# CHAPTER 8

# Mathematical Modeling

## What will you learn?

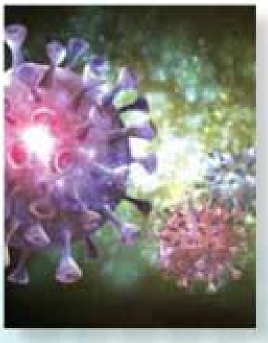
- Mathematical Modeling

## Why study this chapter?

Mathematical modeling is used in various disciplines in the real world. Engineers use mathematical modeling to analyse the traffic flow on a bridge. Telecommunication companies use mathematical modeling to determine the charges for the calls that consumers make. Scientists also use mathematical modeling to predict the trends in population growth and the spread of infectious diseases to ensure the well-being of human race.

## Do you know?

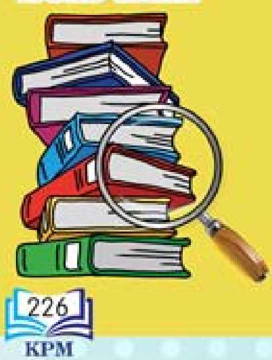
The Covid-19 pandemic has hit the world in 2019. In this regard, the Malaysian government implemented the Movement Control Order (MCO) in an effort to flatten the epidemiological curve of Covid-19 cases in our country. Mathematical modeling can be used to make epidemic predictions.



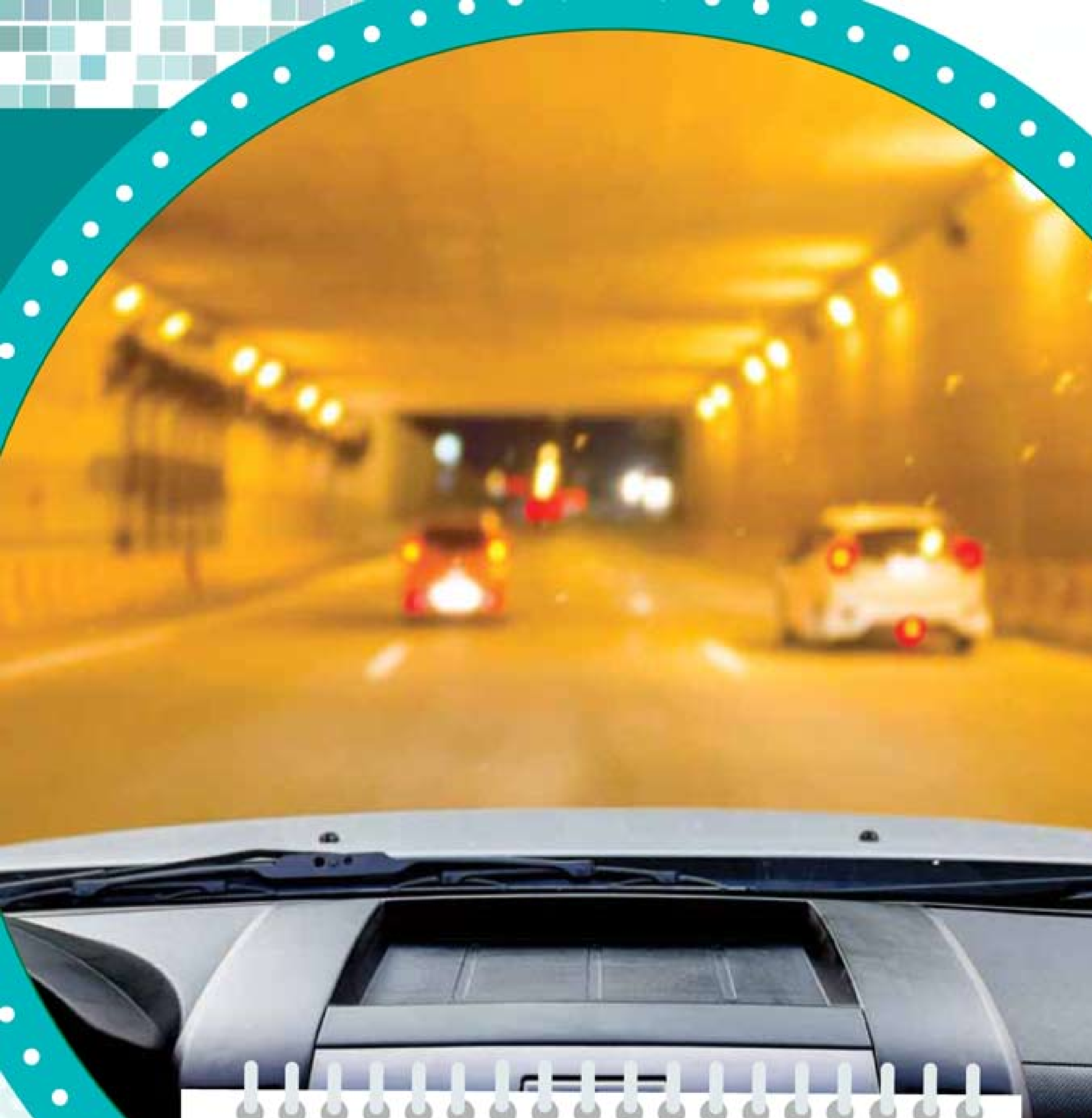
**For more information:**

bit.do/DoYouKnowChap8

## WORD BANK



| | |
|---|---|
| exponent | *eksponen* |
| exponential function | *fungsi eksponen* |
| quadratic | *kuadratik* |
| linear | *linear* |
| mathematical modeling | *pemodelan matematik* |



The construction of the SMART (Stormwater Management and Road Tunnel) tunnel aims to drain flood water during heavy rain in Kuala Lumpur. However, the tunnel also has an additional function in reducing traffic congestion. The unique design and innovative way of operating the tunnel presented challenges to the engineers at the beginning of its construction. Do you know that mathematical modeling was used to demonstrate the four different operation modes of the SMART tunnel?

## 8.1 Mathematical Modeling

### What is mathematical modeling?

**Learning Standard**

Explain mathematical modeling.

The world around us is filled with various important questions that are not answered. For example,

What is the effect of rising sea levels on the coastal regions of Malaysia?

How much will it cost to go to college or university in 10 years?

Will the population in Malaysia surpass 40 million?

Is it possible to identify the personality trait of cybercrime victims by studying the behaviours of the web surfers?

Answers to these questions are constantly being studied by researchers. Will they be able to find the answers? Maybe. The only thing one can say with certainty is that any attempt to find the solution requires the use of mathematics, most likely through the creation, application and refinement of **mathematical models**.

A mathematical model is a mathematical relation that describes the real-world situation. For example, the formula $I = Prt$ is a relation between simple interest and the other three quantities, that is, the principal, the interest rate and the time.

In this chapter, you will be introduced to the process of constructing mathematical models, which is called **mathematical modeling**. In mathematical modeling, a real-world problem will be translated into a mathematical problem. We then solve the mathematical problem and interpret its solution in terms of the real-world problem.

A **mathematical model** is a representation of a system or scenario that is used to gain qualitative and/or quantitative understanding of some real-world problems and to predict future behaviour.

Consider the following problems.

**1** The population of town $A$ is 40 000. It is known that 45% of the residents in the town practise the habit of recycling used plastic drinking bottles. If each person in the town uses 5 plastic drinking bottles per week, how many bottles are recycled each week in the town?

This type of question is often asked in Mathematics books to reinforce the concept of percentage. This is an example of a problem solving question. The problem explicitly gives us all the information we need. We only need to determine the appropriate computations to arrive at one correct answer. This kind of problem solving question can be used to help us understand a particular mathematical concept and reinforce important mathematical skills.

Let us look at the following question.

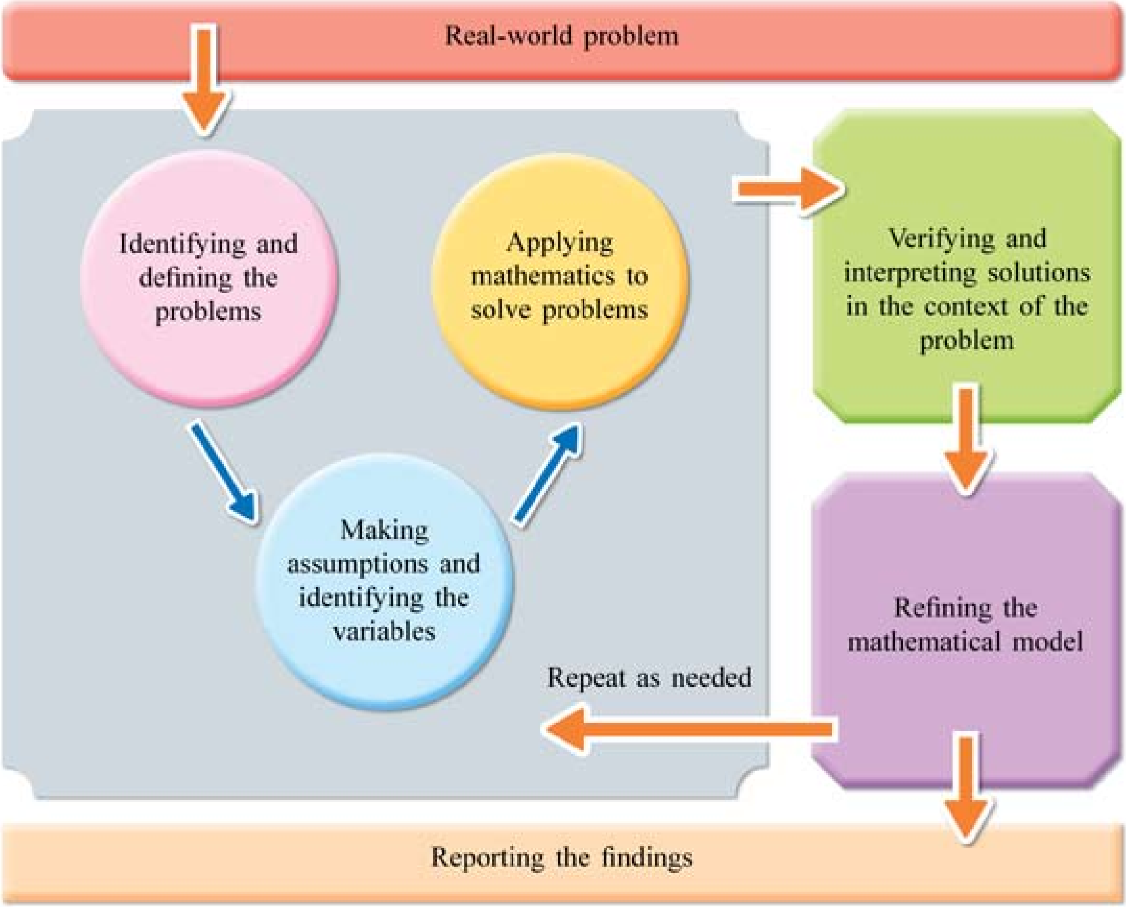
**2** How long does it take to clean up an oil spill in the ocean?

The second question is quite different. We do not have enough information to answer this question. It is an open-ended question. We usually do not have complete information when trying to solve real-world problems. These real-world problems demand us to use both mathematics knowledge and creativity to solve them. In this case, mathematical modeling is perfect for solving such open-ended questions.

There are a few important components in mathematical modeling:

1. Identifying and defining the problems
2. Making assumptions and identifying the variables
3. Applying mathematics to solve problems
4. Verifying and interpreting solutions in the context of the problem
5. Refining the mathematical model
6. Reporting the findings

The modeling process can be simplified as shown in the diagram below.




Let us look at one mathematical modeling example that involves all the above components.

## Real-world Problem

The grocery store which is far from your house sells a pack of 10 kg rice at a lower price than the one closer to your house. Is it worth the drive for the cheaper deal?

### Identifying and defining the problems

- The price of a pack of 10 kg rice and the cost of petrol.
- Determine the distance from the two stores to your house respectively.
- Find the information needed from the internet, for example the price of a pack of 10 kg rice from each store, the distance of each store from your house, the current petrol price, the car's petrol consumption rate and so on.

### Making assumptions

- The route between the house, store A and store B is a perfect straight line
- Driving car to the stores
- Purchasing the same number and brand of rice at store A and store B


**Smart TiPS**

The assumptions made usually start with the simplest variables. Once the problem is solved, more complex assumptions can be considered.

### Identifying the variables

- Let
  - $s$ = the distance between store A and store B
  - $P_1$ = the price of a pack of 10 kg rice at store A
  - $P_2$ = the price of a pack of 10 kg rice at store B
  - $m$ = the petrol consumption rate of the car in km per litre
  - $n$ = the number of packs of 10 kg rice to be purchased
  - $H$ = the current petrol price in RM per litre
  - $S$ = the difference in price in RM for purchasing the pack of 10 kg rice at store B as compared to store A
  - $T$ = the difference in the cost of petrol in RM for driving to store B as compared to store A

**MEMORY BOX**

A variable is a quantity with an unknown value.

### Applying mathematics to solve problems

**The price of a pack of 10 kg rice**

Store A: RM25.95
Store B: RM23.99
Let the quantity of 10 kg rice to be purchased be 2 packs.

**The price to be paid**

Store A: 2 × RM25.95 = RM51.90
Store B: 2 × RM23.99 = RM47.98

Therefore, by purchasing the rice at store B, one can save
RM51.90 – RM47.98 = RM3.92

Let the distance between store A and store B be 6 km,
the petrol consumption rate of a car be approximately 17.6 km per litre,
the current petrol price be RM2.08 per litre.

Petrol needed for the 6 km route = $\frac{6 \text{ km}}{17.6 \text{ km/litre}}$ = 0.341 litre

Cost of petrol for the 12 km (two ways) route = 0.341 litre × $\frac{\text{RM2.08}}{\text{litre}}$ × 2 = RM1.42

Therefore, we save RM3.92 – RM1.42 = RM2.50 by purchasing two packs of 10 kg rice at store B.

## Verifying and interpreting solutions in the context of the problem

The resulting mathematical model is as follows.

$S = (P_1 - P_2) \times n$

$T = \frac{s}{m} \times H \times 2$

If $S > T$, then it is sensible to drive to store B to purchase rice and save more money.
If $S \leqslant T$, then we should not drive to store B to purchase rice.

Example of using the model:

$$\begin{aligned} S &= (P_1 - P_2) \times n \\ &= (25.95 - 23.99) \times 2 \\ &= \text{RM}3.92 \end{aligned}$$

$$\begin{aligned} T &= \frac{s}{m} \times H \times 2 \\ &= \frac{6}{17.6} \times 2.08 \times 2 \\ &= \text{RM}1.42 \end{aligned}$$

When verifying and interpreting solutions, we consider:
- Are the solutions able to solve the problem?
- Is it rational to translate the solutions to real-world situation?

Since $S > T$, you save more money by purchasing rice at store B. Hence it is sensible to drive to store B. Thus, the mathematical model is able to solve the problem.

From the constructed mathematical model, further exploration can be made.

1. If all the variables remain the same, how far can store B be located so that it is an optimum choice?

$$(P_1 - P_2) \times n > \frac{s}{m} \times H \times 2$$

$$(25.95 - 23.99) \times 2 > \frac{s}{17.6} \times 2.08 \times 2$$

$$s < 16.6 \text{ km}$$

This means that if the distance between the two stores is less than 16.6 km, then it is **sensible to drive to store B** to purchase rice.

**Info Bulletin**

Optimum means the best or the most profitable.

2. What is the maximum price of a pack of 10 kg rice at store B that will attract a person drive to store B to purchase it?

$$(P_1 - P_2) \times n > \frac{s}{m} \times H \times 2$$

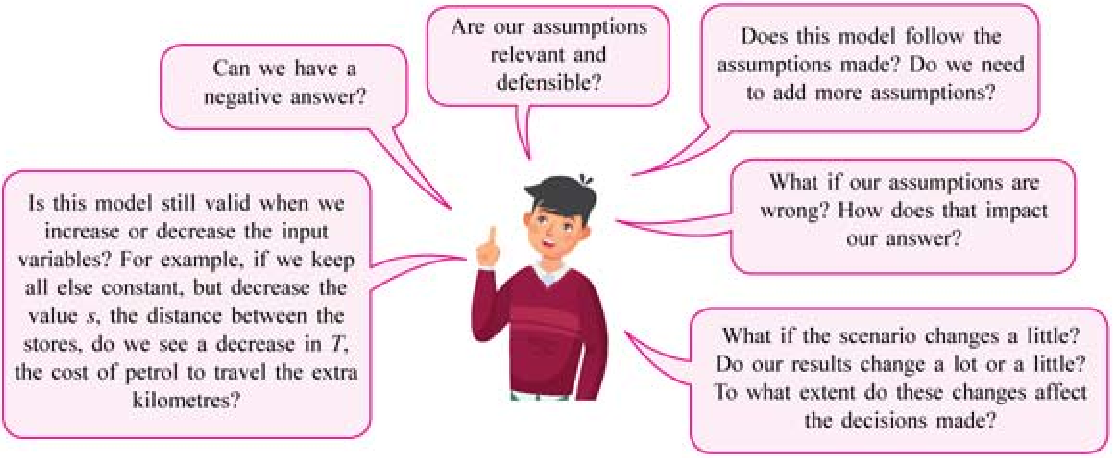
$$(25.95 - P_2) \times 2 > \frac{6}{17.6} \times 2.08 \times 2$$

$$P_2 < \text{RM}25.24$$

This means that if the price of a pack of 10 kg rice at store B is less than RM25.24, then it is worth the trip.

Therefore, the above mathematical model allows us to not only answer the question of "is it worth it" for the given situation, but also allows us to determine other factors that support or oppose the decision to drive further to purchase the rice.

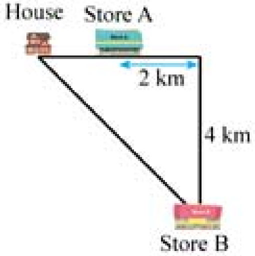
We now reflect on the model and the questions that arise as follows.




These reflections enable us to think of the needs to refine the mathematical model.

## Refining the mathematical model

The assumption made regarding the route between the house, store A and store B is a straight line needs to be revised. If the route between the three places is not a straight line, how does this impact our model? If new assumptions are being made, then the model needs to be revised to reflect this change.




## Reporting the findings

Use symbols and diagrams when necessary to report the findings. The symbols and diagrams will depict the whole modeling process until it leads to results. Every model has strengths and weaknesses. What is important is that the model identifies those strengths and weaknesses in the report.

For example,

- the value of a person's time is not considered in this model. Is it worth driving an extra 12 km for someone to save RM2.50? According to this model, saving 5 sen is considered worthwhile.
- the model did not account for any environmental considerations. Is it environmentally responsible to drive an extra 12 km just to save RM2.50? This model did not account for environmental effort such as efforts to reduce carbon emissions.

We have now seen how the mathematical modeling process works by one example as shown above. We should emphasise that we cannot use this example as a template to solve other problems. In solving problems through mathematical modeling, every individual or every group has different ideas, skills and perceptions. Open-ended questions such as these may even generate completely different mathematical models which are valid yet result in different answers.

**Example 1**

Jeremy saves RM4 000 at Bank Bunga Raya with a simple interest rate of 1.5% per annum. Jeremy wants to buy a computer worth RM4 455 with his savings. How long does Jeremy need to save?

(a) Identify and define the problem.

(b) Determine the assumptions that need to be made and identify the variables in solving the problem.

**MEMORY BOX**

Simple interest is a reward given to the depositor at a certain rate on the principal amount for a certain period of time. Simple interest can be calculated using the formula $I = Prt$ where $I$ is the interest, $P$ is the principal, $r$ is the interest rate and $t$ is the time in years.

**Solution:**

(a) In this problem, we know the principal and the interest rate. The interest from the savings is the amount that Jeremy needs besides the RM4 000 to buy the computer. We need to determine how long Jeremy needs to keep his savings in the bank.

(b) **Assumptions:**

- We need to assume that the interest rate does not change during the period of our interest calculation. Otherwise, the formula $I = Prt$ will not be appropriate.
- We also need to assume that the price of the computer does not change when Jeremy has raised the required amount of money.

**Variables:**

The variables involved are $I$ for interest, $P$ for principal, $r$ for interest rate and $t$ for time in years.

**Self Practice 8.1a**

1. The journey of a boat moving upstream which covers two places located on the riverbank takes 6 hours. The return journey of the boat moving downstream takes 5 hours. If the speed of the river current is 2 km $h^{-1}$, what is the speed of the boat on calm water?
   (a) Identify and define the problem.
   (b) Determine the assumptions that need to be made and identify the variables in solving the problem.

2. In a group, read the question below and find information on the internet.

   > You have just completed a degree in marketing and received offers to become a marketing executive by two companies. Company A is in your hometown and offers a lower starting salary and a lower annual salary increment than company B that is in a big city. Which is the better choice?

   Explain how you use the mathematical modeling process to solve the above problem.

## How to solve real life problems through mathematical modeling?

**Learning Standard**

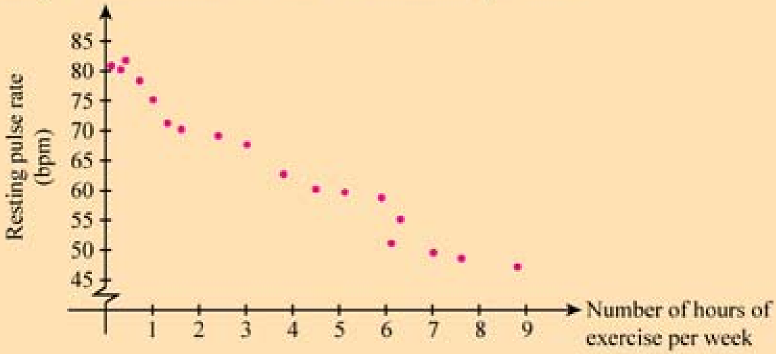
Solve real life problems through mathematical modeling which involves the following functions:
(i) Linear
(ii) Quadratic
(iii) Exponential
and communicate the mathematical modeling process implemented.

**MIND MOBILISATION 1** Pairs

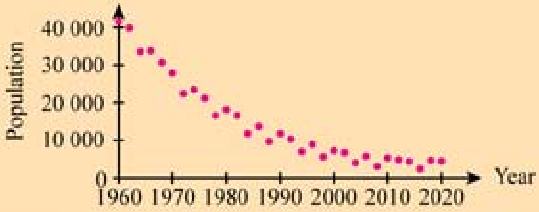
**Aim**: To apply mathematics in problem solving.

**Steps**:

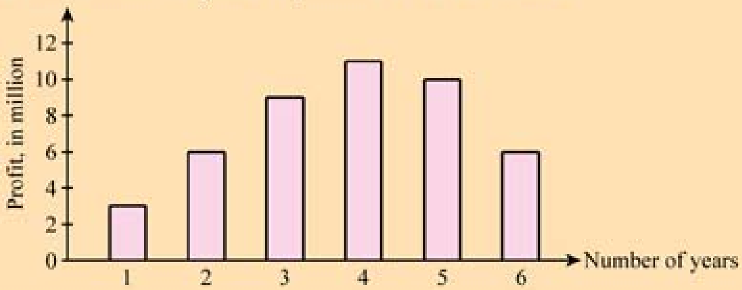
1. Based on the information and diagrams given, determine the type of function (linear, quadratic or exponential) that may be used as a model for the data.
2. Discuss your selection.
   (a) A doctor uses the data to study the resting pulse rate of a normal adult according to the number of hours of exercise per week.




   (b) A scientist uses the data to study the population of tigers.




   (c) A marketing manager studies the data which shows the relationship between the profit and the number of years a product is in the market.

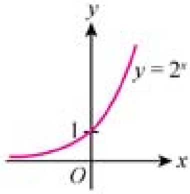



The results of Mind Mobilisation 1 show that we can determine the types of functions that may be used as a model for data by **studying graph patterns**.

Linear function: A graph resembles a straight line.

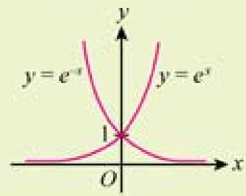
Quadratic function: A curved graph in the form of a parabola. The curve may rise and then fall or fall and then rise.

Exponential function: A curved graph illustrating an increase or decrease in values of data at a sharp rate. The exponential function can be written in the form of equation $y = Ca^x$, where $a$ is a positive real number, $a \neq 1$ and $C$ is an initial value. For example, if $C = 1$ and $a = 2$, graph $y = 2^x$ is shown as the diagram.




**Info Bulletin**

The diagram below shows the graphs of two exponential functions with base $e$ where $e$ = 2.7182818284




**Example 2**

Amin travelled 405 km on 45 litres of petrol in his car. If Amin wishes to go to a place which is 198 km away by car, how much petrol, in litres, does he need? Solve this problem through mathematical modeling.

**Solution:**

**Identifying and defining the problem**

- Determine the volume of petrol required for 198 km.
- We know that the farther we travel, the more petrol we require. Thus, the amount of petrol varies directly with the distance we travelled.

**Making assumptions and identifying the variables**

- Assume that the driving speed for both 405 km and 198 km routes are the same
- Let $x$ represents the distance travelled and $y$ represents the amount of petrol required
- $y$ varies directly with $x$, hence $y = kx$ where $k$ is a constant

**Applying mathematics to solve problems**

Substitute $y = 45$ and $x = 405$ into $y = kx$,

$$45 = k(405)$$

$$k = \frac{45}{405} = \frac{1}{9}$$

Therefore, $y = \frac{1}{9}x$

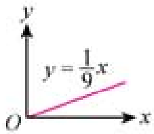
This equation describes the relationship between the amount of petrol required and distance travelled.

When $x = 198$, $y = \frac{1}{9}(198)$

$$= 22 \text{ litres}$$

Hence, 22 litres of petrol is required to travel 198 km.

### Verifying and interpreting solutions in the context of the problem




We may not be able to use the linear function model $y = \frac{1}{9}x$ in all situations. For example, if the 405 km route is through towns and cities, and the 198 km route is through highway. Thus, the car will use up petrol at a faster rate in the first route compared to the second route. When this is translated to the real-world situation, the linear function model obtained is not suitable to solve this problem.

### Refining the mathematical model

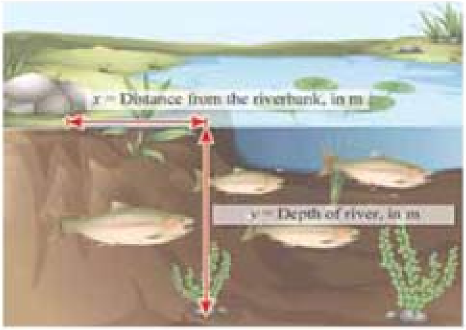
In this problem, we are not able to refine the model due to limited information given.

### Reporting the findings

Report the findings of the problem solving based on the interpretation of solutions as shown in the preceding sections.

**Example 3**

The diagram below shows the cross section of a river. A hydrologist measures the depth of the river, $y$ m, at different distance, $x$ m from the riverbank. The results obtained are given in the following table.




| Distance from the riverbank, $x$ m | Depth of the river, $y$ m |
|---|---|
| 0 | 0 |
| 4 | 1.5 |
| 8 | 2.3 |
| 12 | 2.9 |
| 18 | 2.9 |
| 25 | 1.7 |
| 30 | 0 |

Show how the hydrologist uses the data above to determine the depth of the river through mathematical modeling.

**Solution:**

### Identifying and defining the problem

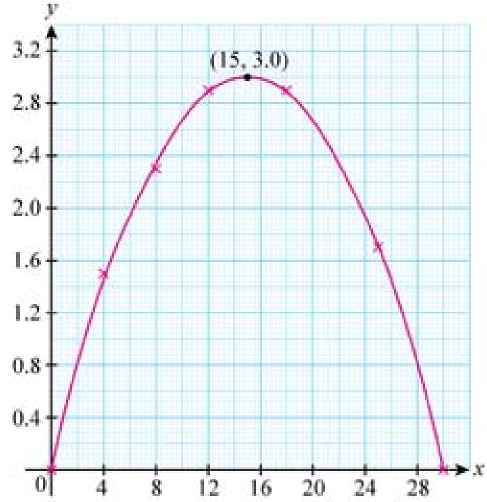
How to determine the depth of the river?


A hydrologist is a scientist who researches the circulation, distribution and quality of water in an environment.

### Making assumptions and identifying the variables

- Assume the river is the deepest in the middle with the depth decreasing to 0 at the edges.
- The two variables involved in this study are the depth of the river, $y$ m, and the distance from the riverbank, $x$ m.

## Applying mathematics to solve problems




- Write the distance from the riverbank and the depth of the river as a set of ordered pairs $(x, y)$ and draw a graph for the data.
- The data seem to rise and fall in a manner similar to a quadratic function.
- The graph drawn shows the curve of best fit and **resembles the graph of a quadratic function**.
- In mathematical modeling to represent the actual situation, the approximate value is used.
- Based on the graph, the depth of the river is 3 m when the distance from the riverbank is 15 m (approximate).

## Verifying and interpreting solutions in the context of the problem

Determine the related quadratic function of the form $y = ax^2 + bx + c$. Determine the constants $a$, $b$ and $c$ by substituting any three data, for example (0, 0), (25, 1.7) and (30, 0) into the equation.

$$0 = a(0)^2 + b(0) + c \qquad\qquad 0 = c$$
$$1.7 = a(25)^2 + b(25) + c \qquad\qquad 1.7 = 625a + 25b + c$$
$$0 = a(30)^2 + b(30) + c \qquad\qquad 0 = 900a + 30b + c$$

Since $c = 0$, the system of two linear equations in two variables is:

$1.7 = 625a + 25b$ ........................ ①

$0 = 900a + 30b$ ........................ ②

From ②, $b = -30a$ ........................ ③

Substitute ③ into ①, $1.7 = 625a + 25(-30a)$

$1.7 = -125a$

$a = -0.0136$

Substitute $a = -0.0136$ into ③, $b = -30(-0.0136)$

$= 0.408$

Hence, the possible quadratic function is $y = -0.0136x^2 + 0.408x$

Substitute $x = 15$, $y = -0.0136(15)^2 + 0.408(15)$

$= 3.06$ (approximate to the answer obtained from the graph)

**Info Bulletin**

A system of linear equations refer to two or more linear equations involving the same set of variables.

## Refining the mathematical model

- For this model, we assume that the river is the deepest in the middle. This may not be true for some other rivers. A new model will be needed if we have new assumptions.
- The answer will be more accurate if more data have been collected.

## Reporting the findings

Write a full report following the above modeling framework structure.

**Example 4**

Compound interest is interest that is calculated based on the original principal and also the accumulated interest from the previous period of savings. At the beginning of a year, Mr Gomez saves RM20 000 in his savings account with an interest rate of 4% per annum and the interest is compounded once a year. Derive a mathematical model for Mr Gomez's total savings after $t$ years of saving. Solve this problem through mathematical modeling.

**Solution:**

**Identifying and defining the problem**

- Mr Gomez is given compound interest which is compounded once a year.
- His principal is RM20 000.
- The yearly interest rate is 4%.
- Derive a mathematical model for Mr Gomez's total savings at the end of $t$ years.

**Making assumptions and identifying the variables**

- Assume Mr Gomez did not withdraw or raise his savings throughout the period of saving.
- The variables are principal amount, RM$P$, yearly interest rate, $r$, number of times the interest is compounded, $n$, and time, $t$ years.

**Applying mathematics to solve problems**

Keep in mind that when calculating the compound interest, each year we have 100% of the principal, plus 4% of the previous balance. We construct a table as follows.

| Year | Principal (RM) | Interest received (RM) | Principal + Interest (RM) | Amount of savings (RM) |
|---|---|---|---|---|
| 1 | 20 000 | $20\ 000 \times 0.04$ | $20\ 000 + 20\ 000 \times 0.04$<br>$= 20\ 000(1 + 0.04)$ | $20\ 000(1.04)$ |
| 2 | $20\ 000(1.04)$ | $20\ 000(1.04) \times 0.04$ | $20\ 000(1.04) + 20\ 000(1.04) \times 0.04$<br>$= 20\ 000(1.04)(1 + 0.04)$ | $20\ 000(1.04)^2$ |
| 3 | $20\ 000(1.04)^2$ | $20\ 000(1.04)^2 \times 0.04$ | $20\ 000(1.04)^2 + 20\ 000(1.04)^2 \times 0.04$<br>$= 20\ 000(1.04)^2(1 + 0.04)$ | $20\ 000(1.04)^3$ |
| 4 | $20\ 000(1.04)^3$ | $20\ 000(1.04)^3 \times 0.04$ | $20\ 000(1.04)^3 + 20\ 000(1.04)^3 \times 0.04$<br>$= 20\ 000(1.04)^3(1 + 0.04)$ | $20\ 000(1.04)^4$ |
| 5 | $20\ 000(1.04)^4$ | $20\ 000(1.04)^4 \times 0.04$ | $20\ 000(1.04)^4 + 20\ 000(1.04)^4 \times 0.04$<br>$= 20\ 000(1.04)^4(1 + 0.04)$ | $20\ 000(1.04)^5$ |

**Verifying and interpreting solutions in the context of the problem**

Based on the table above, at the end of every year, the amount of savings is a power of 1.04 times the principal, RM20 000, and the power corresponds to the number of years of saving.

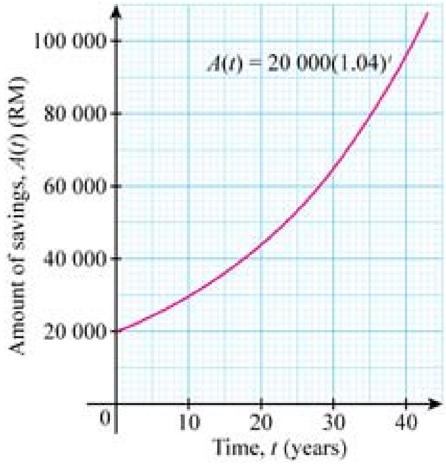
We can generalise this pattern to a mathematical model by letting $P$ represents the principal of the savings, $t$ represents the number of years, and $r$ represents the yearly interest rate. Hence, the mathematical model is

$$A(t) = P(1 + r)^t,$$ ← Note that $1.04 = 1 + 0.04 = 1 + r$

where $A(t)$ is the amount of savings after $t$ years.

This mathematical model is an exponential function that allows us to calculate the amount of savings if the interest is compounded once a year for $t$ years.

The graph on the right shows an exponential function $A(t) = 20\ 000(1.04)^t$. The graph of the amount of savings after $t$ years, $A(t)$, shows an exponential growth as time, $t$, increases.




## Refining the mathematical model

The mathematical model $A(t) = P(1 + r)^t$ is used for annual compounding. In reality, interest is often compounded more frequently, for example, compounded semi-annually or quarterly.

If the interest is compounded monthly, the interest rate $r$ is divided amongst the 12 months because $\frac{1}{12}$ of the rate is applied each month. The variable $t$ in the exponent is multiplied by 12 because the interest is calculated 12 times in a year. Hence, the mathematical model becomes $A(t) = P\left(1 + \frac{r}{12}\right)^{12t}$. In general, if the interest is compounded $n$ times per year, the mathematical model is $A(t) = P\left(1 + \frac{r}{n}\right)^{nt}$.

## Reporting the findings

Write a full report following the above modeling framework structure.

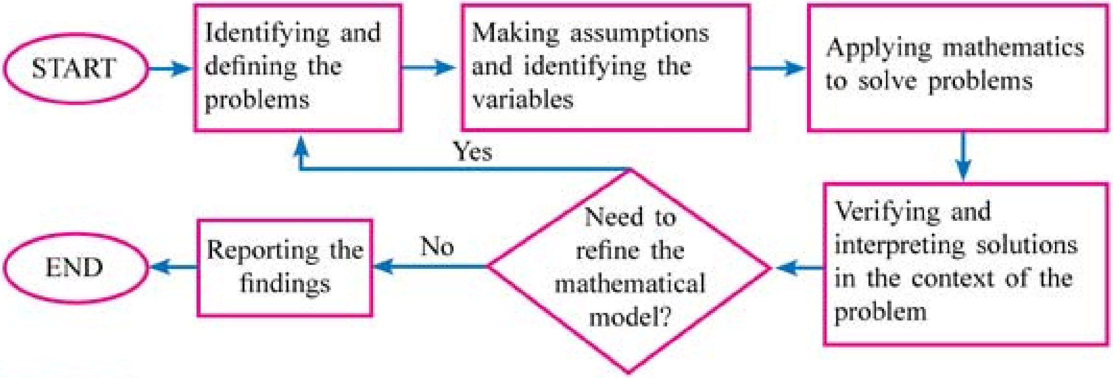
## Self Practice 8.1b

1. A runner is training for a marathon, running a total of 32 km per week on a regular basis. The runner plans to increase his running distance $D(x)$, in km, each week by 10%, where $x$ represents the number of weeks of training. Derive a mathematical model for his running distance, $D(x)$. Solve this problem through mathematical modeling.

2. Hyperthermia is a condition where an individual's body temperature is elevated beyond normal. Causes of hyperthermia include dehydration, an excessive loss of body fluids. Why the risk of hyperthermia higher among young children compared to adults in hot weather? Investigate this problem through mathematical modeling.

# Summary Arena

## MATHEMATICAL MODELING PROCESS




## Reflection

At the end of this chapter, I can

| | | |
|---|---|---|
| explain mathematical modeling. | | |
| solve real life problems through mathematical modeling which involves the following functions:<br>(i) linear<br>(ii) quadratic<br>(iii) exponential<br>and communicate the mathematical modeling process implemented. | | |

## MINI PROJECT

Open the worksheet for this project.

1. Estimate the length of Malaysia's coastline using the 200 km unit and the 100 km unit.
2. Predict the result if the length of coastline is measured using the 50 km unit. Explain.
3. If you continue to decrease the size of the unit of measurement, what do you think will happen to the length of the coastline?
4. Will you ever reach a point where you will find the exact length of the coastline?

Scan the QR code or visit bit.do/MPChap8 for the worksheet.

After you have determined the length of Malaysia's coastline by the method above, find the actual length of Malaysia's coastline from a reliable source. Also find information on how a geographer determine the length of a coastline of a country. Prepare a written report for the mathematical modeling process that you carry out in this project.

UNDERSTAND

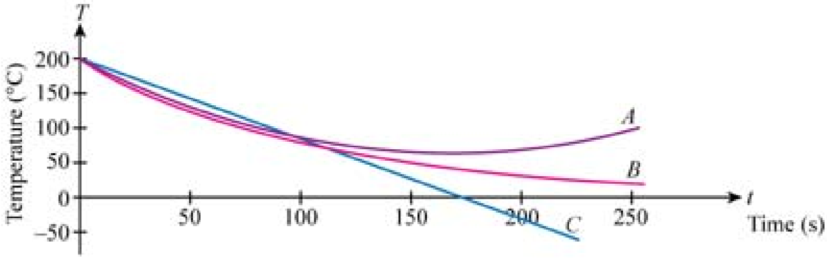
**1.** State all the components in mathematical modeling.

MASTERY

**2.** A scientist is studying the cooling patterns of a particular material over time. His research requires heating a sample of the material up to 200°C. Then, he records the temperature of the sample as it is cooled to room temperature. The following table shows the data collected during the first 2 minutes of the cooling process.

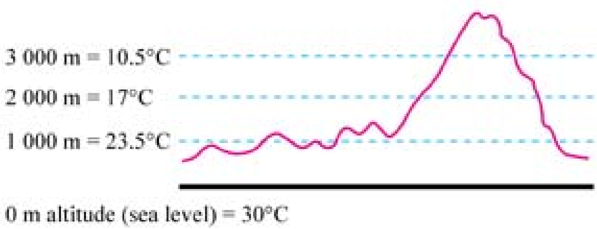
| Time (s) | 0 | 40 | 80 | 120 |
|---|---|---|---|---|
| Temperature (°C) | 200 | 140 | 100 | 75 |

The diagram below shows three possible models for the data, a linear model, a quadratic model and an exponential model.




(a) Which among the graphs of $A$, $B$ and $C$, is a linear model, a quadratic model and an exponential model?
(b) Which among the models of $A$, $B$ and $C$ best describes the temperature of the sample for the range of times $0 \leqslant t \leqslant 250$? Explain why the other models do not fit very well for the range of times given.

**3.** As altitude increases, the temperature usually decreases. On average, the rate at which the temperature changes with height in the troposphere (the first layer of the Earth's atmosphere) is 6.5°C per 1 000 m. The diagram below shows the cross section of Mount Kinabalu (4 095 m) in Sabah with various temperatures at different altitudes. The temperature at an altitude of 0 m (sea level) is 30°C.

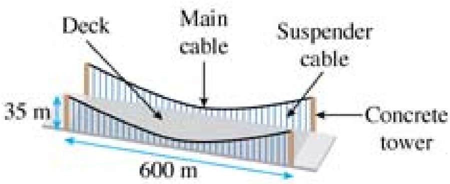



(a) Write an equation that expresses the temperature, $T$°C as a function of the altitude, $x$ m, for the situation shown in the diagram above.

(b) Verify that your equation in (a) is reasonable by using the data provided in the diagram.
(c) Draw a graph that shows the relationship between $x$ and $T$. Show a clearly defined scale on each axis and label your axes.
(d) What is a reasonable range for the function $T(x)$? Justify your answer.
(e) What is the gradient of the graph drawn? What is the meaning of the gradient of the graph in terms of temperature and altitude?
(f) What is the $y$-intercept of the graph drawn? What is the meaning of the $y$-intercept in terms of temperature and altitude?
(g) Is it reasonable to assume that temperature always decreases at a linear rate as altitude increases? Explain your answer.

**CHALLENGE**

**4.** The diagram on the right shows a suspension bridge. The shortest distance from the main cables to the deck is 5 m. Show how an engineer can use the information given to solve the following problems through mathematical modeling.




(a) Determine the function that represents the arc of the main cable.
(b) If the suspender cables are 20 m apart, determine the number of suspender cables needed on both sides of the bridge.
(c) Determine the type of material that should be used to make the suspender cables such that the total cost of the cables is minimum.

| Distance between the suspender cables (m) | Type of material | Cost per m (RM) | Total cost of the suspender cables (RM) |
|---|---|---|---|
| 15 | *A* | 750 | |
| 20 | *B* | 1 000 | |
| 25 | *C* | 1 200 | |

**EXPLORING MATHEMATICS**

Your class is organising an end of the school year party. You are to order the kind of pizza that suits your classmates most. The things you need to decide on are the pizza outlet, the delivery distance, the cost of the pizza, the size of the pizza, the mode of delivery (self-pick-up, food ordering mobile apps, etc.), the kind of pizza (vegetarian, gluten-free, etc.) and the types of pizza toppings (tomato, cheese, onion, etc.).

Solve this problem through mathematical modeling. You can use variables to represent the changing quantities and use equations to represent the relationship between these quantities. For qualitative types of questions, for example, for the types of pizza toppings, you can create a scoring system that includes calorie values and nutrients. In the process of modeling, you can also make assumptions by restricting the choices to narrow down the scope of your modeling.