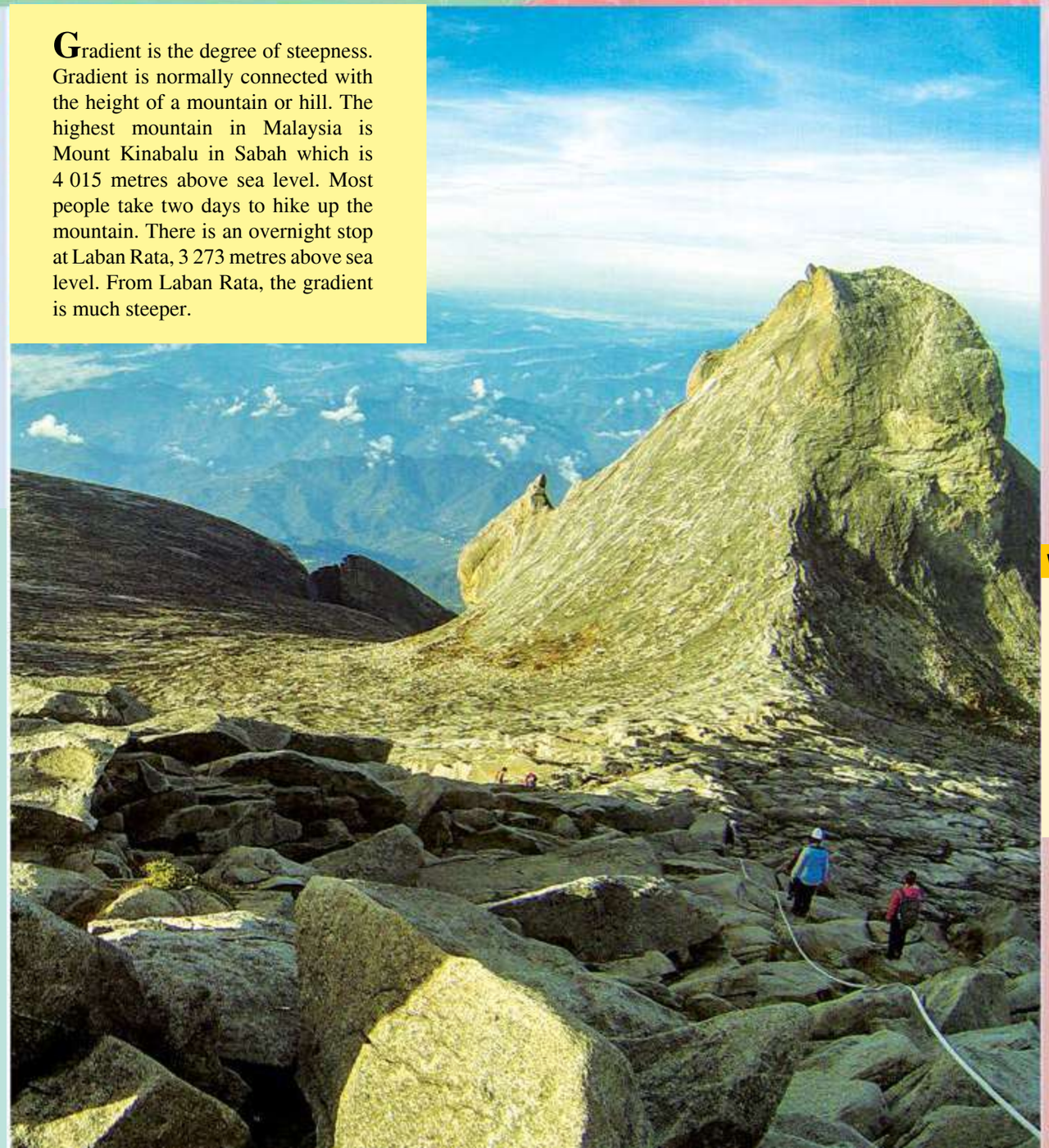
CHAPTER 10

Gradient of a Straight Line

Gradient is the degree of steepness. Gradient is normally connected with the height of a mountain or hill. The highest mountain in Malaysia is Mount Kinabalu in Sabah which is 4 015 metres above sea level. Most people take two days to hike up the mountain. There is an overnight stop at Laban Rata, 3 273 metres above sea level. From Laban Rata, the gradient is much steeper.



WHAT WILL YOU LEARN?

10.1 Gradient

WORD LINK

- Steepness — *Kecuraman*
- Straight line — *Garis lurus*
- Intercept — *Pintasan*
- Inclination — *Kecondongan*
- Ratio — *Nisbah*
- Vertical distance — *Jarak mencancang*
- Horizontal distance — *Jarak mengufuk*
- Gradient — *Kecerunan*

WALKING THROUGH TIME

Edwin Bedwell Wilson (1879-1964) was a mathematician who applied the concept of straight line to gradient. He was a vector analysis expert who published his famous book entitled 'Vector Analysis' in 1901. The gradient concept is applied in vector calculation where the vector's gradient will explain the change.

For more information:

http://rimbunanilmu.my/mat_t2e/ms189

WHY STUDY THIS CHAPTER?

- Learning about gradient will open doors to the careers in mathematics and physics. The formulae used can provide exact calculations to problems in product design.
- In addition, construction engineers, especially those involved in land surveying, use gradient to determine the stability or elevation of a building area.

CREATIVE ACTIVITY

Aim: Understanding the concept of gradient

Materials: Manila card measuring 20 cm x 9 cm, five to six erasers and one marble

Steps:

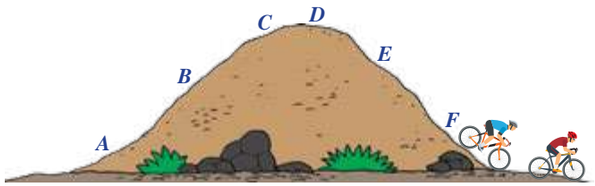
1. Form groups of 3 or 4.
2. Fold the manila card measuring 9 cm as shown in the diagram.
3. Stack three erasers on top of each other and place them at one end under the folded manila card and another eraser under the other end of the manila card.
4. Place the marble at the higher end of the manila card and let it roll along the route.
5. Increase the height of the manila card by adding more erasers to the higher end. Repeat this at the lower end.
6. Take note of the movement of the marble moving along the route.
7. Your friends and you can explore the movement of the marble when the height of both ends of the manila card is of the same level.

The activity above shows the movement of the marble at different speeds. The marble's speed depends on the height of its launch site. When the height is increased the speed increases.

10.1 Gradient

10.1.1 Steepness and inclination area

View the picture. Which area is steeper? Why?



LEARNING STANDARD

Describe gradient and direction of inclination based on real life situations, and then explain the meaning of gradient as a ratio of vertical distance to horizontal distance.

The diagram shows a hilly area travelled by Farid and Afif when they went mountain biking. They found it difficult to continue their ride when they were going up route *C*. However, when they were going down route *E*, their bicycles moved faster. Why did this situation occur?

COGNITIVE STIMULATION Group

Aim: Identifying the area of steepness and inclination

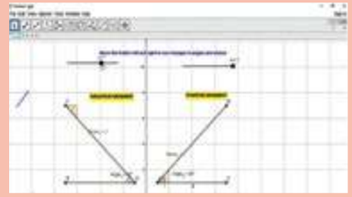
Material: Dynamic geometry software

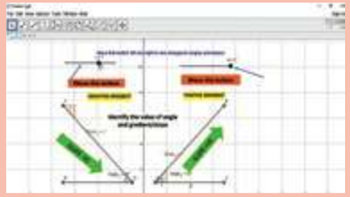
Steps:

1. Open the file MS190.
2. Move the round button *j* and *n* to the left and right.
3. Observe the value of angle and value of gradient.

QR CODE

Scan the QR Code or visit http://rimbunanilmu.my/mat_t2e/ms190 to watch the animated video on gradient.





TIPS

The value of an angle is always measured from the *x*-axis.

Discussion:

(i) Does the value of angle *D* and *A* affect the gradient value?

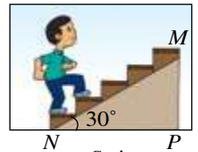
(ii) What is the relationship between the steepness of the line and the direction of the slant.

(iii) Does the negative value of gradient indicates that the slope is decreasing?

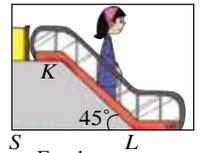
The steepness of a straight line is determined by its gradient value. The greater the gradient value, the steeper the slope of the straight line. The negative or positive gradient value determines the direction of the slope of the straight line.

EXAMPLE 1

Look at the diagram on the side. Compare the direction of the tilt and the steepness of the *MN* line with *KL*. Draw conclusions from both diagrams.



Stairs



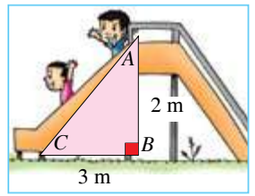
Escalator

Solution:

The line *KL*, has a higher inclination than the line *MN*. The larger the angle value, the higher the gradient value. Thus, line *KL* is steeper than *MN*.

Gradient is the ratio of the vertical distance to the horizontal distance

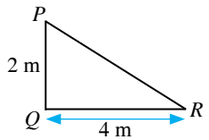
The diagram on the right shows children going down a slide at a playground. The length of the straight line which connects point *A* and point *B* is 2 m. The length of the straight line that connects point *C* and point *B* is 3 m. *CB* is the **horizontal distance** and *AB* is the **vertical distance**.



EXAMPLE 2

In each of the diagrams below, state the horizontal distance and the vertical distance between point *P* and point *R*.

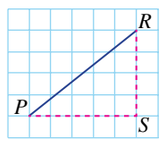
(a)



Solution:

Vertical distance, $PQ = 2$ m
Horizontal distance, $QR = 4$ m

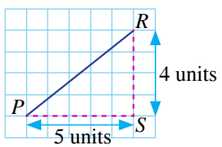
(b)



Solution:

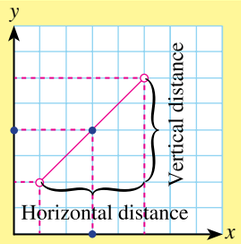
Vertical distance, $RS = 4$ units

Horizontal distance, $PS = 5$ units



THINK SMART

Drivers of heavy lorry will step on the accelerator when going up a steep hill. They will also accelerate from the base of the hill before the climb. Why?



Change in y = Vertical distance
Change in x = Horizontal distance

Gradient, $m = \dfrac{\text{Vertical distance}}{\text{Horizontal distance}}$

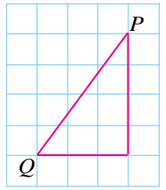
TIPS

m represents gradient of a straight line.

EXAMPLE 3

From the following diagram, specify the gradient of the straight line PQ and BC. Describe the steepness of line PQ and BC.

(a)



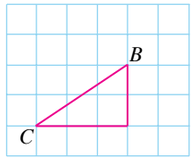
Solution:

Vertical distance is 4 units. Horizontal distance is 3 units.

$$\frac{\text{Vertical distance}}{\text{Horizontal distance}} = \frac{4}{3}$$

Thus, the gradient of PQ is $\frac{4}{3}$.

(b)



Solution:

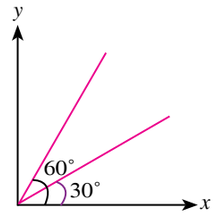
Vertical distance is 2 units.
Horizontal distance is 3 units

$$\frac{\text{Vertical distance}}{\text{Horizontal distance}} = \frac{2}{3}$$

Thus, the gradient of BC is $\frac{2}{3}$.

DO YOU KNOW ?

Line that has a steeper gradient has a greater angle.



10.1.2 Formula of gradient of a straight line on a Cartesian plane

In the Cartesian coordinate system, the gradient of a straight line that passes through two points $A\ (x_1, y_1)$ and $B\ (x_2, y_2)$ can be calculated using the ratio between the vertical distance to the horizontal distance.

LEARNING STANDARD

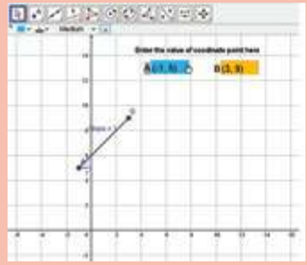
Derive the formulae for gradient of a straight line in the Cartesian plane.

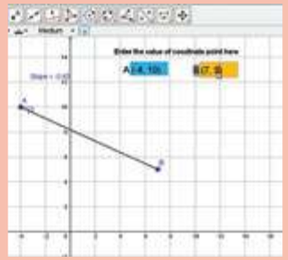
QR CODE

Scan the QR Code or visit http://rimbunanilmu.my/mat_t2e/ms192 to watch the video on gradient.

COGNITIVE STIMULATION Group

Aim: Identifying straight lines on the Cartesian plane
Material: Worksheets
Step:
1. Open the file MS192.
2. Change the corresponding coordinate point values as in the table given in the coordinate spaces.
3. Take note of the changes that occur on each of the straight line.
4. Specify the gradient value for both points.
5. State whether the straight line passes through the origin, parallel to the x-axis or parallel to the y-axis.





Coordinate		Value of gradient	Position of straight line
A	B		
(3,1)	(3, 9)		
(3, –3)	(–2, 2)		
(–1, 5)	(7, 5)		
(4, 4)	(0, 0)		
(0, 6)	(–2, 0)		
(0, 2)	(3, 0)		
(x_1, y_1)	(x_2, y_2)		

Discussion:

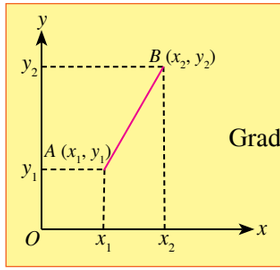
(i) Identify the straight line intersecting the x-axis and the y-axis.

(ii) Prove by using the formula:

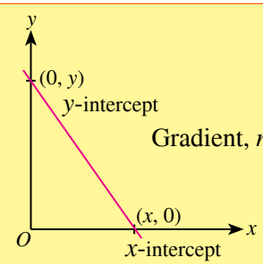
$$m = \frac{y_2 - y_1}{x_2 - x_1} \quad \text{and} \quad m = -\frac{y\text{-intercept}}{x\text{-intercept}}$$

that your gradient value is the same as displayed.

The intersection point between the straight line and the x-axis is called x-intercept, while the intersection point between the straight line and the y-axis is called y-intercept.



Gradient, $m = \dfrac{y_2 - y_1}{x_2 - x_1}$



Gradient, $m = -\dfrac{y\text{-intercept}}{x\text{-intercept}}$

EXAMPLE 4

Determine the gradient of the following coordinate points.

(a) A (3, 1) and B (6, 7)

Solution:

$x_1\ y_1 \qquad x_2\ y_2$

(a) A (3, 1) and B (6, 7)

$$\text{Gradient} = \frac{y_2 - y_1}{x_2 - x_1}$$
$$= \frac{7 - 1}{6 - 3}$$
$$= \frac{6}{3}$$
$$= 2$$

(b) P (4, −1) and Q (3, 5)

Solution:

$x_1\ y_1 \qquad x_2\ y_2$

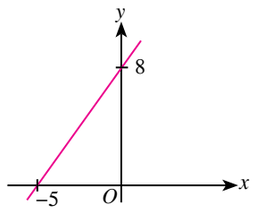
(b) P (4, −1) and Q (3, 5)

$$\text{Gradient} = \frac{y_2 - y_1}{x_2 - x_1}$$
$$= \frac{5 - (-1)}{3 - 4}$$
$$= \frac{6}{-1}$$
$$= -6$$

EXAMPLE 5

Determine the gradient of the following points of coordinate.

(a)



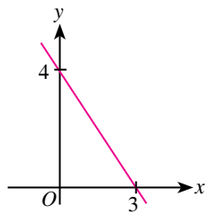
Solution:

y-intercept = 8

x-intercept = −5

$$\text{Gradient} = -\frac{8}{(-5)}$$
$$= \frac{8}{5}$$

(b)



Solution:

y-intercept = 4

x-intercept = 3

$$\text{Gradient} = -\frac{4}{3}$$

EXAMPLE 6

Determine the gradient of the coordinates given.

(a) L (4, 0) and M (0, 8)

Solution:

(a) y-intercept = 8

x-intercept = 4

$$\text{Gradient} = -\frac{8}{4} = -2$$

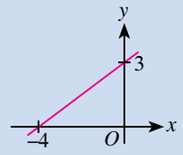
(b) G (−3, 0) and K (0, 9)

(b) y-intercept = 9

x-intercept = −3

$$\text{Gradient} = -\frac{9}{(-3)} = 3$$

TIPS

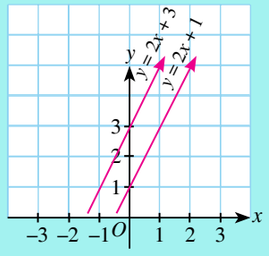


y-intercept coordinates are (0, 3).

x-intercept coordinates are (−4, 0).

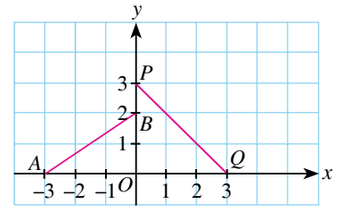
THINK SMART

A straight line is represented by $y = mx + c$, m is the gradient while c is the y-intercept. State the gradient and the y-intercept of the line below and the relationship between the two straight lines.



EXAMPLE 7

Calculate the gradient of a straight line AB and PQ based on the diagram on the right.



Solution:

$$\text{Gradient, } m = -\frac{y\text{-intercept}}{x\text{-intercept}}$$

(i) Gradient $AB = -\dfrac{2}{(-3)}$

$$= \frac{2}{3}$$

Then, gradient AB is $\frac{2}{3}$.

(ii) Gradient $PQ = -\dfrac{3}{3}$

$$= -1$$

Then, gradient PQ is −1.

10.1.3 Gradient for a straight line

LEARNING STANDARD

Make generalisation for the gradient of a straight line.

COGNITIVE STIMULATION Group

Aim: Identifying slant

Materials: Graph paper and card with coordinates

P (1, 1) Q (3, 5)	R (−2,−2) S (−2, 8)	W (−4, 1) V (−7, 8)	T (−4, 3) U (6, 3)

Steps:

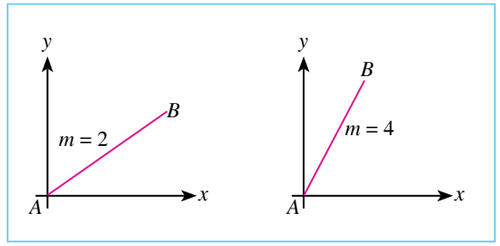
1. Student A constructs a graph with a scale of 1 cm to 1 unit on the x-axis, and 1 cm to 1 unit on the y-axis.
2. Student B matches the value of the point on the card by plotting the coordinates on the Cartesian plane.
3. Student C draws a straight line and determines the gradient of each pair of coordinate points given.
4. Student D will complete the table below. Other friends will discuss and review.

Straight line	Gradient	Direction of inclination right or left	Value of gradient positive or negative
PQ			
RS			
WV			
TU			

Discussion:

(i) The relationship between the gradient value and the direction of inclination.

(ii) Arrange the gradient of the straight line in descending order.

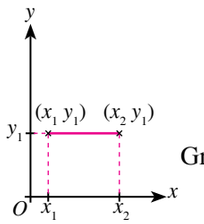


TIPS

Relationship of gradient and straight line.

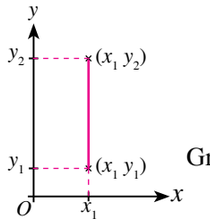
The more the straight line AB approaches the y-axis, the greater the gradient value and vice-versa. Thus, the greater the absolute value of the gradient, the steeper the straight line.

Any point in a straight line that is parallel to the x-axis has the common y-coordinate. Thus, the gradient is zero.



Gradient = 0

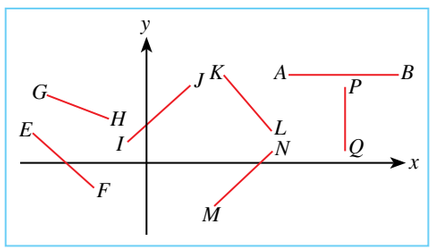
The x–coordinate for any two points in a straight line that is parallel to the y–axis is the same. This will give an undefined gradient.



Gradient is undefined (∞)

EXAMPLE 8

Identify the straight line that has the gradient value of positive, negative, zero or undefined in the diagram below. Justify.



Solution:

Gradient of straight lines IJ and MN are positive because the lines are inclined to the right.

Gradient of straight lines EF, GH and KL are negative because the lines are inclined to the left.

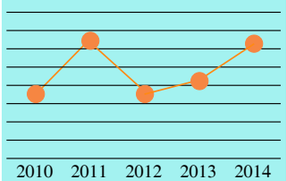
Gradient of line AB is zero because the line is parallel to the x-axis.

Gradient of line PQ is undefined because the line is parallel to the y-axis.

THINK SMART

Based on the graph below. Observe on which year does the inflation rate shows a negative gradient? Discuss.

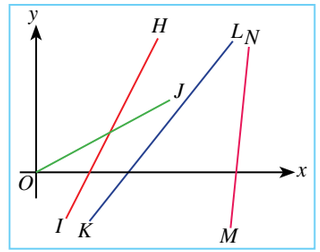
Inflation Rate (%) in Malaysia (2010-2014)



Source: World Bank
https://www.imoney.my/articles/realiti-tentang-inflasi

EXAMPLE 9

Determine the gradient of the four straight line in the diagram on the right. State the line with the highest and lowest gradient. Give reasons.



Solution:

Straight line MN is the straight line that has the highest gradient and it is more vertically inclined.

Straight line OJ is the straight line that has the smallest gradient because it is more horizontally inclined.

10.1.4 Determining gradient

LEARNING STANDARD

Determining the gradient of a straight line.

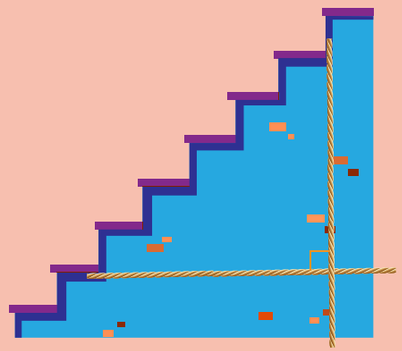
COGNITIVE STIMULATION Group

Aim: Determining gradient

Materials: Stairs, ropes, measuring tape

Steps:

1. In group, determine the gradient of the stairs in your school.
2. Choose any two suitable stairs.
3. Choose two suitable points as shown in the diagram.
4. Use ropes to determine the vertical and horizontal length. Make sure the angle where the two strings meet is at 90°.
5. Repeat steps 3 and 4 for the second stairs.



Discussion:

(i) State the vertical and the horizontal distance of the stairs.

(ii) Calculate the gradient of the two stairs.

(iii) What is the relationship between the ratio of 'the vertical to the horizontal distance' of the two stairs?

(iv) The ratio of 'the horizontal distance to the vertical distance' is not used to determine gradient. Discuss.

The ratio of 'the vertical distance to the horizontal distance' is used to determine the gradient of a straight line. The greater the gradient, the steeper the straight line.

QR CODE

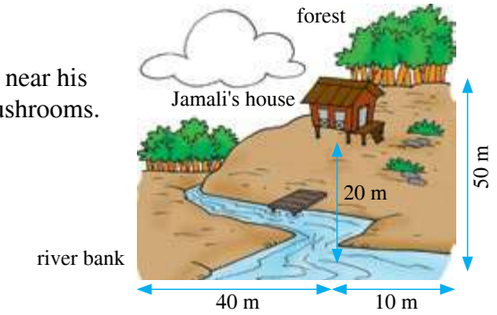
Scan the QR Code or visit http://rimbunanilmu.my/mat_t2e/ms197 to view the activity on gradient.

EXAMPLE 10

Every day Jamali fetches water from the river bank near his house and later goes into the forest to search for mushrooms. Calculate the gradient of

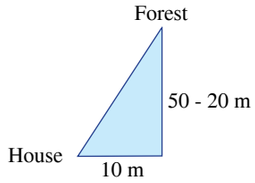
(i) his house to the forest

(ii) the river bank to his house



Solution:

(a)

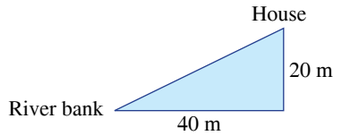


$$\text{Gradient} = \frac{\text{Vertical distance}}{\text{Horizontal distance}}$$

$$= \frac{30}{10} = 3$$

Thus, the gradient from Jamali's house to the forest is 3.

(b)



$$\text{Gradient} = \frac{\text{Vertical distance}}{\text{Horizontal distance}}$$

$$= \frac{20}{40} = \frac{1}{2}$$

Thus, the gradient from the river bank to his house is $\frac{1}{2}$.

10.1.5 Solving problems

LEARNING STANDARD

Solve problems involving the gradient of a straight line.

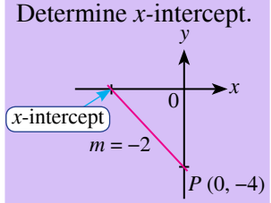
EXAMPLE 11

(a) Determine the x-intercept of a straight line that passes through the point P (0, –4) with the gradient –2.

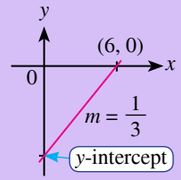
(b) Determine the coordinate of y-intercept of a line that passes through the point Q (6, 0) and the gradient $\frac{1}{3}$.

Solution:

(a)

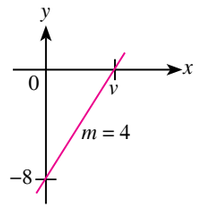
Understanding the problem	**Planning the strategy**	**Implementing the strategy**	**Conclusion**
Determine x-intercept.  	Use the formula: $x\text{-intercept} = -\frac{(y\text{-intercept})}{m}$ $y\text{-intercept} = -4$	Substitute the value and calculate. $x\text{-intercept} = -\frac{-4}{-2} = -2$	Thus, x-intercept is –2 with the coordinates (–2, 0).

(b)

Understanding the problem	**Planning the strategy**	**Implementing the strategy**	**Conclusion**
Determine the position of y-intercept.  	Using the formula: $y\text{-intercept} = -m \times x\text{-intercept}$	Substitute the value and calculate. $y\text{-intercept} = -\frac{1}{3} \times 6 = -2$	Thus y-intercept is –2 with the coordinates (0, –2).

EXAMPLE 12

Calculate the value of v in the diagram on the right.



Solution:

Understanding the problem	**Planning the strategy**	**Implementing the strategy**	**Conclusion**
Determine the position of v. $v = x\text{-intercept}$	Use the formula: $x\text{-intercept} = -\frac{(y\text{-intercept})}{m}$	$v = -\left(\frac{-8}{4}\right)$ $= 2$	Thus, x-intercept is 2.

EXAMPLE 13

Given A (–9, 2), B (–7, 2), C (– 4, 3) and D (–6, –1) are the vertices of a quadrilateral. Determine the type of the quadrilateral.

Solution:

Understanding the problem

Determine the type of the quadrilateral.

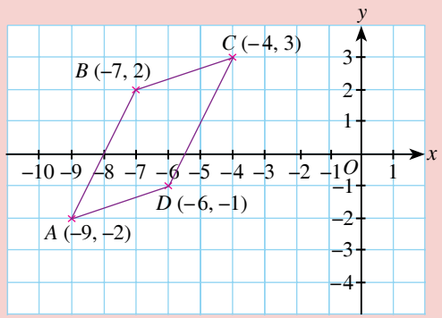
Planning the strategy

- Determine the gradient of the straight line AD, BC, AB and DC by using the formula $m = \frac{y_2 - y_1}{x_2 - x_1}$.
- Draw the quadrilateral.

CHAPTER 10

Implementing the strategy

- Draw the graph.



Gradient of the straight line AD

$$m_1 = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-1-(-2)}{-6-(-9)} = \frac{1}{3}$$

Gradient of the straight line DC

$$m_3 = \frac{y_2 - y_1}{x_2 - x_1} = \frac{3-(-1)}{-4-(-6)} = 2$$

Gradient of the straight line BC

$$m_2 = \frac{y_2 - y_1}{x_2 - x_1} = \frac{3-2}{-4-(-7)} = \frac{1}{3}$$

Gradient of the straight line AB

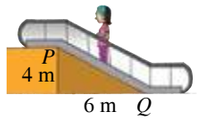
$$m_4 = \frac{y_2 - y_1}{x_2 - x_1} = \frac{2-(-2)}{-7-(-9)} = 2$$

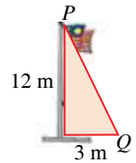
Conclusion

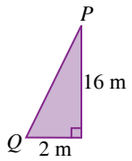
- Straight line AD is parallel to the straight line BC, $m_1 = m_2$.
- Straight line AB is parallel to the straight line DC, $m_3 = m_4$.
- $ABCD$ is a parallelogram.

SELF PRACTICE 10.1

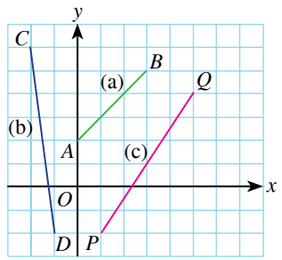
1. Determine the vertical distance and the horizontal distance of point P and point Q.

(a) 

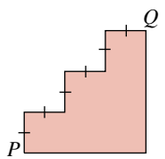
(b) 

(c) 

2. Determine the vertical distance and the horizontal distance of line AB, CD and PQ in the following Cartesian plane.



3. Calculate the vertical distance and horizontal distance in metres between P and Q in the diagram if the distance of each stair is 12 cm.

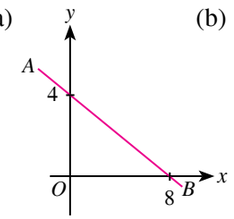


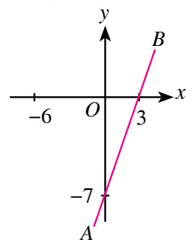
4. State the vertical distance and horizontal distance of the two pairs of points given.

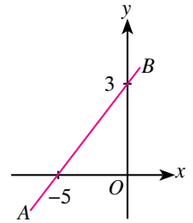
(a) (3, 0) and (–2, 6) (b) (1, 1) and (6, 5) (c) (3, 1) and (1, 5)

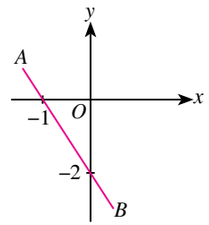
(d) (0, 0) and (4, 4) (e) (1, –2) and (2, 4) (f) (3, 6) and (6, –3)

5. State the value of the x-intercept and the value of the y-intercept of the straight line AB.

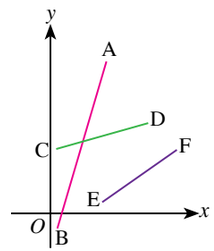
(a) 

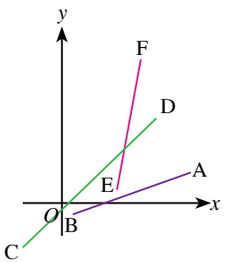
(b) 

(c) 

(d) 

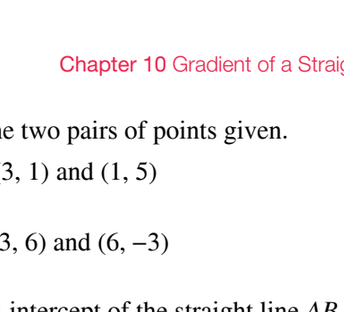
6. Identify the straight line that has the highest gradient in each of the diagrams below.

(a) 

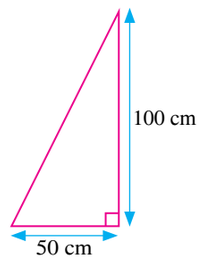
(b) 

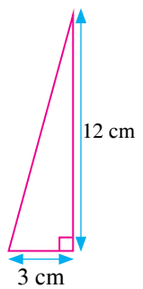
7. Based on the diagram, state whether the gradient is positive or negative.

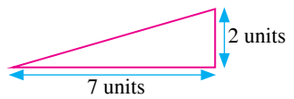
(a) LM
(b) MN
(c) NO
(d) OQ



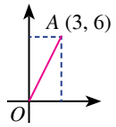
8. State the gradient of the straight line for each of the following.

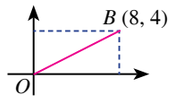
(a) 

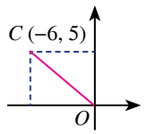
(b) 

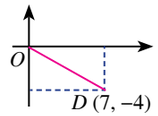
(c) 

9. Calculate the gradient of the straight line for each of the following.

(a) 

(b) 

(c) 

(d) 

10. Calculate the gradient of a straight line that passes through the following pairs of points.

(a) A (4, 5) and B (3, 2)
(b) E (−1, −2) and F (0, 7)
(c) C (6, 6) and D (3,1)
(d) G (2, 4) and H (6, 1)

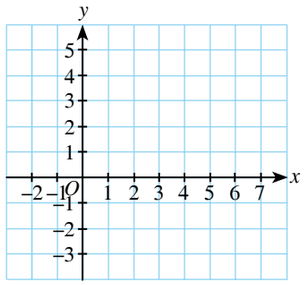
11. Calculate the gradient of the following straight line,

(a) x-intercept = 4, y-intercept = 1
(b) x-intercept = 9, y-intercept = 10
(c) x-intercept = −3, y-intercept = 8
(d) x-intercept = −5, y-intercept = −5

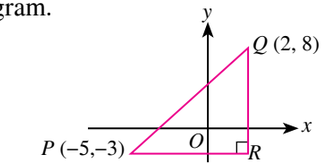
GENERATING EXCELLENCE

1. Draw the straight line that passes through the given pairs of coordinates on the Cartesian plane. Determine whether the gradient of the straight line is positive or negative.

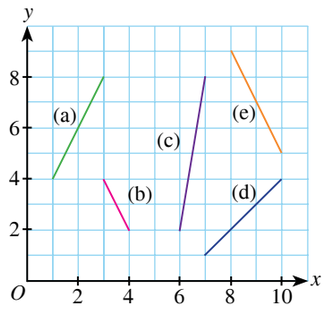
(a) (−1, 0) and (−2, 5)
(b) (0, 1) and (3, 5)
(c) (1, −3) and (2, 4)
(d) (7, −2) and (2, 2)
(e) (0, 1) and (5, 3)
(f) (0, 3) and (5, 0)
(g) (0, 0) and (6, 5)



2. Determine the gradient of the straight line PQ in the diagram.



3. Calculate the gradient of all the straight lines in the following diagram. Compare and determine the line with the steepest gradient.



4. Complete the table below.

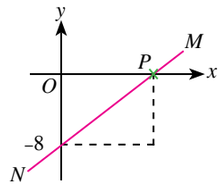
	x-intercept	y-intercept	Gradient
(a)	4		2
(b)	−2	4	2
(c)	−4		−3
(d)	−1	4	4
(e)	−1	2	
(f)	5		1

5. Given the gradient of the straight line that passes through M (1, k) and N (−2, 3) is −2, determine the value of k.

6. The gradient of straight line PQ is −1 with point P (2, −1) and the vertical distance of point Q is 3 units to the left of y-axis. State the coordinate Q.

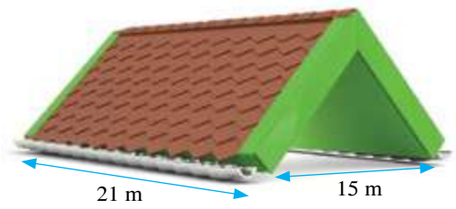
7. If the gradient of a straight line is 2 and the y-intercept is −18, determine x-intercept of the straight line.

8. Calculate the gradient of the straight line MN, if the horizontal distance of P from the y-axis is 6 units.



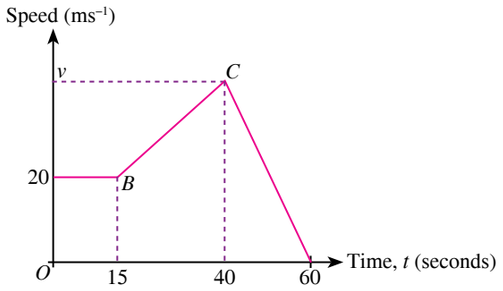
9. If points A and B are on the same straight line with the gradient $\frac{4}{3}$ and point A (0, 8), determine the coordinate of B if B is x-intercept.

10.

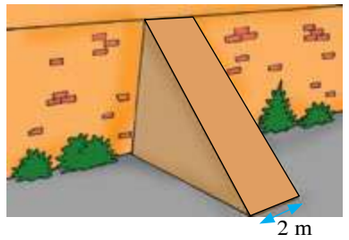


The diagram above is the roof of a terrace house. If the height of the roof is 5 m, calculate
(a) the gradient of the roof
(b) the length of the slant of the roof

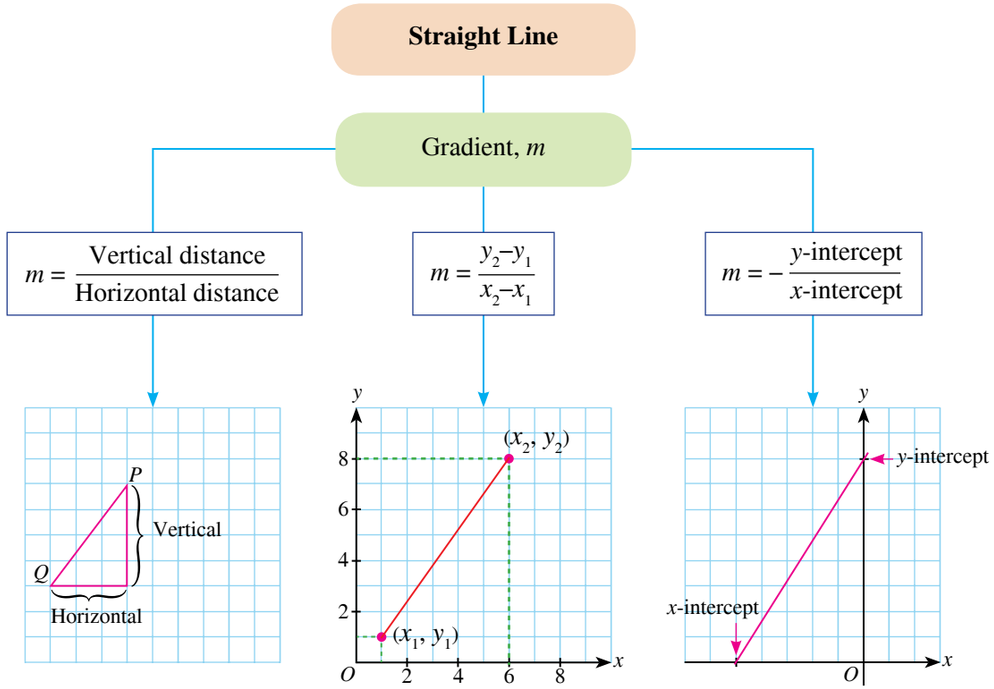
11. The diagram shows the journey of a motorcycle in 60 seconds.
    (a) State the speed of the motorcycle at the constant phase.
    (b) Calculate the value of v if the motorcycle moves at 0.88m/s when $t = 15$ s.



12. The cross section area of a brick wall that is shaped as right-angled triangle is $12m^2$ and the height is 6 metre. Calculate the gradient and the area of the slanted surface of the stone wall.



CHAPTER SUMMARY



SELF REFLECTION

At the end of the chapter, I am able to:

1. Describe the steepness and inclination directions based on daily situations, and further explaining the meaning of gradient as a ratio for vertical distance and horizontal distance.	
2. Derive the formulae for gradient of a straight line in the Cartesian plane.	
3. Make generalisation for the gradient of a straight line.	
4. Determine the gradient of a straight line.	
5. Solve problems involving the gradient of a straight line.	

MINI PROJECT

Students are required to search for information on mountains in Malaysia. The information should be on

(i) the height above sea level

(ii) the horizontal distance

You can calculate the gradient of each mountain and arrange the value of gradients from the highest value to the lowest value. Compare your information with your friends. You can also research on the mountains of South East Asia.


Gunung Tahan, Pahang


Gunung Korbu, Perak


Gunung Mulu, Sarawak