CHAPTER 4

Polygon

In our daily life, there is a combination of polygons around us especially in the designs of buildings. The combinations of polygons produce interesting and diverse forms of art. This geometric pattern can be seen at Tanjung Bungah Floating Mosque, Penang whereby it has a unique combination of local and Western Asian architecture.

WHAT WILL YOU LEARN?

WORD LINK

• Polygon	• *Poligon*
• Regular polygon	• *Poligon sekata*
• Irregular polygon	• *Poligon tak sekata*
• Axis of symmetry	• *Paksi simetri*
• Side	• *Sisi*
• Interior angle	• *Sudut pedalaman*
• Exterior angle	• *Sudut peluaran*
• Supplementary angle	• *Sudut penggenap*
• Origami	• *Origami*

WALKING THROUGH TIME

Polygon refers to the words 'poly' which means many and 'gon' which means angle. Polygon is named by the number of sides. For larger polygons, mathematicians name the polygon according to the number of sides for example 17-gon.

For more information:

http://rimbunanilmu.my/mat_t2e/ms055

WHY STUDY THIS CHAPTER?

- Creating logos, murals on school walls and creating symmetry on drawing.
- In the field of technology, knowledge of polygon is used in building architecture, roofing, interior designing, fabric design and more.
- Careers involved in this field are surveyors, technicians, engineers, architects, graphic designers and many others.

CREATIVE ACTIVITY

Aim: Producing a pentagon using paper folding technique (origami)

Materials: Square paper and scissors

Steps:

1. Fold the square paper into two sections.
2. Label each vertex with *PQRS*.
3. Fold *P* towards line *QR*. Press the fold down. Open the fold.
4. Fold *Q* towards line *PS*. Press the fold down. Open the fold.
5. There should be fold marks shaped *X* in the middle. Label it as *X*.
6. Bring *S* to the centre marked *X*. Press the fold down.
7. Bring the vertex that touches *X* and fold it back so that this side rests on the furthest side.
8. Take *P* and fold it to line *TU*. Fold this shape back to get Diagram *D*.
9. Now, cut off the top as shown in Diagram *D*.
10. Open the folds. State the shape of the origami.

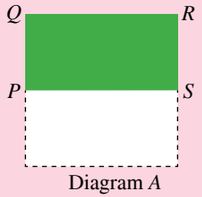


Diagram *A*

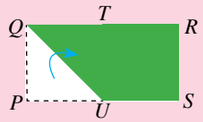


Diagram *B*

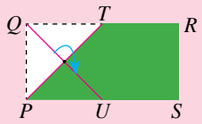


Diagram *C*

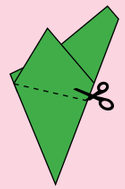


Diagram *D*

QR CODE

Scan the QR Code or visit http://rimbunanilmu.my/mat_t2e/ms056 to view the tutorial video of the pentagon shape origami.

4.1 Regular Polygon

4.1.1 Geometric properties of regular polygon

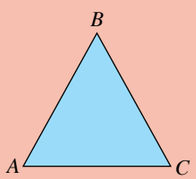
A regular polygon is a polygon that has sides with equal length and interior angles of the same size.

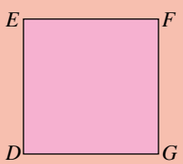
Identifying regular polygon

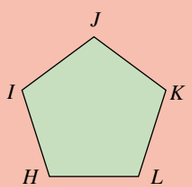
COGNITIVE STIMULATION Individual

Aim: Exploring geometrical characteristics of regular polygon

Materials: Ruler and compasses







LEARNING STANDARD

Describe the geometric properties of a regular polygons using various representations.

DO YOU KNOW

Origami originated from Japan that means
'ori' = art, 'gami' = paper

FLASHBACK

A polygon is a closed form on a plane that is bounded by three or more straight lines as the sides.

Steps:

1. Measure the length of the side and interior angle for all the polygons.
2. Complete the table below.

Triangle *ABC*			
Length of side		Measurement of angle	
AB		∠*CAB*	
BC		∠*ABC*	
CA		∠*BCA*	

Square *DEFG*			
Length of side		Measurement of angle	
DE		∠*GDE*	
EF		∠*DEF*	
FG		∠*EFG*	
GD		∠*FGD*	

Pentagon *HIJKL*			
Length of side		Measurement of angle	
HI		∠*HIJ*	
IJ		∠*IJK*	
JK		∠*JKL*	
KL		∠*KLH*	
LH		∠*LHI*	

Conclusion: _____

Conclusion: _____

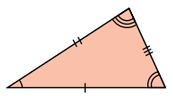
Conclusion: _____

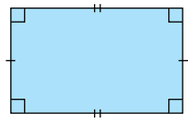
Discussion:
Your findings from the activity above.

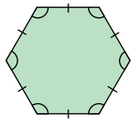
Regular polygons are polygons for which all sides are equal and all interior angles are of the same size. Regular polygons have congruent interior angles. **Irregular polygons** are polygons with irregular sides.

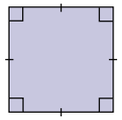
EXAMPLE 1

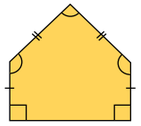
Based on the diagram, which one is a regular polygon and which is an irregular polygon?

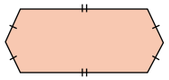
(a) 

(b) 

(c) 

(d) 

(e) 

(f) 

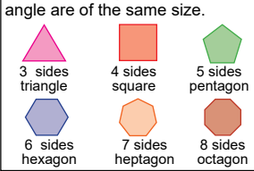
Solution:

(a) Irregular polygon
(b) Irregular polygon
(c) Regular polygon
(d) Regular polygon
(e) Irregular polygon
(f) Irregular polygon

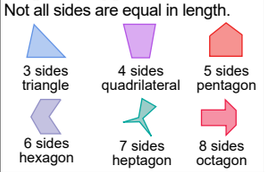
DO YOU KNOW

Determining types of polygon.
A polygon can have 3 or more sides.

Regular polygon
All sides are equal. All interior angle are of the same size.



Irregular polygon
Not all sides are equal in length.

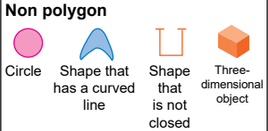


Concave polygon
Has at least one angle that is more than 180°.

Convex polygon
No interior angle more than 180°.

Complex polygon
Has lines that intersects in the polygon.

Non polygon



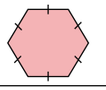
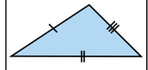
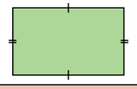
Determining axis of symmetry

COGNITIVE STIMULATION Group

Aim: Describing the axis of symmetry of a regular polygon
Materials: Dynamic geometry software and scissors
Steps:

1. Open the file MS058A and print the worksheet.
2. Divide the class into two groups.
3. The first group cuts the regular polygons and second group cuts the irregular polygons.
4. By folding the polygon, determine the axis of symmetry for all the regular polygons and irregular polygons.
5. Complete the table below.

QR CODE
Scan the QR Code or visit http://rimbunanilmu.my/mat_t2e/ms058a to print the worksheet.

		Number of sides	Number of axis of symmetry
Regular polygons			
			
			
			
Irregular polygons			
			

Discussion:
(i) What is the relationship between the number of sides with the number of axis of symmetry?
(ii) Conclusion from the findings of the first group and second group.

QR CODE
Scan the QR Code or visit http://rimbunanilmu.my/mat_t2e/ms058b to find out names of the multi-sided polygon.

The number of axis of symmetry for a regular polygon is equals to the number of sides of the polygon.

For irregular polygons the number of axis of symmetry should be explored using the folding method.

4.1.2 Constructing a regular polygon

LEARNING STANDARD
Construct regular polygons using various methods and explain the rationales for the steps of construction.

Regular polygon can be constructed by using various methods. Explore the activity below.

COGNITIVE STIMULATION Individual

Aim: Creating regular polygon
Materials: Dynamic geometry software, paper and scissors
Steps:

1. Open the file MS059A.
2. Click on the *polygon* instructions and choose *regular polygon*.
3. Click any points on the Cartesian plane.
4. Click any second point.
5. On the window of the *regular polygon*, at the *vertices* enter the number of edges that has to be built. For example, pentagon has five vertices.
6. Repeat the same steps for regular hexagon, regular heptagon, regular octagon and regular nonagon.
7. Print out the shapes.
8. Paste your work in your book.

Discussion:
Your findings from the activity above.

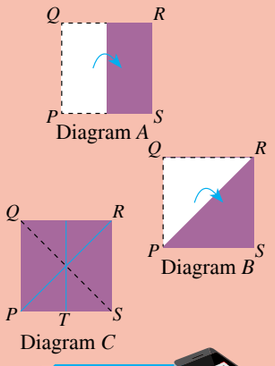
QR CODE
Scan the QR Code or visit http://rimbunanilmu.my/mat_t2e/ms059a for cognitive stimulation.

COGNITIVE STIMULATION Individual

Aim: Producing a regular octagon using origami
Materials: Dynamic geometry software, printer, square shaped coloured paper and scissors
Steps:

1. Open the file MS059B to watch the tutorial on making an octagon shaped origami.
2. Fold the paper into two parts. Open the fold.
3. Fold the diagonal part of the paper into two parts.
4. Take the centre point of the folded line and bring it close to the diagonal line through the centre point.
5. Cut away the extra paper.
6. Open the fold, then an octagon is produced.

Discussion:
Your findings from the activity above.

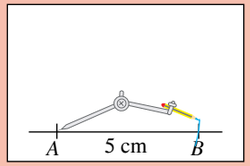


QR CODE
Scan the QR Code or visit http://rimbunanilmu.my/mat_t2e/ms059b to watch the tutorial on creating an octagon shaped origami.

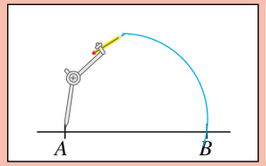
COGNITIVE STIMULATION Individual

Aim: Building a regular polygon using geometry tools
Materials: Pencil, ruler, A4 paper and compasses

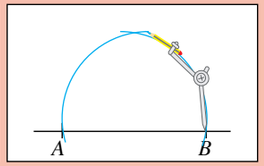
Activity 1:
Construct an **equilateral triangle** with the sides 5 cm



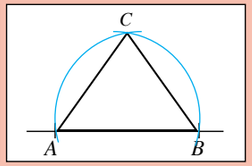
(a) Construct a line segment AB with length 5 cm.



(b) Construct a curve with the radius 5 cm from point A.

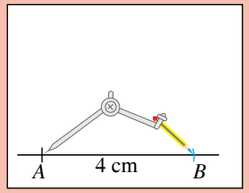


(c) Construct a curve with radius 5 cm from point B so that it intersects with the first curve. The intersection point is labelled C.

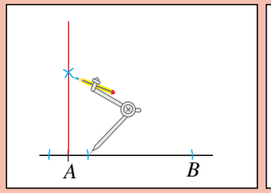


(d) Draw a line from A to C and B to C. An equilateral triangle is created.

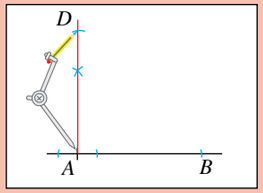
Activity 2:
Construct a **square** with sides 4 cm



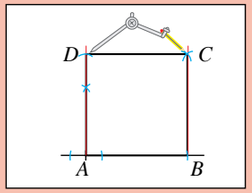
(a) Construct a line segment AB with the length 4 cm.



(b) Construct a perpendicular line with AB through point A.

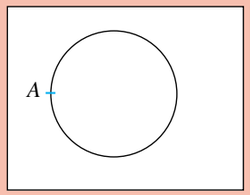


(c) Construct a curve with a distance 4 cm from point A so that it intersects with the perpendicular line. The intersection point is labelled D.

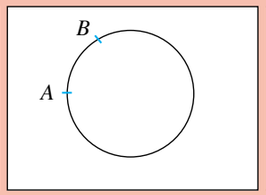


(d) Construct two curves with a distance of 4 cm from B and D so that both the curves intersect. The intersection point is labelled C.

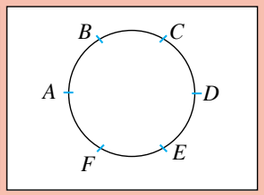
Activity 3:
Construct a regular **hexagon** with sides 3.5 cm



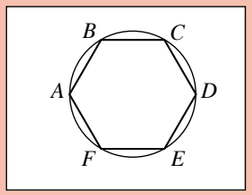
(a) Construct a circle with the radius of 3.5 cm. Mark one point at the circumference and label it A.



(b) Construct a curve with radius 3.5 cm from A and label it B.



(c) Construct a curve with the distance 3.5 cm from B and label it C. Repeat the steps until F.



(d) Draw line AB, BC, CD, DE, EF and FA to form a regular hexagon.

Discussion:
Your findings from the activity above.

From all the activities that have been done, the most accurate method in constructing regular polygons is by using dynamic geometry software.

TIPS

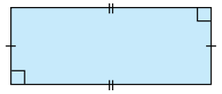
Regular polygons can also be constructed by dividing the angles at the centre of a circle according to the number of sides.

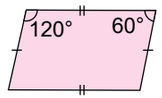
QR CODE

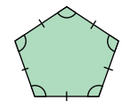
Scan the QR Code or visit http://rimbunanilmu.my/mat_t2e/ms060 to construct regular polygon using geometry tools.

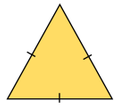
SELF PRACTICE 4.1

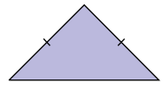
1. Determine whether each polygon is a regular polygon or irregular polygon.

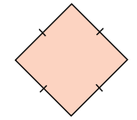
(a) 

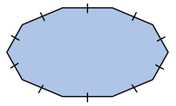
(b) 

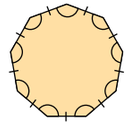
(c) 

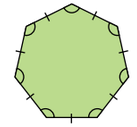
(d) 

(e) 

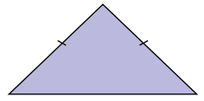
(f) 

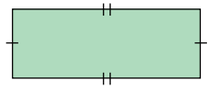
(g) 

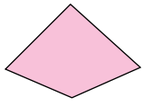
(h) 

(i) 

2. Trace the following diagrams. Determine the number of axis of symmetry for each diagram.

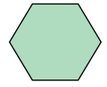
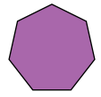
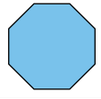
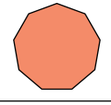
(a) 

(b) 

(c) 

(d) 

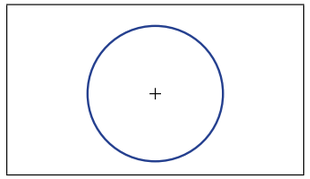
3. Complete the table below with the characteristics of the polygons.

Regular polygon	Polygon name	Number of sides	Number of vertices	Number of axis of symmetry
				
				
				
				

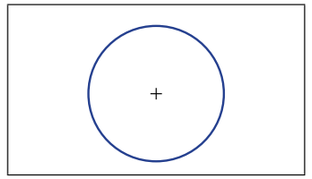
4. Construct the following regular polygons by using a ruler and compasses.
   (a) Equilateral triangle with side length 3.4 cm.
   (b) Square with sides 3.6 cm.
   (c) Regular hexagon with sides 4 cm.
   (d) Regular heptagon with sides 4.2 cm.
   (e) Regular octagon with sides 4.5 cm

5. Draw the following regular polygon by dividing the vertices at the centre equally.

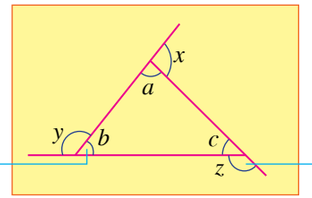
(a) Regular pentagon



(b) Regular hexagon



4.2 Interior Angles and Exterior Angles of Polygons



Interior angle is an angle that is shaped by two adjacent sides of a polygon.

Angle a, b, and c are interior angles.

Exterior angle is an angle that is formed when one side of the polygon is extended. It is the supplementary angle to the interior angle of the adjacent side.

Angle x, y, and z are exterior angles.

4.2.1 Total sum of an interior angle

There is a relationship between the number of vertices of a polygon with the sum of the interior angle. Explore the activity below:

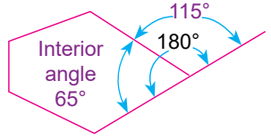
COGNITIVE STIMULATION Individual

Aim: Exploring the number of triangle in a polygon
Materials: Paper and protractor
Steps:

1. Open the file MS062 to obtain information about polygon shapes.
2. Print the riangle, square, pentagon, hexagon, heptagon, octagon and nonagon.

DO YOU KNOW ?

Exterior angle + Interior angle = 180°.



FLASHBACK

Total sum of interior angles of a triangle is 180°.

$a + b + c = 180°$

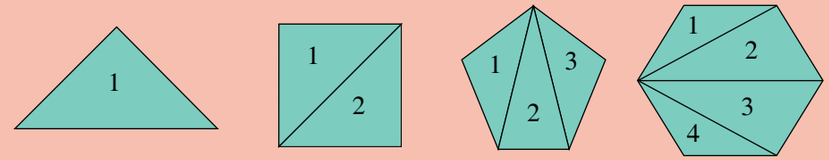
LEARNING STANDARD

Derive the formula for the sum of interior angles of a polygon.

QR CODE

Scan the QR Code or visit http://rimbunanilmu.my/mat_t2e/ms062 to obtain the worksheets on polygon shapes.

3. Connect the edges of each polygon to form a triangle in the polygon as shown below.



4. Complete the table below.

Polygon	Number of sides (n)	Number of triangles	Total sum of interior angles
Triangle	3	1	$1 \times 180° = 180°$
Square	4	2	$2 \times 180° = 360°$
Pentagon			
Hexagon			
Heptagon			
Octagon			
Nonagon			
Decagon			

Discussion:

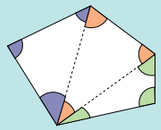
(i) What is the relationship between the number of sides, n with the number of triangles?
(ii) What is the relationship between the number of sides in a triangle with the total sum of interior angles?

5. Total sum of interior angles of a polygon
= number of triangles × 180°
= ☐ × 180°

the n^{th} term

THINK SMART

Pentagon can be divided into 3 triangles. State the number of interior angles of a pentagon.



Total sum of interior angles of a polygon = $(n - 2) \times 180°$.

EXAMPLE 2

State the number of triangles formed for each of the following polygon.

(a) 13 sided polygon (b) 18 sided polygon

Solution:

(a) Number of triangles = 13 – 2
= 11

(b) Number of triangles = 18 – 2
= 16

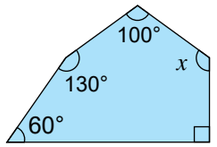
DO YOU KNOW ?

Number of sides	Polygon Name
12	dodecagon
13	tridecagon
14	tetradecagon
15	pentadecagon
16	hexadecagon
17	heptadecagon
18	octadecagon
19	enneadecagon
20	icosagon

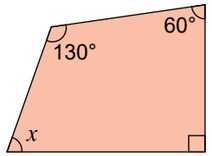
EXAMPLE 3

Calculate the value x for the following.

(a)



(b)



Solution:

(a) Total sum of interior angles,

$= (n - 2) \times 180°$

$= (5 - 2) \times 180°$

$= 540°$

$x + 100° + 130° + 60° + 90° = 540°$

$x + 380° = 540°$

$x = 540° - 380°$

$x = 160°$

(b) Total sum of interior angles,

$= (n - 2) \times 180°$

$= (4 - 2) \times 180°$

$= 360°$

$x + 130° + 60° + 90° = 360°$

$x + 280° = 360°$

$x = 360° - 280°$

$x = 80°$

4.2.2 Total sum of exterior angles of polygons

LEARNING STANDARD

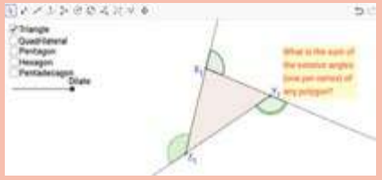
Make and verify conjectures about the sum of exterior angles of a polygon.

COGNITIVE STIMULATION Group

Aim: Exploring the total sum of exterior angle

Material: Dynamic geometry software

Polygon	n	The output angle enhancement	
		Conjecture	Verify (Yes / No)



Steps:

1. Open the file MS064 and print the file.
2. Do a conjecture for each of the polygon in the space provided in worksheet.
3. Open the file MS064 to view total sum of exterior angle.
4. Explore each polygon that is available.
5. Drag the slider *dilate* to change the size of the polygon sides that is being displayed.
6. State the total sum of the exterior angles of a polygon.

Discussion:

The sum of exterior angles of polygon.

QR CODE

Scan the QR Code or visit http://rimbunanilmu.my/mat_t2e/ms064 to obtain the worksheet.

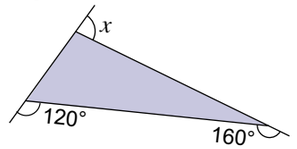
TIPS

A conjecture is a proposition or a theorem that looks right. Conjecture decisions are not formally proven. The conjecture allows students to speculate based on a mathematical situation. For example, if we add 2 positive numbers then the result is always greater than the number.

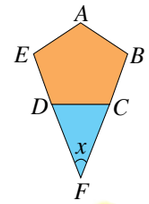
The total sum of exterior angles of a polygon is 360°.

EXAMPLE 4

(a) Calculate the value x for the diagram below.



(b) In the diagram below, $ABCDE$ is a regular pentagon. BCF and EDF are straight lines. Calculate the value x.



Solution:

(a) Total sum of exterior angles = 360°

$x + 160° + 120° = 360°$

$x + 280° = 360°$

$x = 360° - 280°$

$x = 80°$

(b) $\angle FCD = \frac{360°}{5}$

$= 72°$

$x = 180° - 72° - 72°$

$= 36°$

TIPS

Exterior angle of a regular polygon $= \frac{360°}{n}$

Interior angle = 180° − exterior angle

4.2.3 Total sum of exterior angles of polygons

LEARNING STANDARD

Determine the values of interior angles, exterior angles and the number of sides of a polygon.

EXAMPLE 5

Calculate the value of the interior angle for a regular hexagon.

Solution:

Number of sides of regular hexagon, $n = 6$

Total sum of interior angles $= (n - 2) \times 180°$

$= (6 - 2) \times 180°$

$= 4 \times 180°$

$= 720°$

$$\text{Interior angle} = \frac{\text{Total sum of interior angles}}{\text{Number of sides}}$$

$$= \frac{720°}{6}$$

$= 120°$

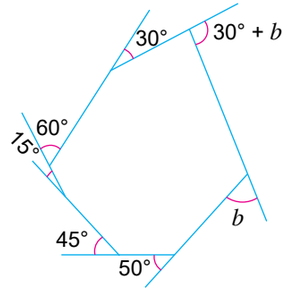
TIPS

Interior angle of regular polygon

$= \frac{(n - 2) \times 180°}{n}$

EXAMPLE 6

Calculate the value of b for the diagram on the right.



Solution:

$360° = (30° + b + b + 50° + 45° + 15° + 60° + 30°)$

$360° = 230° + 2b$

$2b = 360° - 230°$

$2b = 130°$

$b = 65°$

EXAMPLE 7

Calculate the value of the exterior angle of a regular octagon.

Solution:

Number of sides of a regular octagon, $n = 8$

Total sum of exterior angles $= 360°$

Exterior angle $= \dfrac{360°}{8}$

$= 45°$

EXAMPLE 8

Calculate the number of sides of the following regular polygon when given the value of interior angle.

(a) $108°$ (b) $144°$

Solution:

(a) Exterior angle $= 180° - 108°$

$= 72°$

Number of sides, $n = \dfrac{360°}{\text{exterior angle}}$

$n = \dfrac{360°}{72°}$

$n = 5$

(b) Exterior angle $= 180° - 144°$

$= 36°$

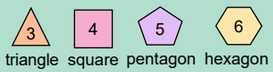
Number of sides, $n = \dfrac{360°}{\text{exterior angle}}$

$n = \dfrac{360°}{36°}$

$n = 10$

ATTENTION

POLYGON



Total sum of interior angles

$(n - 2) \times 180°$

Number of side

$4 \times 180° = 540°$

Interior angle

$\dfrac{\text{Total sum of interior angles}}{\text{Number of sides}}$

or

$180° - \text{exterior angle}$

Exterior angle

$\dfrac{360°}{\text{Number of sides}}$

or

$180° - \text{interior angle}$

4.2.4 Solving problems

LEARNING STANDARD

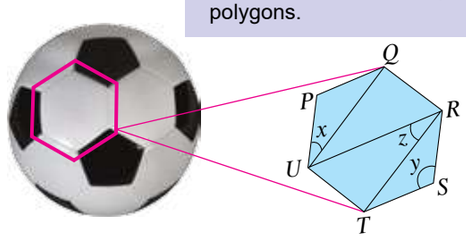
Solve problems involving polygons.

EXAMPLE 9

The diagram on the right is a regular hexagon that is enlarged from the design of a football.

(a) Calculate angle y.

(b) Calculate the difference between y and $(x + z)$.



Solution:

Understanding the problem

Calculating angle y using formula $\dfrac{(n - 2) \times 180°}{n}$

Angle x is in the equilateral triangle

$\angle UPQ = \angle TSR = y$

$\dfrac{180° - \angle UPQ}{2}$

Planning the strategy

(a) $y = \dfrac{(6 - 2) \times 180°}{6}$

$y = 120°$

(b) $x = \dfrac{180° - 120°}{2}$

$x = 30°$

$z = 30°$ (alternate angle)

Implementing the strategy

(b) Difference between y and $(x + z)$

$= 120° - (30° + 30°)$

$= 60°$

Conclusion

(a) $y = 120°$

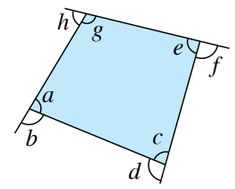
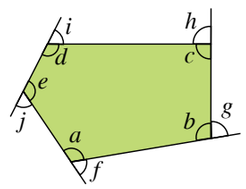
(b) $y - (x + z) = 60°$

SELF PRACTICE 4.2

1. State the number of triangles that can be found in the polygon below and calculate the total sum of the exterior angles.

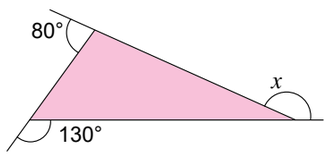
Polygon	Number of triangles in the polygon	Total sum of exterior angles
Pentagon		
Hexagon		
Heptagon		
Octagon		
Nonagon		

2. Name all the interior angles and exterior angles for each of the following polygons.

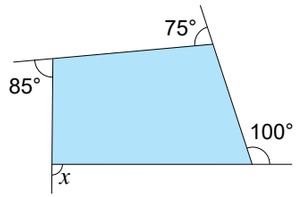
(a)	(b)
	
Interior angles:	Interior angles:
Exterior angles:	Exterior angles:

3. Calculate the value x for each of the following diagrams.

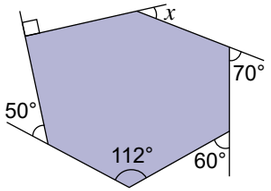
(a)



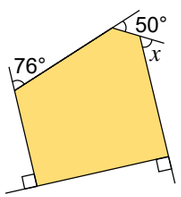
(b)



(c)

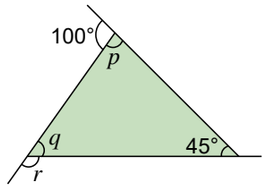


(d)

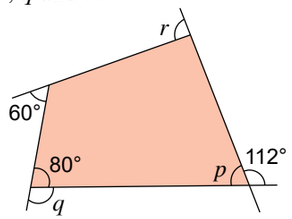


4. For each of the diagram below, calculate the value p, q and r.

(a)

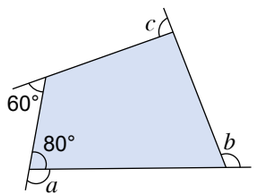


(b)

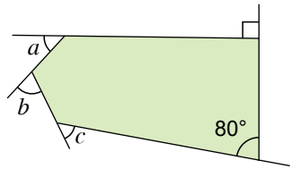


5. Calculate the value $a + b + c$.

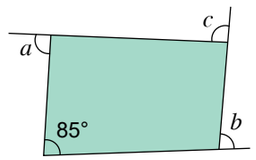
(a)



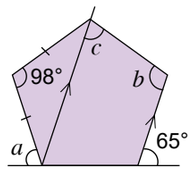
(b)



(c)



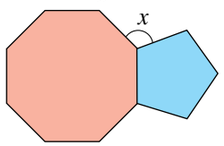
(d)



6. Determine the number of sides for a polygon if the total sum of exterior angles is
   (a) 900° (b) 1 080° (c) 1 260°

7. Zaidi has a vegetable garden that is shaped like a regular polygon. The dotted lines is the axis of symmetry of his garden.
   (a) What is the actual shape of Zaidi's garden?
   (b) Calculate the value y.

8. The diagram shows two swimming pools at a sports centre in the shape of a regular octagon and pentagon. What is the value of angle x?

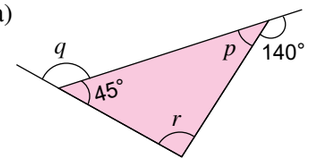


GENERATING EXCELLENCE

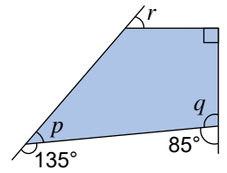
1. Construct the following polygons using compasses and a ruler.
   (a) Equilateral triangle ABC with sides 4 cm.
   (b) Square $PQRS$ with sides 3 cm.

2. Calculate the value p,q and r in the following.

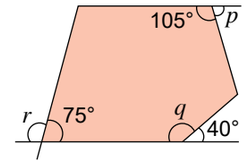
(a)



(b)

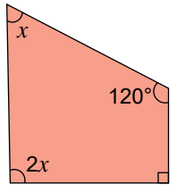


(c)

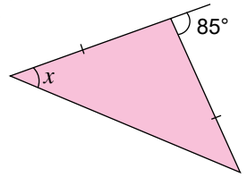


3. Calculate the value x for the following.

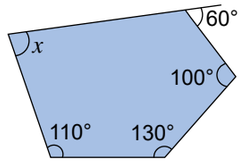
(a)



(b)

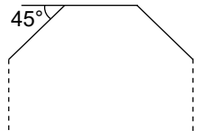


(c)

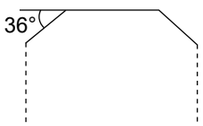


4. Calculate the number of sides for the following.

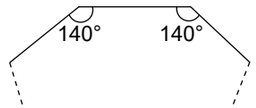
(a)



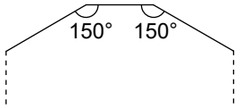
(b)



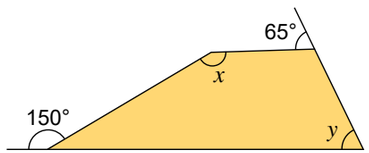
(c)



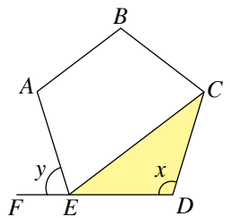
(d)



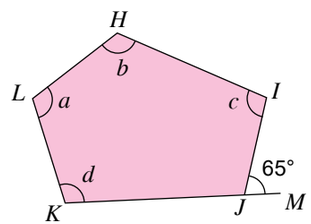
5. (a) Calculate the value $x + y$ in the diagram below.



(b) The diagram shows a logo in the shape of a regular pentagon. FED is a straight line. Calculate the value $x + y$.



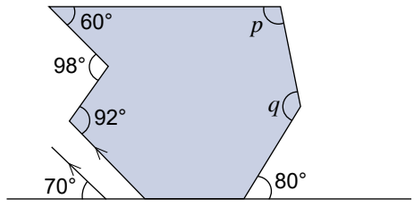
(c) In the diagram below, $HIJKL$ is a pentagon. KJM is a straight line. Calculate the value $a + b + c + d$.



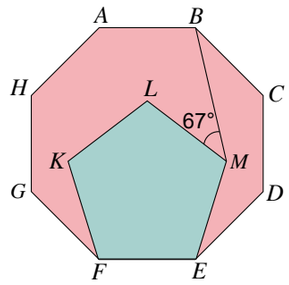
6. Azreen wants to draw a logo for the Peers Counselling Club at her school. She chooses to draw a regular hexagon with the radius 4 cm. Help Azreen draw her logo using a ruler, protractor and compasses.

7. Total sum of all interior angles of a regular polygon is 2 700°. State the number of sides of this polygon.

8. In the diagram below, calculate the value $p + q$.

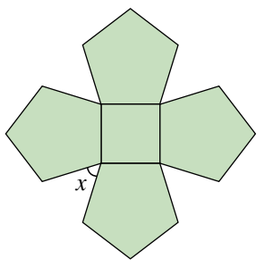


9. In the diagram below, *ABCDEFGH* is a regular octagon and *EFKLM* is a regular pentagon. Calculate $\angle CBM$.



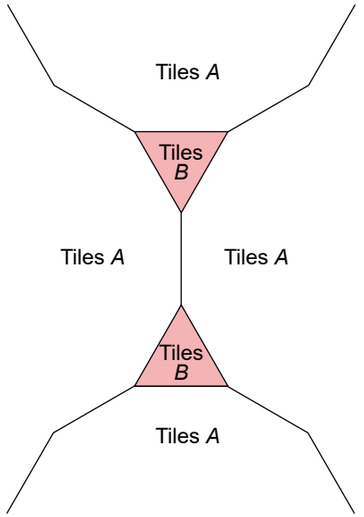
10. The exterior angle of a regular polygon is $2h$ and the interior angle of the same polygon is $7h$.
    (a) Calculate the value of h.
    (b) Calculate the value of the interior angle and exterior angle.
    (c) Calculate the number of sides of the polygon and name the polygon.

11. The diagram below shows 4 regular pentagons and a square. Calculate the value x.

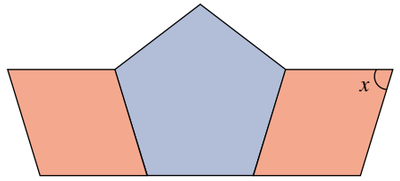


12. Bahar wants to construct a polygon that has an interior angle of 300°. Can Bahar construct the polygon? Justify your answer.

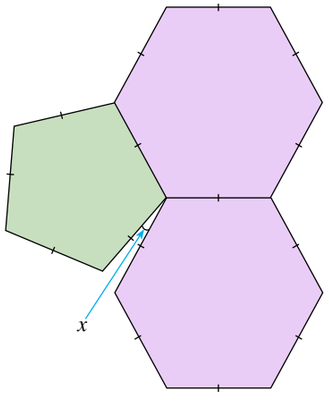
13. The diagram below shows a partial design that has been formed from combining two tiles. There are two types of tiles. They are tile *A* and tile *B* that are regular polygons. Calculate the number of sides of tile *A*.



14. Devaa is a graphic design student at a local university. Help Devaa calculate the value of x to construct a photo frame that has the characteristics of combined polygons.



15. Calculate the value of x.



CHAPTER SUMMARY

The number of axis of symmetry in a regular polygon with n side and n axis of symmetry.

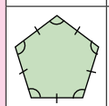
Regular polygon

- Interior angle $= \frac{(n-2) \times 180°}{n}$
- Exterior angle $= \frac{360°}{n}$

Total sum of interior angles $= (n-2) \times 180°$

Total sum of exterior angles $= 360°$

Regular polygon is a polygon where all its sides have the same length and all interior angles are equal. Examples

	Exterior angle	Interior angle
	$= \frac{360°}{3}$	$= \frac{(3-2) \times 180°}{3}$
	$= \frac{360°}{4}$	$= \frac{(4-2) \times 180°}{4}$
	$= \frac{360°}{5}$	$= \frac{(5-2) \times 180°}{5}$

Irregular polygon

Exterior angle of a polygon is a complement to an interior angle of the polygon.

Exterior angle + Interior angle = 180°

Total sum of exterior angle = 360°

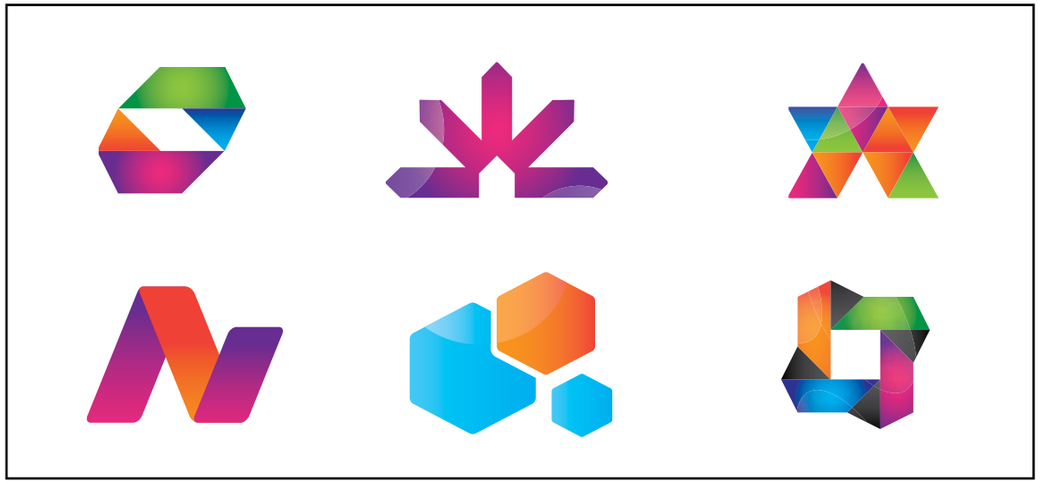
Irregular polygon is a polygon where not all sides are of the same length.

SELF REFLECTION

At the end of this chapter, I will be able to:	
1. Describe the geometric properties of regular polygons using various representations.	
2. Construct regular polygons using various methods and explain the rationales for the steps of construction.	
3. Derive the formula for the sum of interior angles of a polygon.	
4. Make and verify a conjecture about the sum of exterior angles of a polygon.	
5. Determine the values of interior angles, exterior angles and the number of sides of a polygon.	
6. Solve problems involving polygons.	

MINI PROJECT

You are a food stall owner. Create your business logo using the combination of two or three polygons. You can use the dynamic geometry software, geometry tools or origami to design your logo. Present the rationale for the choice of your logo in the class.



Examples of logo