CHAPTER 3

Algebraic Formulae

WHAT WILL YOU LEARN?

3.1 Algebraic Formulae

WORD LINK

- Algebraic formula — *Rumus algebra*
- Variable — *Pemboleh ubah*
- Coefficient — *Pekali*
- Subject of formula — *Perkara rumus*

A wholesale store sells clothes for RMy. During the festive season, the store discounted the price of the clothes as shown below.

As a computer programmer, you are asked to develop a programme that contains the formula for calculating the selling price of the clothes.

WALKING THROUGH TIME

Al-Khwarizmi introduced negative and decimal numbers. He also founded a mathematical programme using a set of instructions to complete a complex calculation.

For more information:

http://rimbunanilmu.my/mat_t2e/ms043

WHY STUDY THIS CHAPTER?

- The algebraic formulae is applied by engineers, statisticians, mathematicians and astronomers in their respective jobs.

CREATIVE ACTIVITY

Aim: Identifying formula
Material: School Calendar
Steps:

1. Students carry out the activity in pairs.
2. Calculate the amount of money that can be saved from the following situations (assume that the calculation starts from first to the last day of each month).

   Situation 1
   Badrul is a form 2 student who likes to save. On each school day, he receives RM5 as his pocket money and spends RM4.50. What is the amount of Badrul's savings in January?

   Situation 2
   Sedthu saves RM15 per month. If he receives RM10 as pocket money, calculate Sedthu's expenditure for one day in April.

3. State the method of calculating the savings.

From the situations above, write an equation for the total savings in relation to pocket money, money spent and to the number of days using basic mathematical operations to get the total amount of savings. Pocket money, money spent and number of days are variables. You can determine the amount saved by changing the value of the variables.

3.1 Algebraic Formulae

Algebraic expression is a combination of two or more algebraic terms. The algebraic formulae combines an algebraic expression using addition, subtraction, multiplication or division and is written in the form of an equation.

3.1.1 Forming formula

LEARNING STANDARD

Write a formula based on a situation.

COGNITIVE STIMULATION Group

Aim: Forming algebraic formulae
Material: Worksheets
Steps:

1. Students carry out this activity in groups.

   A cultural club will perform at a school-level cultural night. The table shows the number of dancers according to the types of dance and race represented by an alphabet.

Types of dance	Race		
	Malay	Chinese	Indian
Sumazau	a	$2c$	$2a$
Kuda Kepang	$2b$	b	$5b$
Lion	$2c$	$3a$	c

The alphabet a, b and c are known as variables.

2. Write a formula for each of the following subject.
   (a) s, number of Chinese dancers.
   (b) d, number of Kuda Kepang dancers.
   (c) w, number of Indian and Malay dancers.

Discussion:
(i) Difference in formula between the groups in your class.
(ii) Conclusion from the activity above.

The formula is expressed as $s = 2c + b + 3a$, $d = 8b$, $w = 3a + 7b + 3c$. From the activity above, the formula is formed by the relationship among a few variables.

EXAMPLE 1

Suzi sold two types of cakes of different prices. The chocolate cake sold at RM3 a slice. The cheese cake was sold at twice the price of the chocolate cake. In conjunction with the opening of a new branch, she gave 10% discount for all cakes. Determine a formula to calculate the selling price of the cake, if m slices of chocolate cake and n slices of cheese cake were sold.

Solution:

Price of cheese cake = twice the price of the chocolate cake
$= 2 \times \text{RM3}$
$= \text{RM6}$

Selling price, z = [(number of chocolate cake × price) + (number of cheese cake × price)] × discount
$= [(m \times \text{RM3}) + (n \times \text{RM6})] \times (100\% - 10\%)$
$= (\text{RM3}m + \text{RM6}n) \times 90\%$
$= (3m + 6n) \times 0.9$

Let, z = Selling price
m = number of chocolate cake
n = number of cheese cake

The algebraic formula is $z = 0.9\,(3m + 6n)$
$= 2.7m + 5.4n$

DO YOU KNOW?

The Sumazau dance is known as the traditional dance of the Kadazan Dusun tribe in Sabah. The Sumazau dance is performed during the Tadau Kaamatan festival and is celebrated every year in May.

http://www.jkkn.gov.my/pemetaanbudaya/

TIPS

In the activity on the left, s, d and w are subjects of formula. They can be written on the left or right side.

TIPS

A variable in a formula can be represented by letters a to z (in example 1, m and n represents variables). z, written on the left is known as subject of formula.

THINK SMART!

Is this equation called a formula?

(i) $a \times (b + c) = (a \times b) + (a \times c)$

(ii) $\frac{p}{a} + \frac{q}{a} = \frac{b}{a}$

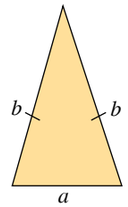
Discuss.

3.1.2 Changing the subject of formula

LEARNING STANDARD

Change the subject of formula of an algebraic equation.

The subject of a formula can be a variable for the algebraic formula and the variable can be the subject of an algebraic formula.



Perimeter, P for an equilateral triangle can be expressed in a and b.
Hence, $P = a + 2b$

The subject of formula of the equation above can be changed as shown.

(i) $a = P - 2b$

(ii) $b = \dfrac{P - a}{2}$

TIPS

Coefficient for the subject of formula must be 1.

FLASHBACK

$1 \times p = p$

$-1 \times p = -p$

$0 \times p = 0$

$\dfrac{1}{3} \times p = \dfrac{p}{3}$

$-\dfrac{1}{3} \times p = -\dfrac{p}{3}$

EXAMPLE 2

State m, as the subject of formula.

(a) $q = m + p$

(b) $b = 2s - m$

(c) $a = \dfrac{5}{2m}$

(d) $t = \dfrac{m - n}{-3}$

Solution:

(a) $m + p = q$

$m + p - p = q - p$

Then, $m = q - p$

m in terms of p and q

(b) $2s - m = b$

$2s - 2s - m = b - 2s$

$-m = b - 2s$

$\dfrac{1}{-1} \times (-m) = \dfrac{1}{-1}(b - 2s)$

$m = -b + 2s$

m in terms of b and s → Then, $m = 2s - b$

TIPS

Subject of formula should be on the left side of the equation.

(c) $a = \dfrac{5}{2m}$

$a \times 2m = \dfrac{5}{2\cancel{m}} \times \cancel{2m}$ (1, 1)

$2am = 5$

$\dfrac{\cancel{2a}m}{\cancel{2a}} = \dfrac{5}{2a}$ (1, 1)

Then, $m = \dfrac{5}{2a}$ ← m in terms of a

(d) $\dfrac{m - n}{-3} = t$

$\dfrac{m - n}{\cancel{-3}} \times \cancel{-3} = t \times (-3)$ (1, 1)

$m - n = -3t$

$m - n + n = -3t + n$

$m = -3t + n$

Then, $m = n - 3t$

m in terms of n and t

FLASHBACK

You have learned to solve linear equation using the following three methods:
(a) Trial and improvement
(b) Application of equality concept
(c) Back track

EXAMPLE 3

State p, as the subject of formula.

(a) $q = \sqrt{p}$

(b) $s = p^2$

(c) $w = \sqrt{\dfrac{p}{3}}$

(d) $t = \dfrac{1}{p^2}$

Solution:

(a) $\sqrt{p} = q$

$(\sqrt{p})^2 = (q)^2$ ← Both sides of the equation are squared

$p = q^2$

(b) $p^2 = s$

$\sqrt{p^2} = \sqrt{s}$

$p = \sqrt{s}$

(c) $\sqrt{\dfrac{p}{3}} = w$

$\left(\sqrt{\dfrac{p}{3}}\right)^2 = w^2$

$\dfrac{p}{3} = w^2$

$\dfrac{p}{\cancel{3}} \times \cancel{3} = w^2 \times 3$ (1, 1)

$p = 3w^2$

(d) $t = \dfrac{1}{p^2}$

$t \times p^2 = \dfrac{1}{\cancel{p^2}} \times \cancel{p^2}$ (1, 1)

$tp^2 = 1$

$p^2 = \dfrac{1}{t}$

$p = \sqrt{\dfrac{1}{t}}$

TIPS

$(\sqrt{a^2})^2 = a^2$

$\sqrt{a^2} = a$

ATTENTION

$\sqrt{x} = x^{\frac{1}{2}}$

$(\sqrt{x})^2 = (x^{\frac{1}{2}})^2$

$= x^{\frac{1}{2} \times 2}$

$= x$

ATTENTION

Reciprocals

$\dfrac{1}{x} = a,\ x = \dfrac{1}{a}$

Squares

$(\sqrt{x})^2 = a^2,\ x = a^2$

Square root

$\sqrt{x^2} = \sqrt{a},\ x = \pm\sqrt{a}$

3.1.3 Determining the value of variable

LEARNING STANDARD

Determine the value of a variable when the value of another variable is given.

The value of a subject of a formula can be obtained when all variable values are given. On the other hand, the value of a variable can be obtained when the value of subject of the formula and variable is given.

EXAMPLE 4

Given $w = 7t - 5u$, calculate the following.

(a) value w when $t = 3$ and $u = -2$

(b) value t when $w = 15$ and $u = 4$

Solution:

(a) Substitute $t = 3$ and $u = -2$ into the formula.

$w = 7(3) - 5(-2)$

$= 21 + 10$

$= 31$

FLASHBACK

$-a + a = 0$

$-a - a = -2a$

$-a \times a = -a^2$

$(-a) \times (-a) = a^2$

$-a \div a = -1$

$(-a) \div (-a) = 1$

(b) Substitute $w = 15$ and $u = 4$ into the formula.

$$7t - 5u = w$$
$$7t - 5(4) = 15$$
$$7t = 15 + 20$$
$$t = \frac{35}{7}$$
$$t = 5$$

EXAMPLE 5

Given $m = \frac{1}{4}(p-q)^2$, calculate the value q if $m = 16$ and $p = 3$.

Solution:

$$m \times 4 = \frac{1}{4}(p-q)^2 \times 4$$
$$4m = (p-q)^2$$
$$\sqrt{4m} = \sqrt{(p-q)^2}$$ ← Square root both sides of the equation
$$p - q = \sqrt{4m}$$
$$-q = \sqrt{4m} - p$$
$$(-q) \times \frac{1}{-1} = (\sqrt{4m} - p) \times \frac{1}{-1}$$ ← Multiply both sides of the equation by $\frac{1}{-1}$
$$q = -\sqrt{4m} + p$$
$$q = p - \sqrt{4m}$$
$$q = 3 - \sqrt{4(16)}$$ ← Replace $m = 16$ and $p = 3$
$$q = 3 - 8$$
$$q = -5$$

FLASHBACK

Algebraic formulae

Variables
Variables is a quantity where the value is not known or can be changed.

Constant
Constant is a quantity where the value is fixed.

Algebraic formulae
Algebraic formulae involve equations that connect a few variables.

Subject of formula
Subject of formula is a dependant variable expressed in terms of an independent variable of a formula. The subject of formula always has coefficient 1. The algebraic formulae involves
(a) One of the basic mathematical operations.
(b) Squares and square root
(c) A combination of basic and square operations or square root.

TIPS

Alternative method

Substitute $m = 16$ and $p = 3$

$$16 = \frac{1}{4}(3-q)^2$$
$$64 = (3-q)^2$$
$$\sqrt{64} = (3-q)$$
$$8 = 3 - q$$
$$q = 3 - 8$$
$$q = -5$$

3.1.4 Solving problems

LEARNING STANDARD

Solving problems involving formulae.

EXAMPLE 6

The price of a fried chicken at a school canteen is twice the price of a bun. With RM5, Azman bought two buns and a piece of chicken. The balance of RM1 is saved. If Azman has RM12 and decides to buy the same number of buns, how many pieces of fried chickens will he be able to buy?

Solution:

Understanding the problem

Number of fried chicken that can be bought by Azman for RM12.

Planning the strategy

Determine the price of a bun.

(a) Represent the price of bun and chicken with x.
Price of bun = RMx
Price of chicken = RM2x

(b) The total price of bun + The total price of chicken + RM1 = Total expenditure

$$2(\text{RM}x) + \text{RM}2x + \text{RM}1 = \text{RM}5$$
$$2x + 2x + 1 = \text{RM}5$$
$$4x + 1 = 5$$
$$x = \frac{5-1}{4}$$
$$= 1$$

Thus, the price of a bun is RM1 and the price of a piece of chicken is RM2.

Conclusion

Azman gets to buy 5 pieces of fried chicken.

Implementing the strategy

(a) Represent the number of fried chicken with y.

(b) Total price of bun + Total price of chicken = RM12
$$(\text{RM}1 \times 2) + (\text{RM}2 \times y) = \text{RM}12$$
$$2 + 2y = 12$$
$$y = \frac{12-2}{2}$$
$$= 5$$

SELF PRACTICE 3.1

1. Express the letters in the brackets as subject of formula.

(a) $z = m - qp$ [m]
(b) $v = u + 2$ [u]
(c) $3y = \frac{7w}{x}$ [x]
(d) $3a = \frac{4}{5+b}$ [b]
(e) $5q = \frac{3}{u} - 5$ [u]
(f) $2w = -4 + \frac{5}{v}$ [v]
(g) $2a = \sqrt{3b} + 5$ [b]
(h) $(-5t)^2 = \frac{25w^2}{36}$ [w]
(i) $(-3m)^2 = 4p - 8$ [m]
(j) $\sqrt{(9r^2)} = 4s - 7$ [r]

2. The price of a shirt is RM35, while the cost of a pair of trousers is RM45. A discount of 15% is given on the price of a shirt, while a discount of 10% is given on the price of a pair of trousers. Write the formula for the total expenditure, z, if Syamsul wants to buy x shirts and y trousers.

3. Solve the following.

(a) Given $c = 4d + 8$, calculate
   (i) value c when $d = 2$
   (ii) value d when $c = 10$

(b) Given $4p = 18 - 5q$, calculate
   (i) value p when $q = 2$
   (ii) value q when $p = 2$

(c) Given $\frac{1}{3}m = \frac{2}{3}n + 8$, calculate
   (i) value m when $n = -15$
   (ii) value n when $m = 30$

(d) Given $\sqrt{4m} = \frac{n^2 - 5}{2}$, calculate
   (i) value n when $m = 4$
   (ii) value m when $n = 2$

(e) Given $3u = 4r + s$, calculate
   (i) value u when $r = 5$ and $s = -2$
   (ii) value r when $u = 3$ and $s = 3$
   (iii) value s when $u = 2$ and $r = \frac{1}{2}$

(f) Given $\frac{3}{5}p = \frac{2}{3}q - \frac{1}{4}r$, calculate
   (i) value p when $q = 3$ and $r = 8$
   (ii) value q when $r = -12$ and $p = 10$
   (iii) value r when $p = -15$ and $q = -15$

(g) Given $\sqrt{3a} = 9b - 4c$, calculate
   (i) value a when $b = \frac{1}{3}$ and $c = \frac{1}{2}$
   (ii) value b when $c = 3$ and $a = 12$
   (iii) value c when $a = 3$ and $b = 3$

(h) Given $1\frac{1}{2}s = \frac{3}{5}t^2 + \frac{1}{3}u^2$, calculate
   (i) value s when $t = -5$ and $u = 3$
   (ii) value t when $u = -6$ and $s = 28$
   (iii) value u when $s = \frac{4}{6}$ and $t = \frac{5}{6}$

4. Write the algebraic formula based on the following situations.

(a) The total price RMz that needs to be paid by a buyer who bought x workbook and y geometry set. The workbook and the geometry set each costs RM5.90 and RM3.60 respectively.

(b) In a class party, a teacher buys p carton of canned drinks to be distributed to the q students. From the total number of canned drinks, seven cans were distributed to the subject teachers. If a carton contains 24 cans of drinks, calculate the number of cans received by each student, b in terms of p and q.

(c) Shoe A is sold at RM35 a pair, while shoe B costs RM76 a pair. Beautiful Shoe Shop offers a 15% discount on purchases of two pairs of shoes. Mei Ling buys m pairs of shoe A and n pairs of shoe B. Calculate the price payable, P in terms of m and n.

(d) A car is able to travel as far as 10 km with a litre of petrol. Express the cost, RMx of the petrol that needs to be filled for s km if a litre of petrol costs RMt.

GENERATING EXCELLENCE

1. Write the algebraic formula from the following situation.
   (a) A represents area, x represents the length of a square. Write the formula that relates A to x.

(b) The rental fee of a sepak takraw court is RM5 for the first hour. Payment for the next hour is RM3. Write the formula that relates the amount of payment p, and the hours used, h.

(c) Acceleration, a is defined as the difference between the final velocity, v_2 and initial velocity v_1 divided by time, t. Write the relationship between a, v_2, v_1 and t.

2. Express the letters in the brackets as a subject of formula.

(a) $m = -3q + p$ [q]
(b) $x = -p - w$ [w]
(c) $2e = 4g + 3h$ [g]
(d) $\frac{3}{4}m - 6p = \frac{3}{4}q$ [q]
(e) $w = 3v^2$ [v]
(f) $2m = \frac{3}{4}n^2$ [n]
(g) $3w = \frac{(v+1)^{\frac{1}{2}}}{2}$ [v]
(h) $\frac{5}{4}f = \frac{5}{\sqrt{k-7}}$ [k]

3. Calculate the following value.

(a) Given $w = \frac{x+y}{1+x}$, calculate the value
   (i) w, when $x = 2$ and $y = -8$
   (ii) x, when $w = 20$ and $y = 5$
   (iii) y, when $w = 5$ and $x = 6$

(b) Given $6b = \sqrt{\frac{c-d^2}{9}}$, calculate the value
   (i) b, when $c = 20$ and $d = 2$
   (ii) c, when $b = \frac{1}{9}$ and $d = 2$
   (iii) d, when $b = \frac{1}{2}$ and $c = 90$

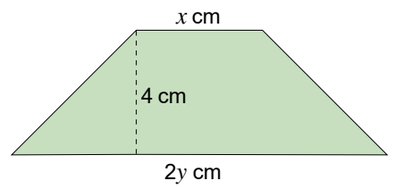
(c) Given $-2p = \frac{(q+1)}{(r+q)}$, calculate the value
   (i) p, when $q = 3$ and $r = 3q$
   (ii) q, when $p = 3$ and $r = 2q$
   (iii) r, when $p = -\frac{1}{3}$ and $q = 2p$

(d) Given $4s^2 = \left(\frac{3t - 4u}{5}\right)^2$, calculate the value
   (i) s, when $t = s - 1$ and $u = 2s$
   (ii) t, when $s = -5u$ and $u = 3$
   (iii) u, when $s = \frac{1}{3}t$ and $t = 2 - u$

4. The salary of fast food store branch manager is 3 times more than a part time employee salary, RMx per day. Working hours for part time employees are half of the manager's working time, y within a month. If they work 26 days in a month, write the formula for the difference in salary, RMz between the two workers in terms of x and y.

5. Julia takes 40 seconds to walk as far as 50 metres. Write a formula to help Julia calculate the duration of the trip, t in minutes from her home to the school that is s kilometre away.

6. The area of the trapezium below is 36 cm^2. If $x + y = 11$ cm, calculate the value of x and y.



CHAPTER SUMMARY

Algebraic Formulae

Algebraic formulae connect the algebraic expressions through addition, subtraction, multiplication and division in an equation form.

1. $y = 3x - 5$
2. $w = \frac{6 - 7v}{v}$
3. $L = \frac{1}{2}th$
4. $A = \pi r^2$

Subject of formula is represented by a letter.

Subject of formula can be changed according to the value of the variable.

$$w = -6 - 8t$$

$$t = \frac{-6 - w}{8}$$

A variable in the subject of formula can be obtained when the value of the other variables is given.

Example:

Given $Q = \frac{2v}{-v + u}$, calculate value u,

if $v = 2,\ Q = 4$

Thus, $u = 3$

Solving problem involves changing the subject of a formula, combination of basic mathematical operations, square and square root.

SELF REFLECTION

At the end of the chapter, I am able to:

1. Write a formula based on a situation.	○	○	○
2. Change the subject of formula for an algebraic equation.	○	○	○
3. Determine the value of a variable when the value of another variable is given.	○	○	○
4. Solve problems involving formulae.	○	○	○

MINI PROJECT

Title: Counting board

Materials: Manila card, used card board, coloured paper, glue and scissors

Steps:

1. Create a counting board to calculate the price that needs to be paid by the student to purchase three items.
2. Example of the things that needs to be purchased are pen, mineral water, and note book.
3. Price of the pen, mineral water and note book is determined by the students according to the current price.

Items			
Number	☐ *a*	☐ *b*	☐ *c*
Price	*a* × RM ☐	*b* × RM ☐	*c* × RM ☐
Total	☐ (i)	☐ (ii)	☐ (iii)
Overall total	☐ (i) + ☐ (ii) + ☐ (iii)		

Example of a counting board