CHAPTER 5

Circles

The clockwise movement of the hand of a clock forms a circle as it completes a 360° rotation. This is called 'kirkos' in Greek, which means to rotate and to become a curve.

WHAT WILL YOU LEARN?

5.1 Properties of Circles

5.2 Symmetrical Properties of Chords

5.3 Circumference and Area of a Circle

WORD LINK

• Circle	• *Bulatan*
• Circumference	• *Lilitan*
• Radius	• *Jejari*
• Centre	• *Pusat*
• Diameter	• *Diameter*
• Chord	• *Perentas*
• Segment	• *Tembereng*
• Sector	• *Sektor*
• Minor sector	• *Sektor minor*
• Major sector	• *Sektor major*
• Minor segment	• *Tembereng minor*
• Major segment	• *Tembereng major*
• Symmetry	• *Simetri*

WALKING THROUGH TIME

A circle is a curved pathway locus of a point that is equidistant from a fixed point. This fixed point is known as the centre and the distance from this fixed point to the pathway is called the radius. A circle is also a curve that is joined which is known as circumference. A mathematician named Euclid was the first person to study circles. He is also known as the 'Father of Geometry' due to his research.

For more information:

http://rimbunanilmu.my/mat_t2e/ms075

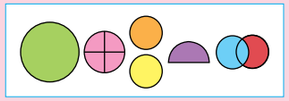
WHY STUDY THIS CHAPTER?

- The application of this chapter is in the field of architecture, astronomy, design and astrology.

CREATIVE ACTIVITY

Aim: Getting to know circles
Materials: Coloured paper, glue, scissors, string and punch
Steps:

1. Students form groups.
2. Each group is required to draw circles of various sizes.
   Examples are like the ones in the diagram on the right.



3. The circles will be used to decorate the class.
4. Write down the mathematical formulae of area of a rectangle, area of triangle, volume of cube, volume of cuboid, Pythagoras theorem and so on in the circles.

5.1 Properties of Circles

5.1.1 Getting to know the parts of the circle

LEARNING STANDARD

Recognise parts of a circle and explain the properties of a circle.

COGNITIVE STIMULATION Group

Aim: Knowing parts of a circle
Material: Dynamic geometry software
Steps:

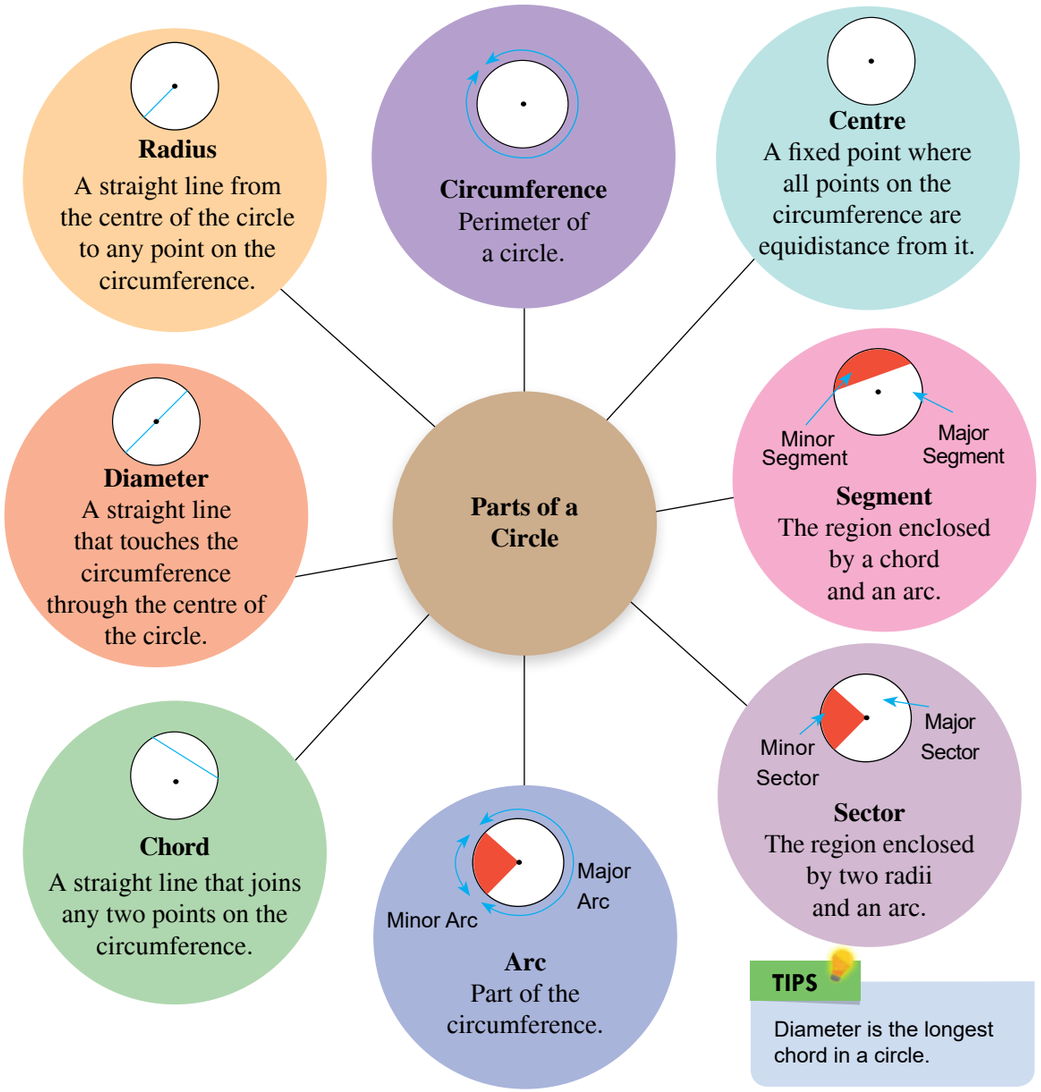
1. Open the file MS076.
2. The perimeter of a circle is called ____ .
3. Drag point *A* in the all directions.
   (i) Point *A* is called the ____ of the circle.
4. Drag point *B* around the circle.
   (i) The line from the centre of the circle to any point on the circumference of the circle is called ____ .
5. Drag point *C* around the circle.
   (i) The *CC*' line that goes through the centre and touches the circumference is called ____ .
6. Drag point *D* and the point *E* around the circle.
   (i) The line that connects two points on the circumference is called ____ .
   (ii) The region is called ____ .
7. Drag points *C* and *D*.
   (i) Name two lines generated. Lines *AC* and ____ .
   (ii) The region bounded by these two radius is called ____ .

QR CODE

Scan the QR Code or visit http://rimbunanilmu.my/mat_t2e/ms076 to explore parts of the circle.

Discussion:
Make a conclusion on your explorations.

From the activity above, several parts of the circle have been identified as in the diagrams on the next page.



TIPS

Diameter is the longest chord in a circle.

TIPS

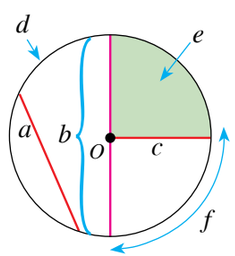
A circle is a curved pathway of a point that is equidistant from a fixed point.

THINK SMART

Why are balls, globes and marbles not considered as circles?

EXAMPLE 1

Identify the following parts of a circle.



Solution:

a, Chord
b, Diameter
c, Radius
d, Circumference
e, Sector
f, Arc

5.1.2 Constructing a circle

LEARNING STANDARD

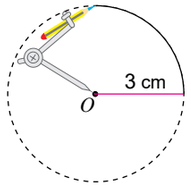
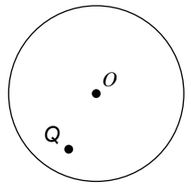
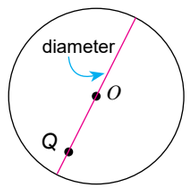
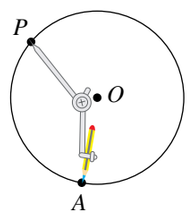
Construct a circle and parts of the circle based on the conditions given.

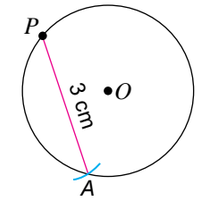
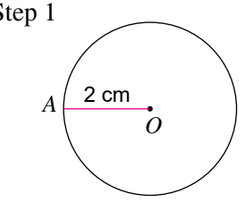
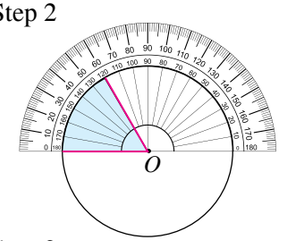
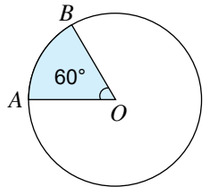
COGNITIVE STIMULATION Group

Aim: Constructing a circle and parts of the circle based on the conditions given

Materials: Compasses, protractor, ruler and pencil

Steps:

Conditions	Steps	Solution
(a) Construct a circle with a radius of 3 cm from the centre O.	1. Mark point O. 2. Using compasses measure 3 cm on a ruler. 3. Place the sharp point of the compasses on point O and draw a circle with the radius of 3 cm.	
(b) Construct a diameter that passes through point Q in a circle with the centre O.	1. Join points O and Q with a straight line using a ruler. 2. Extend the line until it touches the circumference. The extended straight line that passes through Q and centre O is the **diameter**.	Step 1  Step 2 
(c) Construct two chords of 3 cm in length from point P on a circle.	1. Using compasses measure 3 cm on a ruler. 2. Place the sharp point of the compasses on point P. 3. Draw the arc that cuts on the circumference and label it as point A.	Step 1 

	4. Join point P to point A that has been marked on the circumference. 5. Thus, the line PA is a **chord**.	Step 2 
(d) Construct a sector with an angle of 60° at the centre of a circle with a radius of 2 cm.	1. Draw a circle with the radius OA 2 cm. 2. Measure 60° with a protractor as shown. 3. Draw the radius OB. AOB is the **sector of the circle**.	Step 1  Step 2  Step 3 

Discussion:

From the activity above, what parts of the circle has been constructed?

From the activities above, students are able to

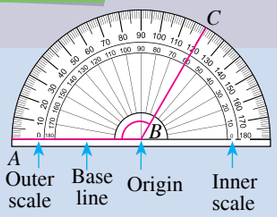
(a) construct a circle when the radius or diameter is given.

(b) construct a diameter through a given point in a circle.

(c) construct a chord through a given point when the length of the chord is given.

(d) construct a sector when angle of the sector and the radius is given.

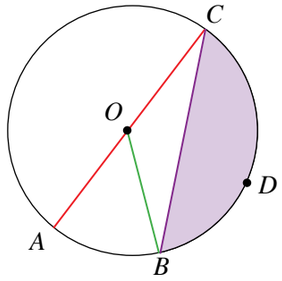
TIPS



To measure the ABC angle, place the protractor's centre point on the vertex of that angle. Make sure the line with the value 0 is located on the AB line. Read the angle using the external scale. Thus, the angle for ABC is 120°.

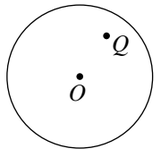
SELF PRACTICE 5.1

1. Name the following parts
   (i) point O
   (ii) line AOC
   (iii) dector AOB
   (iv) line OA
   (v) arc AB
   (vi) line BC
   (vii) the shaded area BCD.

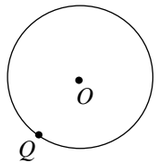


2. Construct a circle with radius
   (a) 3 cm
   (b) 4.5 cm
   (c) 2.5 cm
   (d) 6 cm

3. Construct a diameter that passes through point Q for each of the circles with the centre O.
   (a)



   (b)



4. Construct the chord of a circle with radius and length given below.

	Radius	Length of Chord
(a)	3 cm	4 cm
(b)	4.5 cm	6.7 cm

5. Using a protractor, construct the sector AOB with O as the centre of the circle. The radius and $\angle AOB$ as given below.

	Radius	$\angle AOB$
(a)	3 cm	70°
(b)	3.6 cm	120°

CHAPTER 5

5.2 Symmetry and Chords

5.2.1 Features in a circle

COGNITIVE STIMULATION Group

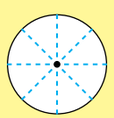
Aim: Verifying
(i) properties of the diameter of a circle.
(ii) the relationship of a radius with chords.

Material: Dynamic geometry software

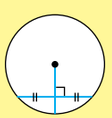
Steps:
1. Open the file MS081.
2. Click on the *Activity* box.
3. Drag point Q to points P, T, U, B_1, V and Z.
   (i) Name the diameter of the circle. Lines [ ] .
   (ii) Observe the value of angle at the centre when the diameter QQ' is moved. Does it produce the same value? Are the resulting shapes similar?
   (iii) If you fold the circle on the line QQ', do the shapes overlap each other perfectly?
   (iv) The diameter of a circle is known as [ ] .
4. Click the *Activity* box for the next activity.
5. Drag the slider *Drag Me* until the end.
   (i) Radius that bisects a chord is [ ] to the chord.
   (ii) Radius that is perpendicular to the chord [ ] the chord.
   (iii) Equal chords produce [ ] arc.

Discussion:
State the conclusions for all the activities above.

The diameter of a circle is the axis of symmetry of the circle.



A radius which is perpendicular to the chord bisects the chord.



LEARNING STANDARD

Verify and explain that
(i) diameter of a circle is an axis of symmetry of the circle;
(ii) a radius that is perpendicular to a chord bisects the chord and vice versa;
(iii) perpendicular bisectors of two chords intersect at the centre;
(iv) chords that are equal in length produce arcs of the same length and vice versa;
(v) chords that are equal in length are equidistant from the centre of the circle and vice versa.

DO YOU KNOW?

The circle has an infinite number of axes of symmetry because any straight line that passes through the centre is the axes of symmetry of the circle.

QR CODE

Scan the QR Code below or visit http://rimbunanilmu.my/mat_t2e/ms081 for the properties of symmetric chord 1.

TIPS

The diameter is the chord that passes through the centre of the circle.

CHAPTER 5

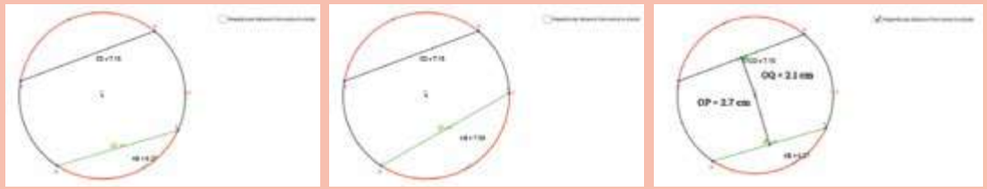
COGNITIVE STIMULATION

Individual

Aim: Verifying
(i) properties of the bisector of the two chords.
(ii) properties of equal chords in a circle.

Material: Dynamic geometry software

Steps:
1. Open the file MS082.
2. Drag point *A* where, *AB* = *CD*.
3. Click on the box *length of the perpendicular line from the centre of the circle*.
4. Repeat steps 1 and 2 to get the others values.



Discussion:

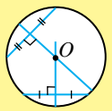
(i) Where do lines *OP* and *OQ* meet?

(ii) Is the length of arc *AGB* and *CID* the same?

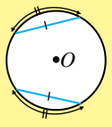
(iii) If the length of *AB* = *CD*, the distance of *OP* = the distance of ☐.

(iv) Is the distance of *OP* and *OQ* the same?

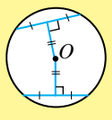
Perpendicular bisectors of two chords meet at the centre of the circle.



Equal chords or chords of the same length produce arc of the same length.



Equal chords are equidistant from the centre of the circle.



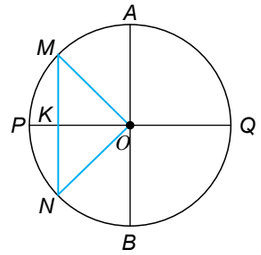
QR CODE

Scan the QR Code or visit http://rimbunanilmu.my/mat_t2e/ms082 for the properties of symmetric chord 2.

THINK SMART

How many axes of symmetry are there in half a circle?

EXAMPLE 2



The diagram above shows a circle with centre *O* and the line *MN* is the chord.

(a) Name the axes of symmetry of this circle.

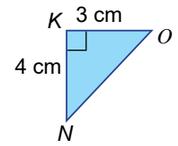
(b) Given *OK* = 3 cm and *NK* = 4 cm, calculate length of *ON*.

(c) Name the angle that is equal to ∠*ONK*.

Solution:

(a) *AOB* and *POQ*

(b)



$ON^2 = 4^2 + 3^2$

$ON = \sqrt{(16 + 9)}$

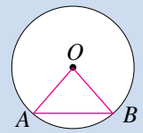
$ON = \sqrt{25}$ ← *ON* = *OM*

$ON = 5$

Therefore, length of *ON* is 5 cm.

(c) ∠*OMK*

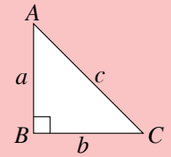
TIPS



Two radii and a chord forms an isosceles triangle.

FLASHBACK

Pythagoras theorem

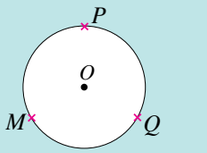


$AB^2 + BC^2 = AC^2$

or

$a^2 + b^2 = c^2$

THINK SMART



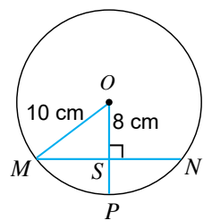
O is the centre of the circle. What is the relationship between *OP*, *OQ* and *OM*?

EXAMPLE 3

The diagram on the right shows a circle with the radius *OP* that is perpendicular to the chord *MN*.

(a) Is the length *MS* equal to length of *SN*? Explain.

(b) If the radius of the circle is 10 cm and *OS* = 8 cm, calculate the length of the chord *MN*.



Solution:

(a) Yes, *MS* = *SN*
The radius *OP* which is perpendicular bisects *MN*.

(b) $MS = \sqrt{10^2 - 8^2}$

$MS = \sqrt{100 - 64}$

$MS = \sqrt{36}$

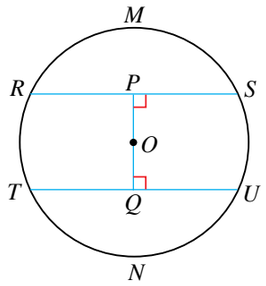
$MS = SN = 6$

Therefore, *MN* = 12 cm.

CHAPTER 5

EXAMPLE 4

The diagram on the right shows two equal chords, RS and TU. POQ is a straight line passing through the centre of the circle O. Given $OP = 5$ cm and $RS = 24$ cm.
(a) Calculate the length of PR.
(b) Are minor arcs RMS and TNU equal in length? Explain.
(c) Calculate the radius of the circle.



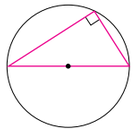
Solution:

(a) A radius that is perpendicular to the chord bisects the chord into two equal lengths. Length of $PR = 24 \div 2$ cm $= 12$ cm.

(b) Yes, chords that are equal in length produce arc of the same length.

(c) $OR = \sqrt{PR^2 + OP^2}$ ← Chords RS and TU are equal in length
$= \sqrt{12^2 + 5^2}$
$= \sqrt{144 + 25}$ ← OR, OS, OT and OU are radii of a circle
$= \sqrt{169}$
$= 13$ cm

DO YOU KNOW ?



Angle at the circumference of a semicircle is 90°.

5.2.2 Centre and radius of a circle

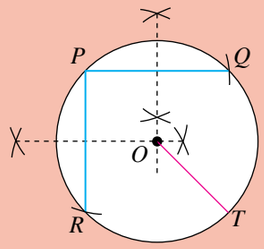
LEARNING STANDARD

Determine the centre and radius of a circle by geometrical construction.

COGNITIVE STIMULATION Group

Aim: Determining the centre and radius of the circle
Materials: Compasses, rulers, pencils and rounded material
Steps:
1. Trace a circle on a piece of paper.
2. Construct two chords, PQ and PR from point P.
3. Construct a perpendicular line for the chords PQ and PR.
4. The intersection point of two perpendicular lines is indicated by O.
5. Draw a line from O to the circumference and label it as OT.

Discussion:
(i) Properties of point O.
(ii) Properties of line OT.



A perpendicular bisector for any chord will always intersect at the centre of the circle.

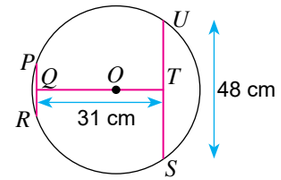
5.2.3 Solving problems

LEARNING STANDARD

Solve problems involving symmetrical properties of chords.

EXAMPLE 5

A blacksmith was asked to build a round-shaped window frame as shown below. The window is 50 cm in diameter. Three iron rods, PR, US and QT that are not equal in length are used to support the window. Calculate the length of PR.



Solution:

Understanding the problem
Diameter of window = 50 cm
$QT = 31$ cm
$US = 48$ cm
Calculate length PR.

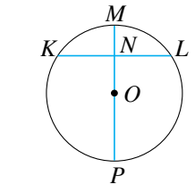
Planning the strategy
Radius $= \frac{\text{diameter}}{2}$
$= \frac{50}{2}$
$= 25$ cm
$OT = \sqrt{OU^2 - UT^2}$
$OQ = QT - OT$
$PQ = \sqrt{OP^2 - OQ^2}$
$PR = PQ \times 2$

Conclusion
Therefore, PR is 14 cm.

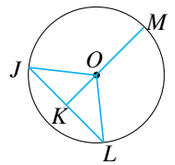
Implementing the strategy
$OT = \sqrt{25^2 - 24^2}$
$= \sqrt{625 - 576}$
$= \sqrt{49}$
$= 7$ cm
$OQ = 31 - 7$
$= 24$ cm
$PQ = \sqrt{25^2 - 24^2}$
$= \sqrt{625 - 576}$
$= \sqrt{49}$
$= 7$ cm
$PR = 7 + 7$
$= 14$ cm

SELF PRACTICE 5.2

1. In the diagram on the right, O is the centre of the circle. $MNOP$ and KNL are straight lines. Given that $MN = 8$ cm and $NP = 18$ cm. Calculate the length of KL.



2. The diagram on the right shows a circle with the centre O. JKL and KOM are straight lines. Given that $JK = KL = 15$ cm and radius of the circle is 25 cm. Calculate the length of KOM.



5.3 Circumference and Area of a Circle

5.3.1 Relationship between circumference and diameter

LEARNING STANDARD

Determine the relationship between circumference and diameter of a circle, and hence define π and derive the circumference formula.

Circumference is the measurement around a circle. The diagram shows a round table that needs to be lined with skirting for a wedding. What is the length of the skirting needed?

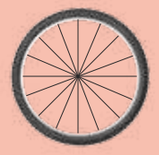
The length for the skirting can be calculated using the formula that involves π(pi).

COGNITIVE STIMULATION Group

Aim: Determining the relationship between circumference and diameter
Materials: Stopwatch, pail, bicycle tyre, measuring tape, pencil or any circular material.
Steps:
1. Measure the circumference of the stopwatch, pail and bicycle tyre with the measuring tape. Record the results in the table below.
2. Measure the diameter of the three items and record them in the table.
3. Complete the table below.

Material	Circumference (cm)	Diameter (cm)	$\frac{\text{Circumference}}{\text{Diameter}}$
1. Stopwatch			
2. Pail			
3. Bicycle tyre			





Discussion:
(i) The relationship between diameter and circumference.
(ii) What is the ratio of the circumference to the diameter?

From the above activities, the ratio of circumference to diameter, is π of a circle which is 3.142 or $\frac{22}{7}$.

$$\frac{\text{Circumference}}{\text{Diameter}} = \pi$$

REMEMBER

Diameter = 2 × radius

The circumference of a circle is π multiplied by the diameter.

$$\text{Circumference} = \pi \times \text{diameter}$$
$$= \pi d$$

The circumference of a circle can also be expressed as follows.

$$\text{Circumference} = \pi \times 2 \times \text{radius}$$
$$= 2\pi r$$

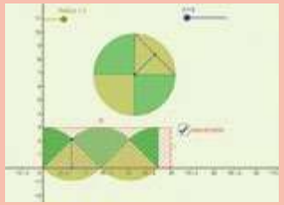
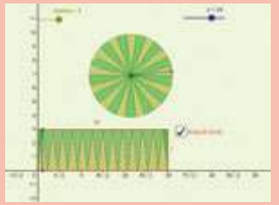
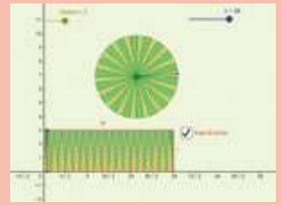
5.3.2 Formula for area of a circle

LEARNING STANDARD

Derive the formula for the area of a circle.

COGNITIVE STIMULATION Group

Aim: Expressing formula of a circle
Material: Dynamic geometry software
Steps:
1. Open the file MS087.
2. Drag the radius up to value 3, and drag the n until it reaches the value of 6. Take note of the changes.
3. Repeat step 2 by changing the value of radius and n. Take note of the changes.

Discussion:
(i) The [ ] the sector of the circle is divided into the clearer is the rectangular shape produced.
(ii) Height of rectangles = [ ] of the circle.
(iii) Rectangular base = [ ] of the circle.

QR CODE

Scan the QR Code or visit http://rimbunanilmu.my/mat_t2e/ms087 to explore area of a circle.

From the activity above,
Area of circle = area of rectangle

$$= \text{base} \times \text{height}$$
$$= \frac{1}{2} \times \text{circumference} \times \text{height}$$
$$= \frac{1}{2} \times 2\pi r \times r$$
$$= \pi r^2$$

Therefore, area of circle $= \pi r^2$

CHAPTER 5

5.3.3 Circumference, area of a circle, length of arc and area of sector

LEARNING STANDARD

Determine the circumference, area of a circle, length of arc, area of a sector and other related measurements.

To determine the circumference of a circle

EXAMPLE 6

Calculate the circumference of a circle, if

(a) diameter, $d = 14$ cm (Use $\pi = \frac{22}{7}$)

(b) Radius, $r = 21.3$ cm (Use $\pi = 3.142$)

Solution:

(a) Radius $= \pi d$

$= \frac{22}{7} \times 14$

$= 44$ cm

(b) Circumference $= 2\pi r$

$= 2 \times 3.142 \times 21.3$

$= 133.85$ cm

EXAMPLE 7

(a) Given the circumference of a circle is 88 cm, calculate the diameter of the circle in cm. (Use $\pi = \frac{22}{7}$)

(b) Given the circumference of a circle is 36.8 cm, calculate the radius of the circle in cm and round off the answer to two decimal places. (Use $\pi = 3.142$)

Solution:

(a) Circumference $= \pi d$

$88 = \frac{22}{7} \times d$

$d = 88 \times \frac{7}{22}$

$d = 28$ cm

(b) Circumference $= 2\pi r$

$2\pi r = 36.8$

$2 \times 3.142 \times r = 36.8$

$r = \frac{36.8}{6.284}$

$r = 5.86$ cm

To determine area of a circle

EXAMPLE 8

Calculate the area of a circle with

(a) diameter 10 cm (Use $\pi = \frac{22}{7}$)

(b) radius 7 cm

REMEMBER

radius, $r = \frac{\text{diameter}}{2}$

diameter, $d = 2r$

Solution:

(a) Area $= \pi r^2$

$= \frac{22}{7} \times \left(\frac{10}{2}\right)^2$

$= 78.57$ cm^2

(b) Area $= \pi r^2$

$= \frac{22}{7} \times 7^2$

$= 154$ cm^2

EXAMPLE 9

Given the area of a circle is 616 cm^2, calculate the radius and diameter. (Use $\pi = \frac{22}{7}$)

Solution:

Area $= \pi r^2$

$\pi r^2 = 616$

$\frac{22}{7} \times r^2 = 616$

$\frac{1\cancel{22}}{\cancel{7}_1} \times \frac{1\cancel{7}}{\cancel{22}_1} \times r^2 = 616 \times \frac{7}{22}$

$r^2 = 616 \times \frac{7}{22}$

$r^2 = 196$

$r = \sqrt{196}$

$r = 14$ cm

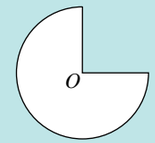
Diameter $= 2 \times 14$

$= 28$ cm

THINK SMART

(a) Calculate the area of quadrant if the radius is 7 cm.

(b) Calculate the area of the semi circle if the radius is 7 cm.



(c) Calculate the area of the three quadrant if the radius is 7 cm.

EXAMPLE 10

Given the circumference is 66 cm, calculate the area of the circle. (Use $\pi = \frac{22}{7}$)

Solution:

Circumference $= 66$ cm

$2\pi r = 66$

$2 \times \frac{22}{7} \times r = 66$

$r = 66 \times \frac{7}{44}$

$r = 10.5$ cm

Area $= \pi r^2$

$= \frac{22}{7} \times 10.5^2$

$= 346.5$ cm^2

EXAMPLE 11

Given area of circle is 75.46 cm^2. Calculate the circumference. (Use $\pi = \frac{22}{7}$)

Solution:

Area $= \pi r^2$

$\pi r^2 = 75.46$

$\frac{22}{7} \times r^2 = 75.46$

$r^2 = 75.46 \times \frac{7}{22}$

$r^2 = 24.01$

$r = \sqrt{24.01}$

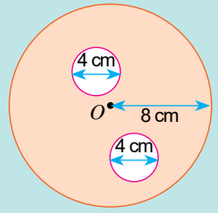
$r = 4.9$ cm

Circumference $= 2\pi r$

$= 2 \times \frac{22}{7} \times 4.9$

$= 30.8$ cm

THINK SMART

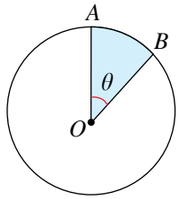


The diagram shows two circles in a bigger circle. Calculate the area of the shaded region.

Determining length of arc in a circle

The arc AB is part of the circumference of the circle. The length of arc is proportional to the angle at the centre of the circle.

$$\frac{\text{Length of arc}}{\text{Circumference}} = \frac{\text{Angle at centre}}{360°}$$



Therefore, $\dfrac{\text{Length of arc}}{2\pi r} = \dfrac{\theta}{360°}$

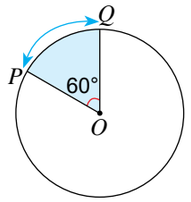
TIPS

The symbol θ is read as theta, a Greek letter used to represent angle.

EXAMPLE 12

The diagram below shows a circle with a radius of 14 cm and centred at O. Calculate the length of minor arc PQ which encloses 60° at the centre. Write your answer to two decimal places.

Solution:



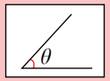
$$\frac{\text{Length of arc}}{2\pi r} = \frac{\theta}{360°}$$

$$\text{Length of arc} = \frac{\theta}{360°} \times 2\pi r$$

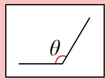
$$\text{Length of arc} = \frac{60°}{360°} \times 2 \times \frac{22}{7} \times 14$$

$$= 14.67 \text{ cm}$$

FLASHBACK

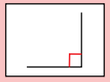


Acute angle
$0° < \theta < 90°$



Obtuse angle
$90° < \theta < 180°$

Reflex angle
$180° < \theta < 360°$

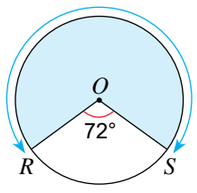


Right angle 90°

EXAMPLE 13

The diagram below shows a circle with a radius of 21 cm and centered at O. $\angle ROS$ is 72°. Calculate the length of major arc RS.

Solution:



$$\text{Angle at centre} = 360° - 72°$$
$$= 288°$$

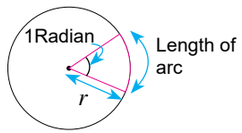
$$\frac{\text{Length of arc}}{2\pi r} = \frac{\theta}{360°}$$

$$\text{Length of arc} = \frac{\theta}{360°} \times 2\pi r$$

$$\text{Length of arc} = \frac{288°}{360°} \times 2 \times \frac{22}{7} \times 21$$

$$= 105.6 \text{ cm}$$

DO YOU KNOW ?



Angles can be measured using radians. 1 radian (1 rad) is the angle at the centre of the circle when the length of the arc is equal to the radius.

EXAMPLE 14

Given the length of the arc of a circle is 11 cm and the angle at the centre of the circle is 45°. Calculate in cm the radius of the circle.

Solution:

$$\frac{\theta}{360°} = \frac{\text{Length of arc}}{2\pi r}$$

$$2\pi r = \text{Length of arc} \times \frac{360°}{\theta}$$

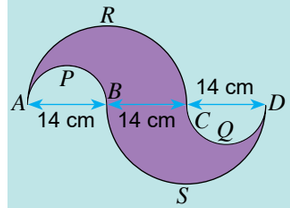
$$2 \times \frac{22}{7} \times r = 11 \times \frac{360°}{45°}$$

$$r = 11 \times \frac{360°}{45°} \times \frac{7}{22} \times \frac{1}{2}$$

$$r = \frac{27\,720}{1\,980}$$

$$r = 14 \text{ cm}$$

THINK SMART

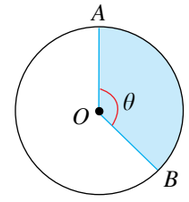


ARC, *APB*, *BSD* and *CQD* are arcs of the circles wheras *AB*, *AC*, *BD* and *CD* are the diameter of the circles. Calculate the area of the shaded region.

To determine area of a sector

The area of a sector is a region bounded by an arc and two radii. The area of the sector is proportional to the area of the circle.

$$\frac{\text{Area of sector}}{\text{Area of circle}} = \frac{\text{Angle at centre}}{360°}$$



Therefore,

$$\frac{\text{Area of } AOB}{\pi r^2} = \frac{\theta}{360°}$$

EXAMPLE 15

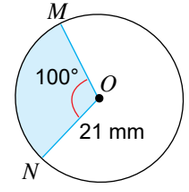
The diagram below shows a circle with centre O and radius 21 mm. Calculate the area of the minor sector MON.

Solution:

$$\frac{\text{Area of sector}}{\pi r^2} = \frac{\theta}{360°}$$

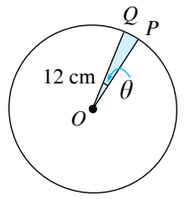
$$\text{Area of sector } MON = \frac{100°}{360°} \times \frac{22}{7} \times 21^2$$

$$= 385 \text{ mm}^2$$



EXAMPLE 16

Given the area of a sector QOP is 18.48 cm^2 and the radius is 12 cm. Calculate the value of θ.



Solution:

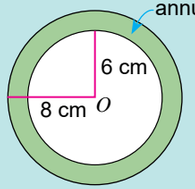
$$\frac{\text{Area of a sector}}{\pi r^2} = \frac{\theta}{360^\circ}$$

$$\frac{\theta}{360^\circ} = \frac{18.48}{\frac{22}{7} \times 12^2}$$

$$\theta = \frac{18.48}{\frac{22}{7} \times 12 \times 12} \times 360^\circ$$

$$\theta = 14.7^\circ$$

THINK SMART



The shaded area is an annulus.Determine the formula to calculate the area of the annulus.

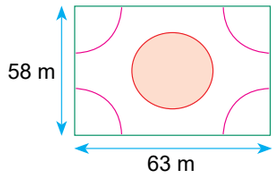
5.3.4 Solving problems

LEARNING STANDARD

Solve problems involving circles.

EXAMPLE 17

Majlis Bandaraya Melaka Bersejarah intends to build a rectangular recreational park with a length of 63 m and a width of 58 m. At every corner of the park, a quadrant with radius of 7 m will be planted with flowers. A circular shaped fish pond with a diameter of 28 m will be built in the middle of the park.The remaining areas will be planted with grass. Calculate the area covered with grass.(Use $\pi = \frac{22}{7}$)



Solution:

Understanding the problem

Radius of quadrant = 7 m

Garden is rectangular.
Length = 63 m
Width = 58 m

Diameter of fish pond = 28 m

Calculate the area covered with grass.

Planning the strategy

Recreational park area = length × width

Flower area = $4 \times \frac{1}{4} \pi r^2$

The fish pond area = πr^2

Area covered with grass
= Recreational area − flower area − fish pond area

Implementing the strategy

(i) Recreational park area = 58×63
$= 3\,654$ m^2

(ii) Flower area = $4 \times \frac{1}{4} \times \pi r^2$
$= \frac{22}{7} \times 7^2$
$= 154$ m^2

(iii) Fish pond area
$= \pi r^2$
$= \frac{22}{7} \times 14^2$
$= 616$ m^2

Conclusion

Thus, the area covered with grass is
$3\,654 \text{ m}^2 - 154 \text{ m}^2 - 616 \text{ m}^2$
$= 2\,884$ m^2

SELF PRACTICE 5.3

1. Calculate the circumference of a circle that has
   (a) a radius of 7 cm.
   (b) a radius of 56 cm.
   (c) a diameter of 9.2 cm.
   (d) a diameter of 98 mm.

   Give answers correct to two decimal places. (Use $\pi = \frac{22}{7}$)

2. Given circumference of a circle is 24.5 cm. Calculate
   (a) the diameter
   (b) the radius

   Give answers correct to two decimal places. (Use $\pi = 3.142$)

3. Calculate the area of the circle with the following radius.
   (a) 21 m
   (b) 56 mm
   (c) 7 cm
   (d) $1\frac{2}{5}$ cm

   Give answers correct to two decimal places. (Use $\pi = \frac{22}{7}$)

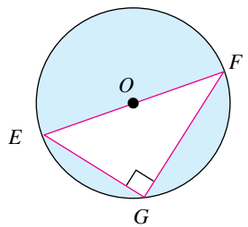
4. The area of a circle is 38.5 cm^2. Calculate.
   (a) the radius
   (b) the circumference of the circle

   Give answers correct to two decimal places. (Use $\pi = \frac{22}{7}$)

5. Calculate the area of a circle, if the circumference is 15.4 cm.

   Give answers correct to two decimal places. (Use $\pi = \frac{22}{7}$)

6. The diagram below shows a circle with centre O. Given $OF = 6.5$ cm and $EG = 5$ cm calculate the area of the shaded region, in cm^2. Give answers correct to two decimal places. (Use $\pi = 3.142$)



7. Calculate the radius when the length of the arc and angles at the centre of the circle are given. State the answer correct to two decimal places.

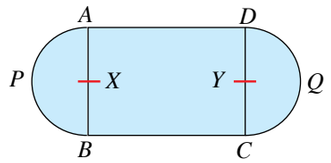
	Length of arc(cm)	Angle at centre
(a)	11	45°
(b)	4.3	35°
(c)	30.8	120°
(d)	110	200°

CHAPTER 5

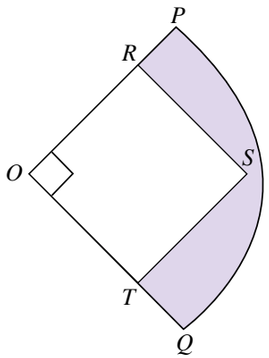
8. Given the radius and area of the circle, calculate the angle at the centre of the circle.

	Radius	Area of sector
(a)	14 cm	18.48 cm^2
(b)	21 m	27.72 m^2
(c)	8.4 cm	15.4 cm^2

9. The diagram below shows a plan for a park. $ABCD$ is a rectangle. APB and DQC are semicircles centred at X and Y. Given $AB = 7$ cm and $AC = 25$ cm. Calculate the perimeter of the park in cm.

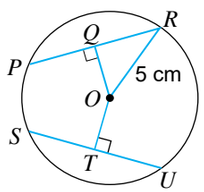


10. The diagram below shows the quadrant OPQ centred at O. $ORST$ is a square. Given $OP = 10$ cm and $OR = 7$ cm. Calculate the area of the shaded region, in cm^2. State the answer in π.

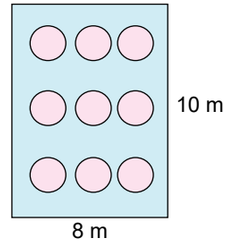


GENERATING EXCELLENCE

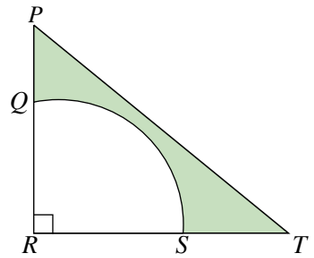
1. The diagram shows a circle with centre O. PQR and STU are straight lines. Given $PQR = STU = 6$ cm, calculate the length.
   (a) PQ
   (b) ST
   (c) OT



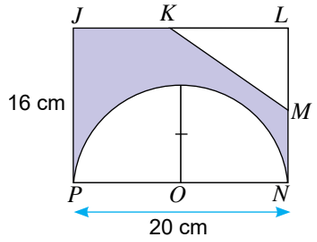
2. The diagram shows a dining hall measuring 10 m long and 8 m wide. It is laid with nine circular carpets, each has a diameter of 200 cm. Calculate the area that is not covered by the carpet in square metres.



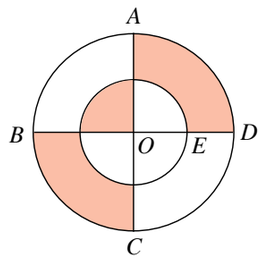
3. The diagram shows a right-angled triangle, PRT. R is the centre for the quadrant. Given $RS = 14$ cm, $ST = 10$ cm and $PQ = 4$ cm. Calculate the perimeter of the shaded area in cm.
   (Use $\pi = \frac{22}{7}$)



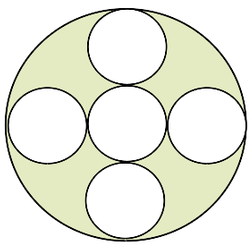
4. The diagram shows a rectangular piece of land owned by Encik Rashid. Encik Rashid divided his land into 3 parts. The first part is a triangle KLM. K is the midpoint of JL and M is the midpoint of LN. The second part is a semicircle. Encik Rashid intends to plant vegetables in the first and second part. Calculate the area that is not planted with vegetables. (Use $\pi = 3.142$)



5. Kevin wants to build a dartboard. The dartboard consists of two circles centred at O and three shaded regions as in the diagram. The diameters BOD and AOC are perpendicular to each other. Given $OE = ED = 10$ cm. Calculate the area of the shaded regions in cm^2. (Use $\pi = \frac{22}{7}$)



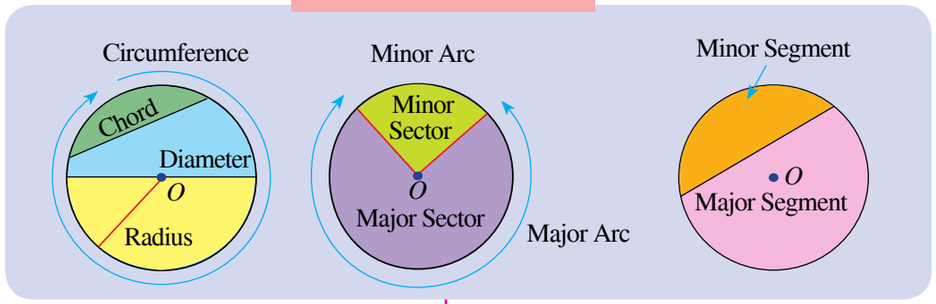
6. In a museum there is a round window decorated with circular rings of the same size as in the diagram. The radius of the window is 45 cm. Calculate the area that is not covered by the decoration. (Use $\pi = 3.142$)



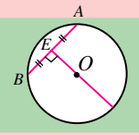
CHAPTER SUMMARY

Circles

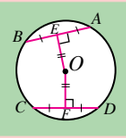
Parts of a Circle



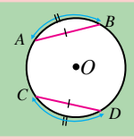
The radius perpendicular to the chord bisects the two chords and vice versa $AE = BE$.



Chords, which are equidistant from a centre of a circle are equal in lengths and vice versa.



Equal chords form arcs with equal lengths and vice versa
$AB = CD$.



Formulae for a circle

Circumference $= \pi d$
$= 2\pi r$
$\rightarrow$ $\dfrac{\text{Length of Arc}}{2\pi r} = \dfrac{\theta}{360°}$

Area of a circle $= \pi r^2$ $\rightarrow$ $\dfrac{\text{Area of sector}}{\pi r^2} = \dfrac{\theta}{360°}$

SELF REFLECTION

At the end of the chapter, I am able to:	☺	😐	☹
1. Recognise parts of a circle and explain the properties of a circle.			
2. Construct a circle and parts of the circle based on the conditions given.			
3. Verify and explain that:			
(a) Diameter of a circle is an axis of symmetry of the circle.			
(b) A radius that is perpendicular to the chord bisects the chord and vice versa.			
(c) Perpendicular bisectors of two chords intersect at the centre.			
(d) Chords that are equal in length produce arcs of the same length and vice versa.			
(e) Chords that are equal in length are equidistant from the centre of the circle and vice versa.			
4. Determine the centre and radius of a circle by using geometrical construction.			
5. Solve problems involving symmetrical properties of chords.			
6. Determine the relationship between circumference and diameter of a circle, and hence define the π and derive the circumference formula.			
7. Derive the formula for the area of a circle.			
8. Determine the circumference, area of a circle, length of arc, area of a sector and other related measurements.			
9. Solve problems involving circles.			

MINI PROJECT

Title: Number board game

You are required to build a number board like the one on the right. The number board consists of four circles with the radius of 5 cm, 15 cm, 20 cm and 25 cm respectively. All four circles share the same centre. The circles should be divided into 20 sectors. Each sector should be labelled with scores/points. This number board can be built using manila card, poster paper or polystyrene board. Arrows can be made from small sticks that are attached to adhesive tape. You can start the games by throwing the arrows towards the board to score points.