

CHAPTER 1

Patterns and Sequences

WHAT WILL YOU LEARN?



- 1.1 Patterns
- 1.2 Sequences
- 1.3 Patterns and Sequences



WORD LINK

- Number pattern
- Odd number
- Even number
- Fibonacci Number
- Pascal's Triangle
- Sequence
- Algebraic expression
- Term
- *Pola nombor*
- *Nombor ganjil*
- *Nombor genap*
- *Nombor Fibonacci*
- *Segi Tiga Pascal*
- *Jujukan*
- *Ungkapan algebra*
- *Sebutan*

The sunflower is a unique flower in terms of the arrangements of its seeds. The seeds are arranged in a spiral pattern and follow a particular direction. The number of seeds in the spirals can be arranged in a number pattern known as Fibonacci Numbers. The seeds are usually arranged into 2 types of spiral patterns. For example, 21 spirals follow the clockwise pattern and 34 spirals follow the anti-clockwise pattern. The numbers 21 and 34 are found in the Fibonacci sequence.



WALKING THROUGH TIME

The Fibonacci Numbers began with a question posed by the Italian mathematician, Leonardo of Pisa or Fibonacci in his book, 'Liber Abaci' about the population of rabbits.

The question posed was that if a pair of female and male rabbits were placed in an enclosed space, how many pairs of rabbits will be reproduced in a year? If every pair of rabbits reproduce a new pair every month, then the increase in the population of the rabbits will produce a number sequence as follows 0, 1, 1, 2, 3, 5, 8, These numbers are known as Fibonacci Numbers. The Fibonacci Numbers are arranged by adding the number before it. For example, the pairs of rabbits are 1 + 1, then the population of the rabbits becomes 2. Consequently, by adding the preceding number 1 and 2, the population of the rabbits becomes 3 and so forth.

For more information



http://rimbunanilmu.my/mat_t2e/ms001

WHY STUDY THIS CHAPTER?

- The concept of number pattern and sequence can be applied in architecture, fashion design, science, astronomy, chemistry, physics and technology.

CREATIVE ACTIVITY

Aim: Recognising patterns

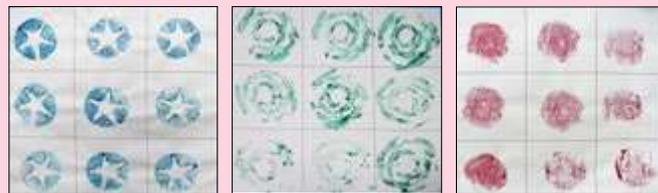
Materials: Potato, onion, mustard stem, drawing paper and water colour

Steps:

1. Take a piece of drawing paper.
2. Cut the potato, onion and mustard stem as shown in the pictures below.



3. Use the materials and do stamping on the drawing paper.
4. Dry the printout.



5. State the pattern produced.

From the activity above, students will be able to recognise the different type of patterns in our natural surroundings. These patterns become attractive formations.

1.1 Patterns

1.1.1 Recognising number patterns

COGNITIVE STIMULATION

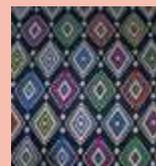


Aim: Recognising patterns

Materials: Batik cloth

Steps:

1. Look at the picture, that shows the patterns on some traditional Malaysian fabric.



Discussion:

- (i) What patterns do you see?
- (ii) What are the arrangements of the patterns like?

LEARNING STANDARD

Recognise and describe patterns of various number sets and objects based on real life situations, and hence make generalisation on patterns.

From the activity above, the patterns seen are repetitions of a polygon.

COGNITIVE STIMULATION

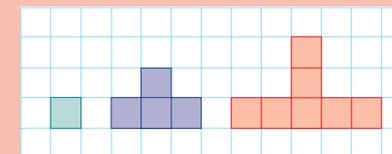


Aim: Recognising patterns

Materials: Colour pencil, ruler, pencil and grid paper

Steps:

1. Work in groups.
2. Open the file MS003 file for the grid paper.
3. Draw and colour the patterns as shown below.
4. Then continue to draw and colour the 4th, 5th and 6th patterns.
5. Fill up the table below.



Pattern Number	1	2	3	4	5	6	7	8
Number of squares	1	4	7					

6. Present your answers.

Discussion:

- (i) State the pattern that you can observe.
- (ii) Calculate the number of squares for pattern number 7 and 8.



Scan the QR Code or visit http://rimbunanilmu.my/mat_t2e/ms003 to get grid paper.



From the activity above, the number of squares in the pattern 1, 4, 7, ... is determined by adding 3 to the number before it. Addition of 3 is the **pattern** for this sequence.

Patterns are list of numbers or objects arranged based on a rule or design.

EXAMPLE 1

Draw the next object. State its pattern.



Solution:



Pattern: Add two dots to the previous object.



Pattern: Add a triangle to the previous object.

EXAMPLE 2

Determine the patterns for the following.

- (a) $-10, -4, 2, 8, \dots$
- (c) $2, 6, 18, 54, \dots$
- (e) $1, \frac{3}{2}, 2, \frac{5}{2}, \dots$

Solution:

- (a) $-10, -4, 2, 8, \dots$



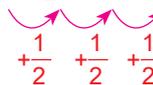
Pattern: Add 6 to the previous number.

- (c) $2, 6, 18, 54, \dots$



Pattern: Multiply the previous number by 3.

- (e) $1, \frac{3}{2}, 2, \frac{5}{2}, \dots$



Pattern: Add $\frac{1}{2}$ to the previous number.

Even and Odd numbers

EXAMPLE 3

Given a series of numbers 7, 12, 17, 22, 27, ..., 67. Identify and state the pattern for the sequence of

- (i) odd numbers
- (ii) even numbers

Solution:

7, 12, 17, 22, 27, 32, 37, 42, 47, 52, 57, 62, 67

- (i) **Odd numbers:** 7, 17, 27, 37, 47, 57 and 67
- (ii) **Even numbers:** 12, 22, 32, 42, 52 and 62



These odd numbers were obtained by adding 10 to the previous number.



These even numbers were obtained by adding 10 to the previous number.

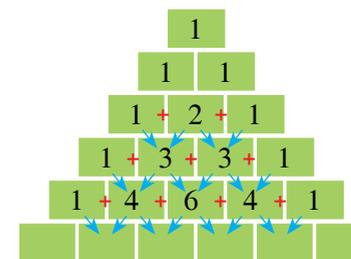
FLASHBACK

Even numbers: numbers that are divisible by 2, e.g. 2, 4, 6, 8, ...

Odd numbers: numbers that are not divisible by 2. e.g. 1, 3, 5, 7, 9, ...

Pascal's Triangle

The diagram below shows a Pascal's Triangle. Based on the triangle, the numbers in the row can be determined by adding the numbers in the previous row.



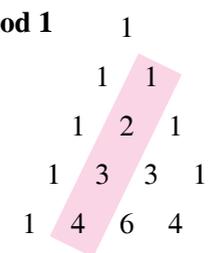
The Pascal's Triangle above starts with the number 1. The next row is 1, 1. All the rows start and end with 1. The other numbers can be obtained by adding the two numbers above.

The number 2 (row 3) is found by adding the two numbers 1, 1 (row 2). Likewise the number 3 on the fourth row is found by adding the number 1 and 2 from the previous row. Number 6 is found by adding the two numbers 3 and 3 from the previous row.

Fill in the last row.

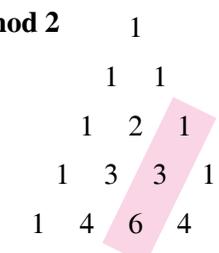
From the triangle above, various number series and certain patterns can be observed:

Method 1



Sequence: 1, 2, 3, 4, ...
Pattern: Add 1

Method 2

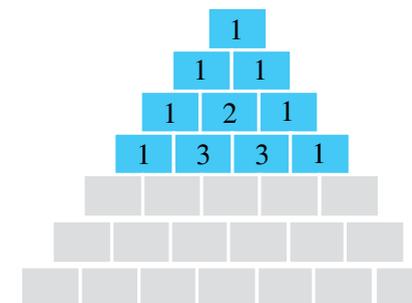


Sequence: 1, 3, 6, ...
Pattern: Add 2, 3, 4, ...

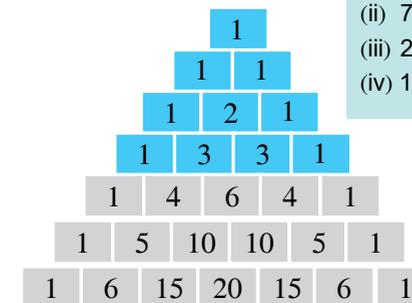
Pattern of a set of numbers is a sequence of numbers that are arranged according to a rule.

EXAMPLE 4

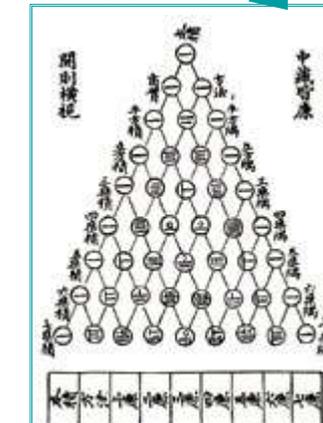
Complete the Pascal's Triangle below.



Solution:



DO YOU KNOW ?



Yang Hui Triangle

Pascal's Triangle was known as Yang Hui's Triangle by the Chinese and is illustrated using magic squares and magic circles.

THINK SMART

1×1	1
11×11	121
111×111	12321
1111×1111	1234321
11111×11111	123454321

Determine the value of the next two terms.

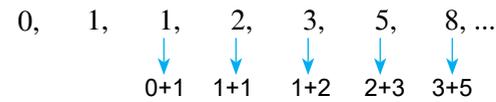
THINK SMART

State the next two terms.

- (i) 3, 8, 15, 24, 35, ...
- (ii) 7, 5, 8, 4, 9, 3, ...
- (iii) 2, 4, 5, 10, 12, 24, 27, ...
- (iv) 1, 4, 9, 18, 35, ...

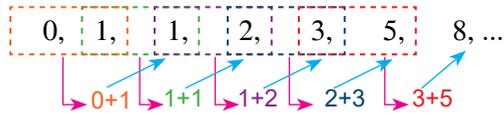
Fibonacci Numbers

Fibonacci Numbers are a pattern of numbers in a sequence.



This sequence starts with 0, 1, 1 and the next term is obtained by adding the previous two terms.

Example:



EXAMPLE 5

Complete the number sequence below .

(a) 0, 1, 1, , , , 8, 13, , ...

(b) 1, 3, , , 11, ...

Solution:

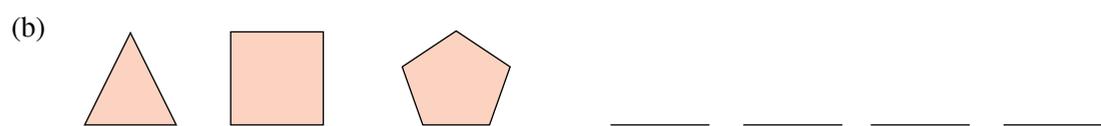
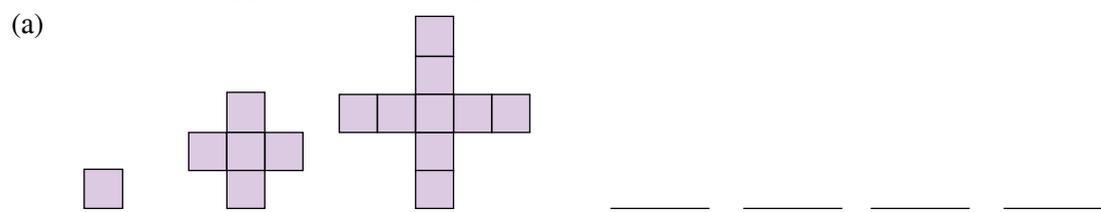
(a) 0, 1, 1, , , , 8, 13, , ...

(b) 1, 3, , , 11, ...

Patterns are list of numbers or objects arranged following a rule or design. A pattern in a list of numbers is obtained from addition, subtraction, multiplication or division of the previous numbers. Whereas pattern in objects is obtained by observing the arrangements of the previous objects.

SELF PRACTICE 1.1

1. Draw the following patterns for the diagrams below.



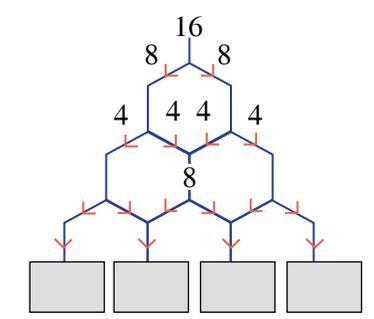
THINK SMART

How will you form more Fibonacci squares?

QR CODE

Scan the QR Code or visit http://rimbunanilmu.my/mat_t2e/ms006 on one of the Fibonacci series.

- State the pattern for the following sequence.
 - 5, 12, 19, 26, ...
 - 1, -4, -7, -10, ...
 - 4, 0, 4, 8, ...
 - 144, 72, 36, 18, ...
 - $\frac{1}{2}, \frac{1}{4}, 0, -\frac{1}{4}, \dots$
 - 11.2, -33.6, 100.8, -302.4, ...
- For the number sequence 28, 37, 46, 55, ... , 145, state the number pattern for
 - odd numbers
 - even numbers
- Complete the following Fibonacci Numbers sequence.
1, , 2, , , , , ...
- Fill in the missing number in the boxes below.



1.2 Sequences

1.2.1 Sequences

COGNITIVE STIMULATION Individual

Aim: Recognising the pattern in a number sequence

Material: Worksheet

Steps:

- Open the file MS007 for the grid paper.
- Complete the table by drawing the next patterns.

Activity 1	1	2	3	4	5
Number	1	2	3	4	5
Pattern					

Activity 2	10	15	22
Number	10	15	22
Pattern			

Activity 3	1	7
Number	1	7
Pattern		

Discussion:

- State the pattern that you found from activity 1, 2 and 3.
- List down the number sequence from activity 1, 2, and 3.

LEARNING STANDARD

Explain the meaning of sequence.

QR CODE

Scan the QR Code or visit http://rimbunanilmu.my/mat_t2e/ms007 to get the worksheet.

Based on the activities, the pattern can be determined by following the previous arrangement. An arrangement of numbers or objects following this pattern is known as **sequence**.

Sequence is a set of numbers or objects arranged according to a certain pattern.

1.2.2 Patterns of a sequence

EXAMPLE 6

Determine whether each set of numbers is a sequence

- (a) $-10, -6, -2, 2, 6, \dots$ (b) $4, 5, -7, 10, -14, \dots$

Solution:

- (a) $-10, -6, -2, 2, 6, \dots$
 $+4 \quad +4 \quad +4 \quad +4$

Pattern: Add 4
 Therefore, the set of numbers is a sequence.

- (b) $4, 5, -7, 10, -14, \dots$
 $+1 \quad -12 \quad +17 \quad -24$

Pattern: None
 Therefore, the set of numbers is not a sequence.

LEARNING STANDARD

Identify and describe the pattern of a sequence, and hence complete and extend the sequence.

DO YOU KNOW?



Astronomers use patterns to predict the path of a comet.

Number sequence

EXAMPLE 7

Complete the number sequences below.

- (a) $7, 13, \square, 25, \square, \square, \dots$

- (b) $88, \square, 64, 52, \square, \square, \dots$

- (c) $\square, 0.3, \square, 0.027, 0.0081, \square, \dots$

- (d) $\square, \square, \frac{1}{3}, \frac{4}{6}, \square, \dots$

Solution:

- (a) $7, 13, \boxed{19}, 25, \boxed{31}, \dots$
 $+6 \quad +6 \quad +6 \quad +6$

- (b) $88, \boxed{76}, 64, 52, \boxed{40}, \boxed{28}, \dots$
 $-12 \quad -12 \quad -12 \quad -12 \quad -12$

- (c) $\boxed{1}, 0.3, \boxed{0.09}, 0.027, 0.0081, \boxed{0.00243}, \dots$
 $\times 0.3 \quad \times 0.3 \quad \times 0.3 \quad \times 0.3 \quad \times 0.3$

- (d) $\boxed{-\frac{1}{3}}, \boxed{0}, \frac{1}{3}, \frac{4}{6}, \boxed{1}, \dots$
 $+\frac{1}{3} \quad +\frac{1}{3} \quad +\frac{1}{3} \quad +\frac{1}{3}$

EXAMPLE 8

Complete the number sequences below based on the given pattern.

- (a) Subtract 4 from the previous number.

$96, \square, \square, \square, \square, \square, \dots$

- (b) Multiply the previous number by 3.

$7, \square, \square, \square, \square, \square, \dots$

- (c) Subtract 8 from the previous number.

$21.3, \square, \square, \square, \square, \square, \dots$

- (d) Divide the previous number by 5.

$400, \square, \square, \square, \square, \square, \dots$

Solution:

- (a) $92, 88, 84, 80, 76, \dots$
 (b) $21, 63, 189, 567, 1\,701, \dots$
 (c) $13.3, 5.3, -2.7, -10.7, -18.7, \dots$
 (d) $80, 16, 3.2, 0.64, 0.128, \dots$

SELF PRACTICE 1.2

1. Determine whether each set of the numbers is a sequence.

- (a) $3, 18, 33, 48, \dots$ (b) $100, 116, 132, 148, \dots$
 (c) $1.0, -1.7, -2.4, 3.1, \dots$ (d) $-15, 30, 60, -120, \dots$
 (e) $\frac{1}{4}, 1\frac{1}{2}, 2\frac{1}{2}, 3\frac{1}{3}, \dots$ (f) $-0.32, -0.16, -0.8, -0.4, \dots$

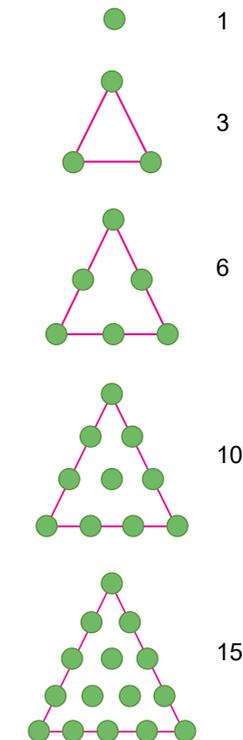
2. Complete the number sequences below.

- (a) $34, 28, \square, 16, \square, \square, \dots$ (b) $\square, \square, 32, 16, \square, 4, \dots$
 (c) $0.07, \square, 1.12, \square, 17.92, \dots$ (d) $1\frac{1}{10}, 1, \square, \square, \square, \dots$
 (e) $0.2, 2.4, 28.8, \square, \square, \dots$ (f) $\square, -80, -16, \square, \square, \dots$
 (g) $\square, \frac{2}{3}, \frac{7}{12}, \square, \square, \dots$ (h) $-8.1, \square, -4.1, -2.1, \square, \dots$

DO YOU KNOW?

Triangular numbers are numbers represented by dots to make an equilateral triangle.

$1, 3, 6, 10, 15, 21, 28, 36, \dots$



3. Complete the number sequences below based on the given pattern.

(a) Add 7 to the previous number.

42, , , , , , ...

(b) Divide the previous number by 2.

96, , , , , , ...

1.3 Patterns and Sequences

1.3.1 Pattern of a sequence using numbers, words and algebraic expressions

EXAMPLE 9

Describe the pattern for the number sequence 1, 9, 17, 25, 33, ... by using numbers, words and algebraic expressions.

Solution:

(i) **Numbers**

1, 9, 17, 25, 33, ...
 $+8 \quad +8 \quad +8 \quad +8$

Therefore, the pattern is + 8.

(ii) **Words**

1, 9, 17, 25, 33, ...
 $+8 \quad +8 \quad +8 \quad +8$

Therefore, the pattern for the above sequence is add 8 to the previous number.

(iii) **Algebraic expressions**

1, 9, 17, 25, 33, ...
 $+8 \quad +8 \quad +8 \quad +8$

$$\begin{aligned} 1 &= 1 + 8(0) \\ 9 &= 1 + 8(1) \\ 17 &= 1 + 8(2) \\ 25 &= 1 + 8(3) \\ 33 &= 1 + 8(4) \end{aligned}$$

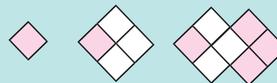
Therefore, the pattern for the number sequence can be expressed as $1 + 8n$ where, $n = 0, 1, 2, 3, 4, \dots$

LEARNING STANDARD

Make generalisation about the pattern of a sequence using numbers, words and algebraic expressions.

THINK SMART

An interior decorator wishes to arrange the tiles on a wall using the patterns below.



What is the pattern?

FLASHBACK

Algebraic expressions is an expression which has a combination of basic mathematical operations on numbers, variables or other mathematical entities.

Example:
 $2ab + 3c, 5a + 2b - 3c$

1.3.2 Terms of a sequence

The n^{th} term in a number sequence and is written as T_n whereby T is the term and n is the position of the term.

$$T_n = n^{\text{th}} \text{ term}$$

For example,

4, 8, 12, 16, ...

From the sequence above...

$$T_1 = 4,$$

$$T_2 = 8,$$

$$T_3 = 12,$$

$$T_4 = 16, \dots$$

EXAMPLE 10

State the 5th term for the following number sequence.

2, 10, 18, ...

Solution:

Step 1: Determine the pattern for the number sequence.

2, 10, 18, ...
 $+8 \quad +8$

Number pattern: Add 8 to the previous number.

Step 2: List down the first 5 terms as shown below.

$$\begin{aligned} T_1 &= 2 & T_4 &= 26 \\ T_2 &= 10 & T_5 &= 34 \\ T_3 &= 18 \end{aligned}$$

Hence, the 5th term is 34.

EXAMPLE 11

Given the number sequence 65, 60, 55, 50, Determine which term in the number sequence is 40.

Solution:

Step 1:

65, 60, 55, 50, ...
 $-5 \quad -5 \quad -5$

Pattern: Subtract 5 from the previous number.

Step 2:

$$\begin{aligned} T_1 &= 65 & T_4 &= 50 \\ T_2 &= 60 & T_5 &= 45 \\ T_3 &= 55 & T_6 &= 40 \end{aligned}$$

Hence, 40 is the 6th term.

LEARNING STANDARD

Determine specific terms of a sequence.

DO YOU KNOW?



The queen bee lays eggs in its nest which has a hexagonal pattern.

THINK SMART

$$\begin{aligned} 2^2 + (2 + 2 + 1) &= 3^2 \\ 3^2 + (3 + 3 + 1) &= 4^2 \\ 4^2 + (4 + 4 + 1) &= 5^2 \\ 5^2 + (5 + 5 + 1) &= 6^2 \end{aligned}$$

- (i) State the next two terms of the sequence.
- (ii) State the n^{th} term.

THINK SMART

Identify the patterns for the sequences below.

- (i) 1, 4, 9, 18, 35
- (ii) 23, 45, 89, 177
- (iii) 5, 7, 12, 19, 31
- (iv) 0, 4, 2, 6, 4, 8
- (v) 4, 7, 15, 29, 59, 117

THINK SMART

1(1) 3(2) 5(5) A C E
 2(1) 4(3) 6(8) B D

State the suitable pairs of numbers for A, B, C, D, E.

1.3.3 Solving problems

EXAMPLE 12



Automatic fish feeder

Specifications

- Container size: Moderate
- Dried Food and pellet maybe used
- A timer is used to arrange feeding time
- Use the latest technology to prevent food from getting moist or stuck in the container
- Can be operated manually or automatically
- Digital screen display

LEARNING STANDARD

Solve problems involving sequences.

The picture shows an automatic fish feeder and its specifications. If Eng Wei decides to feed the fishes 4 times a day with the first feeding time at 7:35 a.m., at what time should he feed the fish for the third feeding?

Understanding the problem	Planning the strategy	Implementing the strategy	Conclusion
Time for third feeding for the fishes.	$1 \text{ day} = 24 \text{ hours}$ each feed $= \frac{24}{4}$ $= 6 \text{ hours}$	Pattern: 6 hours $T_1 = 7:35 \text{ a.m.}$ $T_2 = 7:35 \text{ a.m.} + 6 \text{ hours}$ $= 1:35 \text{ p.m.}$ $T_3 = 1:35 \text{ p.m.} + 6 \text{ hours}$ $= 7:35 \text{ p.m.}$	Hence, fishes are fed for the third time at 7:35 p.m.

SELF PRACTICE 1.3

- State the pattern for the number sequences below in words.
 - 4, 12, 36, 108, 324, ...
 - 256, 128, 64, 32, 16, ...
- Determine the pattern for the number sequences below using algebraic expressions.
 - 2, 4, 8, 16, ...
 - 5, 8, 11, 14, ...
 - 3, 6, 9, 12, ...
 - 3, 1, -1, -3, ...
- Determine the seventh and the eleventh terms for the number sequences below.
 - 3, 5, 13, ...
 - $4, 5\frac{1}{2}, 7, \dots$
 - 3.7, -4.3, -4.9, ...

- The table below shows the timetable for buses travelling from Kuala Lumpur to Pulau Pinang.

Bus	Departure time
A	8:00 a.m.
B	8:30 a.m.
C	9:00 a.m.
D	
E	

Based on the table above, answer the following questions.

- Calculate the interval between departure time of one bus and the next bus.
- What time does Bus E leave?
- What time will Bus E reach Pulau Pinang if the journey takes 5 hours?

GENERATING EXCELLENCE

- Match the term with the suitable statement.

Pascal's Triangle

Odd numbers

Fibonacci Numbers

Even numbers

Numbers that cannot be divided by 2 exactly.

This sequence starts with 0, 1, 1 and the following terms can be determined by adding the previous two terms.

Numbers that can be divided by 2 exactly.

Geometrical arrangements on the binomial coefficients of a triangle.

- Determine the pattern for the given number sequences.

- 7, 13, 19, 25, ...
- 54, 50, 46, 42, ...
- 13, -39, -117, -351, ...
- 1 296, 216, 36, 6, ...

- Complete the table below.

Sequence	Number	Words	Algebraic expressions
(a) 2, 4, 6, 8, ...			
(b) 100, 50, 25, 12.5, ...			

4. Complete the following number sequence.

- (a) 1, 3, 5, , 9, , ...
- (b) , , -20, -10, -5, ...
- (c) 268, , , 169, 136, , ...
- (d) $\frac{1}{2}$, , $\frac{1}{3}$, , $\frac{1}{6}$, ...

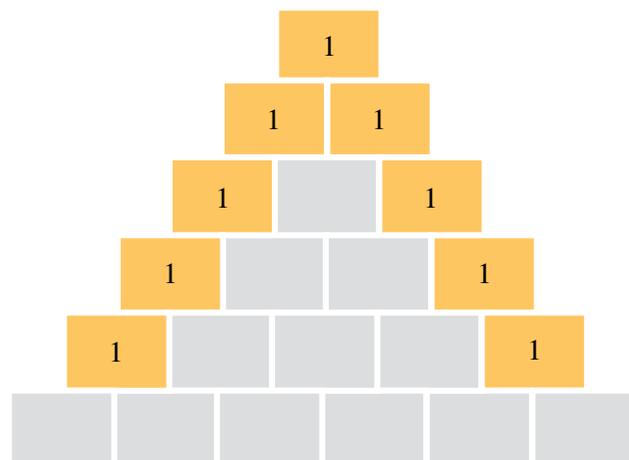
5. The first four terms of a sequence are 9, x , -5, -12, ...

- (a) Calculate the value of x .
- (b) State the pattern of the sequence using
 - (i) Numbers
 - (ii) Words
 - (iii) Algebraic expressions

6. Complete the Fibonacci Numbers shown below.

0, 1, 1, , , , ...

7. The diagram below shows the first five rows of the Pascal's Triangle. Complete the Pascal's Triangle. Explain how the Pascal's Triangle is formed.



8. The first four terms for a sequence are 11, x , -5, -13, ...

- (a) Calculate the value of x .
- (b) State the tenth term, T_{10}

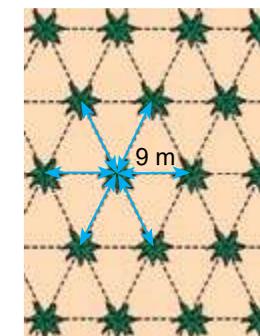


9. Nina arranged some buttons as shown below.



- (a) State the pattern for the number of buttons.
- (b) Determine the sequence for the buttons.
- (c) Draw the fourth term of the arrangement of buttons.
- (d) Calculate the value of T_6 .

10. Encik Hamid wishes to replant the oil palm plants. The distance between each plant is 9 m and the distances are triangular shaped. Encik Hamid sketched a map of the plants as shown below.



If Encik Hamid planted 18 oil palm plants, what is the area of his land?

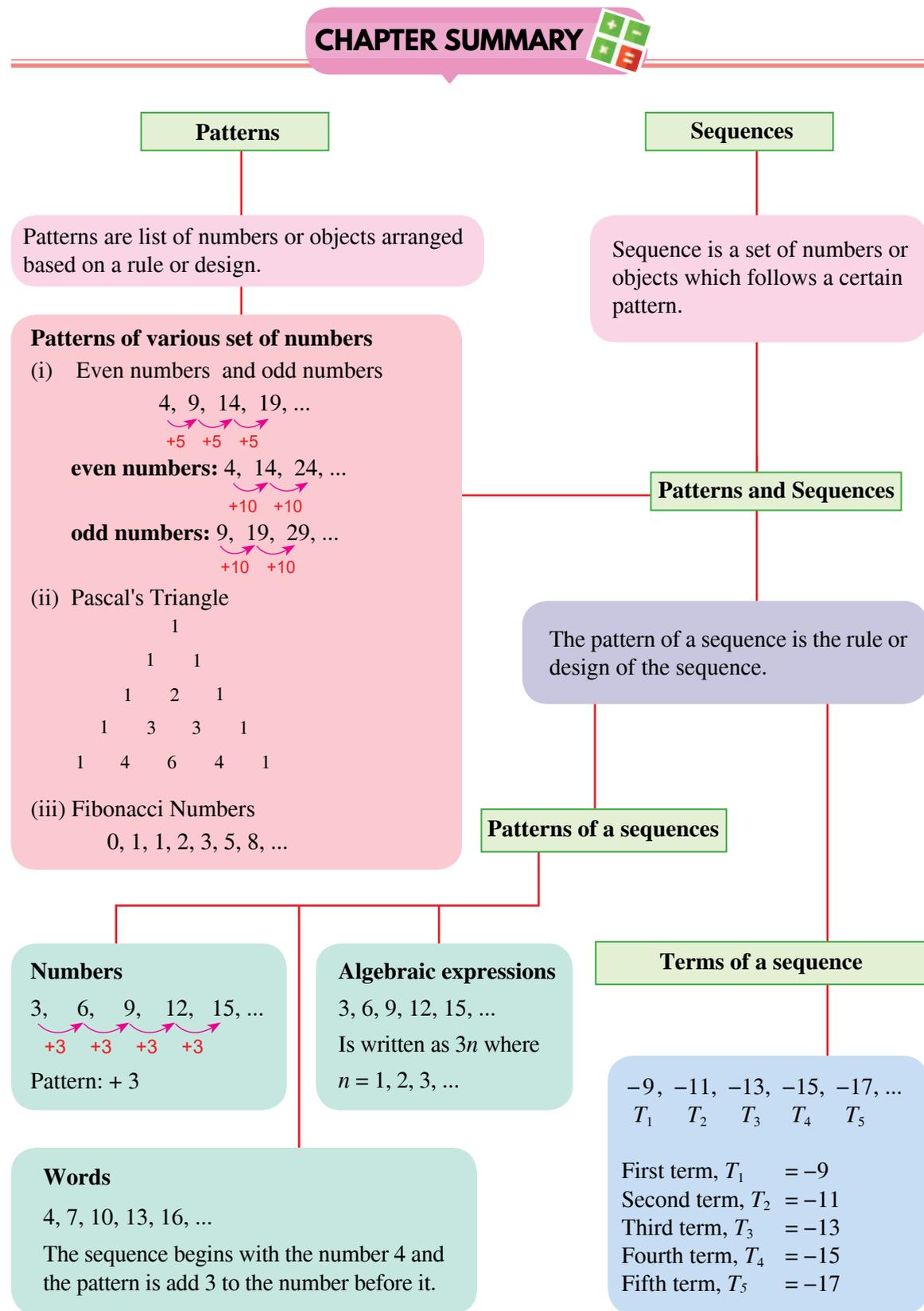
11. Raiyan went to see a doctor because he had been unwell for more than three days. The doctor prescribed three types of medicines which are fever medication, antibiotics and flu medication. Help Raiyan to plot a time table for taking his medication if he starts at 8:30 a.m.



Medicine	1	2	3
Fever			
Antibiotics			
Flu			

Fever = 2 tablets 3 times a day
 Antibiotics = 1 tablet 2 times a day
 Flu = 1 tablet 1 times a day

CHAPTER SUMMARY



SELF REFLECTION

At the end of this chapter, I will be able to:

1. Recognise and describe the patterns of various set of numbers and objects in everyday life.	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>
2. Explain the meaning of sequence.	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>
3. Recognise and describe the pattern of a sequence.	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>
4. Complete and extend a sequence using numbers.	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>
5. Express pattern of a sequence using numbers, words and algebraic expressions.	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>
6. Determine specific terms of a sequence.	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>
7. Solve problems involving sequences.	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>

MINI PROJECT

Title: Futuristic building

Materials: Paper cups, mineral water bottle, glue, ruler and scissors

Each group is to make building which has futuristic features using the materials.

Colour the building and name it.

In group present the product.

