

# CHAPTER 11

## Isometric Transformations



### WALKING THROUGH TIME

The mathematicians Felix Klein (1849-1925) argued that isometry is the balance produced by the movement of a common form or movement by a group of similar forms. Isometry in a pattern is a movement of the same shape. There are four types of isometry; translation, rotation, reflection and glide reflection.

For more information:



[http://rimbunanilmu.my/mat\\_t2e/ms207](http://rimbunanilmu.my/mat_t2e/ms207)

### WHAT WILL YOU LEARN?



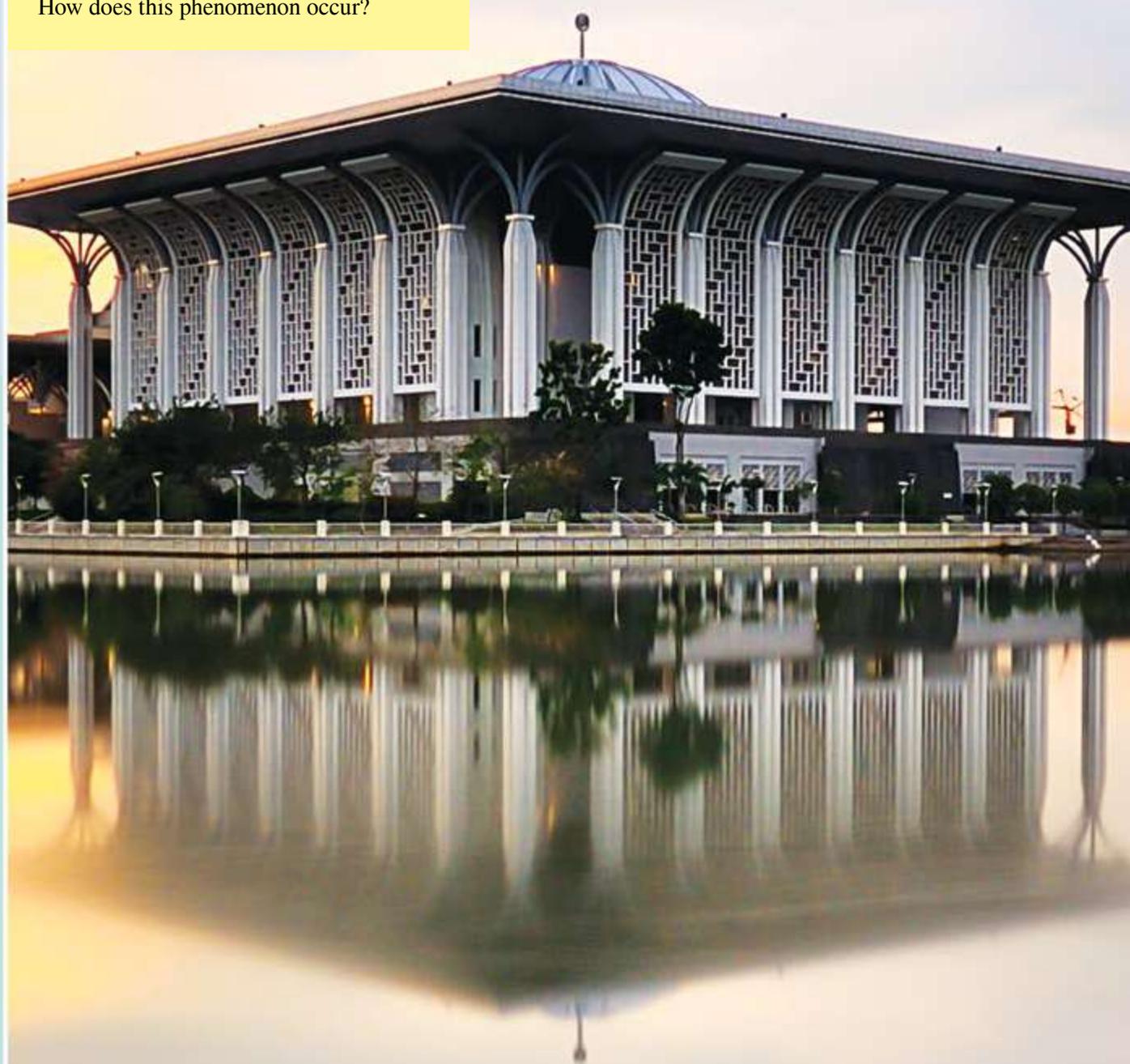
- 11.1 Transformations
- 11.2 Translation
- 11.3 Reflection
- 11.4 Rotation
- 11.5 Translation, Reflection and Rotation as an Isometry
- 11.6 Rotational Symmetry



### WORD LINK

- |                                |                                    |
|--------------------------------|------------------------------------|
| • Object                       | • <i>Objek</i>                     |
| • Image                        | • <i>Imej</i>                      |
| • Clockwise                    | • <i>Ikut arah jam</i>             |
| • Anticlockwise                | • <i>Lawan arah jam</i>            |
| • Isometry                     | • <i>Isometri</i>                  |
| • Congruency                   | • <i>Kekongruenan</i>              |
| • Orientation                  | • <i>Orientasi</i>                 |
| • Axis                         | • <i>Paksi</i>                     |
| • Centre of rotation           | • <i>Pusat putaran</i>             |
| • Reflection                   | • <i>Pantulan</i>                  |
| • Transformation               | • <i>Penjelmaan</i>                |
| • Translation                  | • <i>Translasi</i>                 |
| • Symmetry                     | • <i>Simetri</i>                   |
| • Vector                       | • <i>Vektor</i>                    |
| • Rotation                     | • <i>Putaran</i>                   |
| • Rotational symmetry          | • <i>Simetri putaran</i>           |
| • Order of rotational symmetry | • <i>Peringkat simetri putaran</i> |

Tuanku Mizan Zainal Abidin Mosque was built on 5 April 2004. The mosque is adjacent to the beautiful Putrajaya lakes and clear water. The image of the picture below shows a transformation on the lake. How does this phenomenon occur?



### WHY STUDY THIS CHAPTER?

- Knowledge in the field of transformation can be applied in the manufacturing and fashion design industries. Vehicle designs such as motorcycles, cars and planes require the design of symmetrical objects. Meanwhile, fashion designers will produce different patterns in each of their designs.

**CREATIVE ACTIVITY**

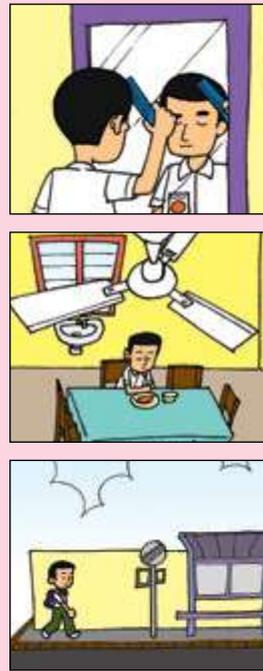
**Aim:** Identifying characteristics of transformation

**Material:** Extracts of stories

Every day before going to school, Akmal brushes his hair in front of the mirror and keeps it neat. While enjoying breakfast, he will sit under a ceiling fan to avoid sweating. After completing breakfast, Akmal walks from his home to the bus stop to go to school.

**Steps:**

- In groups, discuss the actions of:
  - Akmal in front of the mirror.
  - the rotating ceiling fan.
  - Akmal's walk from his home to a bus stop.
- Does the actions change the appearance of Akmal and the fan blades? From these situations, what do you understand about the meaning of transformation in Akmal's daily life?



Transformation is the process of changing the direction, orientation or size of an object's image through translation, reflection and rotation. The image produced by isometric transformation is congruent.

**11.1 Transformations**

**11.1.1 Transformation in the shapes, sizes, directions and orientations of an object**

Transformation involves the transfer of a position on a plane.

**COGNITIVE STIMULATION**

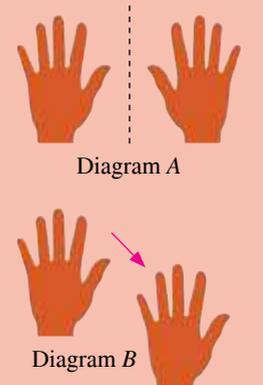


**Aim:** Identifying the transformation through position, size and appearance

**Material:** Manila card, water colour and flashlight

**Steps:**

- Dip both sides of your palms into the water colour and press the palms onto the manila card side by side as shown in Diagram A.
- When the right palm is coloured, press your right palm twice side by side with one side a little lower as shown in Diagram B.



**LEARNING STANDARD**

Describe the changes of shapes, sizes, directions and orientations of an object under a transformation, and hence explain the idea of one-to-one correspondence between points in a transformation.

- Repeat step 2 but the second impression is in a twist as if the hands are waving as shown in Diagram C.
- Direct the flashlight to the palm of your hand and notice the resulting shade on the board. Move the flashlight forward and backwards to see the size of the shadow.



Diagram C

**Discussion:**

Based on the activities above, what is the conclusion from the orientation of the movement, when

- the position of palms are side-by-side.
- one palm is positioned lower than the other.
- the position of the palms are in a waving pose.
- the size of the shadow of your palm varies.

**THINK SMART**

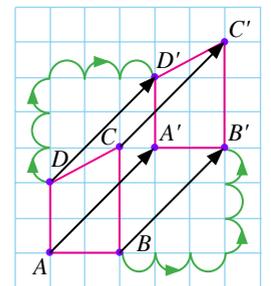


Is the shadow an image?

Before transformation takes place, the original figure is called the **object**. After the transformation, the shape is called the **image**. Transformation is a match of a point on a plane. When the object moves in a transformation, each point of the object follows the same pattern of movement.

Transformation is a movement with a certain orientation and match without changing the shape.

The diagram shows is the movement of an object  $ABCD$  into the image  $A'B'C'D'$  with a movement of three units to the right and three units upwards.



**EXAMPLE 1**

Which of the following shows a transformation and why?

- 
- 
- 
- 

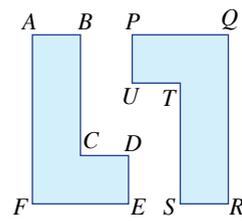
**Solution:**

- Transformation because it does not change the shape.
- Transformation because it only changes position and does not change shape.
- Not a transformation due to a change in appearance.
- Not a transformation due to a change in appearance.

**EXAMPLE 2**

The diagram shows  $ABCDEF$  as an object, while  $PQRSTU$  is an image. State the image of

- (a) point  $C$                       (b) the line  $AB$                       (c)  $\angle BCD$



**Solution:**

- (a) The image of point  $C$  is  $T$  because of the same shape, but the position is different.  
 (b) The image of the line  $AB$  is  $RS$ . This is because the image has the same length.  
 (c) The image for  $\angle BCD$  is  $\angle STU$  because  $\angle BCD$  is of the same size as  $\angle STU$ .

**11.1.2 Congruence**

**COGNITIVE STIMULATION**

**Aim:** Identifying congruency  
**Materials:** Blue and red coloured paper, ruler, protractor and scissors

**Steps:**

- In small groups of 4 to 5, students are required to cut out two triangles.
- Students  $A$  and  $B$  will form triangles using blue coloured paper with measurements of 5 cm, 8 cm and 11 cm.
- Students  $C$  and  $D$  will cut out triangles of the same size using red coloured paper.
- Student  $E$  will combine both triangles.
- Students will measure angles of the triangle using a protractor.

**Discussion:**

- Provide the properties that can be obtained from both forms of the triangle.
- If congruence is the similarity of shape and size, do the triangles meet the criteria of being congruent?

Two objects are congruent if both have the same shape and size, regardless of the orientation of the movement.

**LEARNING STANDARD**

Explain the idea of congruency in transformations.

**QR CODE**

Scan the QR Code or visit [http://rimbunanilmu.my/mat\\_t2e/ms210](http://rimbunanilmu.my/mat_t2e/ms210) to view video on congruency.

**THINK SMART**

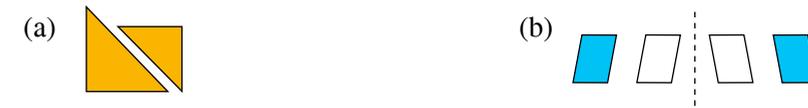
Look at the object above. Are the two objects congruent? If the weight of the two objects is equal, does the number of coins stored in the savings box have the same value? Is that what is meant by congruence?

Regular tools used to determine congruency are rulers, protractors, compasses and tracing paper. You can use these tools to explore the nature of congruency.

Study the 20 sen and 10 sen coins. Can you distinguish them from the shape of the coin? Coins that are round are similar. Do you agree with this statement? All 10 sen coins are congruent, while 20 sen and 10 sen coins are similar but not congruent.

**EXAMPLE 3**

Which of these pairs are congruent? State why.



**Solution:**

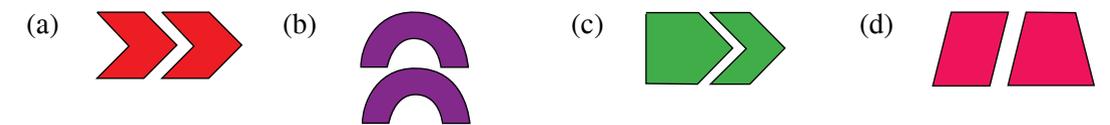
- (a) Not congruent because the size is not the same.  
 (b) Congruent as they are of the same size, although the position and orientation are not the same.

**DO YOU KNOW?**  
 Orientation is something to do. For example, clockwise, counter-clockwise, left and right.

**TIPS**  
 Congruents are the same form and size.

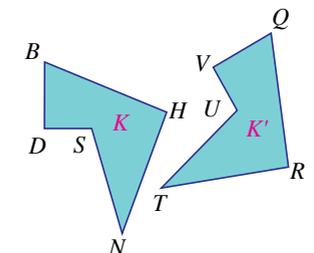
**SELF PRACTICE 11.1**

1. Which of the following diagram does not show a transformation?

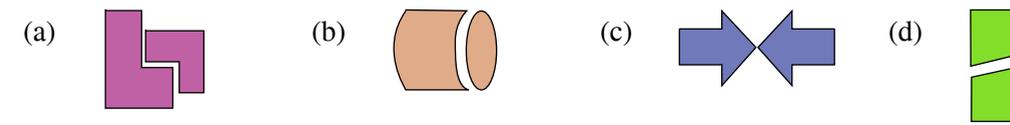


2.  $K'$  is the image of  $K$  under a transformation. Identify

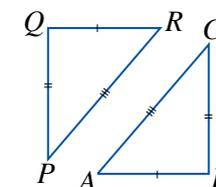
- (a) the image of vertex  $N$     (b) the image of  $BH$     (c) the image of  $\angle SDB$



3. Identify congruent pairs and state why.



4. The diagram below are two congruent triangles. Complete the table below with matching lines and equal angles.



Triangle	Side	Side	Angle	Angle
$PQR$	$QP$		$\angle PQR$	
$CBA$		$AB$		$\angle CAB$

## 11.2 Translation

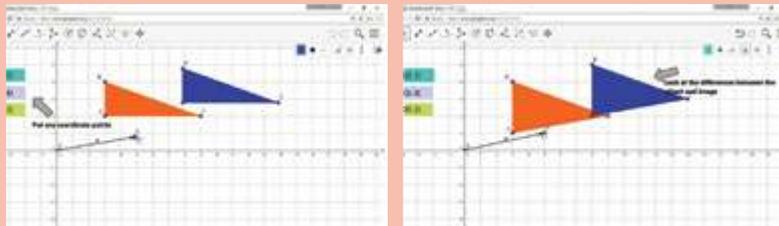
### 11.2.1 Translation

**LEARNING STANDARD**  
Recognise translation.

**COGNITIVE STIMULATION**



**Aim:** Identifying features of translation  
**Material:** Dynamic geometry software  
**Steps:**



1. Open the file MS212.
2. You can explore any coordinates for  $A$ ,  $B$  and  $C$ .
3. Observe the resulting blue image after the point is changed.
4. Movement of the image depends on the assigned arrow  $E$ . You can also move the blue slider to see the movement of the image.

**Discussion:**

- (i) What conclusions can be made from the exploration activities above?
- (ii) How does the features of the image change when the coordinate value at the point of the object changes?

Translation is the transfer of all points on a plane in the same direction and through the same distance.

With a translation, objects and images have the same shape, size and orientation.

**EXAMPLE 4**

Identify which diagram shows translation. Justify.



**Solution:**

- (a) Translation because the shape, size and orientation are the same.
- (b) Not a translation because the orientation is not the same.

**QR CODE**  
Scan the QR Code or visit [http://rimbunanilmu.my/mat\\_t2e/ms212](http://rimbunanilmu.my/mat_t2e/ms212) to view video on translation.

### 11.2.2 Representation of translation in the form of vector

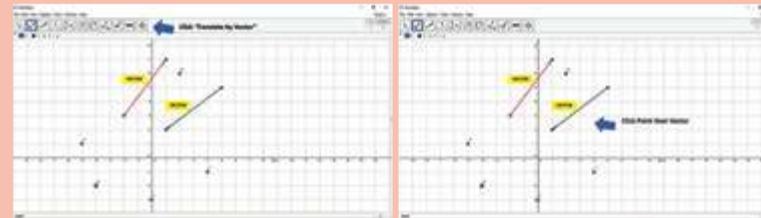
Vector of translation is a movement that has direction and magnitude. This vector is also represented by an arrow. Determining translation is based on the value and direction of a vector.

**LEARNING STANDARD**  
Describe translation by using various representations including vector form.

**COGNITIVE STIMULATION**



**Aim:** Exploring vectors of translations  
**Material:** Dynamic geometry software



**Steps:**

1. Open the file MS213. Follow the instruction of the video.
2. Given Vector 1 and Vector 2, determine the coordinate images of  $A$ ,  $B$ ,  $C$ ,  $D$  and  $E$ .
3. You can select any vector to specify the image for those points.
4. Complete the table below.

Coordinate of Object	Total unit of movements Left/Right $a$	Total unit of movements Upwards/Downwards $b$	Form of vector $\begin{pmatrix} a \\ b \end{pmatrix}$	Coordinate of image
$A ( \quad )$				$A' ( \quad )$
$B ( \quad )$				$B' ( \quad )$
$C ( \quad )$				$C' ( \quad )$
$D ( \quad )$				$D' ( \quad )$
$E ( \quad )$				$E' ( \quad )$

**Discussion:**

- (i) Is the direction of motion of the object equal to the direction of movement of the arrow?
- (ii) How do you write unit of movement for vector of translation

$$\begin{pmatrix} \text{Movement right/left} \\ \text{Movement upwards/downwards} \end{pmatrix} ?$$

**TIPS**  
  
A coordinate system with x and y axes. A vector  $\vec{OP}$  is shown starting from the origin  $O$  and pointing to point  $P$ .  
 $\vec{OP}$  is known as a vector.

**QR CODE**  
Scan the QR Code or visit [http://rimbunanilmu.my/mat\\_t2e/ms213](http://rimbunanilmu.my/mat_t2e/ms213) to view a demonstration on vector of translation.

**FLASHBACK**  
Coordinate  $(x, y)$ . The value of  $x$  is written first followed by the value of  $y$ .

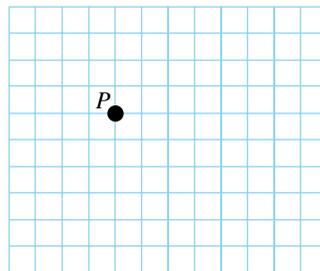
Translation can be described by specifying the direction and distance of the movement. Translation can be described using two methods,

- (a) Direction of movement: to the right, left, upwards, downwards.
- Distance of movement: number of units.

- (b) Write in vector form  $\begin{pmatrix} a \\ b \end{pmatrix}$

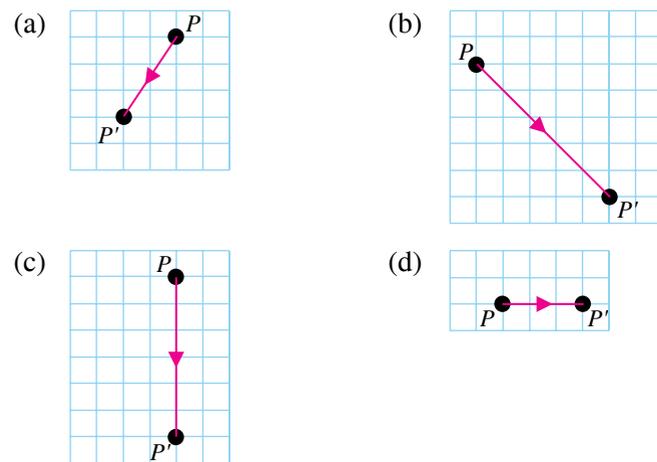
**EXAMPLE 5**

Determine the position of point  $P$  in the given diagram for the translation description and draw the vector translation.



- (a) Point  $P$  moves 2 units to the left and 3 units downwards.
- (b) Point  $P$  moves 5 units to the right and 5 units downwards.
- (c) Point  $P$  moves 6 units downwards.
- (d) Point  $P$  moves 3 units to the right.

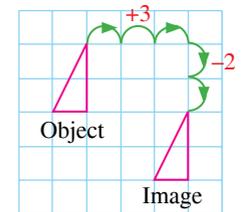
**Solution:**



**DO YOU KNOW ?**

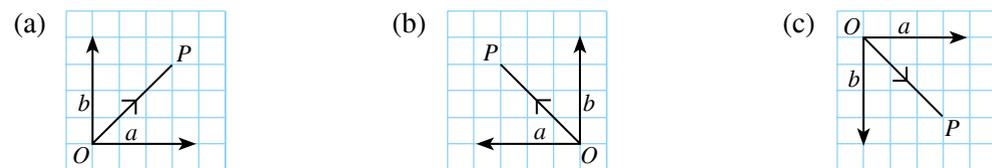
Vector translation is written as  $\begin{pmatrix} a \\ b \end{pmatrix}$ .  
 $a$  represents a movement parallel to the  $x$ -axis which is positive if the object moves to the right and is negative if the object moves to the left.

$b$  represents a movement parallel to the  $y$ -axis which is positive if the object moves upwards and is negative if the object moves downwards. Therefore, the translation of the image generated by the object in the diagram below is  $\begin{pmatrix} 3 \\ -2 \end{pmatrix}$ .



**EXAMPLE 6**

Determine the vector translation of  $OP$  in the diagram below.

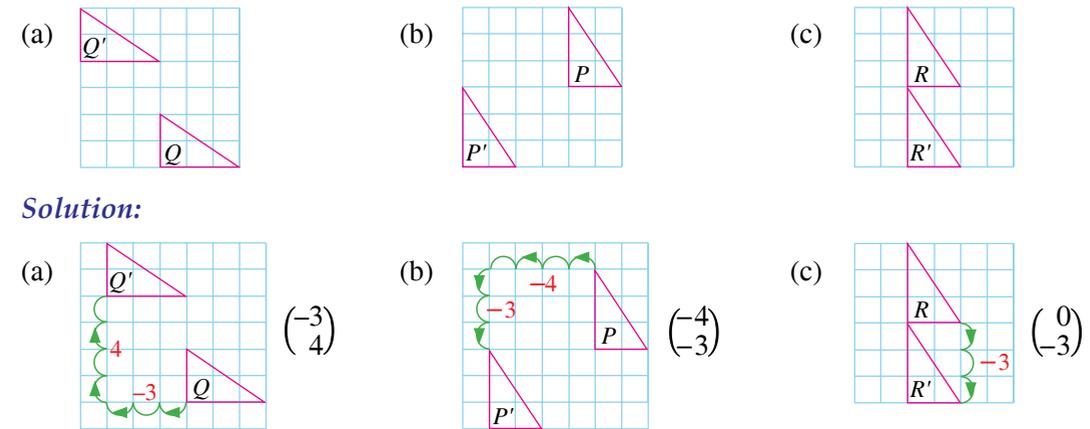


**Solution:**

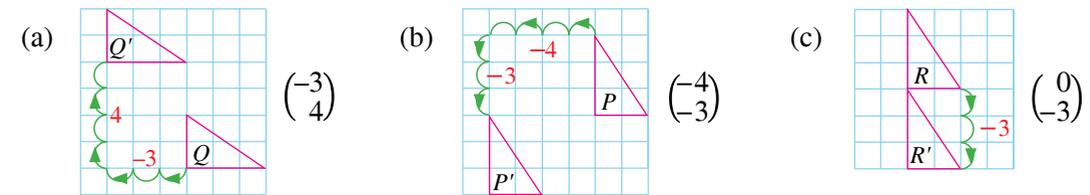
- (a)  $\begin{pmatrix} 3 \\ 3 \end{pmatrix}$
- (b)  $\begin{pmatrix} -3 \\ 3 \end{pmatrix}$
- (c)  $\begin{pmatrix} 3 \\ -3 \end{pmatrix}$

**EXAMPLE 7**

Determine the translation for the diagrams below.



**Solution:**



**11.2.3 Image and object under a translation**

**COGNITIVE STIMULATION**



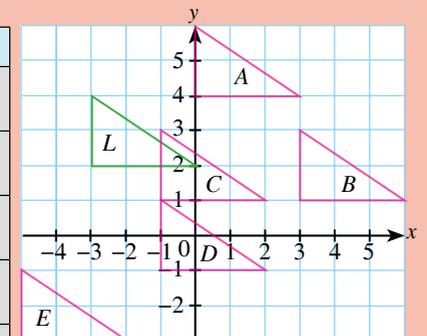
**Aim:** Identifying the image of an object in a translation

**Material:** Worksheet

**Steps:**

- Study the diagram on the right. Identify the image of the object  $L$  for the translation provided.
- Complete the table.

Translation	Image
$\begin{pmatrix} 3 \\ 2 \end{pmatrix}$	
$\begin{pmatrix} 2 \\ -1 \end{pmatrix}$	
$\begin{pmatrix} 6 \\ -1 \end{pmatrix}$	
$\begin{pmatrix} 2 \\ -3 \end{pmatrix}$	
$\begin{pmatrix} -2 \\ -5 \end{pmatrix}$	



**Discussion:**

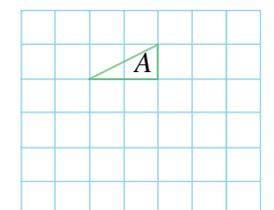
- Compare the length of the sides and the value of angle for the object as well as the image.
- Conclusion on features of translation.

The image of an object in a translation will always be the same in terms of shape, size and orientation.

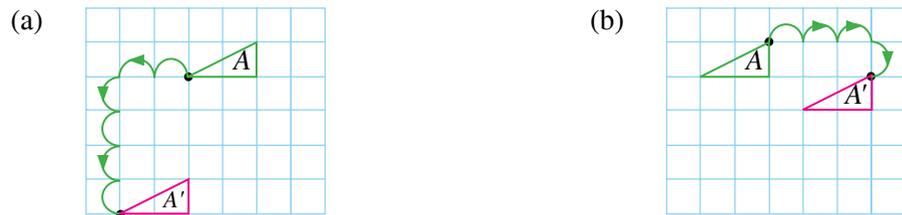
**EXAMPLE 8**

Draw the image of object  $A$  in the diagram with the translation

- (a)  $\begin{pmatrix} -2 \\ -4 \end{pmatrix}$
- (b)  $\begin{pmatrix} 3 \\ -1 \end{pmatrix}$

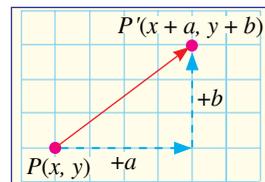


**Solution:**



**Determining the coordinates of image when coordinates of the object is given**

To locate the **image** with translation  $\begin{pmatrix} a \\ b \end{pmatrix}$ , the coordinates of the object  $P(x, y)$  will be mapped to  $P'(x + a, y + b) = P'(x' y')$



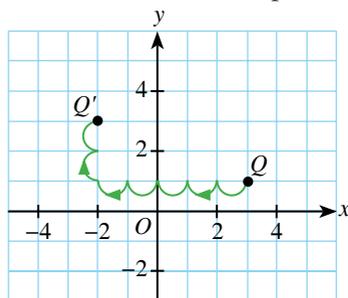
**EXAMPLE 9**

Determine the coordinates of the image for point  $Q(3, 1)$  with translation  $\begin{pmatrix} -5 \\ 2 \end{pmatrix}$ .

**Solution:**

**Method 1:** Draw on a Cartesian plane

**Method 2:** Calculate

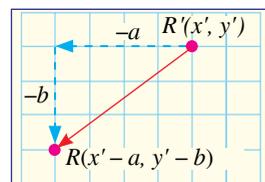


$$\begin{aligned} \text{i. } Q(3, 1) &\rightarrow Q'(3 + (-5), 1 + 2) \\ &= (-2, 3) \\ \text{ii. } \begin{pmatrix} 3 \\ 1 \end{pmatrix} + \begin{pmatrix} -5 \\ 2 \end{pmatrix} &= \begin{pmatrix} -2 \\ 3 \end{pmatrix} \end{aligned}$$

Therefore, image for  $Q(3, 1)$  is  $(-2, 3)$ .

**Specifying coordinates of object when coordinates of the image is given**

To locate an **object** with translation  $\begin{pmatrix} a \\ b \end{pmatrix}$ , coordinates of object  $R(x', y')$  will be mapped to  $R(x' - a, y' - b) = R(x, y)$



**EXAMPLE 10**

Object  $A$  undergoes the translation  $\begin{pmatrix} 3 \\ -2 \end{pmatrix}$ . Determine the coordinates of object  $A$  if the image  $A'$  is as follows.

- (a)  $(-6, 1)$                       (b)  $(9, 0)$

**Solution:**

- (a) Coordinate  $A = [-6 - 3, 1 - (-2)] = (-9, 3)$     (b) Coordinate  $A = [9 - 3, 0 - (-2)] = (6, 2)$

**TIPS**

Alternative methods  
 $\begin{pmatrix} a \\ b \end{pmatrix} + \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} a+x \\ b+y \end{pmatrix}$   
 $\begin{pmatrix} a \\ b \end{pmatrix} - \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} a-x \\ b-y \end{pmatrix}$

To change from vector form to ordered pairs.

$$\begin{pmatrix} x \\ y \end{pmatrix} \rightarrow (x, y)$$

**TIPS**

Alternative methods  
 (a)  $\begin{pmatrix} x \\ y \end{pmatrix} + \begin{pmatrix} 3 \\ -2 \end{pmatrix} = \begin{pmatrix} -6 \\ 1 \end{pmatrix}$   
 $\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} -6 \\ 1 \end{pmatrix} - \begin{pmatrix} 3 \\ -2 \end{pmatrix}$   
 $= \begin{pmatrix} -9 \\ 3 \end{pmatrix}$   
 (b)  $\begin{pmatrix} x \\ y \end{pmatrix} + \begin{pmatrix} 3 \\ -2 \end{pmatrix} = \begin{pmatrix} 9 \\ 0 \end{pmatrix}$   
 $\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 9 \\ 0 \end{pmatrix} - \begin{pmatrix} 3 \\ -2 \end{pmatrix}$   
 $= \begin{pmatrix} 6 \\ 2 \end{pmatrix}$

**Defining vector translation when position of image and object is given**

**EXAMPLE 11**

Given  $P'(3, 6)$  is the image of  $P(2, 9)$ , determine the translation.

**Solution:**

$$\begin{aligned} \text{Vector translation} &= \begin{pmatrix} x' - x \\ y' - y \end{pmatrix} \\ &= \begin{pmatrix} 3 - 2 \\ 6 - 9 \end{pmatrix} \\ &= \begin{pmatrix} 1 \\ -3 \end{pmatrix} \end{aligned}$$

**TIPS**

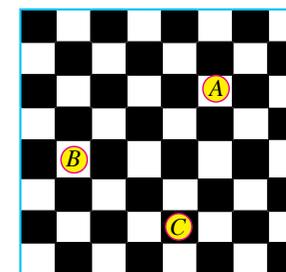
Given object  $(x, y)$  and the image  $(x', y')$ . Vector translation is  $\begin{pmatrix} x' - x \\ y' - y \end{pmatrix}$

**11.2.4 Solving problems**

**EXAMPLE 12**

Agnes moves her checker piece from  $A$  to  $B$  and then to  $C$ . Indicate her movement in the form of translation of the checker piece marked

- (a)  $A$  to  $B$ .  
 (b)  $B$  to  $C$ .



**Solution:**

**Understanding the problem**

Translation movement to the left or right, upwards or downwards.

**Planning the strategy**

- (a) 4 units to the left, 2 units downwards.  
 (b) 3 units to the right, 2 units downwards.

**Conclusion**

- (a) Therefore, vector translation  $A$  to  $B$  is  $\begin{pmatrix} -4 \\ -2 \end{pmatrix}$ .  
 (b) Therefore, vector translation  $B$  to  $C$  is  $\begin{pmatrix} 3 \\ -2 \end{pmatrix}$ .

**Implementing the strategy**

Using  $\begin{pmatrix} a \\ b \end{pmatrix}$   
 (a)  $\begin{pmatrix} -4 \\ -2 \end{pmatrix}$   
 (b)  $\begin{pmatrix} 3 \\ -2 \end{pmatrix}$

**LEARNING STANDARD**

Solve problems involving translation.

**THINK SMART**

A shot putt can be thrown in two different styles. Does the force affect the direction of the throw? Discuss the relationship with the concept of translation.

**TIPS**

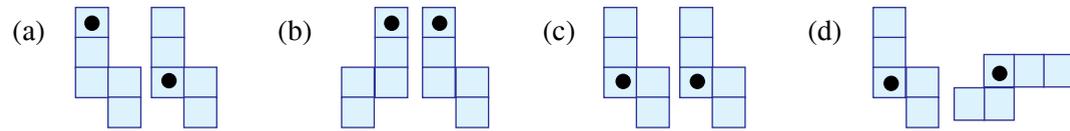
Translation movements always begin from the left or the right, then upwards or downwards.

**THINK SMART**

Object	Translation	Image
$A(-3, 4)$	$\begin{pmatrix} 2 \\ -3 \end{pmatrix}$	
$B(7, 9)$	$\begin{pmatrix} -4 \\ -5 \end{pmatrix}$	
	$\begin{pmatrix} -3 \\ 2 \end{pmatrix}$	$P'(-5, 2)$
	$\begin{pmatrix} 0 \\ 5 \end{pmatrix}$	$Q(4, 1)$

**SELF PRACTICE** 11.2

1. Which of the following pairs is a translation?



2. Determine the coordinates of the image for the object  $(5, -3)$  under translation

- (a)  $\begin{pmatrix} 2 \\ 2 \end{pmatrix}$       (b)  $\begin{pmatrix} 4 \\ 6 \end{pmatrix}$       (c)  $\begin{pmatrix} -3 \\ -1 \end{pmatrix}$       (d)  $\begin{pmatrix} -2 \\ -5 \end{pmatrix}$

3. Determine the coordinates of the object for the image  $(-1, -4)$  under translation

- (a)  $\begin{pmatrix} 1 \\ 4 \end{pmatrix}$       (b)  $\begin{pmatrix} -3 \\ 5 \end{pmatrix}$       (c)  $\begin{pmatrix} -8 \\ 0 \end{pmatrix}$       (d)  $\begin{pmatrix} 7 \\ 2 \end{pmatrix}$

4. State the vector translation for the following points.

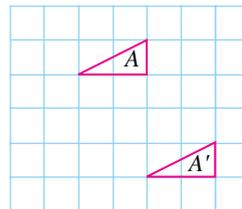
- (a)  $A(1, 2), A'(3, 6)$       (b)  $B(5, 7), B'(-1, -1)$   
 (c)  $C(4, 4), C'(8, 0)$       (d)  $D(6, 4), D'(3, -3)$

5. The object  $L(1, 4)$  is mapped to a position  $L'(3, -5)$  after a translation. Determine the position of an image or object with the same translation for the coordinate points below.

- (a)  $A(3, 1)$       (b)  $S'(4, -2)$   
 (c)  $J'(5, -6)$       (d)  $D(-7, -8)$

6. Using the same orientation with the diagram on the right, determine the coordinate of the image for the following.

- (a)  $(-1, -4)$       (b)  $(5, -5)$



**LEARNING STANDARD**  
 Recognise a reflection.

**11.3 Reflection**

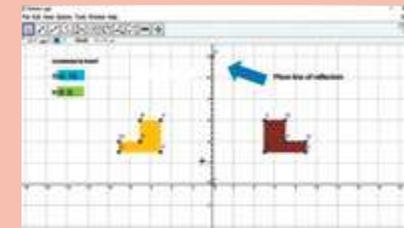
**11.3.1 Reflection**

When Preveena looks at the mirror while brushing her hair, she will be able to see her appearance in the mirror. The image of Preveena in the mirror is the result of reflection. Reflection is a transformation that occurs when all the points on the plane are reversed in the same plane on a line. The line is called the **axis of reflection**.

**COGNITIVE STIMULATION**

Group

**Aim:** Identifying the features of reflection  
**Material:** Dynamic geometry software



**Steps:**

1. Open the file MS219.
2. Observe the line changes when point  $G$  and  $H$  change.
3. See the changes that occur to the image.

**Discussion:**

- (i) What do you understand about the axis of reflection?
- (ii) What will happen to the image in yellow when the axis for line  $GH$  is moved?
- (iii) From the activity, what do you understand about characteristic of symmetry?

**QR CODE**

Scan the QR Code or visit [http://rimbunanilmu.my/mat\\_t2e/ms219](http://rimbunanilmu.my/mat_t2e/ms219) to view a video on features of reflection.



Reflections have certain features which are

- (i) the object and the image are on the opposite sides of the axis of reflection.
- (ii) the object and its image have the same perpendicular distance from the axis of reflection.
- (iii) the shape and size of the image is the same as the object, but the orientation is reversed.
- (iv) the image of a point on the axis of reflection is the point itself.

Symmetry is a type of match in terms of size and shape between one side or one part in the direction of an object. The line of symmetry is a line that divides a form into two congruent parts. This line also divides the perpendicular lines that connect all points which join the object and the image. The line of symmetry is the axis of reflection of the image and object.

**TIPS**

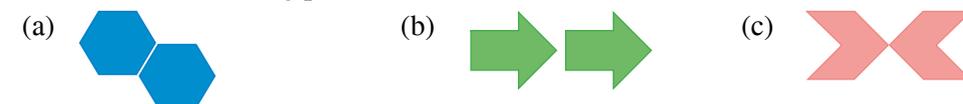
All the points located on the axis of reflection do not change their position during a transformation.

The properties of an image of a reflection are

- (a) the image is the same shape and the same size as the object.
- (b) the image has different orientations, inverted sides and forms mirror images with one another.

**EXAMPLE 13**

Which of the following patterns show the orientation of a reflection?

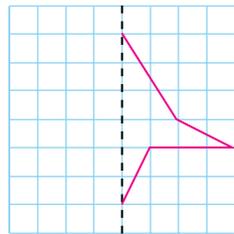


**Solution:**

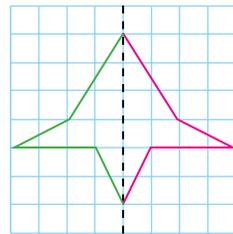
- (a) Yes      (b) No      (c) Yes

**EXAMPLE 14**

Complete the sketch below.



*Solution:*



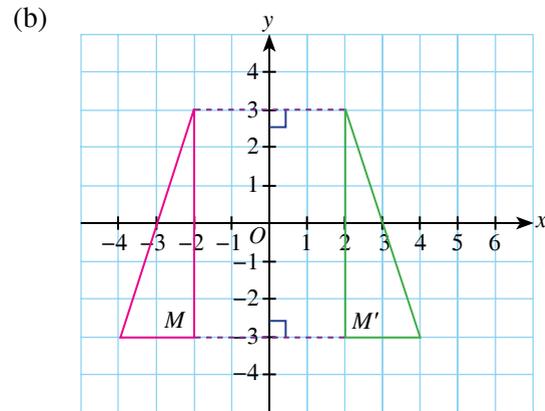
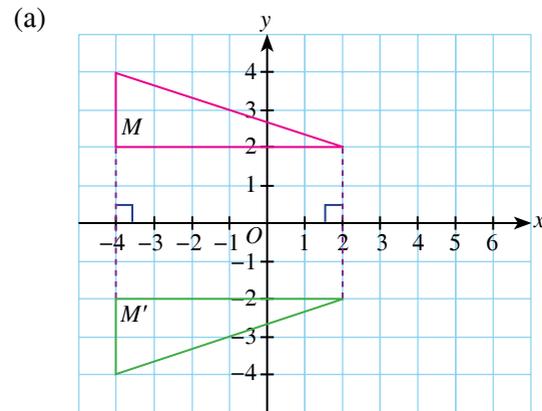
**THINK SMART**

Object	Transformation	Image
(3, 4)	Reflection in $x$ -axis	
(-3, -5)	Reflection in $x$ -axis	
(3, 4)	Reflection in $y$ -axis	
(-3, -5)	Reflection in $y$ -axis	

**11.3.2 Describing reflection**

**EXAMPLE 15**

In the following Cartesian diagram, the triangle  $M'$  is the image of the triangle  $M$  under a reflection. Describe the reflection.



*Solution:*

(a) The object  $M$  is reflected in the  $x$ -axis.

(b) The object  $M$  is reflected in the  $y$ -axis.

**11.3.3 Image of an object**

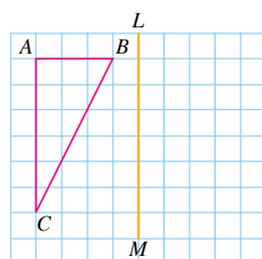
**EXAMPLE 16**

Draw the image of the triangle  $ABC$  with a reflection on line  $LM$ .

*Solution:*

Step 1: Select any vertex and construct a perpendicular line from the vertex to the line  $LM$  and extend beyond the axis of reflection.

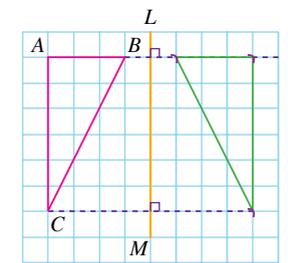
Step 2: Draw parallel lines to all other vertices.



**LEARNING STANDARD**

Determine the image and object under a reflection.

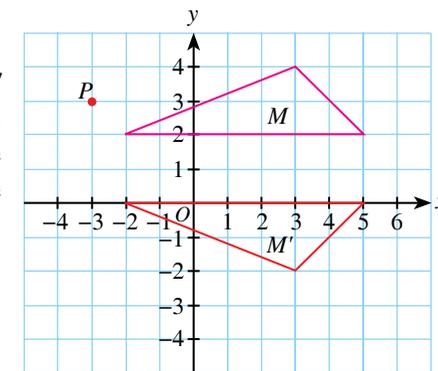
Step 3: Determine the distance of each vertex from the axis of reflection and mark an equal distance from the axis to the same line. Do the same for all vertices.



**11.3.4 Solving problems**

**EXAMPLE 17**

In the diagram on the right,  $M'$  is the image of  $M$  in an axis of reflection. Determine the coordinates of  $P'$  under the same axis of reflection.



*Solution:*

**Understanding the problem**

$M'$  is an image of  $M$ . Determine the axis of reflection.

**Planning the strategy**

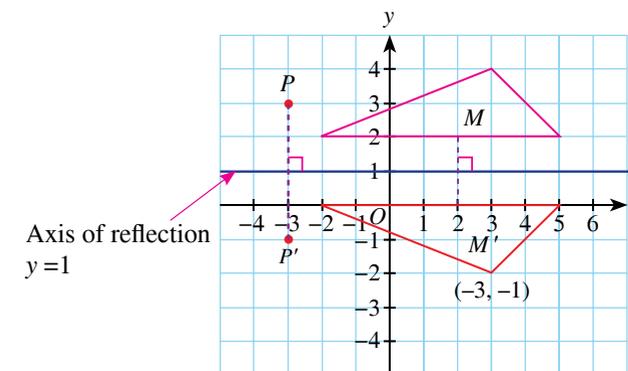
Determine the pair of vertices for the images and objects. Draw a perpendicular line for both pairs of vertices. Construct or determine the bisectors for the perpendicular line.

**Conclusion**

Axis of reflection  $y = 1$ , coordinate of  $P'$  are  $(-3, -1)$ .

**Implementing the strategy**

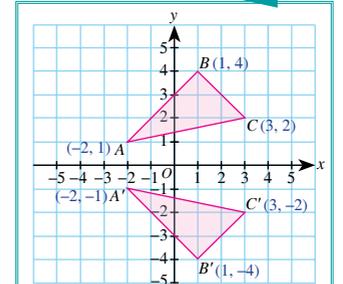
Axis of reflection is at  $y = 1$ . Use this axis to locate  $P'$ .



**LEARNING STANDARD**

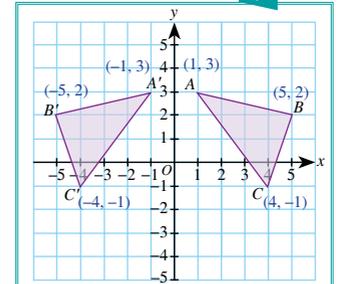
Solve problems involving reflection.

**DO YOU KNOW?**



Reflection of the point  $(x, y)$  in  $x$ -axis is the point  $(x, -y)$

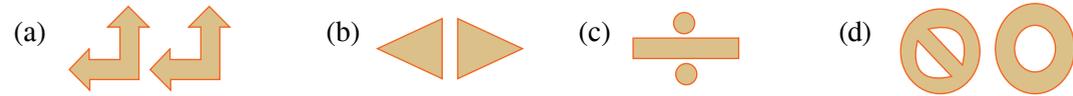
**DO YOU KNOW?**



Reflection of the point  $(x, y)$  in  $y$ -axis is the point  $(-x, y)$

**SELF PRACTICE 11.3**

1. Which of the following is a reflection?



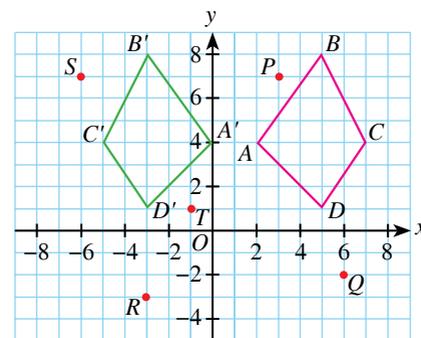
2. Complete the diagram below.



3. Construct an image for the object below under a reflection in the line  $PQ$ .



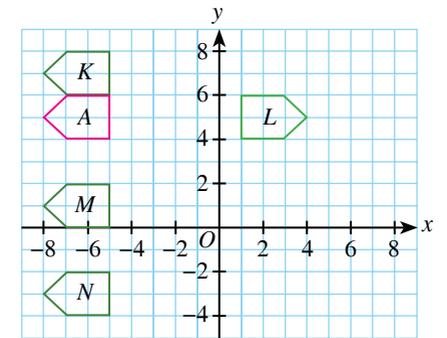
4.  $A'B'C'D'$  is the image for the object  $ABCD$  in an axis of reflection. Determine the coordinate of the image for the points  $P, Q, R$  and  $S$  under the same axis of reflection.



5. Draw the axis of reflection for the following diagrams.



6. Based on the Cartesian plane on the right, describe the mapping of reflection of polygon  $A$  to the polygon  
 (a)  $K$       (b)  $L$       (c)  $M$       (d)  $N$



7. Identify the axis of reflection and describe the representation of reflection for these pair of coordinate points.

- (a)  $A(3, 1)$  and  $A'(-3, 1)$       (b)  $B(-4, 2)$  and  $B'(-4, -2)$
- (c)  $C(5, 6)$  and  $C'(-5, 6)$       (d)  $D(2, 2)$  and  $D'(4, 2)$

8. If  $L(4, 1)$  is mapped to  $L'(4, 5)$  under a reflection, determine

- (a) coordinate of the image for  $(-3, -1)$  in the same axis of reflection.
- (b) coordinate of the object for  $(7, 2)$  under the same reflection.

**11.4 Rotation**

**11.4.1 Rotation**

Take note of the rotating objects around you, such as clocks, ceiling fans and tyres. The hands of a clock makes a full rotation every twelve hours. However the tyre depends on the forward or backward movement. All these movements have a centre of rotation.

**LEARNING STANDARD**  
 Recognise a rotation.

**COGNITIVE STIMULATION**



**Aim:** Recognising rotation

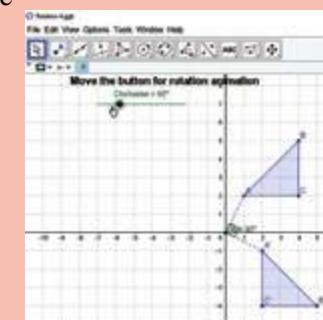
**Material:** Dynamic geometry software

**Steps:**

1. Open the file MS223.
2. Drag the green button and take note of the rotation.
3. Adjust the button to see the object being rotated.

**Discussion:**

- (i) Can you identify the image of the triangle that moves when the angle of rotation is adjusted? What conclusions can you make of the triangle image?
- (ii) What are the properties of the image in the activity above?



**QR CODE**

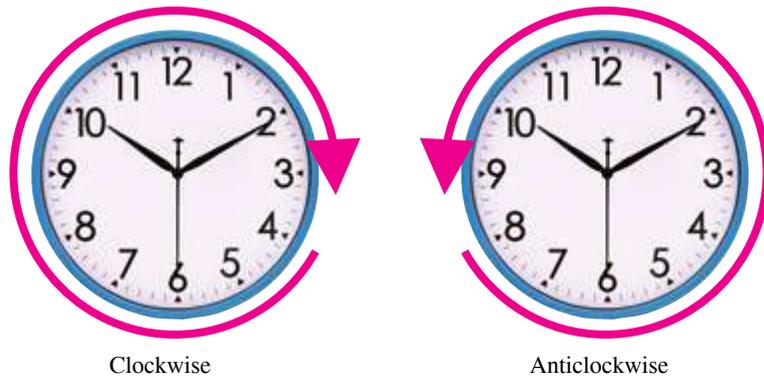
Scan the QR Code or visit [http://rimbunanilmu.my/mat\\_t2e/ms223](http://rimbunanilmu.my/mat_t2e/ms223) to view a video on rotation.

The properties of the image of a rotation:

- (a) The resulting image has the same shape, size and orientation as the object.
- (b) The centre of rotation is a stationary point.
- (c) The distance of all the points of the image to the centre of the rotation is equal to the distance of the object to the centre of the rotation.

### 11.4.2 Rotation in various representations

When we describe a rotation, we need to state the **centre**, the **angle** and the **direction of the rotation** that maps the object to the image.

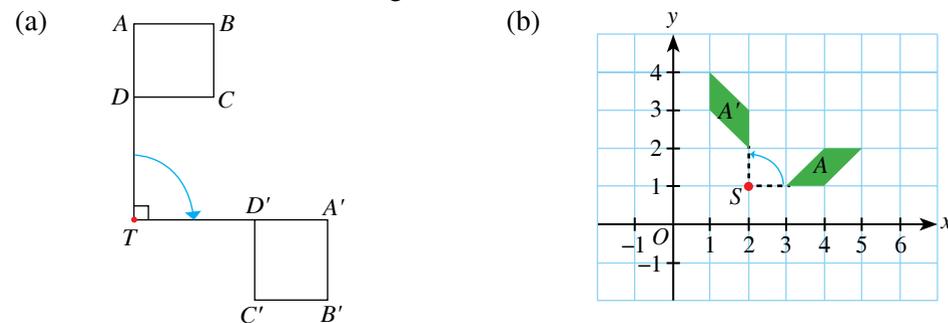


**LEARNING STANDARD**  
Describe a rotation using various representations.

**TIPS**  
The image generated by a rotation of  $180^\circ$  clockwise is equal to the rotation of  $180^\circ$  anticlockwise.

### EXAMPLE 18

Describe the rotation for the diagram below.



**Solution:**

- (a) Clockwise rotation of  $90^\circ$  at point  $T$ .
- (b) Anticlockwise rotation of  $90^\circ$  at point  $S$ .

### 11.4.3 Determining the image and object under a rotation

We can use tracing paper, protractor and compasses to determine the image or object under a rotation.

**LEARNING STANDARD**  
Determine the image and object under a rotation.

### EXAMPLE 19

Determine the image of  $\triangle PQR$  when rotated  $90^\circ$  anticlockwise at point  $M$ .

**Solution:**

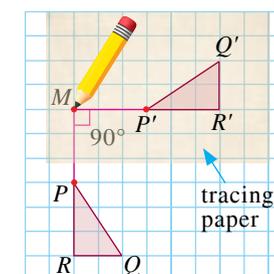
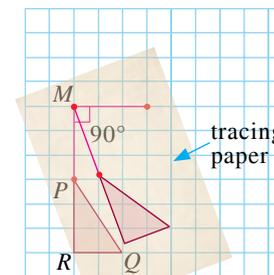
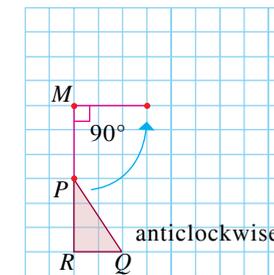
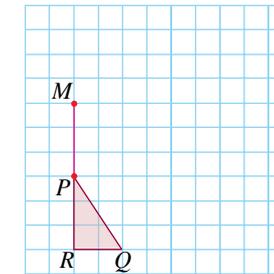
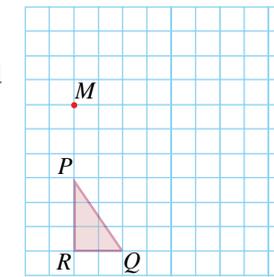
#### Method 1 (Using tracing paper)

Step 1: Draw the line from point  $M$  to point  $P$ .

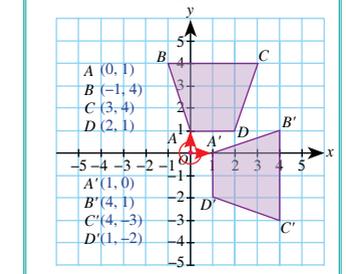
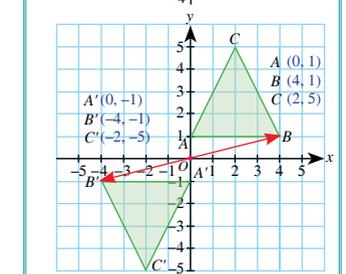
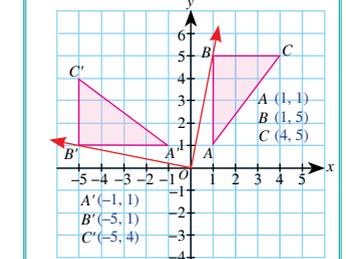
Step 2: Determine the angle of  $90^\circ$  anticlockwise.

Step 3: Redraw the triangle  $PQR$  on tracing paper.

Step 4: Press the tip of a pencil at the point  $M$ , turn the tracing paper  $90^\circ$  anticlockwise.



### DO YOU KNOW?



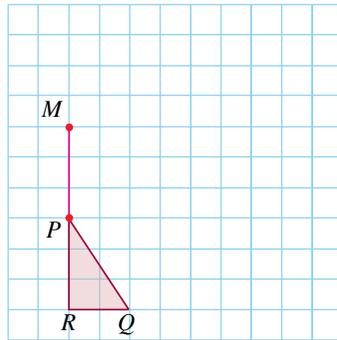
Rotation of $90^\circ$ anticlockwise at origin	$(x, y) \rightarrow (-y, x)$
Rotation of $180^\circ$ at origin	$(x, y) \rightarrow (-x, -y)$
Rotation of $270^\circ$ anticlockwise at origin	$(x, y) \rightarrow (y, -x)$

### THINK SMART

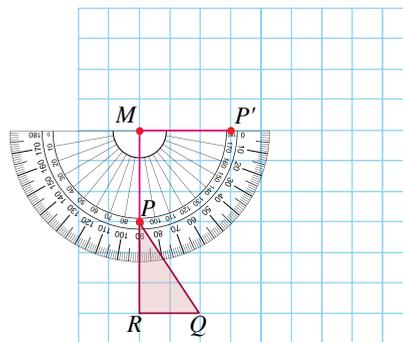
Object	Transformation	Image
(5, 2)	Clockwise rotation of $90^\circ$ at the point (0, 0)	
(-3, 4)	Anticlockwise rotation of $90^\circ$ at the point (2, 1)	
(-4, 7)	Rotation of $180^\circ$ at the point (-1, 3)	

**Method 2 (Using protractor)**

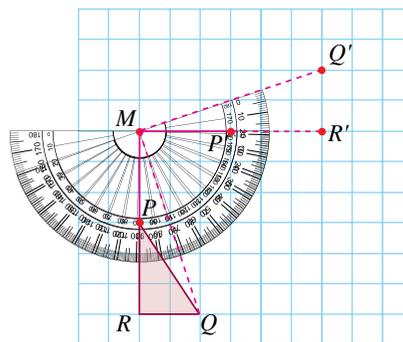
Step 1: Construct the line  $MP$ .



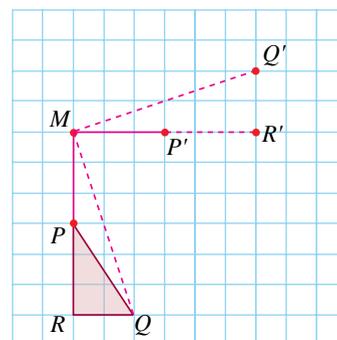
Step 2: Using a protractor, draw a line  $MP'$  in an anticlockwise rotation of  $90^\circ$  with an equal distance to  $MP$ .



Step 3: Repeat step 2 with lines  $MR$  and  $MQ$ .



Step 4: Join all the points  $P'$ ,  $R'$  and  $Q'$  to form a triangle similar to  $PRQ$ .



**THINK SMART**

Object	Transformation	Image
	Clockwise rotation of $90^\circ$ at the point $(-2, 3)$	$(-3, 1)$
	Anticlockwise rotation of $90^\circ$ at the point $(1, 3)$	$(3, 2)$
	Rotation of $180^\circ$ at the point $(-3, 4)$	$(2, 1)$

**TIPS**

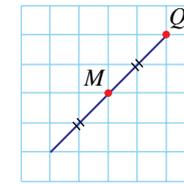
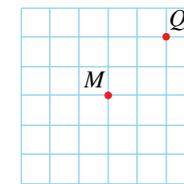
If the question uses a square grid, then you do not have to use protractor for rotations of  $90^\circ$ ,  $180^\circ$  and  $270^\circ$ .

**EXAMPLE 20**

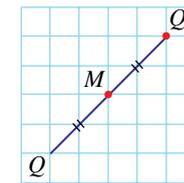
Determine the object for the image point  $Q'$  when rotated  $180^\circ$  clockwise at point  $M$ .

**Solution:**

Step 1: Draw a line to join the points  $M$  and  $Q'$  and extend it to an equal distance to  $MQ'$  in the opposite direction.

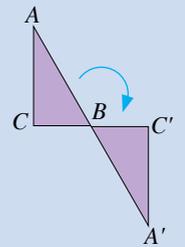


Step 2: Mark point  $Q$  on the extended line with  $MQ = MQ'$ .



**TIPS**

If  $B$  is the centre of rotation, then the position of image  $B$  will not change.



**LEARNING STANDARD**

Solve problems involving rotation.

**11.4.4 Solving problems**

Do you know that if the object and image of a rotation is given, centre, angle and rotational direction can be determined using the method of geometric construction?

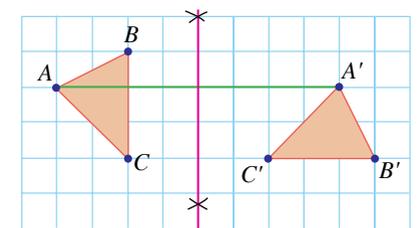
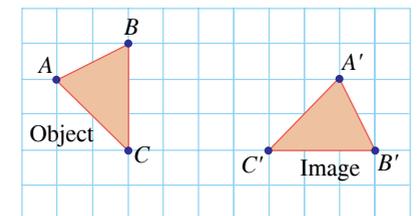
**Determining centre, angle and direction of rotation**

**EXAMPLE 21**

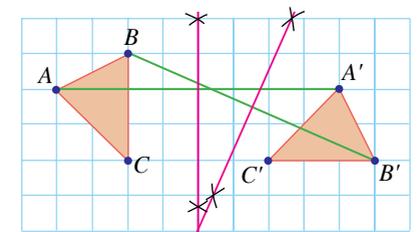
$A'B'C'$  is the image for  $ABC$  after a rotation. Determine the angle, direction and centre under a rotation.

**Solution:**

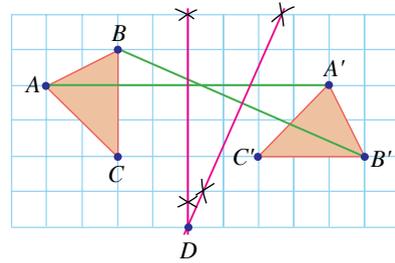
Step 1: Join point  $A$  to  $A'$ . Construct a perpendicular bisector for the line segment  $AA'$ .



Step 2: Repeat step 1 for the lines  $BB'$  or  $CC'$ .

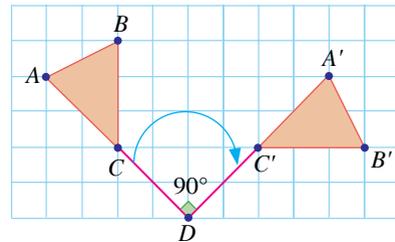


Step 3: The intersection point of two perpendicular lines is the centre of the rotation. Mark the centre of the rotation as  $D$ .



Step 4: Measure the angle  $CDC'$  using a protractor.

Therefore, the image is under rotation of  $90^\circ$  clockwise at point  $D$ .



► Defining coordinates of the image when coordinates of the object are given

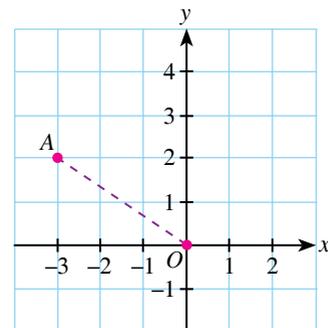
**EXAMPLE 22**

Determine the coordinates of the image of point  $A(-3, 2)$  under a rotation of  $90^\circ$  clockwise at  $O$  origin.

**Solution:**

**Step 1:**

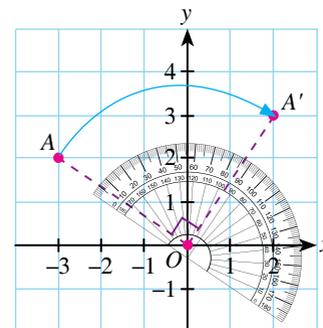
Join the line  $OA$ .



**Step 2:**

Rotate the line  $OA$   $90^\circ$  clockwise at origin  $O$  using a protractor.

From the diagram, the coordinates of the image  $A'$  are  $(2, 3)$ .



► Determining the coordinates of objects when the coordinates of the image is given

**EXAMPLE 23**

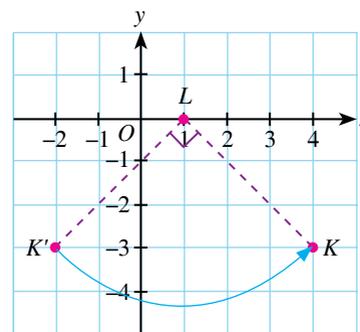
If  $K'(-2, -3)$  is the image of  $K$  with a rotation of  $90^\circ$  clockwise at point  $L(1, 0)$ , state coordinates  $K$ .

**Solution:**

**Step 1:** Reverse the rotation direction to locate the coordinates of the object, that is, the point  $K$ .

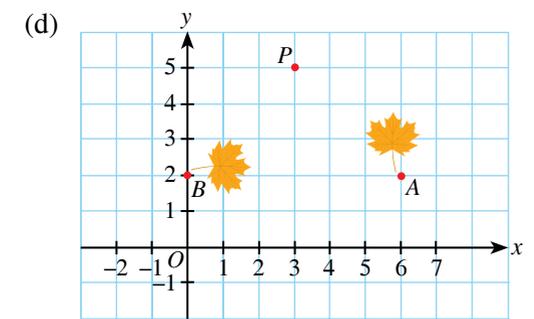
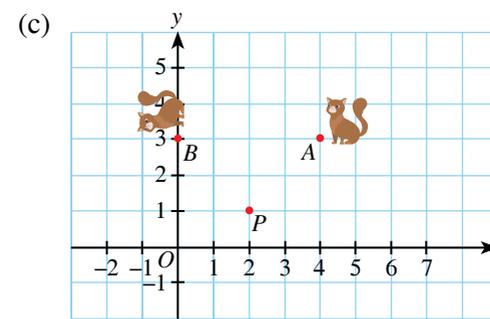
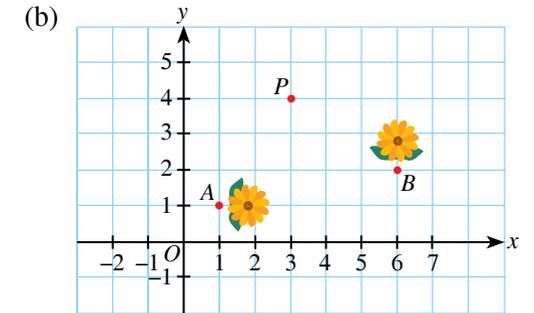
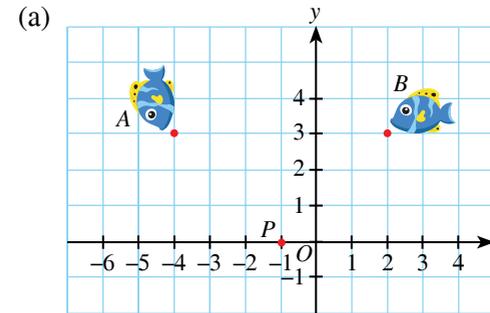
**Step 2:** By using a protractor, rotate the line  $K'L$  at point  $L$ , at  $90^\circ$  anticlockwise.

From the diagram, the coordinates of  $K$  are  $(4, -3)$ .

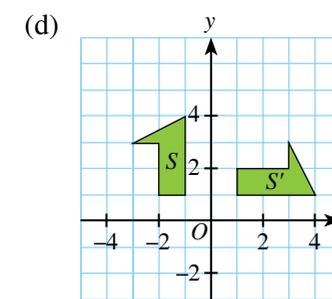
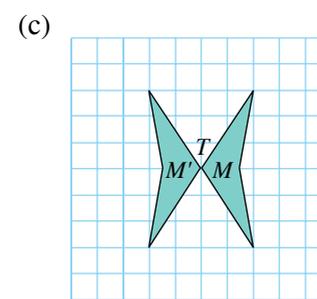
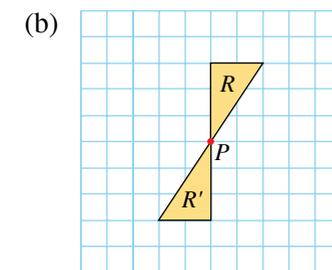
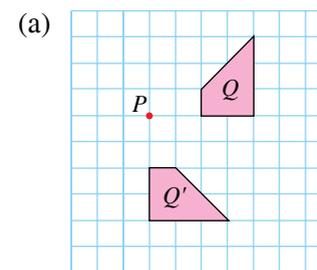


**SELF PRACTICE 11.4**

1. Describe the rotations at centre  $P$  when  $A$  is the object and  $B$  is the image.



2. Describe the rotations that map the object to its image.



3. Draw the image for  $R$  under a rotation for each of the following.

Anticlockwise rotation of  $90^\circ$  at centre  $O$ .

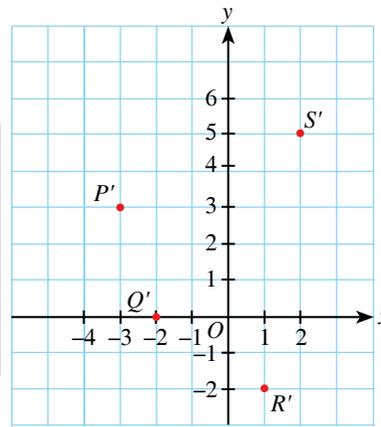


Rotation of  $180^\circ$  at centre  $O$ .



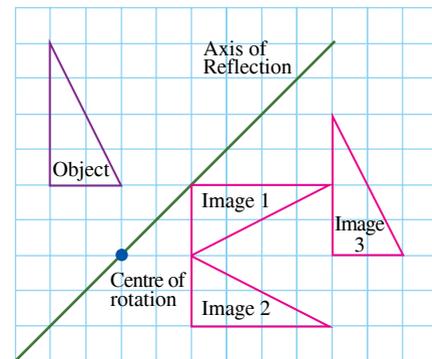
4. Determine the coordinates of objects for the following points at the given rotations below.

Point	Rotation			Coordinate
	Centre	Angle	Direction	
$P$	$(-2, 1)$	$90^\circ$	clockwise	
$Q$	$(0, 0)$	$90^\circ$	anticlockwise	
$R$	$(0, -1)$	$90^\circ$	anticlockwise	
$S$	$(0, 4)$	$90^\circ$	clockwise	



### 11.5 Translation, Reflection and Rotation as an Isometry

#### 11.5.1 Relationship between translation, reflection and rotation with isometry



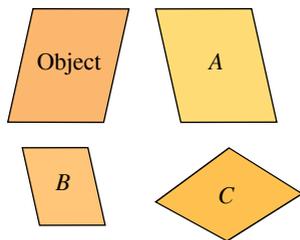
You have learned about the transformation for translation, reflection and rotation for an object. Each one has certain properties. Study the diagram on the left. Are you able to recognise the transformation of Image 1, Image 2 and Image 3? What is the relationship between the distance of the object and the image? If an object is mapped to a congruent image, then it is an isometry. Isometry is a transformation that maintains the distance between any two points on the original object. The isometric transformation will retain the original shape and size of the object.

#### LEARNING STANDARD

Investigate the relationship between the effects of translation, reflection and rotation and the distance between two points on an object and image, and hence explain isometry.

#### EXAMPLE 24

Which of these diagrams A, B and C are the isometric images of the object under an isometry?



#### Solution:

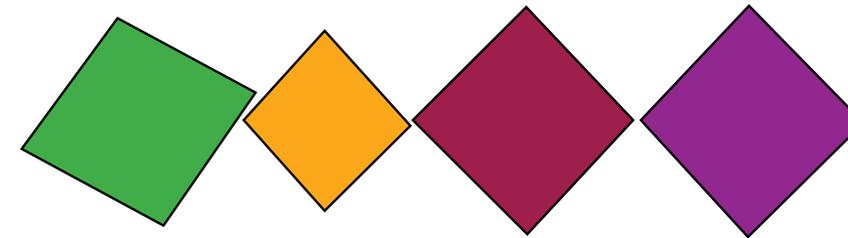
- Figure A: an isometric image as it has the same shape and size.
- Figure B: non-isometric image because it is not of the same size.
- Figure C: non-isometric as the shape and size are not the same.

You are able to recognise that translation, reflection and rotation are isometric transformations.

#### 11.5.2 Relationship between isometry and congruence

Look at the purple object. Can you state the image which is congruent under a transformation of reflection?

Can you determine the axis of reflection for this isometric transformation?

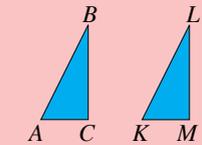


#### LEARNING STANDARD

Explain the relationship between isometry and congruency.

#### FLASHBACK

Two objects are congruent if the shape and size are the same.



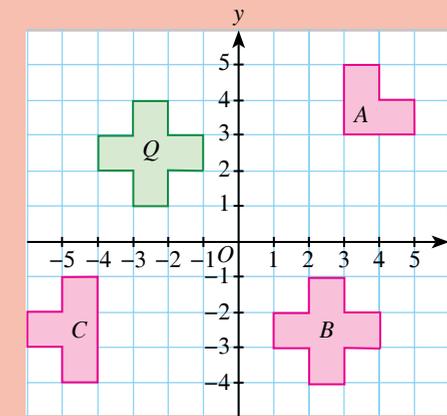
$ABC$  and  $KLM$  are congruent under a translation.

#### COGNITIVE STIMULATION



**Aim:** Identifying the relationship between isometry and congruence

**Materials:** Tracing paper and ruler



#### Steps:

- Study the diagram above.  $Q$  is the object of an image.
- Work together with your friends, to identify congruent images.
- Identify other possible isometry that produces images that are congruent.

#### Discussion:

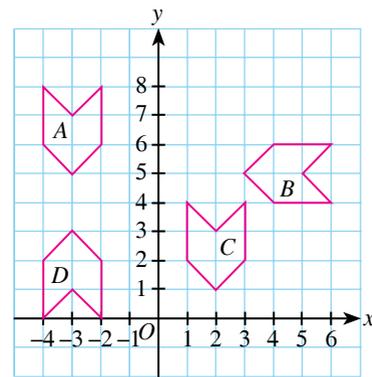
- If the images A and C are non-congruent, are the images an isometry?
- What is the relationship between isometry and congruence?

Under an isometry, objects and images are of equal shape and size. Therefore, objects and images are **congruent**. Isometry is a transformation where the image is congruent with the object.

**EXAMPLE 25**

Objects *A*, *B*, *C* and *D* are congruent. State the isometry of

- (a) object *A* to object *B*.
- (b) object *A* to object *C*.
- (c) object *A* to object *D*.

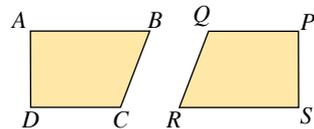


**Solution:**

- (a) Rotation
- (b) Translation
- (c) Reflection

**11.5.3 Solving problems**

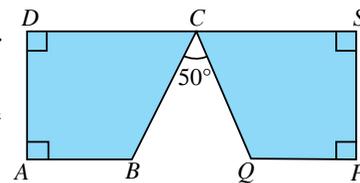
When naming a congruent polygon, the order must be based on the vertex or the corresponding angle.



The rectangles *ABCD* and *SRQP* are congruent.

**EXAMPLE 26**

In the diagram, *ABCD* is the image for *PQCS* under an isometry. Given *DCS* is a straight line, determine  $\angle PQC$ .



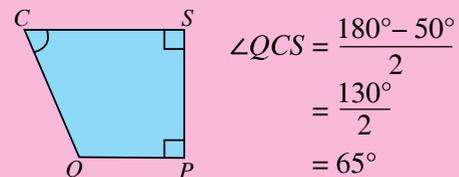
**Solution:**

**Understanding the problem**

*ABCD* is the image of *PQCS*.  
*DCS* is a straight line.  
 $\angle PQC$  is a part of the rectangle *PQCS*.

**Planning the strategy**

Determine



$$\begin{aligned} \angle QCS &= \frac{180^\circ - 50^\circ}{2} \\ &= \frac{130^\circ}{2} \\ &= 65^\circ \end{aligned}$$

**Conclusion**

Therefore,  $\angle PQC$  is  $115^\circ$ .

**Implementing the strategy**

$$\begin{aligned} \angle PQC &= 360^\circ - 90^\circ - 90^\circ - 65^\circ \\ &= 115^\circ \end{aligned}$$

**LEARNING STANDARD**

Solve problems involving isometry and congruency.

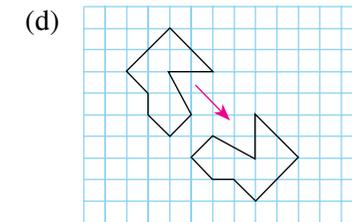
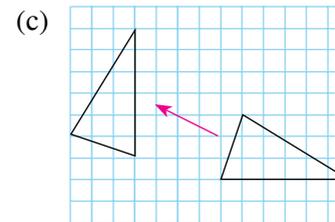
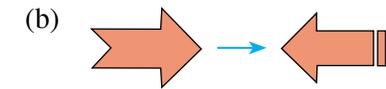
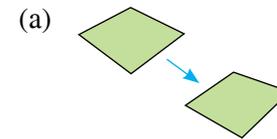
**FLASHBACK**

Scan the QR Code or visit [http://rimbunanilmu.my/mat\\_t2e/ms232](http://rimbunanilmu.my/mat_t2e/ms232) to view an animation on congruence.



**SELF PRACTICE 11.5**

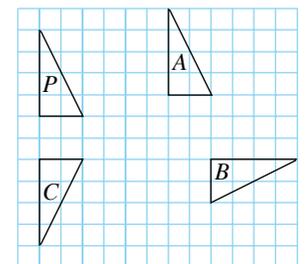
1. Determine whether the following transformation is an isometry.



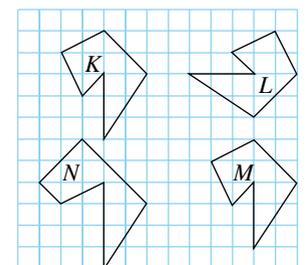
2. Determine whether each of the following transformations is an isometry.

- (a) A reflection followed by another reflection.
- (b) A translation.
- (c) A repeated rotation.

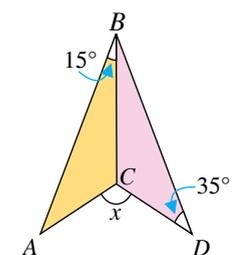
3. In the diagram, *A*, *B* and *C* are images for the object *P*. State the type of transformation.



4. The diagram shows several shapes. State the shapes that are congruent.



5. In the diagram,  $\triangle ABC$  is the image for  $\triangle BCD$ , under an isometric transformation. Calculate the value *x*.



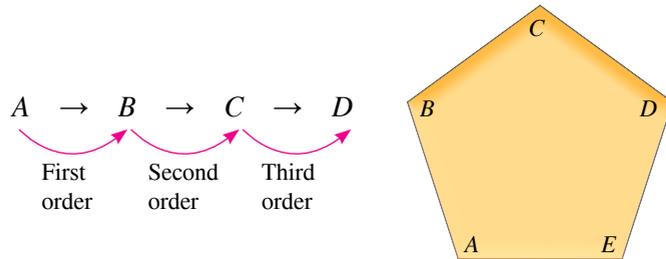
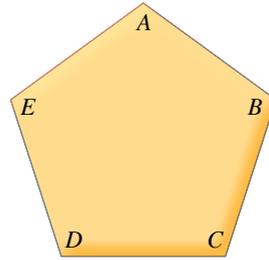


**EXAMPLE 28**

Determine the order of rotational symmetry when the position of  $A$  changes to position  $D$  in the diagram on the right.

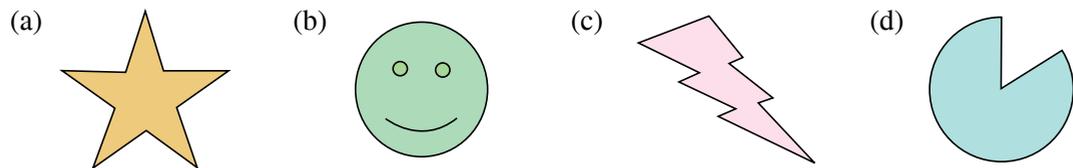
**Solution:**

Using tracing paper, draw and determine the rotation order  $A$  to  $D$ .

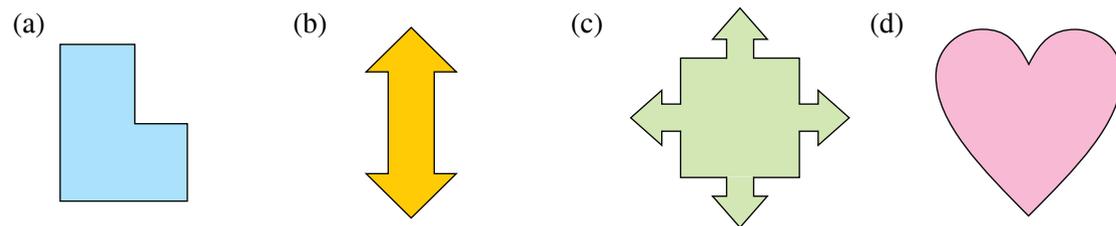


**SELF PRACTICE 11.6**

1. Which of the following objects has rotational symmetry?

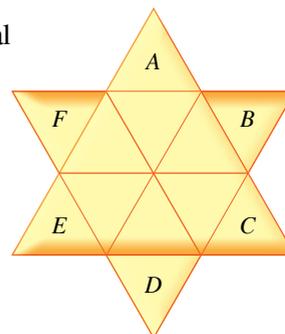


2. Determine the order of rotational symmetry for the following objects.

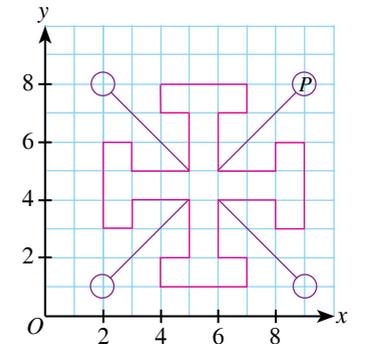


3. The symmetrical object is rotated at a point. State the order of rotational symmetry if

- (i) position  $A$  is at position  $C$ .
- (ii) position  $B$  is at position  $D$ .
- (iii) position  $C$  is at position  $B$ .

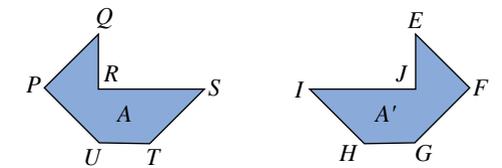


4. The symmetrical object lies on the Cartesian plane. State the coordinates of  $P$  under the third order of symmetry.



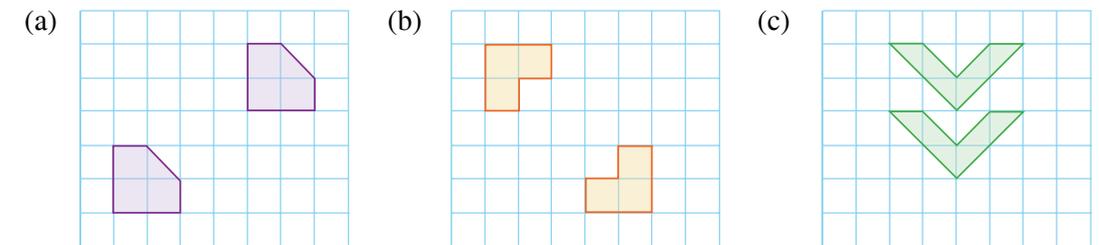
**GENERATING EXCELLENCE**

1. The diagram on the right shows polygon  $A$  mapped to polygon  $A'$  under a reflection. Identify the point that matches

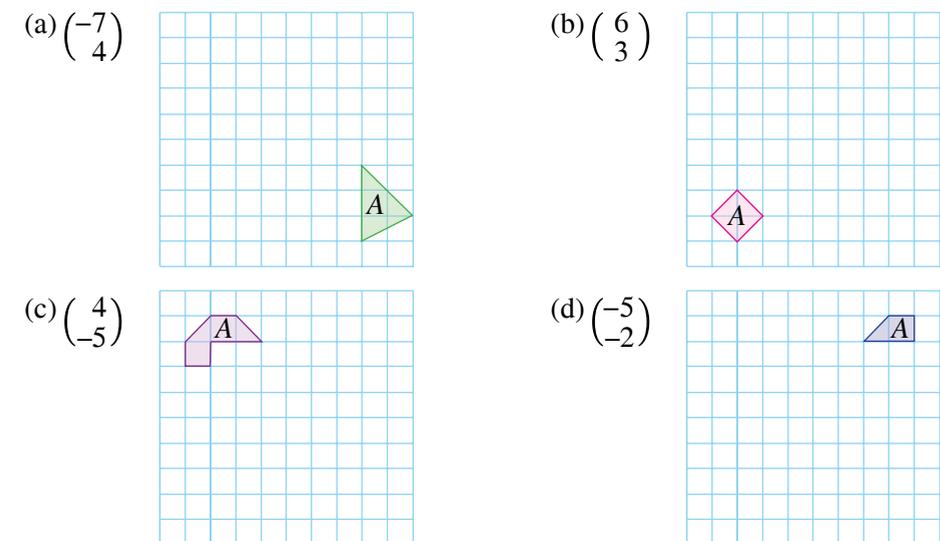


- (a) the image of point  $P$ .
- (b) the object of point  $G$ .

2. Which of the following is a translation and state the reason for your answer.



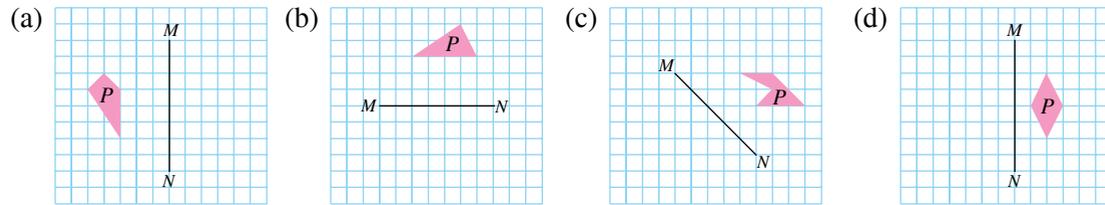
3. Draw the image for object  $A$  under the given translation.



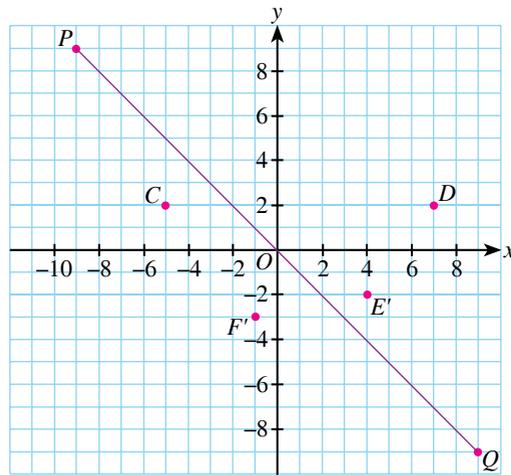
4. If point  $K(-2, -2)$  is the object, identify the image under the following vector translation.

- (a)  $\begin{pmatrix} 0 \\ 2 \end{pmatrix}$  (b)  $\begin{pmatrix} 3 \\ -1 \end{pmatrix}$  (c)  $\begin{pmatrix} -5 \\ 4 \end{pmatrix}$  (d)  $\begin{pmatrix} -3 \\ 4 \end{pmatrix}$  (e)  $\begin{pmatrix} -2 \\ 0 \end{pmatrix}$  (f)  $\begin{pmatrix} 4 \\ -3 \end{pmatrix}$

5. Draw the image  $P'$  of the object  $P$  under the reflection in the line  $MN$ .

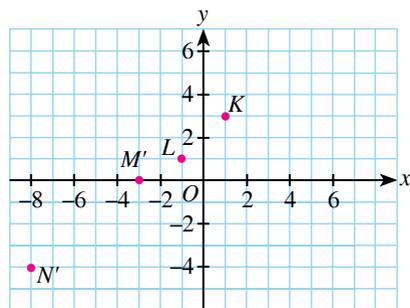


6. Determine the coordinates of the image or object of the following points, under the given axis of reflection.



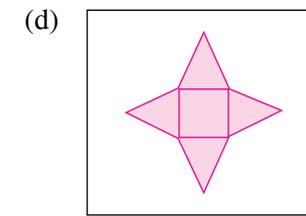
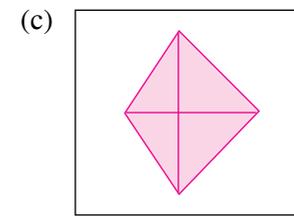
Point	Axis of reflection	Coordinate
$C$	$y$ -axis	$C' ( \quad )$
$D$	$x$ -axis	$D' ( \quad )$
$E'$	Line $PQ$	$E ( \quad )$
$F'$	Line $PQ$	$F ( \quad )$

7. Determine the coordinates of the image or object of the following points, under the given rotation.

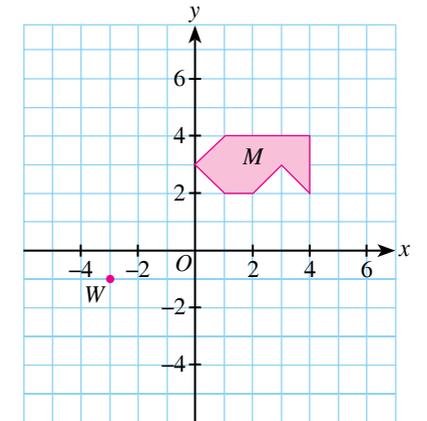


Point	Rotation			Coordinate
	Centre	Angle	Direction	
$K$	$(0, 0)$	$90^\circ$	clockwise	$K' ( \quad )$
$L$	$(0, 2)$	$180^\circ$	clockwise	$L' ( \quad )$
$M'$	$(0, 0)$	$90^\circ$	anticlockwise	$M ( \quad )$
$N'$	$(-3, -4)$	$180^\circ$	clockwise	$N ( \quad )$

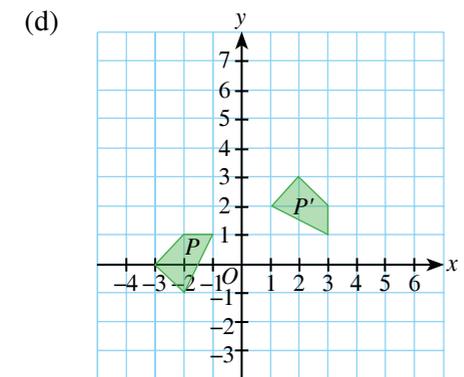
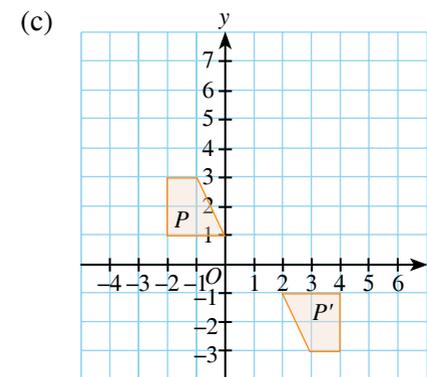
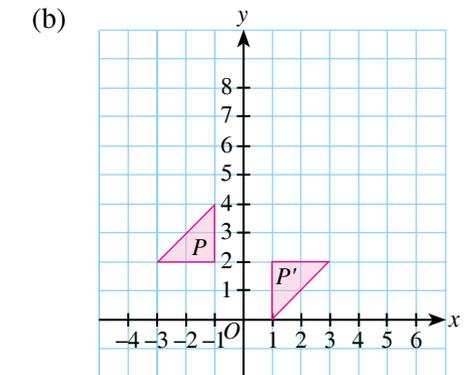
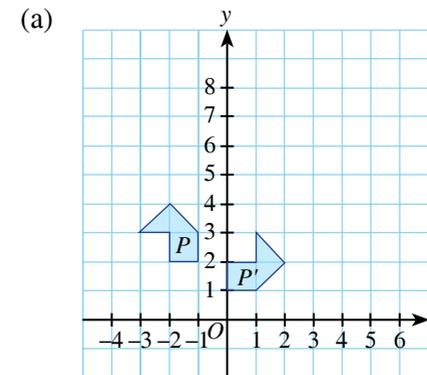
8. Which of the following has rotational symmetry?



9. (i) In the diagram on the right, draw the image of  $M$  under the reflection in the  $x$ -axis.  
 (ii) State the coordinates of the image  $W'$  under the same reflection.

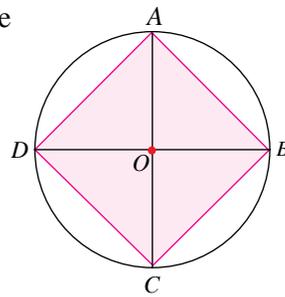


10. Given  $P'$  is the image of  $P$  under rotation, provide a complete description about the rotation.



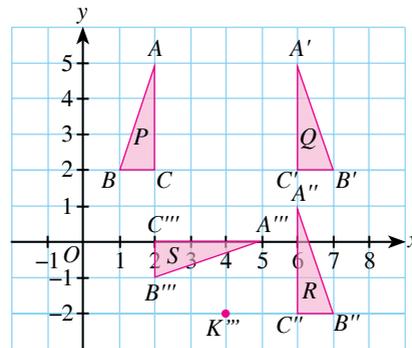
11. In the diagram on the right,  $ABCD$  is a square. State the image of the triangle  $OAB$  under the following rotations.

- (i) Clockwise rotation of  $90^\circ$  at point  $O$ .
- (ii) Rotation of  $180^\circ$  at point  $O$ .
- (iii) Anticlockwise rotation of  $270^\circ$  at point  $O$ .

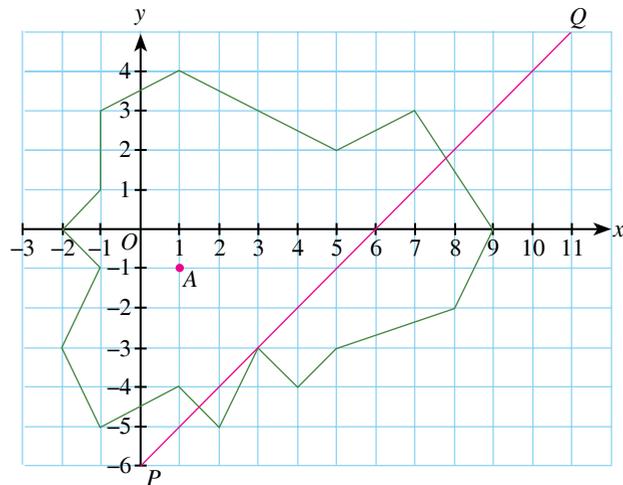


12. The diagram on the right shows triangle  $ABC$  which has three transformations,  $P \rightarrow Q \rightarrow R \rightarrow S$ .

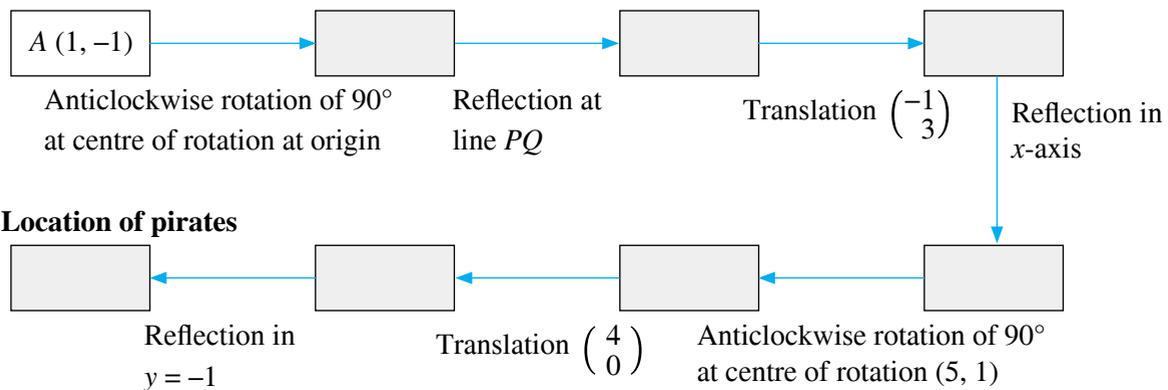
- (a) Describe the transformations.
- (b) If the point  $K'''$  is the image of point  $K$ , state the coordinates of the object  $K$  under the same transformation.



13. The diagram below shows a map of the sea of Kejora.

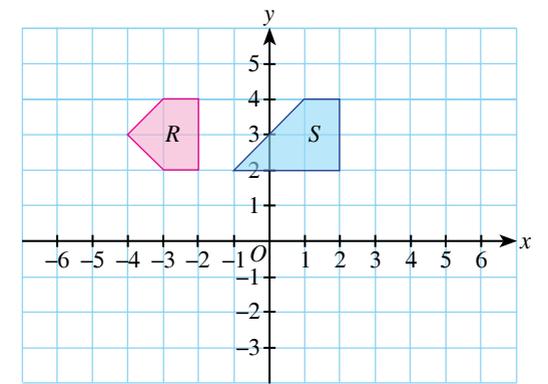


Point  $A$  is the position of Makdis military vessel. Help Makdis soldiers track pirates in the following order of transformations.



14. The diagram on the right shows the object  $R$  and  $S$ .

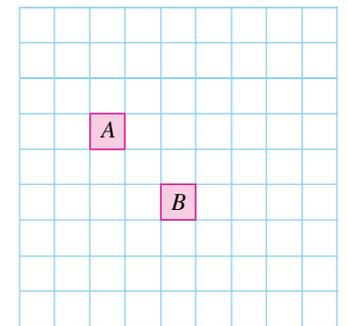
- (a) Draw the image of  $R$  under
  - (i) anticlockwise rotation of  $90^\circ$  at the point of origin followed by the translation  $\begin{pmatrix} -3 \\ 0 \end{pmatrix}$ .
  - (ii) the reflection on the straight line  $x = 1$ .
- (b) Draw the image of  $S$  under the reflection in the  $x$ -axis followed by the translation  $\begin{pmatrix} -4 \\ 1 \end{pmatrix}$  followed by the rotation of  $180^\circ$  at the point  $(0, -1)$ .



Next, name the polygon.

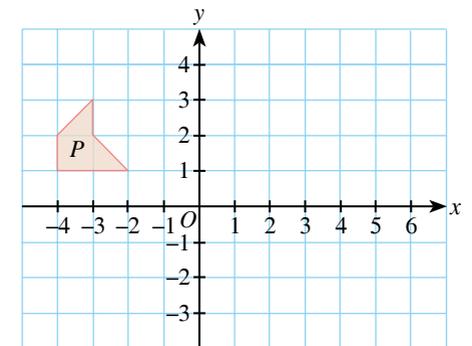
15. The diagram on the right shows rectangles  $A$  and  $B$  drawn on a square grid.

The rectangle  $A$  is the image of rectangle  $B$  under one transformation. Describe the five possible transformations.

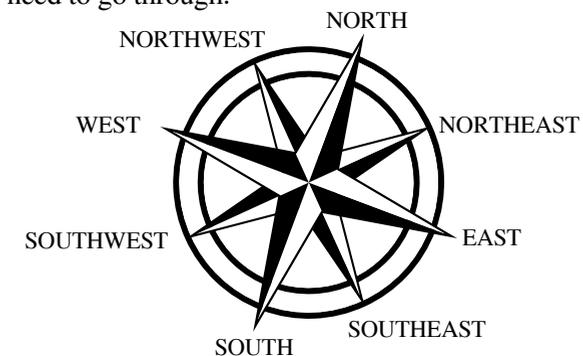


16. Based on the diagram on the right,

- (a) rotate object  $P$  at  $180^\circ$  at point  $(1, 1)$ . Label the rotational image as  $Q$ .
- (b) make a translation  $\begin{pmatrix} -2 \\ -2 \end{pmatrix}$  of  $Q$  and label it as  $R$ .
- (c) describe the other transformations that map the object  $P$  to the image  $R$ .



17. Based on the diagram below, Fauzah and Zainun are located in Southeast and Southwest respectively. If they plan to meet in an area located in the Northeast, state the order of rotational symmetry that they need to go through.



**CHAPTER SUMMARY**

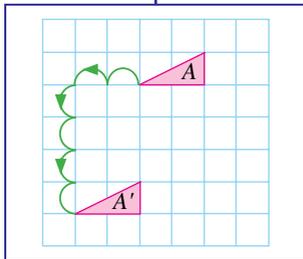
**Isometric Transformations**

**Transfer of points on a plane**

**Translation**

The transfer of all points on a plane in the same direction and magnitude of a vector. The properties of translation are:

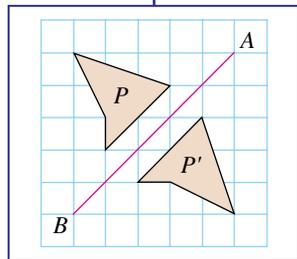
- (i) the image does not change.
- (ii) the image lies in a particular vector of the object.



**Reflection**

The transformation that reverses the points on a plane based on a line known as the axis of reflection. The properties of reflection are:

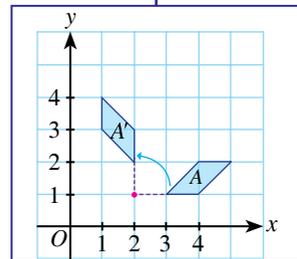
- (i) the object and the image are on the opposite sides of the axis of reflection.
- (ii) the object and its image have the same perpendicular distance from the axis of reflection.
- (iii) the shape and size of the image are the same as the object, but the orientation is reversed.
- (iv) the image of a point on the axis of reflection is the point itself.



**Rotation**

The process of transformation that occurs when each point is rotated at a fixed point through a certain angle and in a certain direction. The properties of rotation are:

- (i) rotates at a specific centre of rotation.
- (ii) has a rotational angle.
- (iii) the image retains the original form but the position changes.



**Isometry**

The transformation that shows the original object and its image is congruent. In isometry, the distance between two points on the original object equals the distance between the two corresponding points in the image. Reflection, rotation and translation are isometric.

**Congruence**

Object and image has the same size and shape.

**Rotational Symmetry**

The shape or image is rotated in less than  $360^\circ$  at a fixed point. Its shape remains the same.

**SELF REFLECTION**

At the end of this chapter, I will be able to:



1. Describe translation, reflection and rotation using various representations.	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>
2. Determine the image and object under a translation, reflection or rotation.	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>
3. Solve problems involving translation, rotation and reflection.	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>
4. Investigate the relationship between the effects of translation, reflection and rotation and the distance between two points on an object and image, and hence explain isometry.	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>
5. Explain the relationship between isometry and congruency.	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>
6. Solve problems involving isometry and congruency.	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>
7. Explain rotational symmetry.	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>
8. Determine the order of rotational symmetry order of an object.	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>



**MINI PROJECT**

You are asked to design a class logo that symbolises cooperation, solidarity, tolerance, respect and determination. These features should be translated in the form of isometric transformation pattern that are diversify yet moderate. Then, explain the meaning of each item you have chosen.

