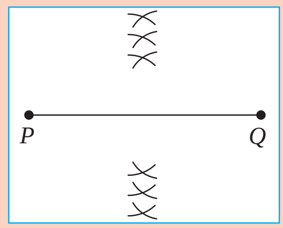


Diagram 3

4. Note the location of the intersecting marks in Diagram 3.
5. Repeat steps 1 to 3 with different distances between point *P* and point *Q*.
   Are your answers the same as the answer in step 4?

Scan the QR code or visit http://bukuteksssm.my/Mathematics/F3/Chapter8LocusfromTwoFixedPoint.mp4 to watch a video that describes the locus of the points that are equidistant from two fixed points.

Discussion:

What is the shape formed by the location of the intersecting marks? Explain.

From Brainstorming 4, it is found that:

The location of the intersecting marks that are equidistant from fixed points *P* and *Q* form a straight line through the midpoint of *PQ*.

In general, The locus of a point that is equidistant from **two fixed points** is the **perpendicular bisector** of the line connecting the two fixed points.

Brainstorming 5

In pairs

Aim: To construct locus of points that are equidistant from two fixed points.

Materials: Dynamic software

Steps:

1. Start with *New Sketch*.
2. Select *Straightedge Tool* to draw a line segment. Select *Text Tool* to label point *A* and point *B*.
3. Select *Construct* menu to construct the midpoint of the line segment.
4. Mark both lines and midpoint segments with *Selection Arrow Tool*.
5. Select *Construct* menu to construct a perpendicular line (Diagram 1).

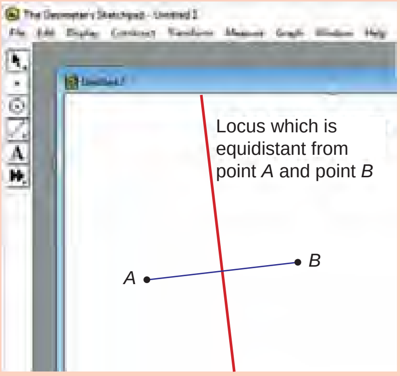


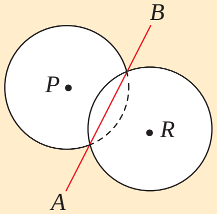
Diagram 1

Discussion:

What is the geometric shape formed? Explain.

FLASHBACK

The line *AB* is known as a bisector.



From Brainstorming 5, it is found that:

The **locus that is equidistant** from two fixed points *A* and *B* is **a straight line perpendicular** to the straight line *AB* and it passes through the midpoint of *AB*.

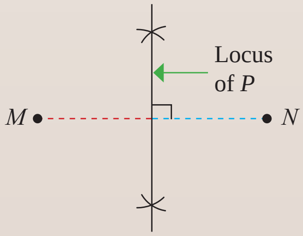
How do you construct the locus of points that are equidistant from two fixed points?

Example 5

Construct the locus of point *P* that is equidistant from two fixed points *M* and *N*.

M •- - - - - - - - - - - - - - - - -• N

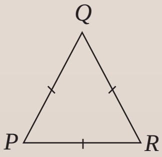
Solution:



1. Mark two small arcs using a pair of compasses with the gap set at more than half of the length of *MN* from the point *M*.
2. With the compasses set at the same gap, mark the intersecting arcs of point *N*.
3. Connect the two points of intersection with a straight line.

Example 6

The diagram on the right shows an equilateral triangle *PQR*. Determine the locus of point *X* that is equidistant from point *P* and point *R*.



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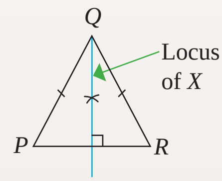
Solution:

Locus of point X that is equidistant from point P and point R is the perpendicular bisector of the line connecting point P and point R.



What is the locus of points that are of constant distance from a straight line?

Brainstorming 6

In pairs

LEARNING STANDARD

Describe the locus of points that are of constant distance from a straight line.

Aim: To determine the locus of points that are of constant distance from a straight line.

Materials: Square grid paper, a ruler, a pencil.

Steps:

1. Draw a straight line MN (Diagram 1).
2. Mark a point ×, which is 3 units from the line MN (Diagram 2).

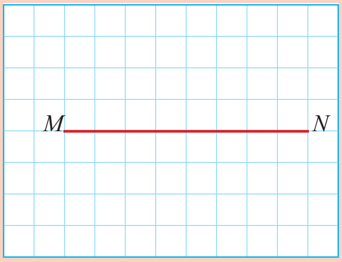


Diagram 1

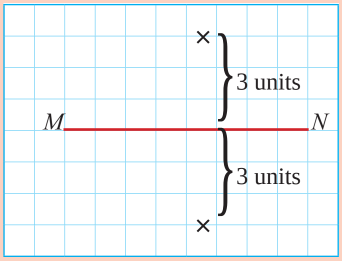


Diagram 2

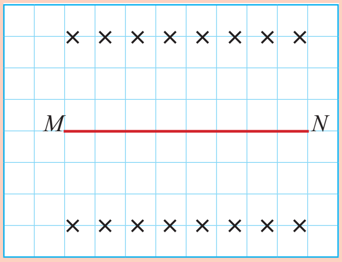


Diagram 3

3. Repeat step 2 with as many × points as possible (Diagram 3).
4. Note the location of the × points in Diagram 3. What do you think about the location of the × points?
5. Repeat steps 1 through 4 with a different unit distance.
6. Repeat steps 1 through 4 with the straight line MN drawn vertically.

Discussion:

What is your conclusion about the location of the points marked equidistantly from the straight line?

From Brainstorming 6, it is found that:

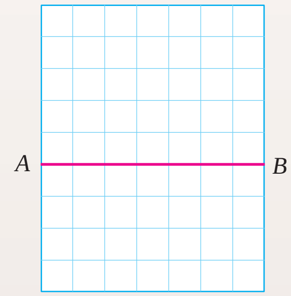
The locus of points that are equidistant from the line MN is a pair of straight lines parallel to MN.

In general,

The locus of points that are of constant distance **from a straight line** are **straight lines parallel** to that straight line.

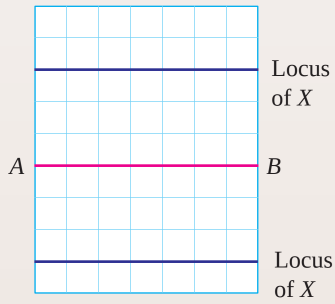
Example 7

The diagram on the right shows a line *AB* drawn on a square grid with sides of 1 unit. On the diagram, draw the locus of the point *X* which always moves at 3 units from the line *AB*.



Solution:

The locus of point *X* moving 3 units from the line *AB* is a pair of lines parallel to *AB* and 3 units from *AB*.



What is the locus of points that are equidistant from two parallel lines?

Brainstorming 7 In groups

LEARNING STANDARD

Describe the locus of points that are equidistant from two parallel lines.

Aim: To determine the locus of points that are equidistant from two parallel lines.

Materials: Plain paper, compasses, a ruler and a pencil.

Steps:

1. Draw two straight lines *PQ* and *MN* that are parallel (Diagram 1).
2. Using compasses, mark the point of intersection from point *P* and point *M*.
3. Repeat steps 2 for point *Q* and point *N* (Diagram 2).
4. Connect all the intersection points marked by drawing a straight line (Diagram 3).

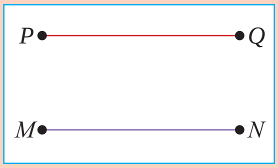


Diagram 1

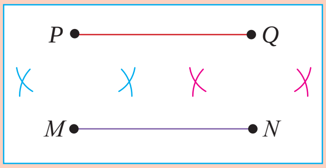


Diagram 2

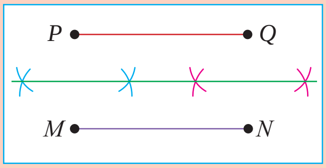


Diagram 3

5. Describe the nature of the straight line that connects all the points of intersection (Diagram 3).

Discussion:

1. Repeat steps 1 to 4 by drawing two vertical straight lines and two inclined straight lines. Ensure that each pair of lines is parallel.
2. Do you get the same result as in step 4?

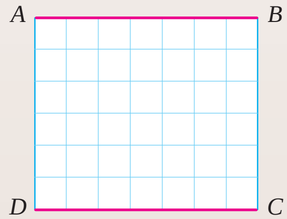
From Brainstorming 7, it is found that:

(a) The locus of points that are equidistant from two parallel lines *PQ* and *MN* is a straight line.
(b) The locus is parallel to the straight lines *PQ* and *MN* and it passes through the midpoints of the lines *PQ* and *MN*.

In general, The locus of points that are **equidistant from two parallel lines** is **a straight line parallel to and passes through the midpoints** of the pair of parallel lines.

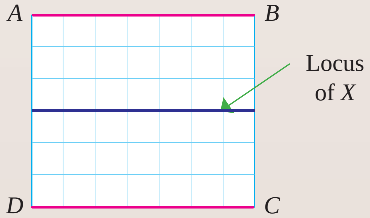
Example 8

The diagram on the right shows the rectangle, *ABCD* drawn on a square grid with sides of 1 unit. Describe and draw the locus of *X* which is equidistant from the lines *AB* and *DC*.



Solution:

The locus of point *X* that is equidistant from the line *AB* and *DC* is a line parallel to *AB* and *DC* and is 3 units from the lines *AB* and *DC*.



What is the locus of points that are equidistant from two intersecting lines?

Brainstorming 8 In groups

LEARNING STANDARD
Describe the locus of points that are of equidistant from two intersecting lines.

Aim: To determine the locus of points that are equidistant from two intersecting lines.

Materials: A square grid paper, a ruler, a pencil and a protractor.

Steps:

1. Draw *x*-axis and *y*-axis on a Cartesian plane on the grid paper (Diagram 1).
2. Mark the coordinates of equal value pairs. For example, (0, 0), (–2, –2), (4, 4) and so on (Diagram 2).
3. Connect all the points with a straight line. Measure $\angle a$, $\angle b$, $\angle c$ and $\angle d$ using a protractor (Diagram 3).

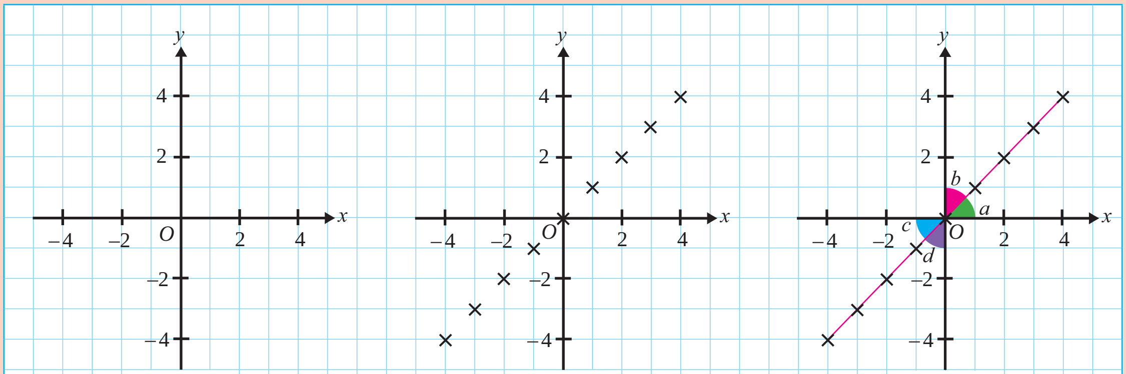


Diagram 1   Diagram 2   Diagram 3

Discussion:

1. What is your conclusion about the values of $\angle a$, $\angle b$, $\angle c$ and $\angle d$ which are the angles formed at the intersection of the x-axis and y-axis?
2. What is the relationship between the straight line that connects equal value pairs of coordinates to the values of $\angle a$, $\angle b$, $\angle c$ and $\angle d$?

From Brainstorming 8, it is found that:

(a) $\angle a = \angle b = \angle c = \angle d = 45°$.

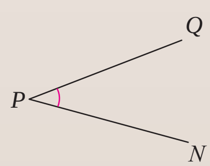
(b) The straight line that connects equal value pairs of coordinates bisects the angle of intersection between the x-axis and y-axis.

In general, The locus of points that are equidistant from **two intersecting lines** is the **angle bisector** of the angles formed by the intersecting lines.

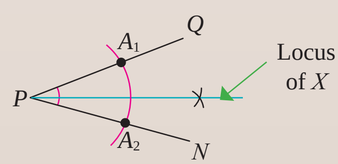
How do you construct a locus of points that are equidistant from two intersecting lines?

Example 9

Construct the locus of point X that is equidistant from two straight lines PQ and PN intersecting at P.



Solution:



1. By using a pair of compasses, draw an arc from the point P which cuts through the straight lines PQ and PN.
2. Mark the points of intersection between the arc and the straight lines PQ and PN as A_1 and A_2 respectively.
3. Construct intersecting mark from A_1 and A_2.
4. Draw a straight line joining the intersecting mark constructed in step 3 and the point P.

CHAPTER 8

Brainstorming 9

In pairs

Aim: To construct locus of a point that is equidistant from two intersecting straight lines.

Materials: Dynamic software

Steps:

1. Start with *New Sketch*.
2. Select *Straightedge Tool* to draw lines AB and BC intersecting at point B.
3. Use *Text Tool* to label point A, followed by point B and then point C (point of intersection must be marked on the second turn).
4. Mark all three points A, B and C with *Selection Arrow Tool*. (Diagram 1)
5. Select the *Construct* menu to construct the bisector of the angle (*Angle bisector*) between the two intersecting lines. (Diagram 2)

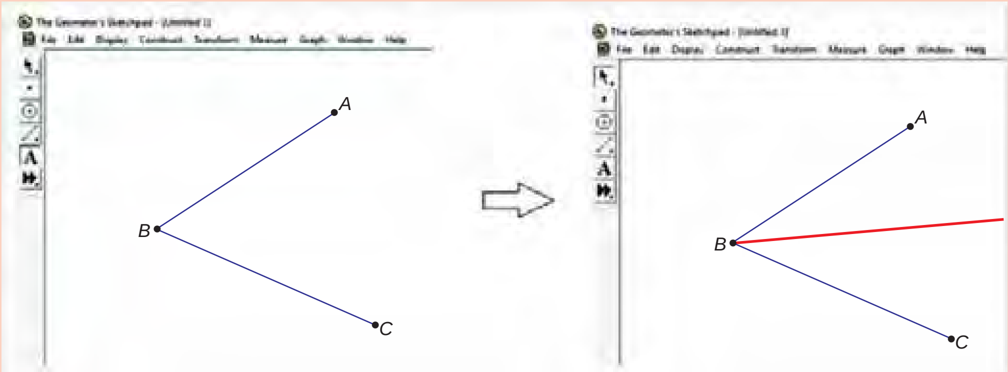


Diagram 1 Diagram 2

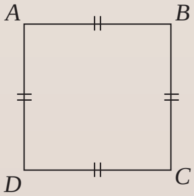
Discussion:

What is your conclusion about the locus of points that are equidistant from two intersecting lines?

From Brainstorming 9, it is found that:

The locus of a point that is equidistant from the two straight lines AB and BC intersecting at the point B is a straight line that bisects $\angle ABC$.

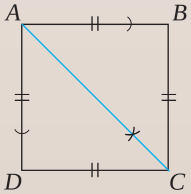
Example 10



The diagram on the right shows a square $ABCD$. Describe and draw the locus of a point which moves at the same distance from the straight lines AB and AD.

Solution:

The locus of a point which moves at the same distance from the line AB and AD is a straight line which bisects the angle BAD.



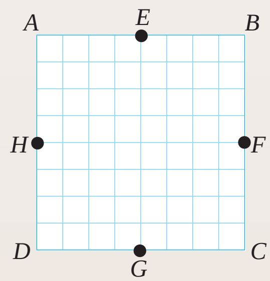
MIND TEST 8.2a

1. The diagram shows a straight line PQ of 5 cm.

(a) X is a point that is always 3 cm from point P. Describe the locus of point X completely.

(b) Y is a point that is always 4 cm from point Q. Describe the locus of point Y completely.

2. The diagram on the right shows a square $ABCD$ drawn on a square grid with sides of 1 unit. P, Q, R, S and T are five points that move in the square $ABCD$. Using the letters in the diagram, state the locus of point



(a) P moving equidistantly from points A and D

(b) Q moving equidistantly from points B and D

(c) R moving such that it is always 4 units from the line BC

(d) S moving equidistantly from the straight lines AB and BC

(e) T moving such that it is always 4 units from the line EG

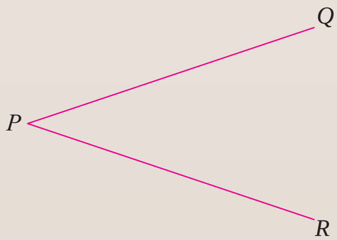
3. The diagram on the right shows the straight line, CD which is 6 cm long. T is a point that is always 1.5 cm from the straight line, CD.

(a) Draw the locus of point T.

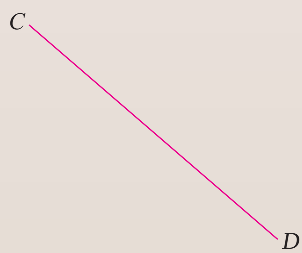
(b) Describe completely, the locus of point T.

4. Construct the locus of point Y for a given situation.

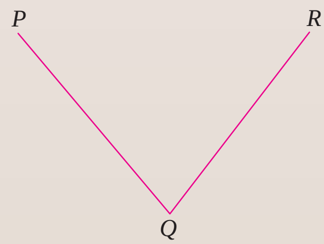
(a) It is equidistant from the straight lines PQ and PR.



(b) $YC = YD$



(c) $\angle PQY = \angle RQY$



5. The picture on the right shows a running track. An athlete always practises by running two lanes away from lane 4 of the track. Draw the locus of the athlete's run.

How do you determine the locus that satisfies two or more conditions?

The intersection of two or more loci can be determined by constructing each specified locus in the same diagram.

LEARNING STANDARD

Determine the locus that satisfies two or more conditions.

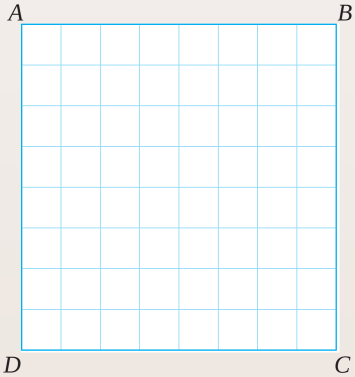
Example 11

The grid on the right shows a square *ABCD* drawn on a square grid with sides of 1 unit. Points *X* and *Y* are two points that move inside the square *ABCD*. On the grid,

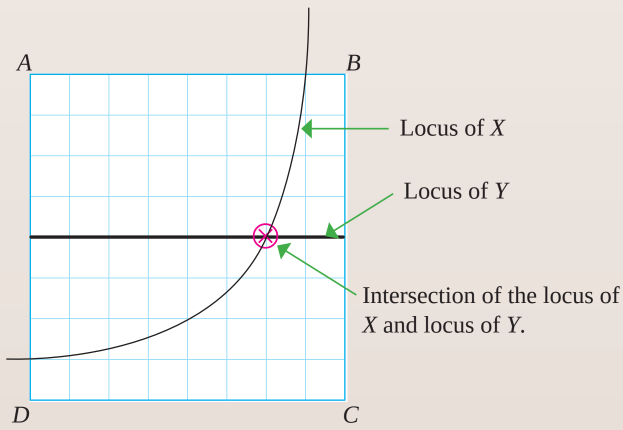
(a) draw the locus of a moving point *X* which is constantly 7 units from *A*

(b) draw the locus of a moving point *Y* which is equidistant from the lines *AB* and *CD*

(c) mark all points of intersection of locus of *X* and locus of *Y* with the symbol ⊗



Solution:



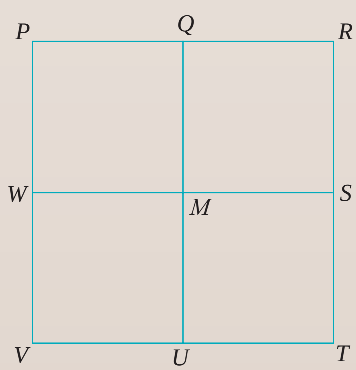
Example 12

The diagram on the right shows four combined squares with sides of 2 cm. *X* and *Y* are two moving points inside the square *PRTV*. On the diagram,

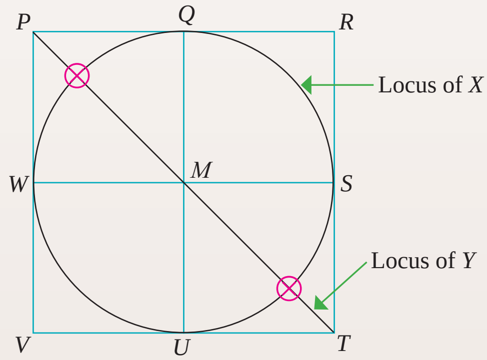
(a) draw the locus of a moving point *X* which is always 2 cm from point *M*

(b) draw the locus of a moving point *Y* which is equidistant from line *PR* and line *PV*

(c) mark all points of intersection of locus of *X* and locus of *Y* with the symbol ⊗



Solution:



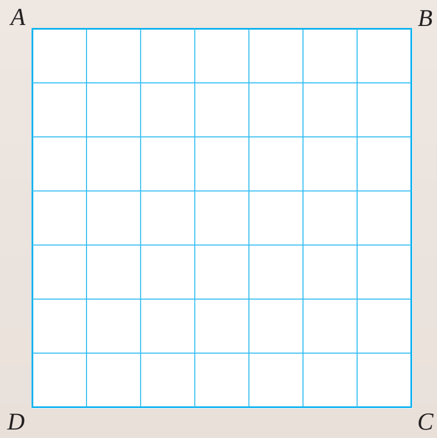
MIND TEST 8.2b

1. In the grid on the right, the rectangle *ABCD* represents a part of a lake. *ABCD* is drawn on a square grid with sides of 1 unit. Points *V* and *W* represent the trips of boat *V* and boat *W*. On the grid,

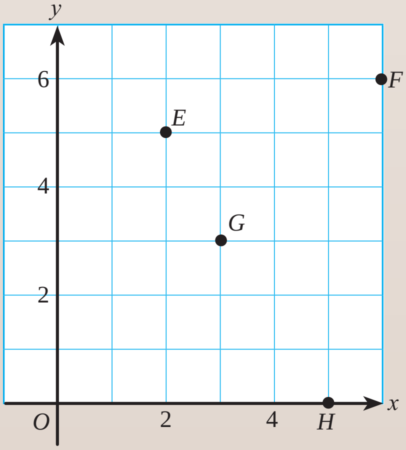
(a) draw the locus of boat *V* which always moves 5 units from point *D*

(b) draw the locus of boat *W* which is 3 units from line *BC*

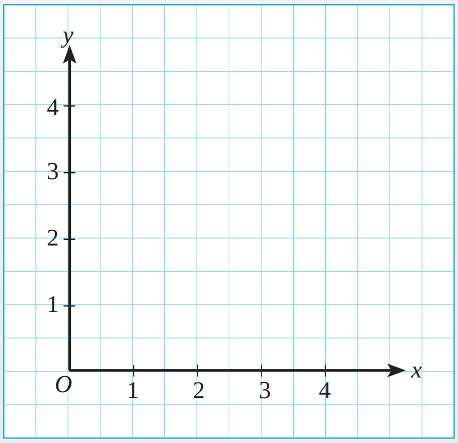
(c) mark the intersection of the paths of boat *V* and boat *W* with the symbol ⊗



2. The diagram on the right shows the Cartesian plane marked with four points *E*, *F*, *G* and *H*. Faruk is at the same distance from *x*-axis and *y*-axis. Faruk's location is also less than 5 units from the centre of *O*. Which of the points *E*, *F*, *G* and *H* is Faruk's location?



3. The diagram on the right shows the Cartesian plane. Point F always moves 3 units from the x-axis while point G always moves 4 units from the origin. Mark all the points of intersection between the locus of F and the locus of G with the symbol ⊗.

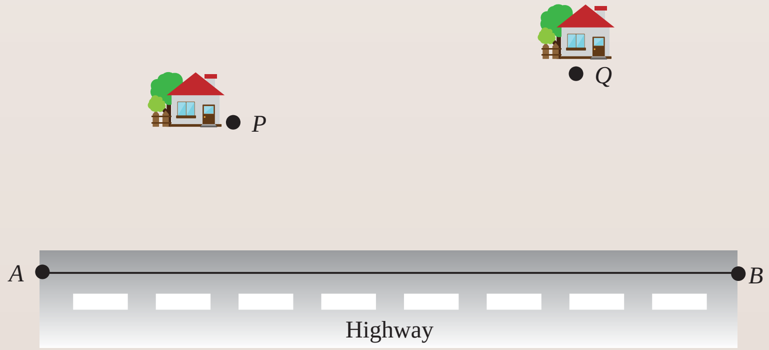


How do you solve problems involving loci?

Example 13

A clinic will be built in a village. The clinic should be equidistant from house P and house Q, as well as 600 metres away from the highway AB. Determine the possible location of the clinic.
(scale 1 cm = 600 metres)

LEARNING STANDARD
Solve problems involving loci.



Solution:

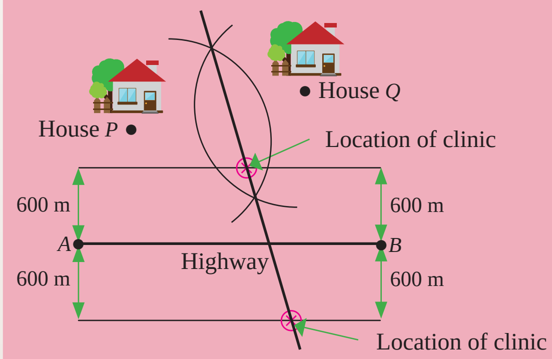
Understanding the problem

The clinic is equidistant from P and Q. Therefore the locus is the bisector of the straight line connecting points P and Q. The clinic is 600 metres from the highway AB. There are two lines parallel to the highway AB.

Planning a strategy

To draw using a pair of compasses and a ruler.

Implementing the strategy



Making a conclusion

Two locations that are marked with the symbol ⊗ satisfy the requirements of the location to build the clinic.

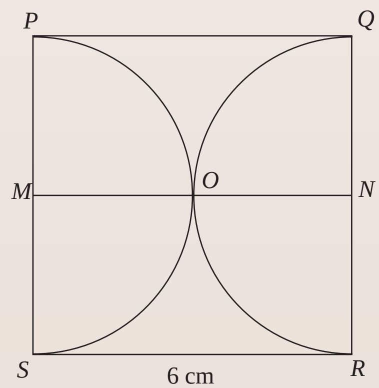
MIND TEST 8.2c

1. The diagram on the right shows square *PQRS* with sides of 6 cm. Two semicircles with centres *M* and *N* are drawn inside square *PQRS*. *M* and *N* are the midpoints of *PS* and *QR*. On the diagram, shade the region that satisfies the following conditions.

(a) The locus of point *X* which moves such that $XM \leqslant 3$ cm and more than 3 cm from line *SR*.

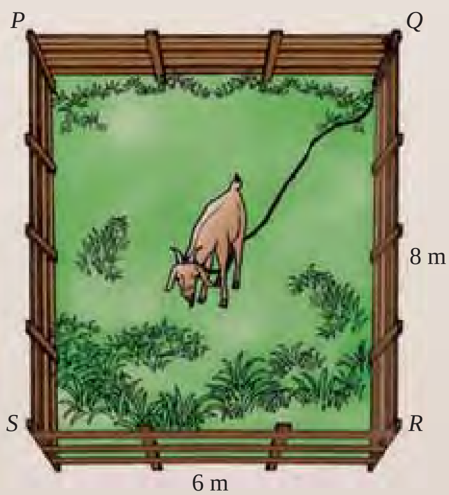
(b) The locus of *Y* which moves such that $YM \geqslant 3$ cm and $YN \geqslant 3$cm.

(c) Describe the intersection between locus of *X* and locus of *Y*.



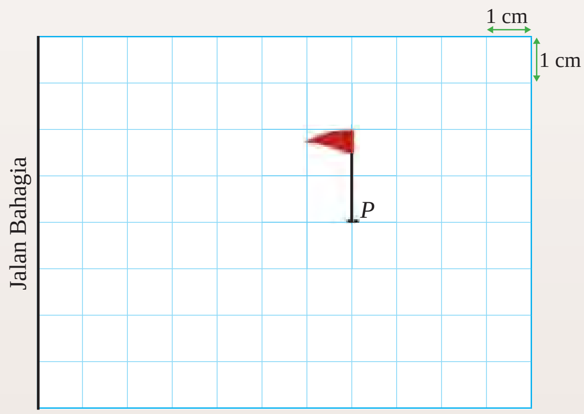
2. The diagram on the right shows a rectangular fenced-up grass field *PQRS* measuring 6 m × 8 m. A goat is tied at point *Q* with a 7-metre long rope.

Shade the region that is reachable by the goat.



3. Khalid draws the plan for a treasure hunt on a square grid with a scale of 1 cm to 1 metre.
On the diagram, draw
(a) the location of the treasure if it is 3 m away from the flagpole *P*
(b) the location of the treasure if it is 5 m from Jalan Bahagia

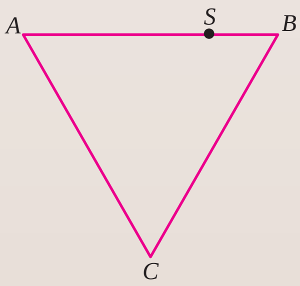
Then, mark the possible locations of the treasure with the symbol ⊗.



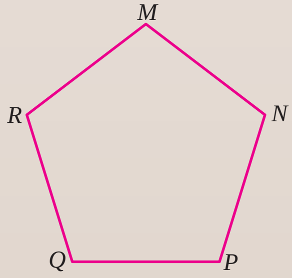
Dynamic Challenge

Test Yourself

1. The diagram below shows an equilateral triangle *ABC*. *S* is a point on line *AB*. *X* and *Y* are two moving points in the diagram. On the diagram,
(a) draw the locus of point *X* such that $AX = AS$
(b) draw the locus of point *Y* such that *Y* is equidistant from *AC* and *BC*
(c) mark all the intersection points for locus of *X* and locus of *Y* with the symbol ⊗



2. The diagram below shows a regular pentagon *MNPQR*. *X* and *Y* are two moving points inside the pentagon. On the diagram,
(a) draw the locus of point *X* such that $RX = XN$
(b) draw the locus of point *Y* such that $RY = RQ$
(c) mark all the intersection points for locus of *X* and locus of *Y* with the symbol ⊗

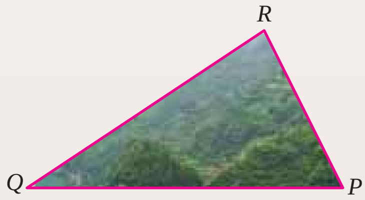


3. The picture below shows the triangular forest area *PQR*. Point *X* and point *Y* are two loci that describe the location of a helicopter that crashed. On the diagram,

(a) draw the locus of point *X* such that it is equidistant from lines *QR* and *QP*

(b) draw the locus of point *Y* such that *YP* = *PR*

(c) mark the possible location for the helicopter with the symbol ⊗



Skills Enhancement

1. The diagram on the right is drawn on a square grid with sides of 1 unit. Point *X*, point *Y* and point *Z* are three points which move in the square.

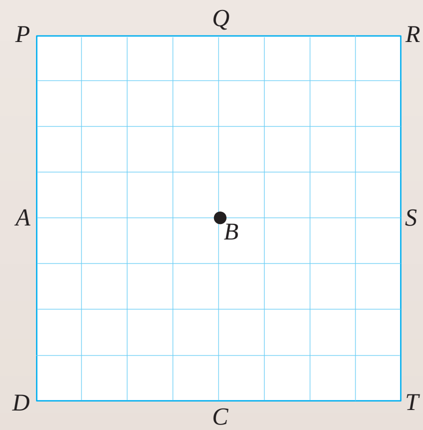
(a) *X* is a point which moves such that it is equidistant from points *Q* and *C*. Using the letters in the diagram, state the locus of point *X*.

(b) On the diagram,

(i) draw the locus of point *Y* which moves such that it is equidistant from the straight lines *PD* and *DT*

(ii) draw the locus of point *Z* which moves such that it is always 5 units from point *S*

(c) Mark the location of all the intersection points for locus of *Y* and locus of *Z* with the symbol ⊗.

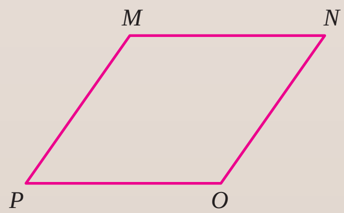


2. The diagram below shows a rhombus *MNOP*. Point *X* and point *Y* are two points that move within the rhombus. On the diagram,

(a) draw the locus of point *X* which moves equidistantly from the straight lines *PM* and *PO*

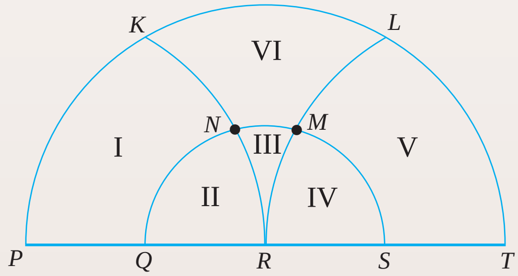
(b) draw the locus of point *Y* which moves such that *YP* = *PO*

(c) mark the location of all the intersection points for locus of *X* and locus of *Y* with the symbol ⊗



Self Mastery

1. The diagram below shows two semicircles, *PKLT* and *QNMS* centred at *R*, with diameters of 8 cm and 4 cm respectively. *KNR* and *RML* are arcs of circles centred at *P* and *T* respectively.



Based on the diagram above, state

(a) the point which is 2 cm from *R* and 4 cm from *P*

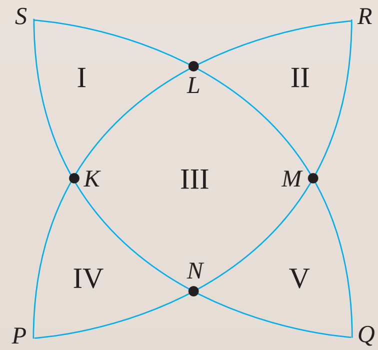
(b) the point which is more than 2 cm from *R* and 4 cm from *T*

(c) the location of a moving point *X* in the diagram such that it is less than 4 cm from *P* and more than 2 cm from *R*

(d) the location of a moving point *Y* in the diagram such that $YR < 2$ cm and $YP < 4$ cm.

(e) the location of a moving point *Z* in the diagram such that $ZT > 4$ cm, $ZP > 4$ cm and $ZR > 2$ cm

2. In the diagram below, *SLMQ*, *PKLR*, *QNKS* and *RMNP* are arcs of circles with radii of 4 cm and centred at *P*, *Q*, *R* and *S* respectively.



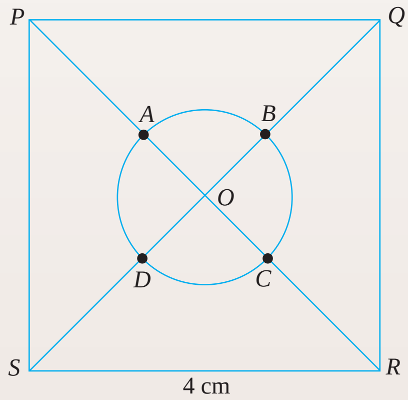
Based on the diagram above, state

(a) the location of a moving point *X* in the diagram such that $XS < 4$ cm, $XP < 4$ cm and $XQ > 4$ cm

(b) the location of a moving point *Y* in the diagram such that $YR > YP$

(c) the location of a moving point *Z* in the diagram such that $ZP < 4$ cm, $ZQ < 4$ cm, $ZR < 4$ cm and $ZS < 4$ cm

3. The diagram below shows a square *PQRS* with sides of 4 cm and a circle centred at *O* with radius of 1 cm. Point *X* and point *Y* are two points that always move inside the square *PQRS*.



Describe the possible movement of the loci of point *X* and point *Y* for the following points of intersection:

(a) *B* and *D*

(b) *A* and *C*

PROJECT

When we look at a clock to tell the time, we can see that the tip of the hour hand always moves with the same pattern, that is always equidistant from the centre of the clock.

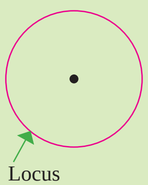
Why is the shape of a circle selected to represent the movement of time on a clock? Gather information about the relationship between hours, minutes and seconds and the shape of a circle. Create a report with illustrations using multimedia applications.

CONCEPT MAP

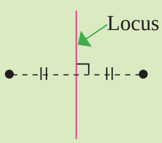
Loci in Two Dimensions

A set of points whose locations satisfy certain conditions.

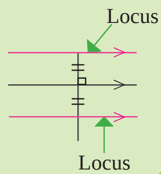
- The locus of points that are of constant distance from a fixed point. → **Circle**



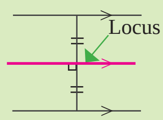
- The locus of points that are equidistant from two fixed points. → **Perpendicular bisector**



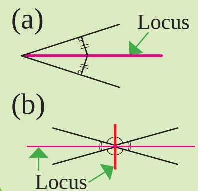
- The locus of points that are of constant distance from a straight line. → **A pair of parallel lines**



- The locus of points that are equidistant from two parallel lines. → **A straight line**



- The locus of points that are equidistant from two intersecting lines. → **Angle bisector**



SELF-REFLECT

At the end of this chapter, I can:

1.	Recognise loci in real life situations and hence explain the meaning of locus.		
2.	Describe the locus of points that are of constant distance from a fixed point.		
3.	Describe the locus of points that are equidistant from two fixed points.		
4.	Describe the locus of points that are of constant distance from a straight line.		
5.	Describe the locus of points that are equidistant from two parallel lines.		
6.	Describe the locus of points that are equidistant from two intersecting lines.		
7.	Determine the locus that satisfies two or more conditions.		
8.	Solve problems involving loci.		

EXPLORING MATHEMATICS

We can sketch an ellipse using the following steps:

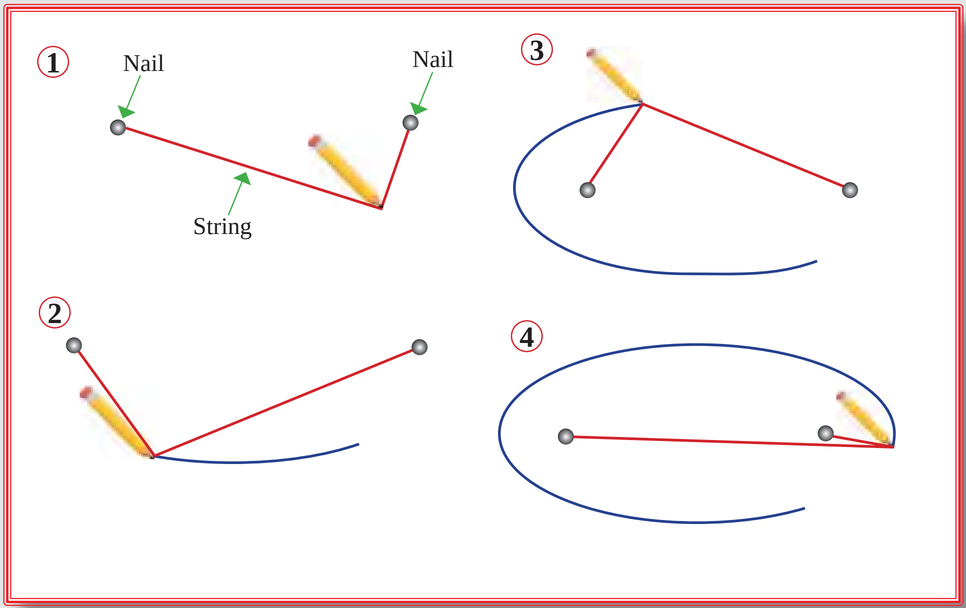
1. Tie two nails with a string (one nail on each end of the string).

2. Place a sheet of paper on a flat piece of board.

3. Fix the two nails onto the piece of board but do not pull the string too tightly. The two nails are called the foci.

4. We can begin to sketch an elliptical shape by using the tip of a pencil to pull the string tightly and draw on the paper, first from one nail to the second nail, and then from the second nail back to the first nail. The curve drawn by the pencil is an ellipse.

5. Observe the elliptical shape formed.



SMART MIND

Why is an ellipse also known as a locus?