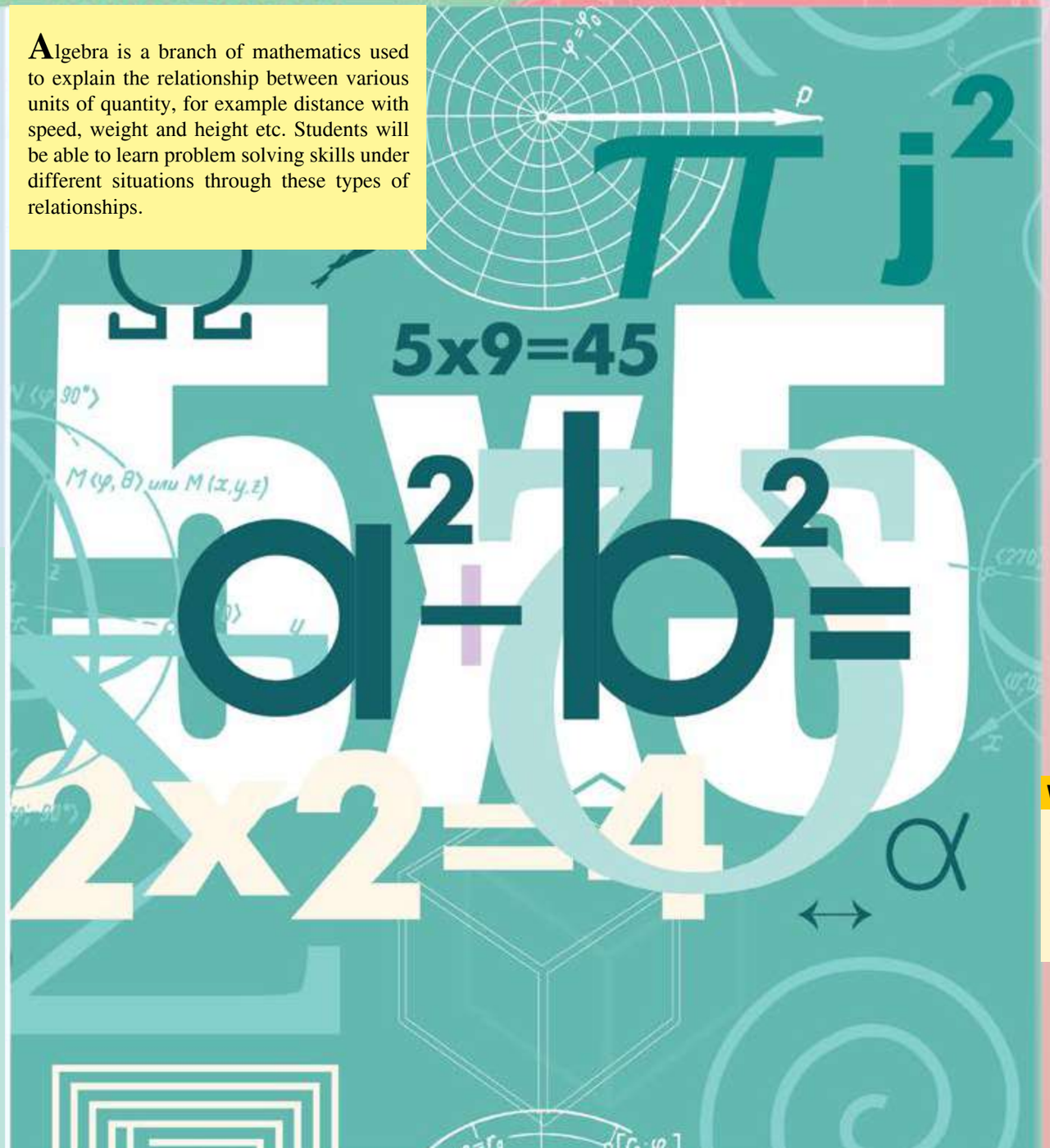
CHAPTER 2

Factorisation and Algebraic Fractions

Algebra is a branch of mathematics used to explain the relationship between various units of quantity, for example distance with speed, weight and height etc. Students will be able to learn problem solving skills under different situations through these types of relationships.



WHAT WILL YOU LEARN?

WORD LINK

- Expansion
- Algebraic expression
- Factor
- Highest Common Factor (HCF)
- Algebraic fraction
- Perfect squares
- Cross multiplication
- Numerator
- Denominator
- Lowest term
- Lowest Common Multiple (LCM)

- *Kembangan*
- *Ungkapan algebra*
- *Faktor*
- *Faktor Sepunya Terbesar (FSTB)*
- *Pecahan algebra*
- *Kuasa dua sempurna*
- *Pendaraban silang*
- *Pengangka*
- *Penyebut*
- *Sebutan terendah*
- *Gandaan Sepunya Terkecil (GSTK)*

WALKING THROUGH TIME

According to the book 'al-Jabr w'al-Muqabalah' written by the Persian Arabian mathematician, Muhammad Ibn Musa al-Khwarizmi, the word algebra originated from 'al-Jabr'. He was also known as the 'Father of Algebra' for his contribution in the field of Algebra.

For more information:

http://rimbunanilmu.my/mat_t2e/ms019

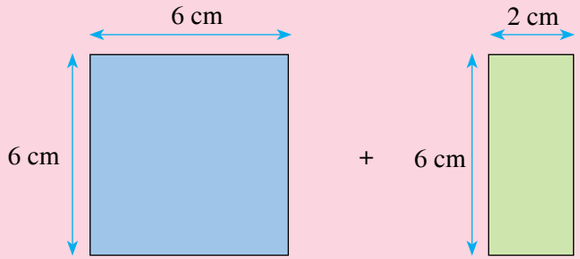
WHY STUDY THIS CHAPTER?

- Algebra is mostly used in price comparison, buying and selling process, measurement, etc.
- Algebra is also used in certain fields of study like Chemistry, Physics and Forensics.

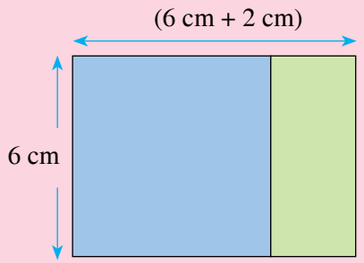
CREATIVE ACTIVITY

Aim: Determining area using algebraic tiles
Materials: Green and blue papers
Steps:
1. Cut the blue paper into a square measuring 6 cm by 6 cm.
2. Cut the green paper into a rectangle measuring 6 cm by 2 cm.
3. Calculate the area of the blue square and green rectangle using method 1 and 2.

Method 1: Area of blue square **+** area of green rectangle.



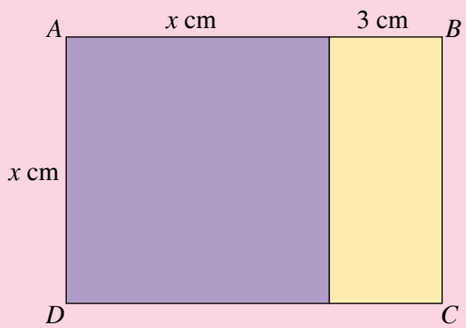
Method 2: Length × (width of the blue square + width of the green rectangle)



DO YOU KNOW ?

Algebra tiles are rectangular and square tiles used to represent algebraic principles.

4. Do the two methods give similar answers? Discuss.
5. Based on the diagram below, calculate the area of the rectangle, *ABCD*.



QR CODE

Scan the QR Code or visit http://rimbunanilmu.my/mat_t2e/ms020 to learn more on algebra tiles.

2.1 Expansion

2.1.1 Expansion of algebraic expressions

LEARNING STANDARD

Explain the meaning of the expansion of two algebraic expressions.

Expansion of algebraic expression is the product of multiplication of one or two expressions in brackets.

FLASHBACK

Algebraic expressions are expressions that combine numbers, variables or mathematical entities using mathematical operations. Example, $2a + 5$.

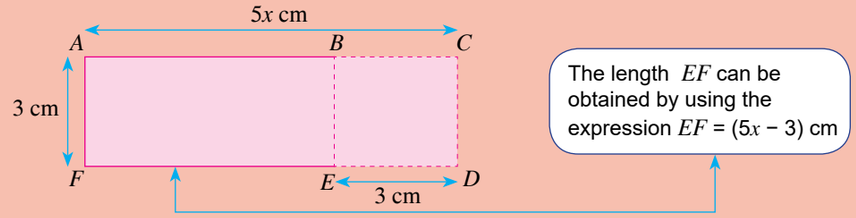
2.1.2 Expansion on two algebraic expressions

LEARNING STANDARD

Expand two algebraic expressions.

COGNITIVE STIMULATION Individual

Aim: Determining the area of rectangle *ABEF*
Material: Worksheet
Steps:
1. Calculate area *ABEF* using the two methods shown below.



Method 1 :
Area *ABEF*
= Area *ACDF* − Area *BCDE*
= ☐ − ☐
= ☐ cm²

Method 2:
Area *ABEF*
= length × width
= *EF* × *AF*
= ☐ × ☐
= ☐ cm²

Discussion:
Is the answer for method 2 the same as the answer for method 1? Explain.

When doing expansion of algebraic expressions, every term within the bracket needs to be multiplied with the term outside the bracket.

FLASHBACK

(+) × (+)	+
(+) × (−)	−
(−) × (+)	−
(−) × (−)	+

EXAMPLE 1

Expand the following expressions.

(a) $6(3 + 4w)$

(b) $3r(r - 2s)$

(c) $-5b(a + 3)$

(d) $-\frac{2y}{3}(9y - 3z + 6x)$

Solution:

(a) $6(3 + 4w)$
$= (6 \times 3) + (6 \times 4w)$
$= 18 + 24w$

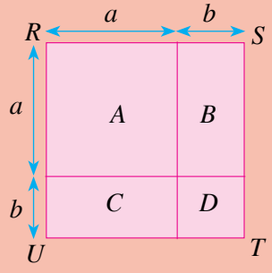
(b) $3r(r - 2s)$
$= (3r \times r) + [3r \times (-2s)]$
$= 3r^2 - 6rs$

(c) $-5b(a + 3)$
$= (-5b \times a) + (-5b \times 3)$
$= -5ab - 15b$

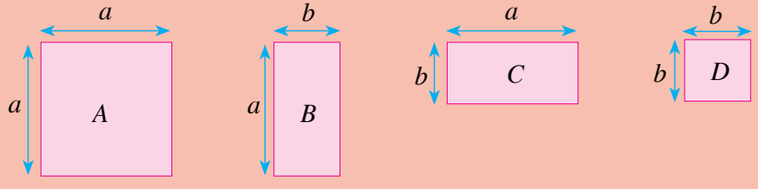
(d) $-\frac{2y}{3}(9y - 3z + 6x)$
$= \left(-\frac{2y}{1\cancel{3}} \times \cancel{9}^{3}y\right) + \left[-\frac{2y}{1\cancel{3}} \times (-\cancel{3}^{1}z)\right] + \left(-\frac{2y}{1\cancel{3}} \times \cancel{6}^{2}x\right)$
$= -6y^2 + 2yz - 4xy$

COGNITIVE STIMULATION Group

Aim: Expanding two algebraic expressions
Material: Worksheet
Steps:
1. Work in pairs.
2. Student *A* calculates the area of square *RSTU* using method 1.
3. Student *B* calculates the area of square *RSTU* using method 2.



Method 1



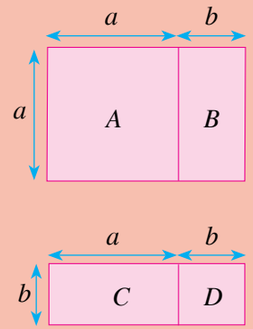
Area for the square *RSTU* = Area *A* + Area *B* + Area *C* + Area *D*

$= \boxed{(\_ \times \_)} + \boxed{(\_ \times \_)} + \boxed{(\_ \times \_)} + \boxed{(\_ \times \_)}$
$= \boxed{} + \boxed{} + \boxed{} + \boxed{}$
$= \boxed{} + \boxed{} + \boxed{}$

Method 2
Separate the square into two parts as follow.

Area of square *RSTU* = Area *A* and *B* + Area *C* and *D*

$= \boxed{(\_\_\_)(a + b)} + \boxed{(\_\_\_)(a + b)}$
$= \boxed{} + \boxed{} + \boxed{} + \boxed{}$
$= \boxed{} + \boxed{} + \boxed{}$



Discussion:
Are your answers for both methods similar? Explain.

When doing expansion on two algebraic expressions in two brackets, every term in the first bracket must be multiplied with every term in the second bracket. For example,

$(a + 2)(a + 1)$
$= a(a + 1) + 2(a + 1)$
$= a^2 + a + 2a + 2$
$= a^2 + 3a + 2$

Like terms can be solved

TIPS

$(a + b)(a + b) = (a + b)^2$
$(a - b)(a - b) = (a - b)^2$
$(a + b)(a - b) = (a \times a) + [a \times (-b)] + (b \times a) + [b \times (-b)]$
$= a^2 - ab + ba - b^2$
$= a^2 - b^2$

EXAMPLE 2

Expand each of the following expressions.

(a) $(y + 1)(y - 3)$ (b) $(4 + 3r)(2 + r)$
(c) $(3r + 4s)(r - 2s)$ (d) $(3p + 2)^2$

Solution:

(a) $(y + 1)(y - 3)$
$= y(y - 3) + 1(y - 3)$
$= y^2 - 3y + y - 3$
$= y^2 - 2y - 3$

(b) $(4 + 3r)(2 + r)$
$= 8 + 4r + 6r + 3r^2$
$= 8 + 10r + 3r^2$
$= 3r^2 + 10r + 8$

ATTENTION

$(a + b)(a - b) = a^2 - b^2$
$(a + b)(a + b) \neq a^2 + b^2$
$(a - b)(a - b) \neq a^2 - b^2$

DO YOU KNOW?

Alternative methods
(i) Cross multiplication

(×) a	$+2$	$2a$	(+)
a	$+1$	a	
a^2	$+2$	$3a$	

Hence, $a^2 + 3a + 2$

(ii) Standard form

$$\begin{array}{r} a + 2 \\ \times \quad a + 1 \\ \hline a + 2 \\ (+)\ a^2 + 2a \\ \hline a^2 + 3a + 2 \end{array}$$

(c) $(3r + 4s)(r - 2s)$
$= 3r(r - 2s) + 4s(r - 2s)$
$= (3r \times r) + [3r \times (-2s)] + (4s \times r) + [4s \times (-2s)]$
$= 3r^2 - 6rs + 4sr - 8s^2$
$= 3r^2 - 2rs - 8s^2$

Like terms can be solved

Like terms
$sr = rs$

(d) $(3p + 2)^2$
$= (3p + 2)(3p + 2)$
$= 9p^2 + 6p + 6p + 4$
$= 9p^2 + 12p + 4$

Like terms can be solved

THINK SMART

Relationship between repeated multiplication of Binomial expression with Pascal's Triangle.

$1 \quad (a+b)^0$
$1a + 1b \quad (a+b)^1$
$1a^2 + 2ab + 1b^2 \quad (a+b)^2$
$1a^3 + 3a^2b + 3ab^2 + 1b^3 \quad (a+b)^3$
$1a^4 + 4a^3b + 6a^2b^2 + 4ab^3 + 1b^4 \quad (a+b)^4$

State the next two terms.

TIPS

Algebraic terms are arranged from the highest power to the lowest power.

QR CODE

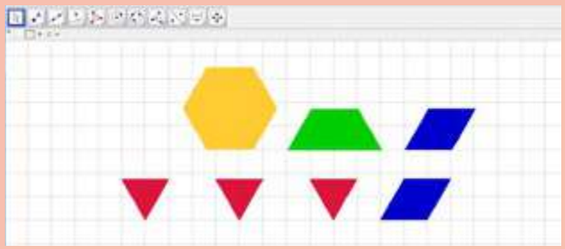
Scan the QR Code or visit http://rimbunanilmu.my/mat_t2e/ms024a to view a video on cross multiplication method.

2.1.3 Combined operations including expansion

Combined operations for algebraic terms must be solved by following the 'BODMAS' rule.

COGNITIVE STIMULATION Group

Aim: Writing algebraic expression using algebra tiles
Material: Dynamic geometry software
Steps:



1. Open the file MS024B to display a yellow hexagon and other coloured shapes of red, blue and green.
2. Choose a combination of coloured shapes of red, blue or green to fit into the yellow hexagon.
3. Write out the algebraic relationship.
4. Choose other combinations of shapes to fit into the green trapezium.

Discussion:
Compare your findings with other groups.

LEARNING STANDARD

Simplify algebraic expressions involving combined operations, including expansion.

QR CODE

Scan the QR Code or visit http://rimbunanilmu.my/mat_t2e/ms024b to construct polygon.

EXAMPLE 3

Simplify.
(a) $(3w - 2)(4w - 1) - 10w$ (b) $(r - 3t)^2 + 4rt$
(c) $(x + y)(x - y) + x(x - 2y)$

Solution:

(a) $(3w - 2)(4w - 1) - 10w = 3w\,(4w - 1) - 2\,(4w - 1) - 10w$
$= 12w^2 - 3w - 8w + 2 - 10w$
$= 12w^2 - 3w - 8w - 10w + 2$
$= 12w^2 - 21w + 2$

(b) $(r - 3t)^2 + 4rt = (r - 3t)(r - 3t) + 4rt$
$= r^2 - 3rt - 3rt + 9t^2 + 4rt$
$= r^2 + 9t^2 - 3rt - 3rt + 4rt$
$= r^2 + 9t^2 - 2rt$

(c) $(x + y)(x - y) + x(x - 2y) = x^2 - xy + xy - y^2 + x^2 - 2xy$
$= x^2 + x^2 - y^2 - xy + xy - 2xy$
$= 2x^2 - y^2 - 2xy$

FLASHBACK

B = Brackets
O = Order
D = Division
M = Multiplication
A = Addition
S = Subtraction

For further information:

Scan the QR Code or visit http://rimbunanilmu.my/mat_t2e/ms025

FLASHBACK

Distributive law is used when expanding bracket.

$a \times (b + c) = a \times b + a \times c$
$a \times (b - c) = a \times b - a \times c$

2.1.4 Solving problems

LEARNING STANDARD

Solve problems involving expansion of two algebraic expressions.

EXAMPLE 4

Puan Maria has a piece of carpet, $(3r - 2)$ metre in length and $(r + 1)$ metre in width. Calculate the area of her carpet.

Solution:

Area = length × width
$= (3r - 2)(r + 1)$
$= 3r^2 + 3r - 2r - 2$
$= 3r^2 + r - 2$

Hence, the size of the carpet is $(3r^2 + r - 2)$ square meters.

EXAMPLE 5

Ramesh received RM50 pocket money for $(y - 8)$ days. Everyday he spends RM$(x - 3)$ for a cup of coffee and RM$(x + 4)$ for a bowl of mee rebus. How much pocket money is he left with?

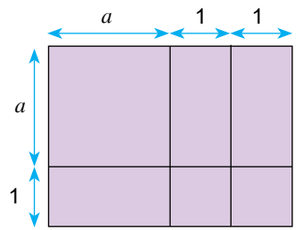
Solution:

Understanding the problem	Planning the strategy	Implementing the strategy	Conclusion
Determine the total price of the coffee and mee rebus. $(x-3)+(x+4)$ $=2x+1$	Determine the total spending for $(y-8)$ days using the expansion method. $(y-8)(2x+1)$ $=2xy+y-16x-8$	Calculate the remainder money using the expansion process. Pocket money – spending $=50-(2xy+y-16x-8)$ $=50-2xy-y+16x+8$ $=58-2xy-y+16x$	Remainder of pocket money RM$(58-2xy-y+16x)$

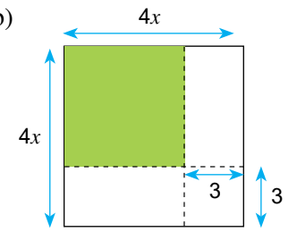
SELF PRACTICE 2.1

1. Based on the following algebra tiles, write out the area of the shaded region in the form of multiplication of two algebraic expressions.

(a)



(b)



2. Expand the following algebraic expressions.

(a) $3(x+2)$ (b) $4(8x-3)$ (c) $2(a+5)$

(d) $p(6p-8)$ (e) $-\frac{r}{8}(2s-8)$ (f) $-2(pr-2pq)$

(g) $3(5bc-6)$ (h) $7(2ef+3e)$ (i) $8g(2+gh)$

3. Expand the following algebraic expressions.

(a) $(a+1)(a+2)$ (b) $(x-5)(x+4)$ (c) $(2+m)(5-m)$

(d) $(3p-2)(4p-1)$ (e) $(3r-2)(4r-1)$ (f) $(2r+s)(4r-3s)$

(g) $(2d-\frac{1}{2}b)(3d-\frac{1}{2}b)$ (h) $(r-3s)^2$ (i) $(4e-3)^2$

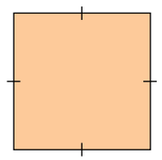
4. Simplify each of the following expressions.

(a) $(5b+3)+4(3b-a)$ (b) $3(4m-5mn)-2(8m+mn)$

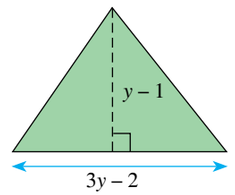
(c) $(h-j)^2-2h(3h-3j)$ (d) $(x+y)(x-y)+2x(x+2y)$

5. Determine the area for the diagrams below using algebraic expressions.

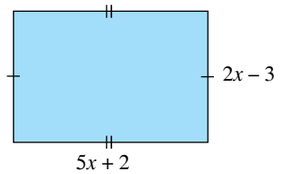
(a)



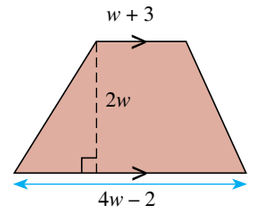
(b)



(c)



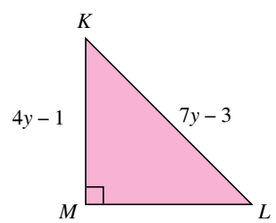
(d)



6. Hadila is 2 years younger than Kai Yee. Kai Yee's father's age is the square of Hadila's age. If Kai Yee is p years old, calculate the total age of the three of them. Express your answer in the form of algebraic expression.

7. The table top is a rectangular shape with length $(5x-2)$ metre and width $(x+2)$ metre. Mr. Phillip wants to put a piece of glass over the table top. The section of the table top not covered with the glass has a width of $(x-3)$ metre. Determine the area of the table top that is not covered in the form of algebraic expressions.

8. Determine the length of LM in terms of y.



2.2 Factorisation

2.2.1 Factors and factorisation concept

LEARNING STANDARD

Relate the multiplication of algebraic expression to the concept of factors and factorisation, and hence list out the factors of the product of the algebraic expressions.

Factorisation is the process of determining the factors of an algebraic expression or algebraic terms and when multiplied together will form the original expression. Factorisation is the reverse process of an expansion.

Example, the factors of $3p$

$1 \times 3p$ $3 \times p$

Therefore, factors of $3p$ are 1, 3, p and $3p$.

Factors, Common Factors and Highest Common Factor (HCF) for the product of algebraic expressions

Common factor is the factor of an algebra term that divides two or more other terms exactly. Highest Common Factor (HCF) is the largest of those common factors.

Examine the expression,

$$4x + 2 = 2(2x + 1)$$

2 is the common factor for the expression $4x$ and 2.

TIPS

Factorisation is the opposite of expansion.

Expansion

$a(a + b) = a^2 + ab$

Factorisation

EXAMPLE 6

List out all the common factors for each of the following terms.

(a) $6h, 4gh$ (b) $9c^2d, 3d^2e, 6def$

Solution:

(a) $6h = \boxed{1} \times 6h$
$\boxed{2} \times 3h$
$3 \times \boxed{2h}$
$\boxed{h} \times 6$
$4gh = \boxed{1} \times 4gh$
$4 \times gh$
$\boxed{2} \times 2gh$
$2g \times \boxed{2h}$
$g \times 4h$
$\boxed{h} \times 4g$

Therefore, common factors for $6h$ and $4gh$ are 1, 2, h and $2h$.

(b) $9c^2d, 3d^2e$ and $6def$
$9c^2d = \boxed{1} \times \boxed{3} \times 3 \times c \times c \times \boxed{d}$
$3d^2e = \boxed{1} \times \boxed{3} \times \boxed{d} \times d \times e$
$6def = \boxed{1} \times 2 \times \boxed{3} \times \boxed{d} \times e \times f$

Common factors for $9c^2d$, $3d^2e$ and $6def$ are 1, 3, d and $3d$. $3d$ is a common factor as it can divide all the above terms exactly.

ATTENTION

'1' is a factor for all algebraic terms.

2.2.2 Factorisation of algebraic expressions

Using HCF

Algebraic expressions can be factorised using Highest Common Factor (HCF).

Example,

$8x$
$12x^2$ → $4x$ is HCF

Therefore, the algebraic expressions, $8x + 12x^2$ can be written as a product of two factors as in,

$$4x(2 + 3x)$$

This is known as **factorisation**.

LEARNING STANDARD

Factorise algebraic expressions using various methods.

FLASHBACK

Factors of 16

$16 \div 1 = 16$ $16 \div 8 = 2$
$16 \div 2 = 8$ $16 \div 16 = 1$
$16 \div 4 = 4$

Hence, factors of 16 are 1, 2, 4, 8 and 16.

EXAMPLE 7

1. Determine the Highest Common Factor (HCF) for each of the following the terms.
   (a) $6h, 4gh$ (b) $9c^2d, 3d^2e, 6def$
2. Factorise the expressions below.
   (a) $3x + 15$ (b) $7m + 21m^2$

Solution:

1. (a)
$2 \mid 6h, 4gh$
$h \mid 3h, 2gh$
$3, 2g$
HCF = $2h$

(b)
$3 \mid 9c^2d, 3d^2e, 6def$
$d \mid 3c^2d, d^2e, 2def$
$3c^2, de, 2ef$
HCF = $3d$

Solution:

2. (a)
$3 \mid 3x + 15$
$x + 5$
HCF = 3
Hence, $3(x + 5)$

(b)
$7 \mid 7m + 21m^2$
$m \mid m + 3m^2$
$1 + 3m$
HCF = $7m$
Hence, $7m(1 + 3m)$

TIPS

$4 \mid 8x, 12x^2$
$x \mid 2x, 3x^2$
$2, 3x$

HCF = $4x$

HCF can be determined by using long division.

Check your answers using the expansion method.

$= 4x(2 + 3x)$
$= 8x + 12x^2$

FLASHBACK

Perfect squares:
1, 4, 9, 16, 25, 36, 49, 64, 81, 100, 121, 144, ...

Using difference of squares of two terms

$x^2 - y^2$ is the difference of squares. $x^2 - y^2$ can be factorised by using difference of perfect squares. This method can only be used if the two algebraic terms are perfect squares.

Examine this expressions,

$$x^2 - 4 = x^2 - 2^2$$
$$= (x + 2)(x - 2)$$

TIPS

Check your answers using the expansion method.
$(x + 2)(x - 2)$
$= x(x - 2) + 2(x - 2)$
$= x^2 - 2x + 2x - 4$
$= x^2 - 4$

EXAMPLE 8

Factorise each of the following expressions.

(a) $b^2 - 1$ (b) $9m^2 - 100$
(c) $3y^2 - 147$ (d) $5k^2 - 80$

Solution:

(a) $b^2 - 1$
$= b^2 - 1^2$
$= (b + 1)(b - 1)$

(b) $9m^2 - 100$
$= (3m)^2 - 10^2$
$= (3m + 10)(3m - 10)$

DO YOU KNOW ?

Odd numbers	Differences of squares
1	$1^2 - 0^2$
3	$2^2 - 1^2$
5	$3^2 - 2^2$
7	$4^2 - 3^2$
9	$5^2 - 4^2$
11	$6^2 - 5^2$
13	$7^2 - 6^2$

(c) $3y^2 - 147$
$= 3(y^2 - 49)$ ← HCF of 3 and 147 is 3
$= 3(y^2 - 7^2)$
$= 3(y + 7)(y - 7)$

(d) $5k^2 - 80$ ← HCF of 5 and 80 is 5
$= 5(k^2 - 16)$
$= 5(k^2 - 4^2)$
$= 5(k + 4)(k - 4)$

An algebraic expression as in $x^2 + 2xy + y^2$ can be factorised as $(x + y)(x + y)$.

TIPS

Factoring Identities

(a) $(x + y)^2$
$= (x + y)(x + y)$
$= x^2 + 2xy + y^2$

(b) $(x - y)^2$
$= (x - y)(x - y)$
$= x^2 - 2xy + y^2$

(c) $x^2 - y^2$
$= (x + y)(x - y)$

Using cross multiplication

Algebraic expressions of $ax^2 + bx + c$, where by $a \neq 0$ and a, b, c are integers that can be factorised.

Examine the example below with its explanation for the factorisation of the algebraic expression $x^2 + 6x + 8$.

Step 1: Compare the coefficients

$1x^2 + 6x + 8$

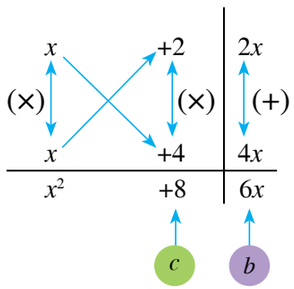
$a\,x^2 + b\,x + c$

Hence, $a = 1$, $b = 6$ and $c = 8$

Step 2: Factors of 8 are 1, 2, 4 and 8. 2 and 4 are selected because they conform to c, i.e. $2 \times 4 = 8$.

Step 3: 2 and 4 are chosen because they conform to b, i.e. $2 + 4 = 6$.

Step 4: Do cross multiplication as shown below.



Step 5: Factors for $x^2 + 6x + 8$ are $(x + 2)(x + 4)$.

QR CODE

Scan the QR Code or visit http://rimbunanilmu.my/mat_t2e/ms030 to learn factorisation methods using algebra tiles.

TIPS

Sum of b	Product of c
$1 + 8 = 9$ $-1 + (-8) = -9$	$1 \times 8 = 8$ $-1 \times (-8) = 8$
$2 + 4 = 6$ $-2 + (-4) = -6$	$2 \times 4 = 8$ $-2 \times (-4) = 8$

DO YOU KNOW?

Factorisation and division

$$x + 2 \overline{)\, x^2 + 6x + 8}$$
$$x + 4$$
$$(-)\ x^2 + 2x$$
$$4x + 8$$
$$(-)\ 4x + 8$$
$$0$$

EXAMPLE 9

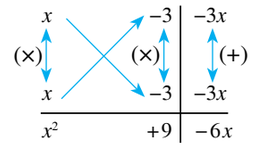
Factorise each of the following expressions.
(a) $x^2 - 6x + 9$
(b) $m^2 - 2m - 8$

Solution:

(a) $x^2 - 6x + 9$

Multiplication of factors of 9:
$(-1) \times (-9)$
$(-3) \times (-3)$

$-3 + (-3) = -6$

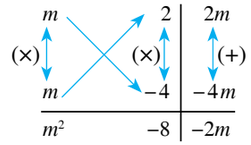


Hence, $x^2 - 6x + 9 = (x - 3)(x - 3)$.

(b) $m^2 - 2m - 8$

Multiplication of factors of 8:
$1 \times (-8)$
-2×4
$2 \times (-4)$

$2 + (-4) = -2$



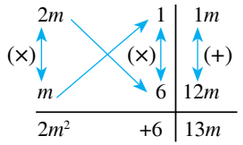
Hence, $m^2 - 2m - 8 = (m + 2)(m - 4)$.

EXAMPLE 10

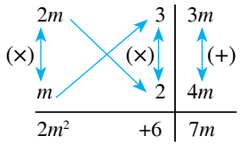
Factorise the following expressions.
$2m^2 + 7m + 6$ ← Multiplication of factors of 6: 1×6, 2×3

Solution:

First trial:



Second trial:



Hence, $2m^2 + 7m + 6 = (2m + 3)(m + 2)$. ← Check your answer with the expansion method

QR CODE

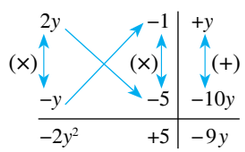
Scan the QR Code or visit http://rimbunanilmu.my/mat_t2e/ms031 to explore factorisation using cross multiplication method.

EXAMPLE 11

Factorise the following expressions.
(a) $-2y^2 - 9y + 5$
(b) $-3x^2 - 8x - 5$

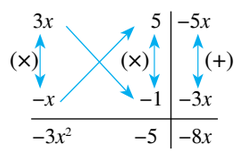
Solution:

(a)



Hence, $-2y^2 - 9y + 5 = (2y - 1)(-y - 5)$.

(b)



Hence, $-3x^2 - 8x - 5 = (3x + 5)(-x - 1)$.

THINK SMART

Solution to $-2y^2 - 9y + 5$ can be written as $(-2y + 1)(y + 5)$. Discuss.

Using common factors involving 4 algebraic terms

$$\begin{aligned} ab + ac + bd + cd &= (ab + ac) + (bd + cd) \\ &= a(b + c) + d(b + c) \\ &= (b + c)(a + d) \end{aligned}$$

← Distributive law

DO YOU KNOW?

Factorisation can be solved as follows.
$2x^2 + 7x + 3$
$= 2x^2 + 6x + x + 3$
$= 2x(x + 3) + (x + 3)$
$= 2x(x + 3) + 1(x + 3)$
$= (2x + 1)(x + 3)$

EXAMPLE 12

Factorise each of the following terms.

(a) $pq + qr + ps + rs$

(b) $2px + 6qy - 4py - 3qx$

Solution:

(a) $pq + qr + ps + rs$
$= (pq + qr) + (ps + rs)$ ← Group terms with common factors in a bracket
$= q(p + r) + s(p + r)$ ← Common factors
$= (q + s)(p + r)$

(b) $2px - 4py - 3qx + 6qy$
$= (2px - 4py) - (3qx - 6qy)$
$= 2p(x - 2y) - 3q(x - 2y)$
$= (x - 2y)(2p - 3q)$

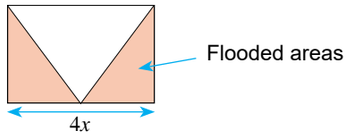
2.2.3 Solving problems

LEARNING STANDARD

Solve problems involving factorisation.

EXAMPLE 13

The area of a rectangular shaped football field is $(4x^2 + 16x)$ square metres. The field was flooded as shown in the diagram below. If the width of the field is $4x$ metre and the two flooded regions are right-angled triangles which are congruent, what is the area of the region that is not flooded?



Solution:

Understanding the problem

Determine the length of the field

$$\text{Length} = \frac{\text{area}}{\text{width}}$$
$$= \frac{4x^2 + 16x}{4x}$$
$$= \frac{\cancel{4x}(x + 4)}{\cancel{4x}}$$
$$= (x + 4)$$

Determine the base of the right-angled triangle

Base of the right-angled triangle
$= 4x \div 2$
$= 2x$

Planning the strategy

Area of two right-angled triangles

$$\text{Area} = 2 \times \left(\frac{1}{2} \times \text{base} \times \text{height}\right)$$
$$= 2 \times \left(\frac{1}{2} \times 2x \times (x + 4)\right)$$
$$= 2x^2 + 8x$$

Implementing the strategy

Area of the region that is not flooded
= Area of the field – area of two right-angled triangles.
$= 4x^2 + 16x - (2x^2 + 8x)$
$= 4x^2 - 2x^2 + 16x - 8x$
$= 2x^2 + 8x$

Conclusion

Area of the region that is not flooded
$= (2x^2 + 8x)$ m^2

SELF PRACTICE 2.2

1. Determine the common factors and HCF for each of the following terms.
   (a) $8y, 12y$ (b) $2b, 3b$ (c) $3w, 5w^2$
   (d) $10m^2, 15mk$ (e) $5bc, 2c^2, 3cd$ (f) $4a^2b, 8b^2c, 6bcd$

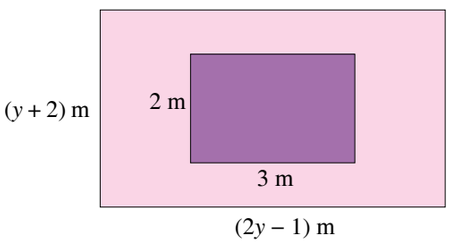
2. Factorise the following algebraic expressions.
   (a) $5e + 10$ (b) $2ab - 8a^2$ (c) $3abc + 6a^2b$
   (d) $4x - 12x^2$ (e) $ef + f^2 + fg$ (f) $2x^2 - 4xy + 6wx$

3. Factorise the following algebraic expressions.
   (a) $b^2 - 81$ (b) $a^2 - b^2$ (c) $x^2 - 1$
   (d) $16y^2 - 49$ (e) $(m + 3)^2 - 16$ (f) $4(x - 1)^2 - 9$

4. Factorise the following algebraic expressions.
   (a) $x^2 + 9x + 14$ (b) $x^2 + 7x - 18$ (c) $x^2 - 5x - 24$
   (d) $m^2 + 11m - 26$ (e) $y^2 - 2y - 15$ (f) $k^2 - 8k + 16$
   (g) $2m^2 - 11m - 6$ (h) $9f^2 - 12f + 4$ (i) $2m^2 + 4m - 16$
   (j) $2x^2 - 5x - 7$ (k) $12y^2 + 8y - 15$ (l) $5p^2 + 6p - 8$
   (m) $-5m^2 - 6m + 8$ (n) $-3p^2 + 8p - 4$ (o) $-6x^2 - x + 15$

5. Factorise the following algebraic expressions.
   (a) $pq - qr - pw + rw$ (b) $x^2 + xy + 6x + 6y$
   (c) $3ab - 9ad + bc - 3cd$ (d) $ah + aj - bh - bj$
   (e) $jm - jn + ym - yn$ (f) $9xy - 3xz + 12py - 4pz$

6. 



A rectangular carpet measuring 3 metre by 2 metre is laid on the rectangular floor of a room.
(a) Calculate the floor area that is not covered by the carpet.
(b) Felisa wants to cover the whole floor area with the carpet of that size. Determine how many pieces of carpets would she need if the value of $y = 2$.

2.3 Algebraic Expressions and Basic Arithmetic Operations

You have learned expansion, factorisation and problem solving. Now try solving the following combined operations with expansion and factorisation.

2.3.1 Addition and subtraction of algebraic expressions

LEARNING STANDARD

Perform addition and subtraction of algebraic expressions involving expansion and factorisation.

EXAMPLE 14

Simplify.

(a) $2x^2 - 2(4x + 5)$ (b) $4w\,(w - 2) - 5$

Solution:

(a) $2x^2 - 2(4x + 5) = 2x^2 - 8x - 10$
$= 2(x^2 - 4x - 5)$
$= 2(x - 5)(x + 1)$

(b) $4w\,(w - 2) - 5 = 4w^2 - 8w - 5$
$= (2w - 5)(2w + 1)$

Addition and subtraction of algebraic fractions with the same denominators

EXAMPLE 15

Simplify each of the following.

(a) $\frac{4a}{5} + \frac{3a}{5}$ (b) $\frac{y}{2x} - \frac{3y}{2x}$ (c) $\frac{x+2}{5w} - \frac{x-5}{5w}$

Solution:

(a) $\frac{4a}{5} + \frac{3a}{5}$
$= \frac{7a}{5}$

(b) $\frac{y}{2x} - \frac{3y}{2x}$
$= \frac{y - 3y}{2x}$
$= -\frac{2y}{2x}$
$= -\frac{y}{x}$

$-\frac{y}{x} = \frac{-y}{x}$
Negative sign is usually not attached to the denominator

(c) $\frac{x+2}{5w} - \frac{x-5}{5w}$
$= \frac{x + 2 - (x - 5)}{5w}$
$= \frac{x + 2 - x + 5}{5w}$
$= \frac{7}{5w}$

$(-) \times (-) = +$

FLASHBACK

Before solving fractions, the first step is to make sure the denominators are of the same value.

(a) $\frac{3}{7} + \frac{2}{7} = \frac{5}{7}$

(b) $\frac{3y}{5} + \frac{8y}{5} = \frac{11y}{5}$

(c) $\frac{7x}{5} - \frac{x}{10}$
$= \frac{7x \times 2}{5 \times 2} - \frac{x}{10}$
$= \frac{14x}{10} - \frac{x}{10}$
$= \frac{13x}{10}$

(d) $\frac{4}{xy^2} - \frac{x}{y}$
$= \frac{4}{xy^2} - \frac{x \times xy}{y \times xy}$
$= \frac{4}{xy^2} - \frac{x^2y}{xy^2}$
$= \frac{4 - x^2y}{xy^2}$

Addition or subtraction of algebraic fractions with different denominators

One of the denominators is a multiple of the other denominators

EXAMPLE 16

Simplify the following expressions.

(a) $\frac{3}{4y} - \frac{1}{2y}$ (b) $\frac{4}{rs} - \frac{2r}{s}$

Solution:

(a) $\frac{3}{4y} - \frac{1 \times 2}{2y \times 2} = \frac{3 - 2}{4y}$
$= \frac{1}{4y}$

Equating the denominators

(b) $\frac{4}{rs} - \frac{2r \times r}{s \times r}$
$= \frac{4 - 2r^2}{rs}$

FLASHBACK

$\frac{1}{2} \frac{\times 2}{\times 2} - \frac{1}{4} = \frac{2 - 1}{4}$
$= \frac{1}{4}$

Denominators of fractions with denominators that are not common factors

EXAMPLE 17

Simplify each of the following expressions.

(a) $\frac{5x}{3} - \frac{3x}{2}$ (b) $\frac{2a}{3} + \frac{b}{2c}$

Solution:

(a) $\frac{5x \times 2}{3 \times 2} - \frac{3x \times 3}{2 \times 3}$
$= \frac{10x - 9x}{6}$
$= \frac{x}{6}$

(b) $\frac{2a}{3} + \frac{b}{2c}$
$= \frac{2a \times 2c}{3 \times 2c} + \frac{b \times 3}{2c \times 3}$
$= \frac{4ac + 3b}{6c}$

FLASHBACK

$\frac{3}{4} - \frac{1}{3} = \frac{3 \times 3}{4 \times 3} - \frac{1 \times 4}{3 \times 4}$
$= \frac{9}{12} - \frac{4}{12}$
$= \frac{5}{12}$

Denominators of fractions that are of common factors.

EXAMPLE 18

Simplify each of the following expressions.

(a) $\frac{1}{4p} + \frac{4}{6p}$ (b) $\frac{m}{4r} - \frac{5m}{14rs}$

Solution:

(a) $\frac{1}{4p} + \frac{4}{6p} = \frac{1 \times 3}{4p \times 3} + \frac{4 \times 2}{6p \times 2}$
$= \frac{3}{12p} + \frac{8}{12p}$
$= \frac{11}{12p}$

$2p \mid 4p, 6p$
$2, 3$
LCM $= 2p \times 2 \times 3 = 12p$

(b) $\frac{m}{4r} - \frac{5m}{14rs} = \frac{m \times 7s}{4r \times 7s} - \frac{5m \times 2}{14rs \times 2}$
$= \frac{7ms - 10m}{28rs}$

$2r \mid 4r, 14rs$
$2, 7s$
LCM $= 2r \times 2 \times 7s = 28rs$

FLASHBACK

LCM or Lowest Common Multiple

2.3.2 Multiplication and division of algebraic expressions

LEARNING STANDARD

Perform multiplication and division of algebraic expressions involving expansion and factorisation.

To multiply and divide algebraic expressions, you need to factorise the expression and simplify similar expressions of the numerators and denominators.

Example,

$(2p+4) \div (p^2-4)$ can be written as $\frac{2p+4}{p^2-4}$.

$$\frac{2p+4}{p^2-4} = \frac{2(p+2)}{p^2-2^2}$$ ← Factorise the numerator

$$= \frac{2(p+2)}{(p+2)(p-2)}$$ ← Simplify similar expressions or terms

$$= \frac{2}{p-2}$$

This process of simplification requires factorisation skills that you have learned.

FLASHBACK

$$\frac{m}{mn} = \frac{1}{n}$$

$$\frac{2s^2}{8sp} = \frac{2(s)(s)}{8(s)(p)}$$

$$= \frac{s}{4p}$$

FLASHBACK

$a^2 + 2ab + b^2 = (a+b)^2$
$a^2 - 2ab + b^2 = (a-b)^2$
$a^2 - b^2 = (a+b)(a-b)$

TIPS

$a + 1 = 1 + a$
$a - b = -(b - a)$
$(p-q)^2 = (q-p)^2$

EXAMPLE 19

Simplify.

(a) $\frac{a^2-1}{2ab} \times \frac{b^2}{1+a}$

(b) $\frac{(h+k)^2}{2k-h} \times \frac{6k-3h}{h^2-k^2}$

(c) $\frac{5a}{a+2b} \div \frac{2ab}{3a+6b}$

(d) $\frac{a^2-b}{10a-5b} \div \frac{(a-b)^2}{8a-4b}$

Solution:

(a) $\frac{a^2-1}{2ab} \times \frac{b^2}{(1+a)}$

$= \frac{(a+1)(a-1)}{2ab} \times \frac{b(b)}{(1+a)}$

$= \frac{b(a-1)}{2a}$ ← Simplify similar expressions

(b) $\frac{(h+k)^2}{2k-h} \times \frac{6k-3h}{h^2-k^2}$ ← Factorise

$= \frac{(h+k)(h+k)}{2k-h} \times \frac{3(2k-h)}{(h+k)(h-k)}$

$= \frac{3(h+k)}{h-k}$ ← Simplify similar expressions

FLASHBACK

$\frac{1}{x} \div \frac{1}{x}$ ← Mutual $\frac{1}{x}$ is $x \div 1$ and change the operation ÷ to ×

$= \frac{1}{x} \times \frac{x}{1}$

$= 1$

TIPS

$\frac{3}{4} \div \frac{5}{4}$

$= \frac{3}{4} \times \frac{4}{5}$

$= \frac{3}{5}$

(c) $\frac{5a}{a+2b} \div \frac{2ab}{3a+6b}$

$= \frac{5a}{(a+2b)} \times \frac{3(a+2b)}{2ab}$

$= \frac{15}{2b}$ ← Simplify similar expressions

(d) $\frac{a^2-b^2}{10a-5b} \div \frac{(a-b)^2}{8a-4b}$

$= \frac{(a+b)(a-b)}{5(2a-b)} \times \frac{4(2a-b)}{(a-b)(a-b)}$

$= \frac{4(a+b)}{5(a-b)}$ ← Simplify similar expressions

2.3.3 Combined operations of algebraic expressions

LEARNING STANDARD

Perform combined operations of algebraic expressions involving expansion and factorisation.

EXAMPLE 20

Solve the following using combined operations.

(a) $\frac{2}{5b}(15a+25b) + \frac{a}{b}$

(b) $\frac{9k^2-12k+4}{(3k+2)(3k-2)}$

(c) $\frac{12m-18m^2}{4n^2-16n} \times \frac{n}{m}$

(d) $\frac{a-b}{3a+b} \div \frac{(a-b)^2}{6a+2b}$

Solution:

(a) $\frac{2}{5b}(15a+25b) + \frac{a}{b}$

$= \frac{2}{5b} \times 5(3a+5) + \frac{a}{b}$

$= \frac{2(3a+5b)}{b} + \frac{a}{b}$

$= \frac{6a+10b}{b} + \frac{a}{b}$

$= \frac{7a+10b}{b}$

(b) $\frac{9k^2-12k+4}{(3k+2)(3k-2)}$

$= \frac{(3k-2)(3k-2)}{(3k+2)(3k-2)}$

$= \frac{3k-2}{3k+2}$

(c) $\frac{12m-18m^2}{4n^2-16n} \times \frac{n}{m}$

$= \frac{6m(2-3m)}{4n(n-4)} \times \frac{n}{m}$

$= \frac{3(2-3m)}{2(n-4)}$

(d) $\frac{a-b}{3a+b} \div \frac{(a-b)^2}{6a+2b}$

$= \frac{a-b}{3a+b} \times \frac{6a+2b}{(a-b)^2}$

$= \frac{(a-b)}{(3a+b)} \times \frac{2(3a+b)}{(a-b)(a-b)}$

$= \frac{2}{a-b}$

ATTENTION

Factoring two, three or four terms:

Two terms

$a^2 - b^2 = (a+b)(a-b)$
Example:
$x^2 - 16$
$= (x+4)(x-4)$

Three terms

Factor in two brackets
( )( )
Example:
$x^2 - 4x - 21$
$= (x-7)(x+3)$

Four terms

$6xy + 2y + 9x + 3$
Example:
$(6xy + 2y) + (9x + 3)$
$= 2y(3x+1) + 3(3x+1)$
$= (2y+3)(3x+1)$

SELF PRACTICE 2.3

1. Simplify each of the following.
   (a) $4(b-1)^2 - 9$
   (b) $(m+3)^2 - 16$
   (c) $(p-5)^2 - 49$
   (d) $7x(x-1) - 3$
   (e) $(2c-1)^2 + 2(4+c)$

2. Simplify each of the following.
   (a) $\frac{3y}{5} + \frac{3y}{5}$
   (b) $\frac{3m+2n}{m-2n} - \frac{m-5n}{m-2n}$
   (c) $\frac{4r-3s}{2r+3s} - \frac{3r-4s}{2r+3s}$

3. Simplify each of the following.
   (a) $\frac{5}{p} - \frac{2}{p^2}$
   (b) $\frac{2s}{3} - \frac{4s}{9}$
   (c) $\frac{3}{x+y} - \frac{3z}{4(x+y)}$

4. Simplify each of the following.
   (a) $\frac{3u}{4} + \frac{5v}{3}$
   (b) $\frac{1}{6s} - \frac{2}{5t}$
   (c) $\frac{2}{r-2} + \frac{4}{3s}$

5. Simplify each of the following.
   (a) $\frac{m}{9}+\frac{n}{12}$ (b) $\frac{3}{3mn}+\frac{n}{6m^2}$ (c) $\frac{4}{d^2g}+\frac{3}{5dg}$

6. Simplify.
   (a) $\frac{x^2-x}{xy}$ (b) $\frac{6a+15}{12}$ (c) $\frac{m+n}{m^2-n^2}$ (d) $\frac{2k-1}{4k^2-1}$ (e) $\frac{c^2-9}{2c+6}$

7. Simplify.
   (a) $\frac{2}{a-3}\times\frac{3}{3+a}$ (b) $\frac{h}{k-2}\times\frac{y}{h+3}$
   (c) $\frac{3m}{(m-n)}\times\frac{2mn}{(n-2m)}$ (d) $\frac{2r}{s-2}\times\frac{s-4}{r+5}$

8. Simplify.
   (a) $\frac{m}{x+2}\times\frac{2(x+2)}{m^2(x-a)}$ (b) $\frac{2r^2}{rs-s^2}\times\frac{5r-5s}{2r-4r^2}$
   (c) $\frac{x}{x+2}\times\frac{x^2+5x+6}{5x^2}$ (d) $\frac{e+2f}{5e-2f}\times\frac{4f^2-10ef}{3e^2-9ef}$

9. Simplify.
   (a) $\frac{5a}{2a+3}\div\frac{3b}{a+b}$ (b) $\frac{4}{n-3}\div\frac{8a}{3n-9}$
   (c) $\frac{6y^2}{x^2+xy}\div\frac{18xy}{x+y}$ (d) $\frac{f-1}{eg+2e}\div\frac{fg-g}{g+2}$

10. Solve the following combined operations.
    (a) $\frac{x^2+x}{x^2-y^2}\times\frac{xy-y^2}{x+y}$ (b) $\frac{4p^2-1}{p^2-1}\times\frac{pq+q}{4p-2}$
    (c) $\frac{pq-pr}{r^2-1}\div\frac{q^2-r^2}{r^2+r}$ (d) $\frac{st+tu}{4t^2-1}\div\frac{s^2-u^2}{4t^2+4t+1}$

GENERATING EXCELLENCE

1. Expand each of the following expressions.
   (a) $\frac{1}{2}(6a+12b)$ (b) $(n+2)(n-5)$ (c) $(a+2b)^2$
   (d) $(4x-y)^2$ (e) $\left(2v-\frac{1}{3w}\right)\left(3v+\frac{2}{3w}\right)$ (f) $(h-k)^2-4h(2k-3h)$

2. Factorise the following expressions.
   (a) $12m-18m^2$ (b) y^2-81 (c) $4ab-8a^2b$
   (d) x^2-16y^2 (e) $(s-3)^2-1$ (f) x^2+4x+3
   (g) $x^2+2x-15$ (h) x^2+6x+8 (i) $6cd-2ce-3bd+be$

3. Simplify each of the following expressions.
   (a) $\frac{a+2}{4v}+\frac{a-b}{2v}$ (b) $\frac{3e}{5ab}-\frac{5d}{4c}$ (c) $\frac{4}{f^2g}-\frac{3}{5fg}$

(d) $\frac{n+2}{m^2}+\frac{n}{mp}$ (e) $\frac{5x}{8yz}+\frac{y-1}{12xz}$ (f) $\frac{rs}{4y}+\frac{2-r}{18yz}$

4. Grandma has a piece of chocolate with a length of (k^2-16) cm and she wants to divide it equally among her $(k-4)$ grandchildren. What length of chocolate will each of her grandchildren receive?

5. Gurdip and Jumrang are part-time workers in a grocery shop. Gurdip is paid RM3 per hour less than twice Jumrang's pay. If Jumrang is paid RMx per hour, how much is Gurdip's pay if he works $(x+2)$ hours and Jumrang's pay if he works $(2x+3)$ hours. State your answers in algebraic form.

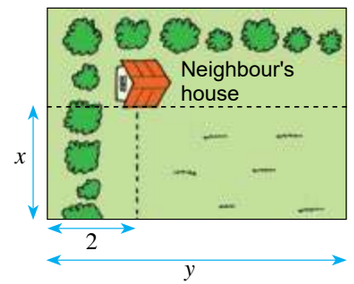
6. The ground area of a piece of land of a supermarket used for parking cars is $25(x^2-8x+16)$ square metres.
   (i) If the area of a parking lot for a vehicle is $(x-4)^2$ square metres, how many cars can be parked there?
   (ii) If 4 units of the parking lot have been booked by the supermarket, how many parking lots are left?

7. Khairul wants to cover a wall measuring $(x+5)$ metres long and $(3x-2)$ metres wide with decorative papers.
   (i) What is the area of the wall that will be covered with decorative paper if there is a door measuring $(x-1)$ metres long by x metres wide.
   (ii) If the cost of the decorative papers is RM $8x$ per square metres, how much will Khairul have to spend?

8. Swee Lee should have finished $(28+16x)$ Mathematics questions in 4 hours.
   (i) How many questions would have been done in 30 minutes?
   (ii) If Swee Lee could only finish $(14+8x)$ questions, how much time did she spend?

9. Azimah bakes a square layered cake measuring $(3x+2)$ cm long and $(x+2)$ cm wide. She cuts the cake into 6 equal parts along the length and 3 equal parts along the width. Determine the area of each piece of cake in the form of algebraic expressions.

10. Encik Hanapi intends to build a single storey bungalow on a piece of land measuring x metres wide and y metres long. He needs to reserve part of the land which is 2 metres wide for a road for his neighbour.
    (i) What is the total area of his piece of land?
    (ii) What is the remaining size of the land after reserving some parts for the road?
    (iii) If the cost of land is RM18 per square metres, what is the total cost of the remaining land?



CHAPTER SUMMARY

Factorisation and Algebraic Factors

Expansion

Multiplication of an algebraic expression with a term or an algebraic expression

- $a(x + y) = ax + ay$
- $(a + b)(x + y) = ax + ay + bx + by$
- $b(c + d) = bc + bd$
- $(b + c)(d + e) = bd + be + cd + ce$
- $(b + c)^2 = b^2 + 2bc + c^2$
- $(b - c)^2 = b^2 - 2bc + c^2$
- $(b + c)(b - c) = b^2 - c^2$

Factorisation

Factorisation is the method of writing an algebraic expression as a product of two or more algebraic terms or algebraic expressions.
Factorisation is the reverse process of an expansion.

- $2a - a^2 = a(2 - a)$
- $a^2 + 4a + 3 = (a + 1)(a + 3)$
- $a^2 - 7a + 10 = (a - 5)(a - 2)$
- $a^2 - 36 = (a^2 - 6^2) = (a - 6)(a + 6)$
- $ab + ac + bd + cd = (b + c)(a + d)$
- $a^2 - 2ab + b^2 = (a - b)^2$

Addition and Subtraction

Before adding or subtracting two algebraic fractions, check the denominators first. If they are not the same, you need to express all fractions in terms of common denominators.

- $\frac{a}{4} + \frac{b}{4} = \frac{a + b}{4}$
- $\frac{1}{a} + \frac{1}{b} = \frac{b + a}{ab}$
- $\frac{1}{2a} - \frac{1}{ab} = \frac{1 \times b}{2a \times b} - \frac{1 \times 2}{ab \times 2}$
  $= \frac{b - 2}{2ab}$

Multiplication and Distribution

Factorise expressions before division or multiplication when it is necessary.

$$\frac{m+n}{x-y} \div \frac{(m+n)^2}{x^2-y^2} = \frac{\cancel{m+n}^{1}}{\cancel{x-y}_{1}} \times \frac{(x+y)(\cancel{x-y}^{1})}{(\cancel{m+n}_{1})(m+n)}$$
$$= \frac{x+y}{m+n}$$

SELF REFLECTION

At the end of this chapter, I will be able to:	
1. Explain the meaning of expansion of two algebraic expressions.	
2. Work out expansion of two algebraic expressions.	
3. Simplify algebraic expressions involving combined operations, including expansion.	
4. Solve problems involving expansion of two algebraic expressions.	
5. Relate the multiplications of algebraic expressions to the concept of factors and factorisation, and hence list out the factors of the product of the algebraic expressions.	
6. Factorise algebraic expressions using various methods.	
7. Solve problems involving factorisation.	
8. Perform addition and subtraction of algebraic expressions involving expansion and factorisation.	
9. Perform multiplication and division of algebraic expression involving expansion and factorisation.	
10. Perform combined operations of algebraic expression involving expansion and factorisation.	

MINI PROJECT

Title: What is the volume of this pail of water?
Materials: A pail of water (labelled z), a few small mineral water bottles (labelled x), a few big mineral water bottles (labelled y) and a funnel

Each group is given a few empty mineral water bottles of different size and a funnel. Students fill up the empty bottles with the water. Then they write out the algebraic expression to express the volume of water. Every group presents their answers. Are they the same? Can you determine the volume of water?