CHAPTER 6

Three-Dimensional Geometrical Shapes

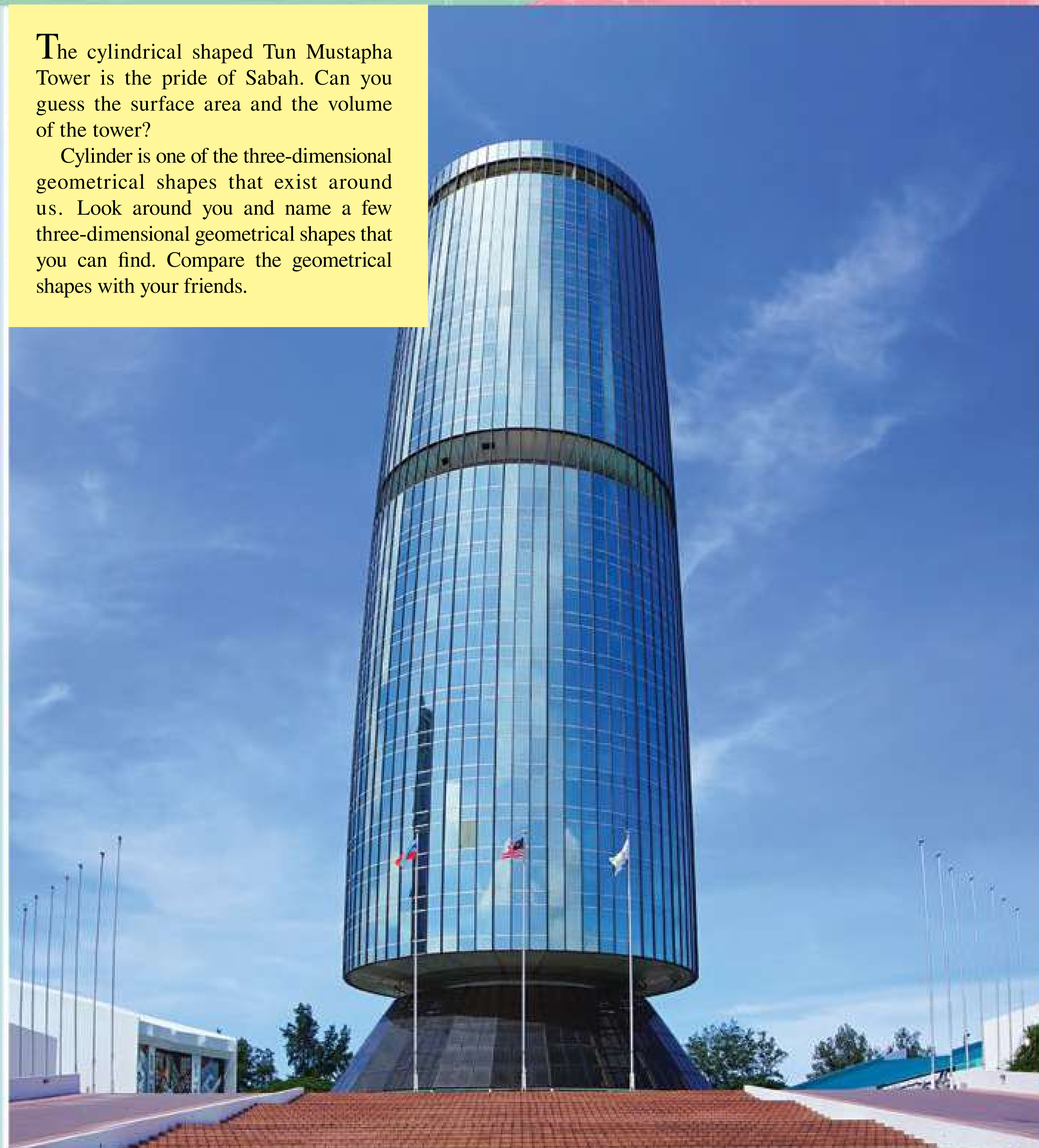
WHAT WILL YOU LEARN?

WORD LINK

• Two-dimensional shape	• *Bentuk dua dimensi*
• Three-dimensional shape	• *Bentuk tiga dimensi*
• Geometrical characteristic	• *Sifat geometri*
• Net	• *Bentangan*
• Surface area	• *Luas permukaan*
• Volume	• *Isi padu*
• Subject of formula	• *Perkara rumus*
• Cross section	• *Keratan rentas*

The cylindrical shaped Tun Mustapha Tower is the pride of Sabah. Can you guess the surface area and the volume of the tower?

Cylinder is one of the three-dimensional geometrical shapes that exist around us. Look around you and name a few three-dimensional geometrical shapes that you can find. Compare the geometrical shapes with your friends.



WALKING THROUGH TIME

The word geometry originated from two Greek words 'geo' meaning earth and 'metria' that means measurement. Euclid who revolutionised the geometrical research is often referred to as the 'Father of Geometry'. His book entitled 'Elements' is used as the main reference in the field of Mathematics, especially geometry in the early 20th Century.

For more information:

http://rimbunanilmu.my/mat_t2e/ms099

WHY STUDY THIS CHAPTER?

- The knowledge and skills in this chapter will help an architect and an engineer in designing and drawing blueprints of a building.
- Interior designers also use knowledge of geometry to create attractive landscape and interior design that optimises the area allocated.

CREATIVE ACTIVITY

Aim: Identifying three-dimensional shapes
Materials:

Steps:
1. Name the geometrical shapes of the objects above.
2. Compare and list the differences between the objects above in terms of:
   (i) Surface properties
   (ii) Shape
3. Discuss your opinions with your friends.

Each of the objects above has its own geometrical characteristics. Two-dimensional geometrical shapes like squares and triangles have width and length, while three-dimensional shapes have width, length and height. However, in a circle, radius is used. We will be discussing, on the geometrical characteristics of three-dimensional shapes in this topic.

6.1 Geometric Properties of Three-Dimensional Shapes

6.1.1 Three-dimensional shapes

LEARNING STANDARD

Compare, contrast and classify three-dimensional shapes including prisms, pyramids, cylinders, cones and spheres, and hence describe the geometric properties of prisms, pyramids, cylinders, cones and spheres.

COGNITIVE STIMULATION Individual

Aim: Exploring the concept of two-dimensional and three-dimensional shapes
Material: Dynamic geometry software
Steps:
1. Open the file MS100.
2. Drag the red slider from *Open* to *Close* indicator. Take note of the differences between the two-dimensional and three-dimensional shapes in the diagram.
3. Repeat step 2 until the blue slider reaches $n = 11$.

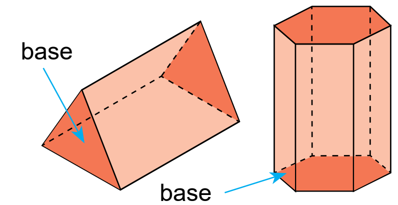
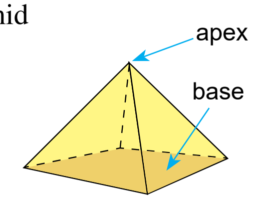
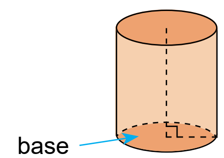
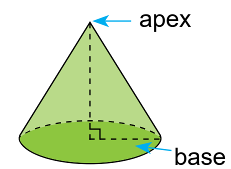
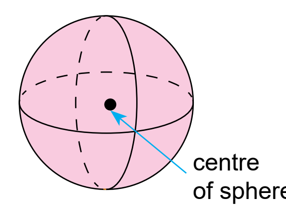
Discussion:
The difference between a two-dimensional shape and a three-dimensional shape.

QR CODE

Scan the QR Code or visit http://rimbunanilmu.my/mat_t2e/ms100 to explore three-dimensional shapes.

From the activity above, it can be concluded that three-dimensional shapes are formed out of two-dimensional shapes.

The table below shows three-dimensional shapes and their characteristics.

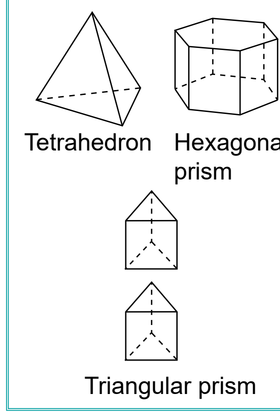
Geometrical shape	Geometrical characteristics
Prism 	• Two flat bases that are polygons which are congruent and parallel. • Flat rectangular shaped side. • Uniform cross section.
Pyramid 	• One flat base that is polygon shaped. • The other sides are triangular shaped that meet at the apex.
Cylinder 	• Two circular bases which are congruent and parallel. • One curved surface.
Cone 	• One circular base. • One apex. • One curved surface that merges the base and the apex.
Sphere 	• All points on the surface are equidistant from the centre of the sphere. • One curved surface.

FLASHBACK

Congruent means an object that has the same size and shape.

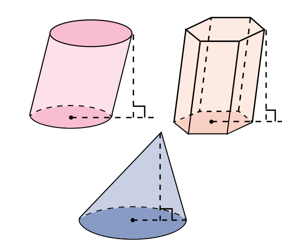
DO YOU KNOW?

The pyramid and prism are named according to the shape of their base.



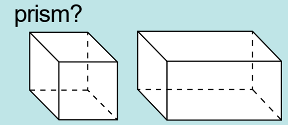
DO YOU KNOW?

Oblique shapes.



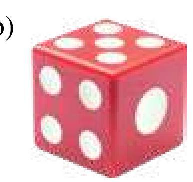
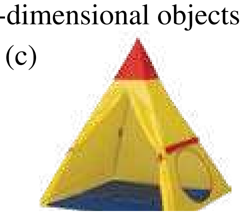
THINK SMART

Is a cube and a cuboid a prism?



SELF PRACTICE 6.1

1. List the geometrical characteristics for the three-dimensional objects below:

(a) 
(b) 
(c) 
(d) 

2. List the three-dimensional shape that has geometrical characteristics as stated below.
   (a) One vertex with one curved surface.
   (b) One vertex with polygonal base.
   (c) Every point on the surface has the same length from the centre of the object.

6.2 Nets of Three-Dimensional Shapes

6.2.1 Nets

Net of a three-dimensional shape is obtained by opening and laying out each surface of a three-dimensional object to become two-dimensional.

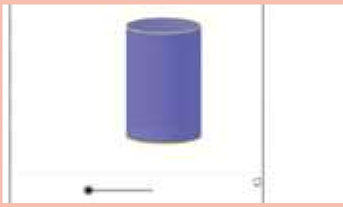
LEARNING STANDARD

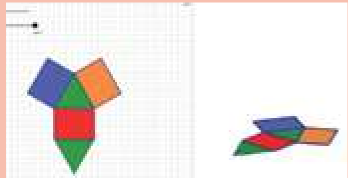
Analyse various nets including pyramids, prisms, cylinders and cones, and hence draw nets and build models.

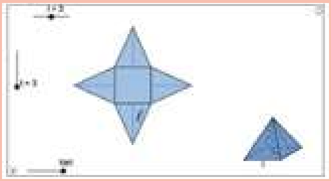
COGNITIVE STIMULATION Group

Aim: Analysing nets of cone, cylinder, prism and pyramid
Materials: Dynamic geometry software, scissors and adhesive tape
Steps:


Nets of cone


Nets of a cylinder


Nets of prism


Nets of pyramid

1. Open the file MS102A.
2. Drag the slider for each layout and observe the nets.
3. Open the file MS102B and print it.
4. Students are required to cut the net.
5. Fold the nets along the dotted lines.
6. Use the adhesive tape to form the three-dimensional shape.
   Example:


Step 4


Step 5


Step 6

QR CODE

Scan the QR Code or visit http://rimbunanilmu.my/mat_t2e/ms102a to view the nets.

QR CODE

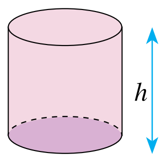
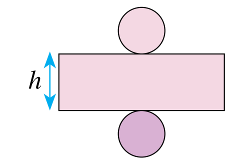
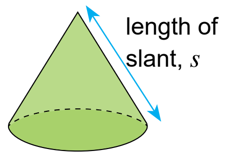
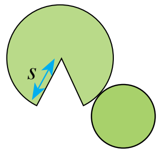
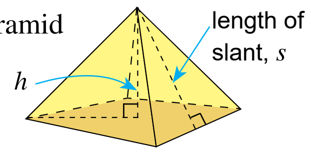
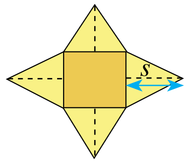
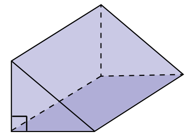
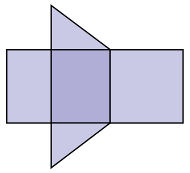
Scan the QR Code or visit http://rimbunanilmu.my/mat_t2e/ms102b to print the layout.

DO YOU KNOW?

A cube can be filled up with six pyramids with the same square base.

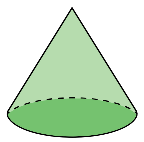
Discussion:
(i) Can the net of a three-dimensional object be customised?
(ii) Sketch the various nets of a cube.

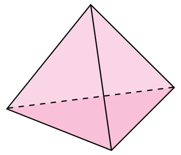
From the activity it can be concluded that the net of three-dimensional object can be vary. The table below shows three-dimensional geometrical shapes and net.

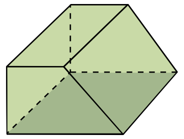
Geometrical shape	Net
Cylinder 	
Cone 	
Square based pyramid 	
Triangular based prism 	

EXAMPLE 1

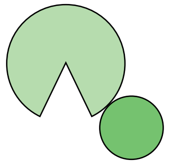
Draw the net for the three-dimensional shapes below.

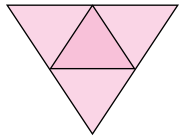
(a) 

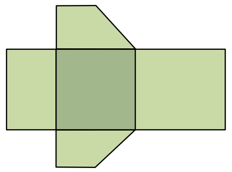
(b) 

(c) 

Solution:

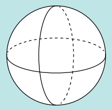
(a) 

(b) 

(c) 

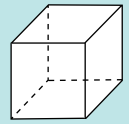
THINK SMART

What is the net of a sphere?



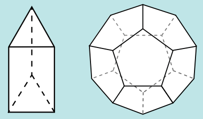
THINK SMART

How many nets are there for a cube?



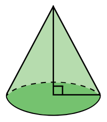
THINK SMART

What are the nets of these prisms?



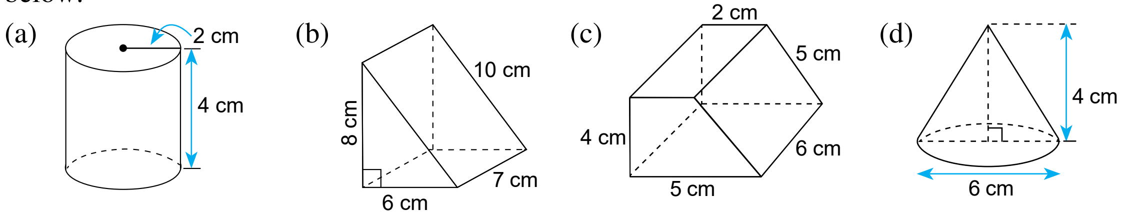
DO YOU KNOW?

A cone is generated by the rotation of a right-angled triangle.

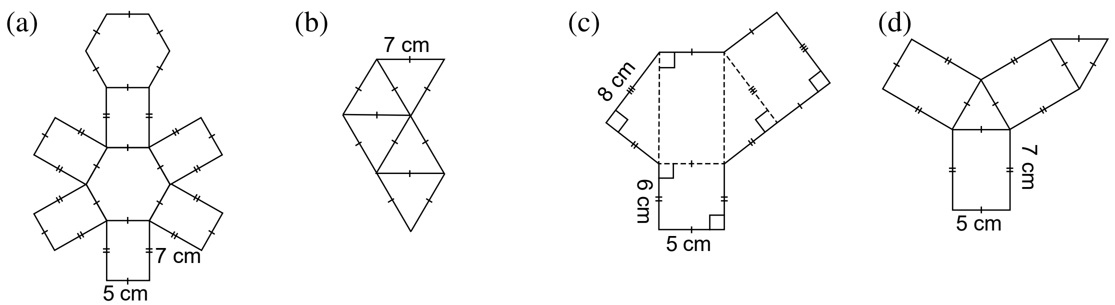


SELF PRACTICE 6.2

1. Using 1 cm grid paper, draw the net and build a model for each of the three-dimensional shapes below.



2. State the three-dimensional shapes that can be built with the following net. Build an actual model.



6.3 Surface Area Three-Dimensional Shapes

6.3.1 Surface area of cube, cuboid, pyramid, prism, cylinder and cone

LEARNING STANDARD

Derive the formulae of the surface areas of cubes, cuboids, pyramids, prisms, cylinders and cones, and hence determine the surface areas of the shapes.

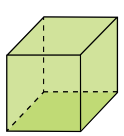
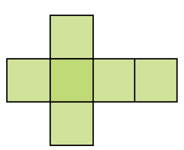
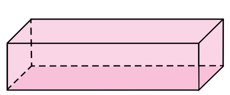
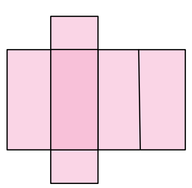
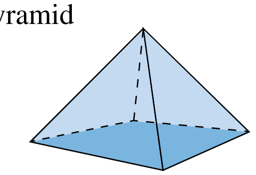
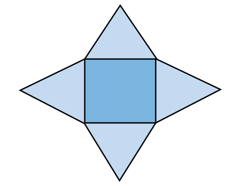
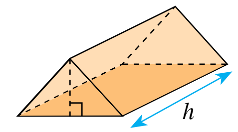
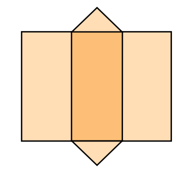
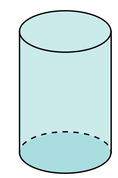
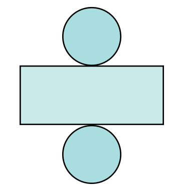
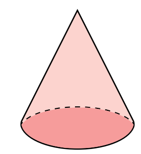
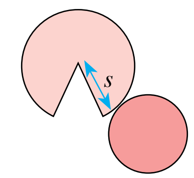
COGNITIVE STIMULATION Individual

Aim: Deriving the surface area of three-dimensional geometrical shapes

Material: Worksheet

Steps:

Fill in the box with the number of surfaces for each of the three-dimensional shapes below.

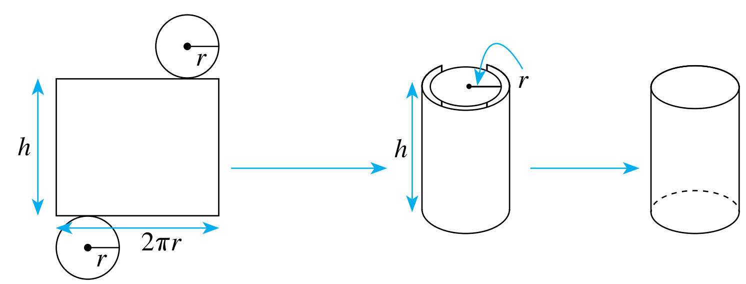
Shape	Net	Surface area
Cube 		☐ × area of a square
Cuboid 		☐ × area of a rectangle + ☐ × area of a square
Pyramid 		☐ × area of a square + ☐ × area of a triangle
Prism 		☐ × area of a triangle + ☐ × area of a rectangle
Cylinder 		☐ × area of a circle + ☐ × area of a rectangle
Cone 		☐ × area of a circle + ☐ × area of a curved surface

Discussion:

Determine the surface area for each of the three-dimensional shapes above.

The surface area of the three-dimensional geometrical shapes can be calculated by adding all the surface area of the net.

The surface area of a closed cylinder



DO YOU KNOW

Cube is also known as hexahedron because a cube has six surfaces.

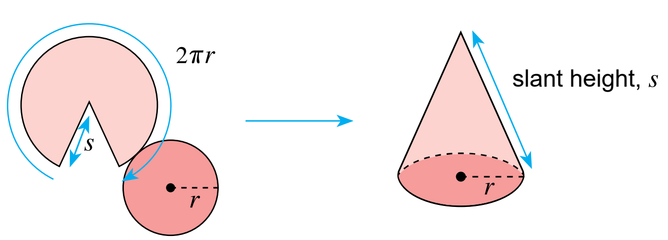
From the net of a cylinder, the length of the rectangle is the circumference of circle and the width of the rectangle is the height of the cylinder.

Surface area of a closed cylinder = (2 × area of circle) + area of rectangle
$= (2 \times \pi r^2) + (2\pi r \times h)$
$= 2\pi r^2 + 2\pi rh$

FLASHBACK

Area of circle = πr^2
Circumference of circle = $2\pi r$

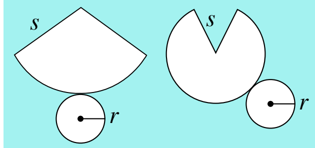
Surface area of a cone is calculated from the cone's net

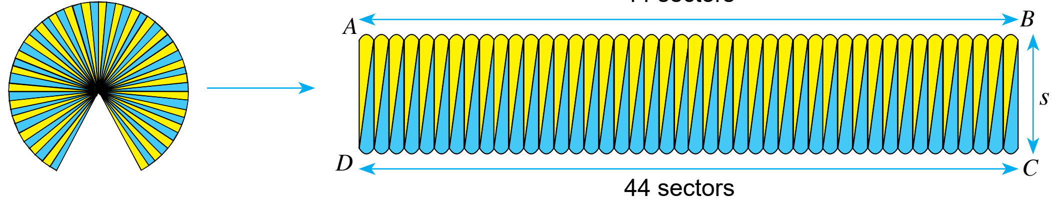


Cut the curved surface into 88 equal sectors. Then arrange them accordingly as in the diagram below.

THINK SMART

What are the differences between the nets below?





A rectangle $ABCD$ is formed. The circumference of the base of the cone is,

$AB + CD$ = circumference of circular base

$= 2\pi r$

Therefore, length AB = Length CD

$= \frac{1}{2} \times 2\pi r$

$= \pi r$

Curved surface area = Area of rectangle $ABCD$

= length × width

$= AB \times BC$

$= \pi r \times s$

$= \pi rs$

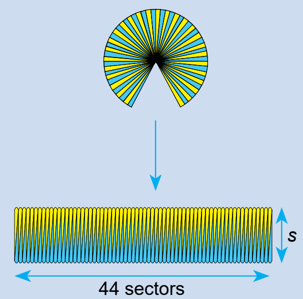
Area of the circular base $= \pi r^2$

Cone surface area = area of circular base + curved surface area

$= \pi r^2 + \pi rs$

TIPS

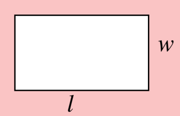
Cut the curved surfaces into 88 equal sectors:



The more sectors are cut, the greater the pieces will resemble a rectangle.

FLASHBACK

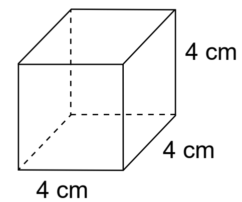
Area of rectangle
= Length × width



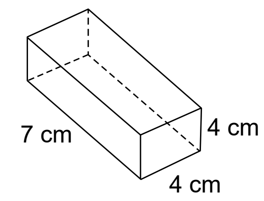
EXAMPLE 2

Calculate the surface area of the geometrical shapes below.

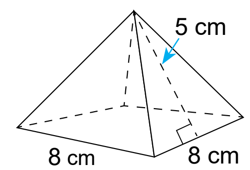
(a)



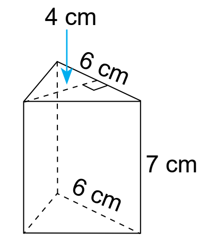
(b)



(c)



(d)



Solution:

(a) Surface area of a cube
= 6 × area of square
= 6 × (4 cm × 4 cm)
= 6 × 16 cm^2
= 96 cm^2

(b) Surface area of a cuboid
= (4 × area of rectangle) + (2 × area of square)
= (4 × 4 cm × 7 cm) + (2 × 4 cm × 4 cm)
= (4 × 28 cm^2) + (2 × 16 cm^2)
= 144 cm^2

(c) Surface area of a pyramid
= (4 × area of triangle) + (area of square)

$= 4\left(\frac{1}{2} \times 8 \text{ cm} \times 5 \text{ cm}\right) + (8 \text{ cm} \times 8 \text{ cm})$

= 80 cm^2 + 64 cm^2
= 144 cm^2

DO YOU KNOW?

The Autocad software can be used to calculate the surface area of a geometrical shape.

TIPS

A two-dimensional shape has two measurements, length and width which will give the surface area. Two-dimensional shapes do not have volume.

A three-dimensional shape has the measurements length, width and height. Three-dimensional shapes have volume.

DO YOU KNOW?

There are two types of solid, polyhedron and non-polyhedron. A solid polyhedron has flat surface with every side being a polygon. Non-polyhedron is a solid object with a curved surface like sphere, cylinder and cone.

CHAPTER 6

(d) Surface area of a prism
$= (3 \times \text{base area of rectangle}) + (2 \times \text{area of triangle})$

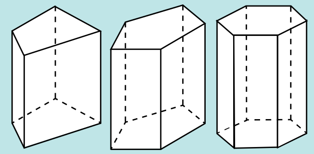
$= \left[(1 \times 6 \text{ cm} \times 7 \text{ cm}) + (2 \times 5 \text{ cm} \times 7 \text{ cm})\right] + 2\left(\frac{1}{2} \times 4 \text{ cm} \times 6 \text{ cm}\right)$

$= 42 \text{ cm}^2 + 70 \text{ cm}^2 + 24 \text{ cm}^2$

$= 136 \text{ cm}^2$

THINK SMART

How do you measure the surface area of the prisms below?



EXAMPLE 3

Calculate the surface area of a cylinder. The radius of the circle is 7 cm. Use $(\pi = \frac{22}{7})$

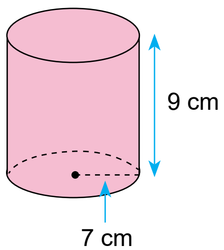
Solution:

Surface area of a cylinder $= 2\pi r^2 + 2\pi rh$

$= \left(2 \times \frac{22}{7} \times 7^2\right) + \left(2 \times \frac{22}{7} \times 7 \times 9\right)$

$= 308 + 396$

$= 704 \text{ cm}^2$



EXAMPLE 4

The diagram below shows a cone. The circle radius is 3 cm. Calculate the surface area of the cone. Use $(\pi = \frac{22}{7})$

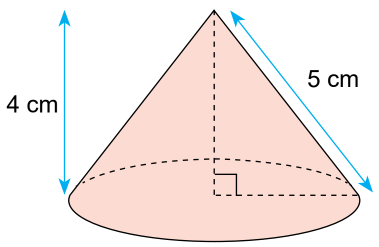
Solution:

Surface area of a cone $= \pi r^2 + \pi rs$

$= \left(\frac{22}{7} \times 3^2\right) + \left(\frac{22}{7} \times 3 \times 5\right)$

$= 28.29 + 47.14$

$= 75.43 \text{ cm}^2$



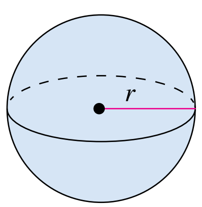
6.3.2 Surface area of a sphere

LEARNING STANDARD

Determine the surface area of spheres using formula.

The surface area of a sphere with radius r cm can be determined by using the formula:

Surface area of a sphere $= 4\pi r^2$



THINK SMART

Many spherical shapes exist in our environment, for example, bubbles and water droplets. Can you think of another example?

EXAMPLE 5

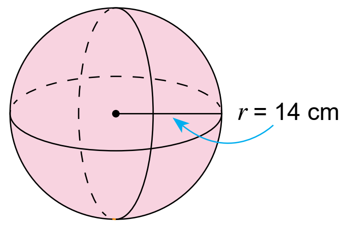
The diagram below shows a sphere with the radius, $r = 14$ cm.
Calculate the surface area of the sphere. (Use $\pi = \frac{22}{7}$)

Solution:

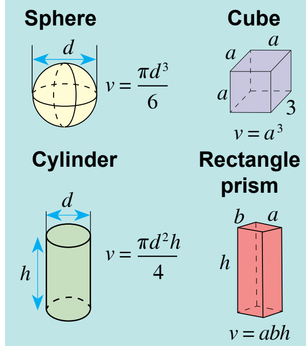
Surface area $= 4\pi r^2$

$= 4 \times \frac{22}{7} \times 14^2$

$= 2\ 464 \text{ cm}^2$



THINK SMART



Can the formulae above be used to calculate volume?

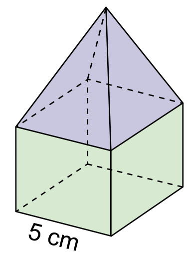
6.3.3 Solving problems

LEARNING STANDARD

Solve problems involving the surface area of three-dimensional shapes.

EXAMPLE 6

The diagram shows an object made up of a pyramid and a cube. The height of the object is 11 cm. Calculate the surface area of the object. State your answer in m^2.



TIPS

1m = 100 cm
$1\text{m}^2 = 1 \times (100 \times 100) \text{ cm}^2$
$= 10\ 000 \text{ cm}^2$

Solution:

Understanding the problem

Calculating the surface area of a combined three-dimensional shape.

Planning the strategy

(i) Identifying the shapes.
(ii) Identifying the surface area formula for each shape.

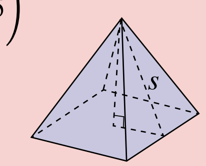
Implementing the strategy

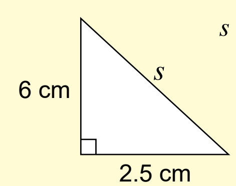
The shapes are pyramid and cube.
Surface area
$= 5 \times (\text{surface area of cube}) + 4 \times (\text{surface area of triangle})$

$= 5(5 \times 5) + 4\left(\frac{1}{2} \times 5 \times 6.5\right)$

$= 125 + 65$

$= 190 \text{ cm}^2$





s = length of slant of the pyramid
$= \sqrt{6^2 + 2.5^2}$
= 6.5 cm

Conclusion

$1 \text{ m}^2 = 10\ 000 \text{ cm}^2$

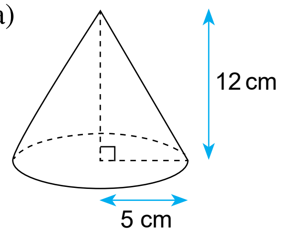
$\therefore \frac{190 \text{ cm}^2}{10\ 000 \text{ cm}^2} \times 1 \text{ m}^2 = 0.019 \text{ m}^2$

Combined surface area is 0.019 m^2.

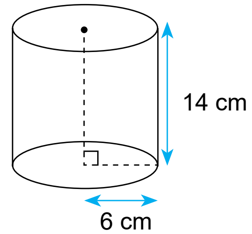
SELF PRACTICE 6.3

1. Calculate the surface area of the three-dimensional objects below.

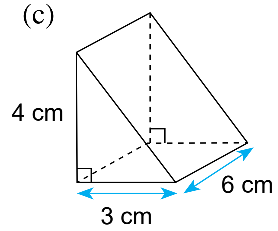
(a)



(b)

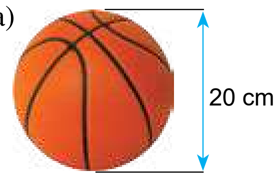


(c)

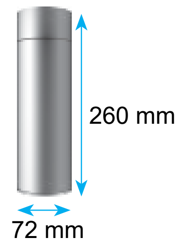


2. Calculate the surface area of the following objects.

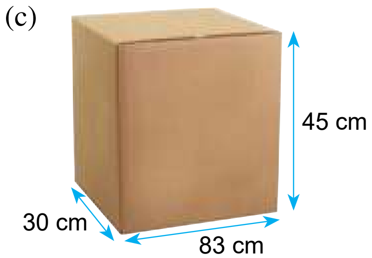
(a)



(b)

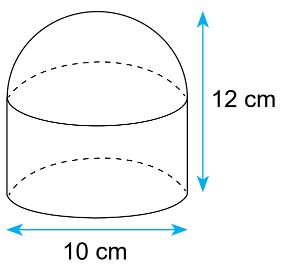


(c)

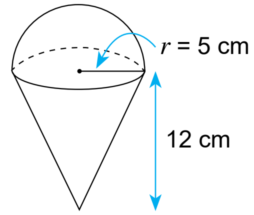


3. Calculate the combined surface area of the following three-dimensional objects.

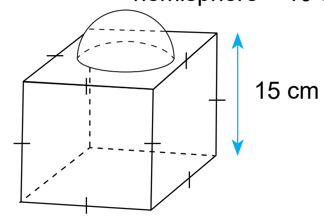
(a)



(b)



(c)



6.4 Volume of Three-Dimensional Shapes

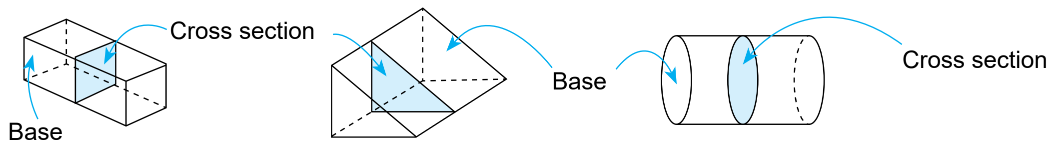
6.4.1 Deriving the formulae

Volume of prisms and cylinders

LEARNING STANDARD

Derive the formulae of the volumes of prisms and cylinders, and hence derive the formulae of pyramids and cones.

The volume of a three-dimensional shape is the measures of the amount of space it occupies. The shape is measured in cubic unit such as cubic millimetre (mm^3), cubic centimetre (cm^3) or cubic metre (m^3). Analyse the three-dimensional shapes below. What is the relationship between the cross-section and the base?



Volume of prism

Analyse the cuboid below.

Volume of a cuboid = length × width × height
= area of base × height

DO YOU KNOW?

Cuboid is a type of prism.

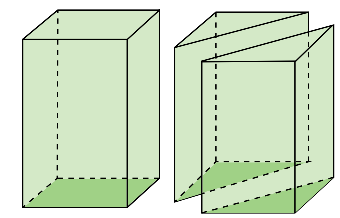
The cuboid is divided into two equal parts. Two triangular prisms are formed. The relationship between the volume of cuboid and the volume of prism is

Volume of a prism $= \frac{1}{2} \times$ cuboid volume

$= \frac{1}{2} \times$ area of base × height

$= \frac{1}{2} \times$ length × width × height

area of triangle



Therefore, **Volume of triangular prism = area of cross section × height**

Volume of cylinder

The diagram above shows a coin in the shape of circle. If 10 coins are arranged upright it will produce a cylinder.

Therefore, volume of cylinder = area of base × height

$= \pi r^2 \times h$

Volume of a cylinder = $\pi r^2 h$

Volume of pyramid

Analyse a cube that has length (l), width (w) and height (h). Six pyramids of equal size can be fitted into the cube with the same base area as the pyramid, just like the base area of a cube and the height of the pyramid is half of the height of cuboid.

Area of base of the pyramid $= l \times w$

Height of pyramid $= \frac{h}{2}$

Height of cube, h = 2 × height of pyramid

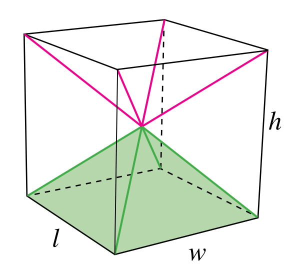
Volume of pyramid $= \frac{\text{Volume of pyramid}}{6}$

$= \frac{l \times w \times h}{6}$

$= \frac{l \times w \times (2 \times \text{height of pyramid})}{6}$

$= \frac{l \times w \times \text{height of pyramid}}{3}$

$= \frac{\text{area of base of pyramid} \times \text{height of pyramid}}{3}$



THINK SMART

Can the same activity be carried out using the rectangular-based pyramid and cuboid?

Therefore,

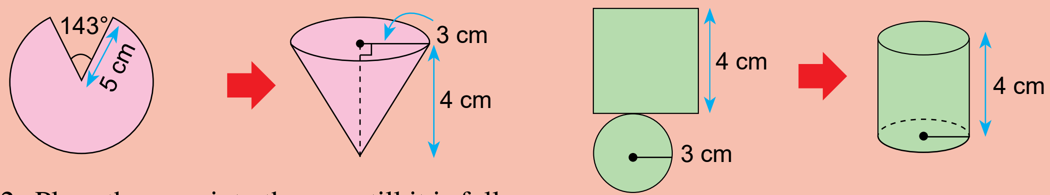
Volume of pyramid,
$= \frac{1}{3} \times$ base area × height

Volume of cone

COGNITIVE STIMULATION Group

Aim: Producing the formula for the volume of cone
Materials: Manila card, scissors, glue and sago
Steps:

1. Using the net below, make a cone and cyclinder. Build an open cone and open cyclinder with the height upright and the base area according to the diagram below.



2. Place the sago into the cone till it is full.
3. Pour the sago from the cone into the cylinder.
4. Repeat steps 2 and 3 until the cylinder is full. How many cones of sago are needed to fill the cyclinder?

Discussion:
(i) Compare your results with your friends.
(ii) The relationship between the volume of cone and cylinder.

From the activity above, you would need 3 cones of sago to fill the cylinder.
Therefore, 3 × volume of cone = 1 × volume of cyclinder

$$\text{Volume of cone} = \frac{1}{3} \times \text{volume of cyclinder}$$

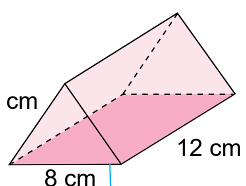
Therefore, $\text{Volume of cone} = \frac{1}{3}\pi r^2 h$

6.4.2 Calculation of volume

LEARNING STANDARD
Determine the volume of prisms, cylinders, cones, pyramids and spheres using formulae.

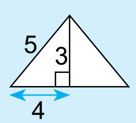
EXAMPLE 7

Calculate the volume of the prism shown.



Solution:

Volume of prism = Cross section area × Height
= Area of triangle × Height
$= (\frac{1}{2} \times 8 \times 3) \times 12$ cm
$= 144 \text{ cm}^3$

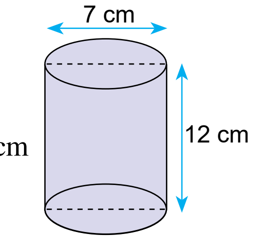


Using Pythagoras theorem:
Height of triangle = $\sqrt{5^2 - 4^2}$
= 3 cm

FLASHBACK
SI unit for:
(i) Area is cm^2 (square centimetre)
(ii) Volume is cm^3 (cubic centimetre)

EXAMPLE 8

Calculate the volume of the cylinder. (Use $\pi = \frac{22}{7}$)

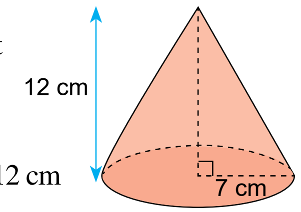


Solution:

Volume of cylinder = Cross section area × Height
$= \pi r^2 h$
$= (\frac{22}{7} \times 3.5 \text{ cm} \times 3.5 \text{ cm}) \times 12 \text{ cm}$
$= 462 \text{ cm}^3$

EXAMPLE 9

Calculate the volume of the cone on the right. (Use $\pi = \frac{22}{7}$)

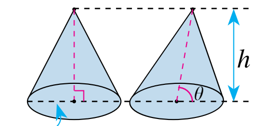


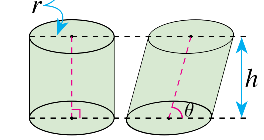
Solution:

Volume of cone = $\frac{1}{3}$ × Area of base × Height
$= \frac{1}{3}\pi r^2 h$
$= \frac{1}{3} \times (\frac{22}{7} \times 7 \text{ cm} \times 7 \text{ cm}) \times 12 \text{ cm}$
$= 616 \text{ cm}^3$

DO YOU KNOW?

Volume of three-dimensional oblique-shaped objects.

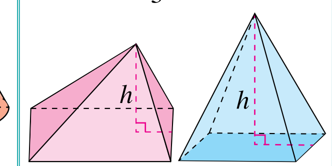




h = height of cone
B = area of base

$V = \frac{1}{3}Bh$

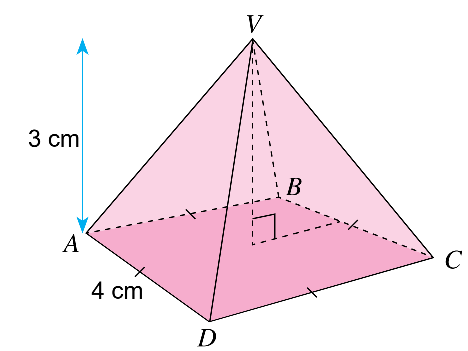
$V = \frac{1}{3}\pi r^2 h$



Volume = $\frac{1}{3}Bh$

EXAMPLE 10

Calculate the volume of the pyramid.



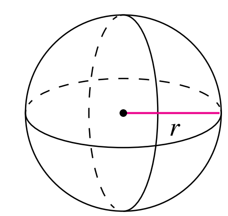
Solution:

Volume of pyramid = $\frac{1}{3}$ × Area of base × Height
$= \frac{1}{3} \times (4 \text{ cm} \times 4 \text{ cm}) \times 3 \text{ cm}$
$= 16 \text{ cm}^3$

Volume of sphere

Sphere is a three-dimensional geometrical shape that has one point known as centre of the sphere. All the points are equidistant from the centre. Volume of the sphere with radius, r is

$$\text{Volume of sphere} = \frac{4}{3}\pi r^3$$



CHAPTER 6

EXAMPLE 11

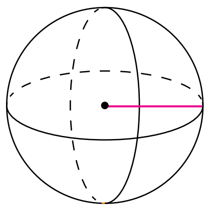
Calculate the volume of the sphere with the radius 7 cm. (Use $\pi = \frac{22}{7}$)

Solution:

Volume of sphere $= \frac{4}{3}\pi r^3$

$= \frac{4}{3} \times \frac{22}{7} \times 7 \text{ cm} \times 7 \text{ cm} \times 7 \text{ cm}$

$= 1437.33 \text{ cm}^3$



EXAMPLE 12

Calculate the volume of hemisphere on the right. (Use: $\pi = \frac{22}{7}$)

Solution:

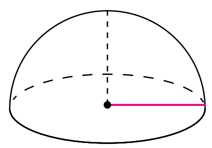
Volume of hemisphere $= \frac{1}{2} \times$ Sphere volume

$= \frac{1}{2} \times \frac{4}{3}\pi r^3$

$= \frac{2}{3}\pi r^3$

$= \frac{2}{3} \times \frac{22}{7} \times 5 \text{ cm} \times 5 \text{ cm} \times 5 \text{ cm}$

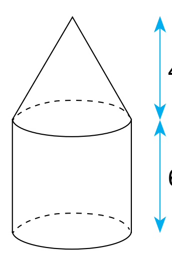
$= 261.90 \text{ cm}^3$



6.4.3 Solving problems

EXAMPLE 13

Salim is an ice cream entrepreneur. He sells his ice creams in a container as shown in the diagram below. If he aims to sell 10 000 containers a month, how many litres of ice cream does he need in a month? Round off the answer to the nearest liters . (Use $\pi = \frac{22}{7}$)



DO YOU KNOW?

The solar system consists of the sun and other planets that are spherical. This includes the planet Earth. Take note of the Earth's position in the solar system.

Radius of each planet,
Mercury = 2 423 km
Venus = 6 059 km
Earth = 6 378 km
Pluto = 1 180 km
Mars = 3 394 km

THINK SMART

A metal ball used in a competition has a radius of 4.9 cm. The density of the metal that is used to make ball is 7.8 g/cm³. Calculate the mass of the metal ball.

LEARNING STANDARD

Solve problems involving the volume of three-dimensional shapes.

DO YOU KNOW?

The Malaysian Health Ministry has organised a healthy eating campaign among Malaysians to consume the right amount of calorie according to the age and the daily needs of an individual. The calorie intake needed by a male aged 13-15 is 2 200 calories a day. Whereas, a female aged 13-15 needs 1 800 calories of food a day.

Solution:

Understanding the problem

To calculate the volume of ice cream needed to produce 10 000 containers of ice cream to the nearest litre.

Planning the strategy

(i) To determine the volume of the container

(ii) To determine the volume of 10 000 containers

Implementing the strategy

Volume of cylinder $= \pi r^2 h$

$= \frac{22}{7} \times 2.5 \times 2.5 \times 6$

$= 117.86 \text{ cm}^3$

Volume of cone $= \frac{1}{3} \times \pi r^2 h$

$= \frac{1}{3} \times \frac{22}{7} \times 2.5 \times 2.5 \times 4$

$= 26.19 \text{ cm}^3$

Therefore, volume of container $= 117.86 + 26.19$

$= 144.05 \text{ cm}^3$

Total volume of 10 000 containers

$= 10\,000 \times 144.05$

$= 1\,440\,500 \text{ cm}^3$

Conclusion

$1 \text{ litre} = 1\,000 \text{ cm}^3$

$1\,440\,500 \text{ cm}^3 = \frac{1\,440\,500 \text{ cm}^3}{1\,000 \text{ cm}^3} \times 1 \text{ litre}$

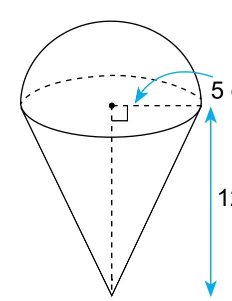
$= 1\,440.5 \text{ litre}$

Then, 1 440.5 litres of ice cream is needed.

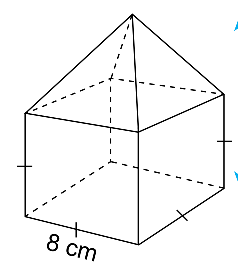
SELF PRACTICE 6.4

1. Calculate the volume of the following.

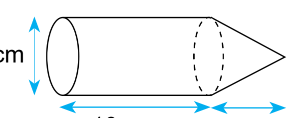
(a)



(b)

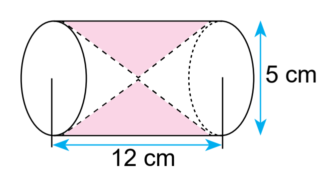


(c)

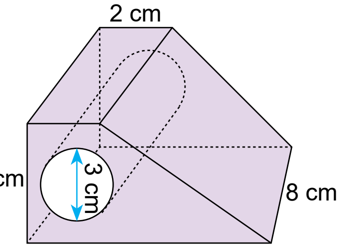


2. Calculate the volume of the shaded region.

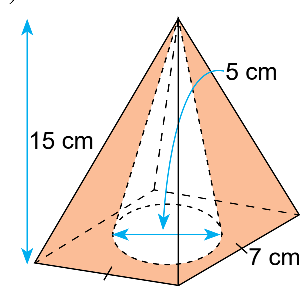
(a)



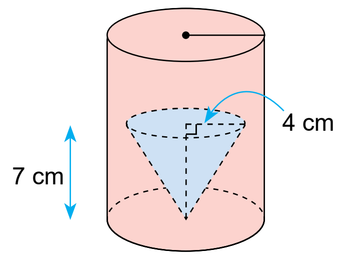
(b)



(c)



3. Ali poured water into a cylindrical container that has a radius of 7 cm and height of 15 cm until it is full. A solid shaped cone is inserted fully into the cylinder as shown in the diagram below. After a while, the solid cone is taken out from the cylinder. Calculate the volume of water that is left in the cylinder.

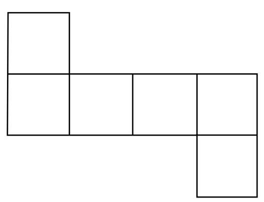


4. A block of metal pyramid with a square base, with side 15 cm and height 10 cm is melted down to form a few balls of spheres with a radius of 5 mm. How many pyramid blocks are needed to form 2 850 balls of spheres?

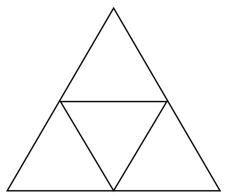
GENERATING EXCELLENCE

1. State the three-dimensional shape of the nets.

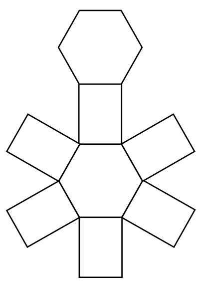
(a)



(b)



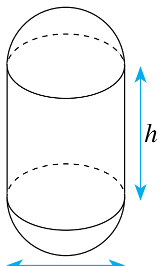
(c)



2. A cylindrical water bottle with a height of 20 cm and diameter of 5.5 cm is filled with water until it is full. Vincent wants to transfer the water in the bottle into a cubic container. State the minimum length of a side of the cube.

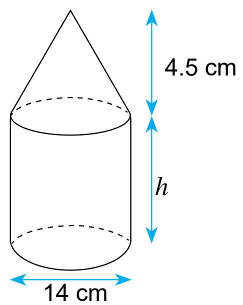
3. Given the volume of the block, calculate the value of h.

(a)



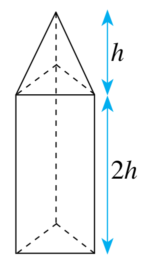
Volume = 122 000 mm³

(b)



Volume = 1 540 cm³

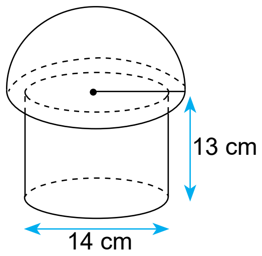
(c)



cross section area of prism = 325 cm²

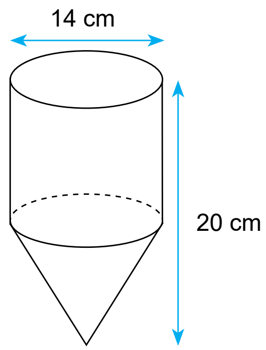
Volume = 6 825 cm³

4. Study the diagram. The diameter of the hemisphere is 22 cm. Calculate
   (a) the volume of the combined shapes.
   (b) the total number of marbles with a volume of 343 mm³ which can be filled into the container.



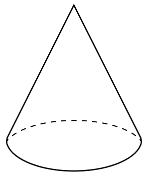
5. An artist wants to do a full painting on the surface of a pottery. The pottery in the shape of a cylinder has the height of 10 cm and a radius of 3.5 cm. If one tube of colour can paint 100 cm² of drawing, how many tubes are needed to paint 10 potteries of the same type?

6. The diagram shows a solid made by combining a cylinder and a cone. $\frac{1}{2}$ kg of sugar can produce 1 litre of syrup to make candies shaped like the solid. If the height of the cylinder is twice the radius of the cylinder, how many candies can be produced using 100 kg of sugar?

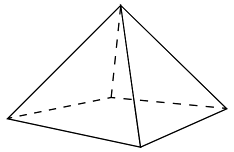


7. A cylinder open at the top with a height twice the radius of the base, is filled with water three quarter full. 539 ml water is needed to fill up the cylinder. Calculate the surface area of the cylinder, in cm². (Use $\pi = \frac{22}{7}$)

8. The diagram shows a block of cone and pyramid. If the volume of the pyramid is three times the volume of the cone, and the surface area of the pyramid is twice the surface of the cone, calculate the height of the cone and the pyramid, if the height of the cone is 18 cm. (Use $\pi = \frac{22}{7}$)

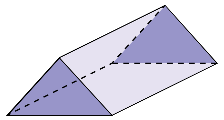
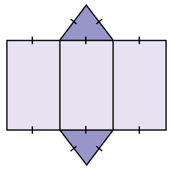
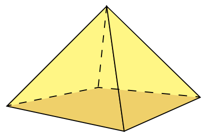
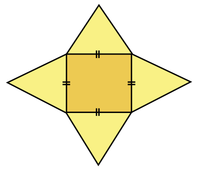
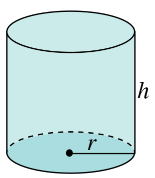
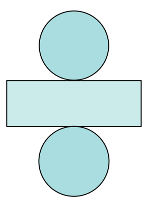
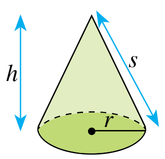
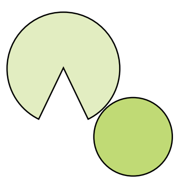
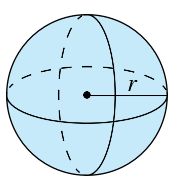


cone



pyramid

CHAPTER SUMMARY

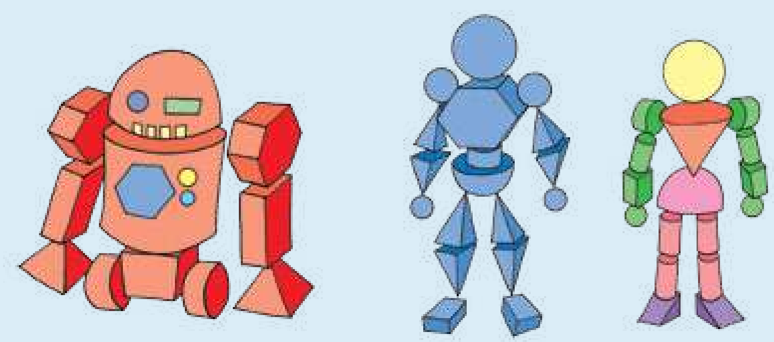
Geometrical shape	Net	Surface area	Volume
Prism 		(2 × area of triangle) + (3 × area of rectangle)	Area of cross section × height
Pyramid 		Area of base + (4 × area of triangle) = (length × width) + $4(\frac{1}{2} \times \text{base} \times \text{height})$	$\frac{1}{3}$ × area of base × height
Cylinder 		$2\pi r^2 + 2\pi rh$	$\pi r^2 h$
Cone 		$\pi r^2 + \pi rs$	$\frac{1}{3}\pi r^2 h$
Sphere 		$4\pi r^2$	$\frac{4}{3}\pi r^3$

SELF REFLECTION

At the end of the chapter, I am able to:	🙂	😐	🙁
1. Compare, contrast and classify three-dimensional shapes including prisms, pyramids, cylinders, cones and spheres, and hence describe the geometric properties of prisms, pyramids, cylinders, cones and spheres.			
2. Analyse various nets including pyramids, prisms, cylinders and cones, and hence draw nets and build models.			
3. Derive the formulae of the surface area of cubes, cuboids, pyramids, prisms, cylinders and cones, and hence determine the surface areas of the shapes.			
4. Determine the surface area of spheres using formula.			
5. Solve problems involving the surface area of three-dimensional shapes.			
6. Derive the formulae of the volumes of a prisms and cylinders, and hence derive the formulae of pyramids and cones.			
7. Determine the volume of prisms, cylinders, cones, pyramids and spheres using formulae.			
8. Solve problems involving the volume of three-dimensional shapes.			

MINI PROJECT

Design a robot with the combination of shapes such as cube, cuboid, prism, pyramid, cylinder, cone and sphere. Students should create the shapes by themselves. You may combine the three-dimensional shapes.



Example of robots

CHAPTER 7

Coordinates

WHAT WILL YOU LEARN?

7.1 Distance in the Cartesian Coordinate System

7.2 Midpoint in the Cartesian Coordinate System

7.3 The Cartesian Coordinate System

WORD LINK

Midpoint	*Titik tengah*
Distance	*Jarak*
Position	*Kedudukan*
Coordinate	*Koordinat*
x-axis	*Paksi-x*
y-axis	*Paksi-y*
Hypotenuse	*Hipotenus*
Origin	*Asalan*
Plots	*Plot*
Cartesian Plane	*Satah Cartes*
Scale	*Skala*

Cartesian Coordinate System is a method to determine the position of a point or object on a plane, or into two or three dimensions.

The position on a plane is determined by the position of the point on a straight line or number. The position of a point in two dimensions is determined by the coordinate system on a Cartesian plane. The position in three dimensions is determined by three numbers.

WALKING THROUGH TIME

The Cartesian Coordinate System was introduced by René Descartes from France or better known as Cartesius. He introduced a coordinate plane which is formed by two perpendicular lines called ‘axis’. Coordinates are a set of numbers that locate a point or a line.

For more information:

http://rimbunanilmu.my/mat_t2e/ms121

WHY STUDY THIS CHAPTER?

- The coordinate system has contributed a lot in the field of archaeology and geography.
- Archaeologists begin their search according to coordinate points on a map digitally.
- Astronomers can determine the position of the stars through this coordinate system.
- A location is determined by a combination of coordinate points which help geographers to identify the area and position on Earth.

CREATIVE ACTIVITY

Aim: Identifying the position of a point
Material: Worksheet
Steps:
1. Open the file MS122A and print out the worksheet.
2. By joining the vertical and horizontal distances, determine the position of the following towns: Batu Pahat, Kluang and Segamat.

QR CODE

Scan the QR Code or visit http://rimbunanilmu.my/mat_t2e/ms122a to get the worksheet.

You have learnt about coordinates of a location on a certain Cartesian plane. A coordinate is a pair of numbers that is used to determine the position of a point on the Cartesian plane. The coordinate of a point is determined based on the distance from x-axis, the distance from y-axis and the origin. Were you able to determine the distance between two points from the activity above?

7.1 Distance in a Cartesian Coordinate System

7.1.1 Distance between two points on the Cartesian plane

LEARNING STANDARD

Explain the meaning of distance between two points on the Cartesian plane.

COGNITIVE STIMULATION Group

Aim: Identifying the distance between two points on a Cartesian plane
Material: Worksheet
Step:

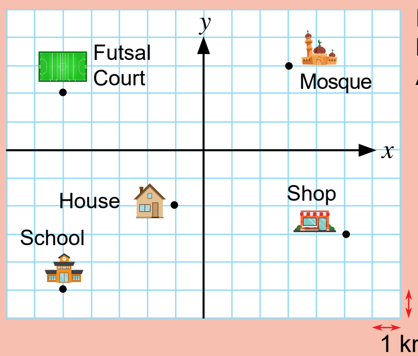
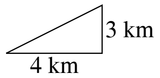


Diagram shows the plan of locations often passed by Azri.

QR CODE

Scan the QR Code or visit http://rimbunanilmu.my/mat_t2e/ms122b to get the worksheet.

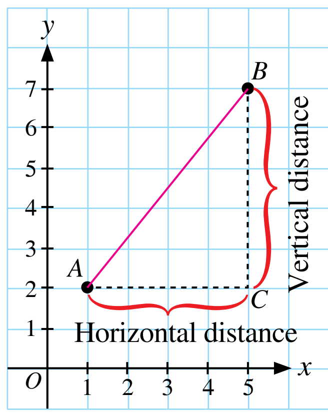
1. Open the file MS122B and print out the worksheet.
2. In pairs, identify Azri's movements to the destinations as in the table.
3. Azri's movement must be drawn in the form of a right angled triangle.
4. Measure the horizontal and vertical distances based on 1 grid box equal to 1 km and fill in the table as show in the example.
5. Add the total distance by completing the table.

Azri's Destination	Triangular representation	Horizontal Distance	Vertical Distance	Total distance travelled = Horizontal distance + Vertical distance
School to house		4 km	3 km	4 km + 3 km = 7 km
House to futsal field				
Mosque to shop				
School to mosque				
School to shop				

Discussion:
(i) From the representation of the right angled triangle, can you identify the nearest distance taken by Azri to a certain destination?
(ii) What is the easiest way to calculate the shortest distance?
(iii) What do you understand about distance on a Cartesian plane?

To determine distance between two points on a Cartesian plane, the right angled triangle representation method is used.
In this method you have to identify the horizontal distance and the vertical distance of two points on a Cartesian plane. This distance can be determined from the scale on the x-axis and the y-axis.

AB is the shortest distance, taken without going through C



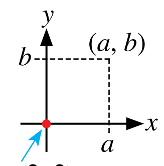
The Pythagoras theorem is used to calculate the distance AB, that is

$$AB^2 = AC^2 + CB^2$$

$$AB = \sqrt{AC^2 + CB^2}$$

DO YOU KNOW

The Cartesian plane has two axes as in the diagram. The horizontal line is the x-axis and the vertical line is y-axis. Both lines will intersect perpendicularly. The intersection point is the origin which is the starting point for both x-axis and y-axis. The value of the numbers will increase when it moves to the right and upwards. However, the value of a number will decrease when it moves to the left and downwards.



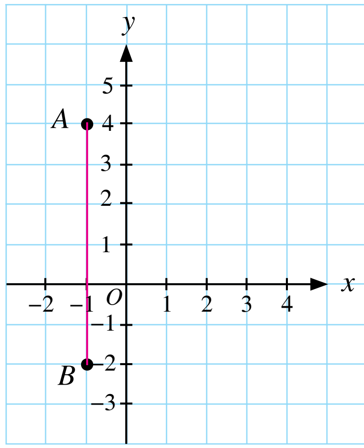
TIPS

In coordinates (x, y), the value of x is written first followed by the value of y.

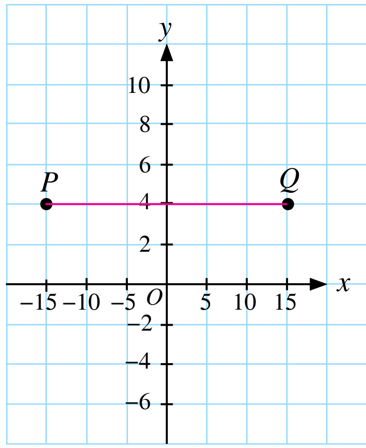
EXAMPLE 1

Determine the distance between two points in the following Cartesian plane.

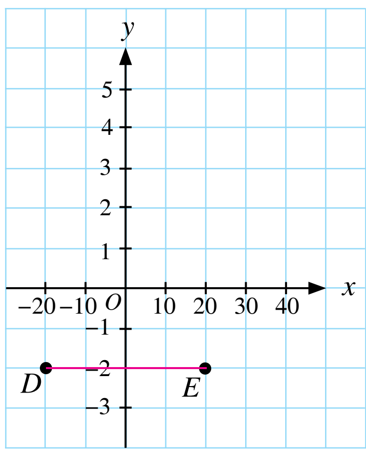
(a)



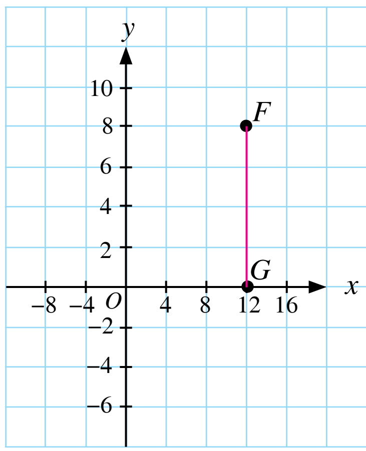
(b)



(c)

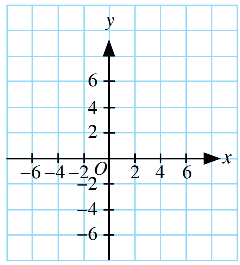


(d)



DO YOU KNOW?

What is a scale? Scales need to be determined in the Cartesian coordinate system. The units that can be written on the x-axis are 1, 2, 3, ... and on the text of the origin are −1, −2, −3, ... The units that can be written on the y-axis are 1, 2, 3, ... and the values below the origin are −1, −2, −3, ... This is how each box is represented as one unit. Apart from that, scales can be written in the sequence of 2, 4, 6, 8, ... or 5, 10, 15, ... on both axes. These conditions depend on the suitability in certain situations.



Scale on x-axis is 2 units
Scale on y-axis is 2 units

Solution:

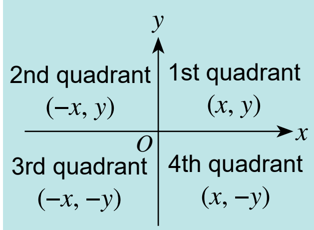
(a) The scale on x-axis and y-axis is 1 unit
Distance of AB
$= 6 \times 1$
$= 6$ units

(b) The scale on x-axis is 5 units and y-axis is 2 units.
Distance of PQ
$= 6 \times 5$
$= 30$ units

(c) The scale on x-axis is 10 units and y-axis is 1 unit.
Distance of DE
$= 4 \times 10$
$= 40$ units

(d) The scale on x-axis is 4 units and y-axis is 2 units.
Distance of FG
$= 4 \times 2$
$= 8$ units

THINK SMART



If (x, y) is (3, 4) in the 1st quadrant, state the coordinates of the point in the 2nd quadrant 3rd quadrant and 4th quadrant. What type of transformation is experienced by the point?

7.1.2 The formula if the distance between two points on the plane

LEARNING STANDARD

Derive the formula of the distance between two points on the Cartesian plane.

COGNITIVE STIMULATION (Group)

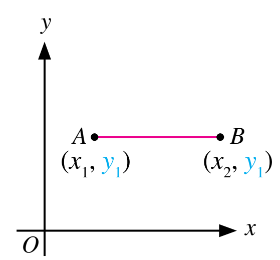
Aim: Determining the distance between two points with the same x-coordinate and y-coordinate.
Material: Printed Worksheet
Steps:
1. With a friend, identify the coordinates on the x-axis and the y-axis.
2. Complete the table by determining the coordinates with common axis.

Example:

Coordinate		Same coordinate	Distance
A (2 , 1)	B (2 , 4)	x-coordinate	4 − 1 = 3 unit
C (–1, 3)	D (7 , 3)		
E (6 , 5)	F (6 , –5)		
G (–7, 2)	H (1 , 2)		

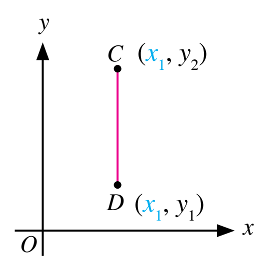
Discussion:
How can you create a simple formula for determining the distance between two points that has
(i) the same x-coordinate?
(ii) the same y-coordinate?

Distance can be determined if,
(i) Two points have the same y-coordinate



Distance for $AB = (x_2 - x_1)$ unit

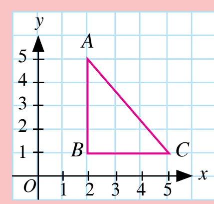
(ii) Two points have the same x-coordinate.



Distance for $CD = (y_2 - y_1)$ unit

FLASHBACK

Look at the triangle on the Cartesian plane below.



The base of the triangle BC is parallel to the x- axis. This makes the y- coordinates the same. This is called common y- axis. It is the same the other way around.

EXAMPLE 2

Calculate the distance between the points.
(a) (2, –3) and (4, –3)
(b) (0, 1) and (0, –2)

Solution:

(a) The distance between the two points is
= 4 – 2
= 2 units ← Horizontal distance = $x_2 - x_1$

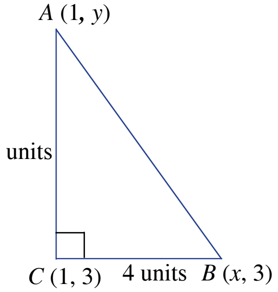
(b) The distance between the two points is
= 1 – (–2)
= 3 units ← Vertical distance = $y_2 - y_1$

QR CODE

Scan the QR Code or visit http://rimbunanilmu.my/mat_t2e/ms126a for the Submarine Target game.

EXAMPLE 3

The diagram shows the distance between two points A and B. Complete the coordinates of A and B.



Solution:

$y - 3 = 5$ units
$y = 5 + 3$
$= 8$ units
Therefore, coordinate A is (1,8).

$x - 1 = 4$ units
$x = 4 + 1$
$= 5$ units
Therefore, coordinate B is (5, 3).

7.1.3 Distance between two points on a plane

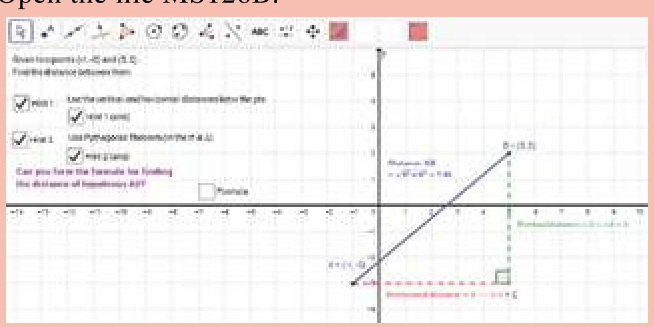
LEARNING STANDARD

Determine the distance between two points on a Cartesian plane.

If the straight line that joins two points on a Cartesian plane is not parallel to the x-axis or y-axis, then the distance between the two can be determined using the Pythagoras theorem.

COGNITIVE STIMULATION Group

Aim: Identifying the distance between two points
Material: Dynamic geometry software
Steps:
1. Open the file MS126B.



QR CODE

Scan the QR Code or visit http://rimbunanilmu.my/mat_t2e/ms126b to identify the distance between two points.

2. Move the coordinates A and B on the Cartesian plane based on the table below.
3. Identify the horizontal distance and the vertical distance for the line AB.
4. Compare the displayed answers with the answers using the formula.
5. Complete the table below with the answers by choosing *Hint*.

	Points		Difference in Distance		Distance AB
	A	B	Horizontal $y_2 - y_1$	Vertical $x_2 - x_1$	$AB = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$
(a)	(1, 5)	(1, 7)	1 – 1 = 0	7 – 5 = 2	
(b)	(4, 1)	(1, 1)			
(c)	(8, 2)	(0, –4)			
(d)	(6, 7)	(2, 4)			

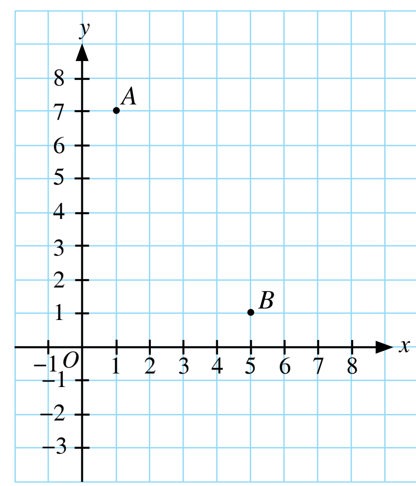
Discussion:
(i) What do you understand about the distance of AB?
(ii) What is the relevance of Pythagoras theorem?

The distance AB is the hypotenuse. The Pythagoras theorem is used to determine the distance between two points on a Cartesian plane.

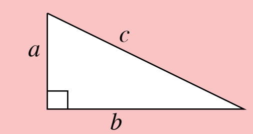
The distance between two points on a Cartesian plane $= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

EXAMPLE 4

Calculate the distance between point A and point B on the Cartesian plane in the diagram below.



FLASHBACK



$c = \sqrt{a^2 + b^2}$

What is this formula?

The theorem shows that for a right-angled triangle, the square of its hypotenuse is equal to the sum of the squares of the other two sides.

Solution:

Method 1

Based on the diagram, draw a right-angled triangle *ACB*.

$AC = 6$ units, $BC = 4$ units

Using Pythagoras theorem

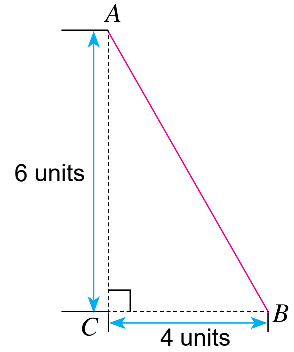
$AB^2 = BC^2 + AC^2$

$AB^2 = 4^2 + 6^2$

$AB^2 = 16 + 36$

$AB = \sqrt{52}$

$= 7.21$ units



Method 2

Distance $= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

Distance $AB = \sqrt{(5-1)^2 + (1-7)^2}$

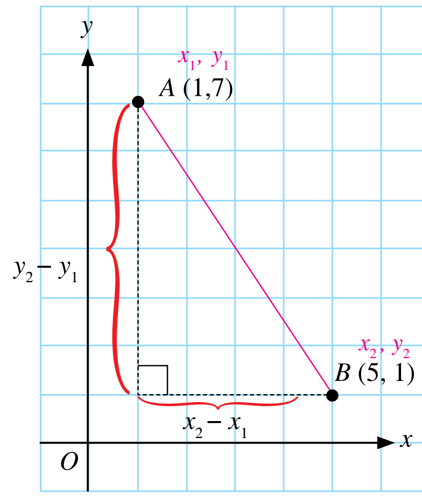
$= \sqrt{4^2 + (-6)^2}$

$= \sqrt{16 + 36}$

$= \sqrt{52}$

$= 7.21$ units

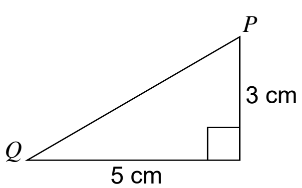
Therefore, the distance of *AB* is 7.21 units.



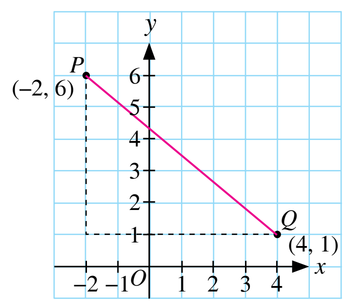
EXAMPLE 5

Calculate the distance between point *P* and point *Q*.

(a)



(b)



Solution:

(a) $PQ^2 = 5^2 + 3^2$

$= 25 + 9$

$PQ = \sqrt{34}$

$= 5.83$ cm

Therefore, the distance of *PQ* is 5.83 cm.

(b) $PQ^2 = \sqrt{[4-(-2)]^2 + (1-6)^2}$

$= \sqrt{6^2 + (-5)^2}$

$= \sqrt{36 + 25}$

$= \sqrt{61}$

$= 7.81$ cm

Therefore, the distance of *PQ* is 7.81 cm.

7.1.4 Solving problems

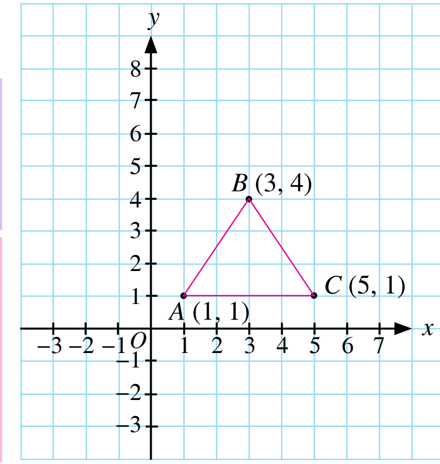
LEARNING STANDARD

Solve problems involving the distance between two points in the Cartesian coordinate system.

EXAMPLE 6

Calculate the perimeter of an isosceles triangle if the vertices for the triangle are *A* (1, 1), *B* (3, 4) and *C* (5, 1).

Solution:



Understanding the problem

ABC is an isosceles triangle with vertices *A* (1, 1), *B* (3, 4) and *C* (5, 1).

Planning the strategy

- Draw and determine the points on a Cartesian plane.
- Perimeter $\Delta ABC = AB + BC + AC$
- The distance of *AC* and *AB*.

Implementing the strategy

Distance $AB = \sqrt{3^2 + 2^2}$

$= \sqrt{9 + 4}$

$= \sqrt{13}$

$= 3.6$ units

$AB = BC$

Conclusion

Therefore, the perimeter of the triangle *ABC* is $3.6 + 3.6 + 4 = 11.2$ units.

TIPS

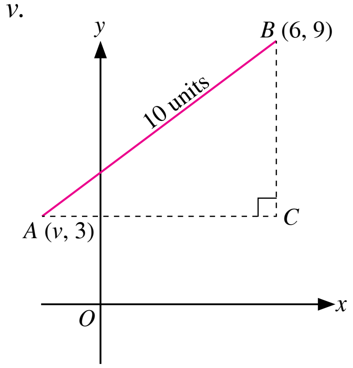
The distance between two points

Distance $= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

Distance is the measurement of length between two points.

EXAMPLE 7

Given that the distance of $AB = 10$ units. Calculate the value of *v*.



Solution:

Understanding the problem

Calculate the value of *v*.

Planning the strategy

Distance $AB = 10$

Formula of distance

$= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

Implementing the strategy

$AB = \sqrt{(6-v)^2 + (9-3)^2}$

$10 = \sqrt{(6-v)^2 + 6^2}$

$10 = \sqrt{(6-v)^2 + 36}$

$10^2 = \left(\sqrt{(6-v)^2 + 36}\right)^2$

$10^2 - 36 = (6-v)^2$

$\sqrt{64} = 6 - v$

$8 = 6 - v$

$v = 6 - 8$

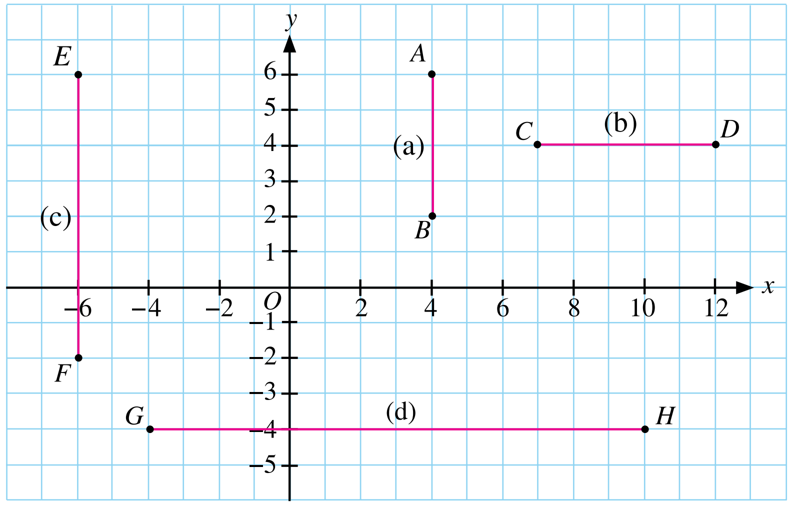
$v = -2$

Conclusion

Therefore, the value of *v* is –2.

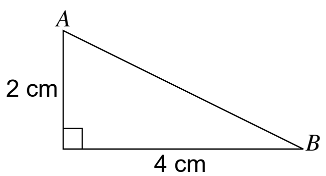
SELF PRACTICE 7.1

1. Determine the distance between two points on the following Cartesian plane.

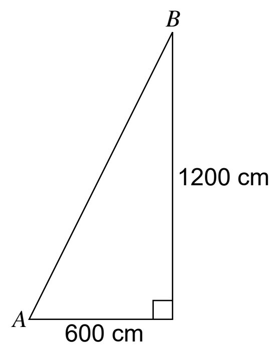


2. Calculate the distance of AB.

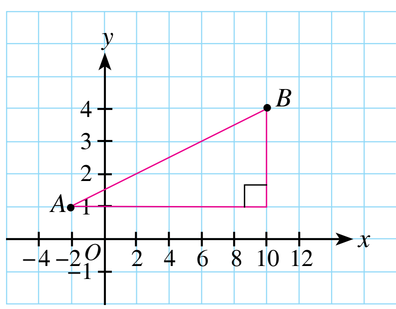
(a)



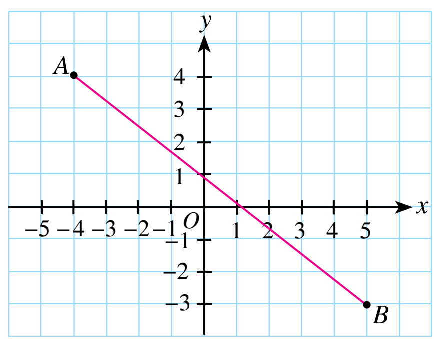
(b)



(c)



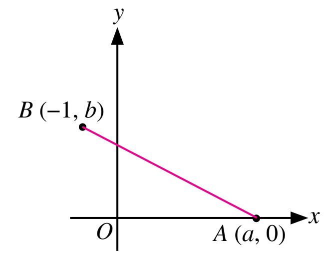
(d)



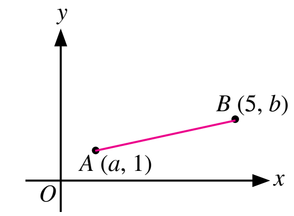
3. State the distance between each set of points below.
   (a) (1, 3) and (1, 7)
   (b) (0, −9) and (0, 9)
   (c) (5, −2) and (−2, −2)
   (d) (7, 4) and (8, 4)

4. Given that the horizontal distance is 4 units and the vertical distance is 3 units for the points A and B, calculate the values of a and b.

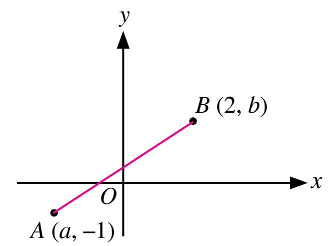
(a)



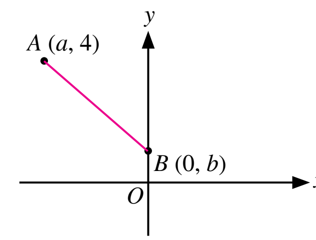
(b)



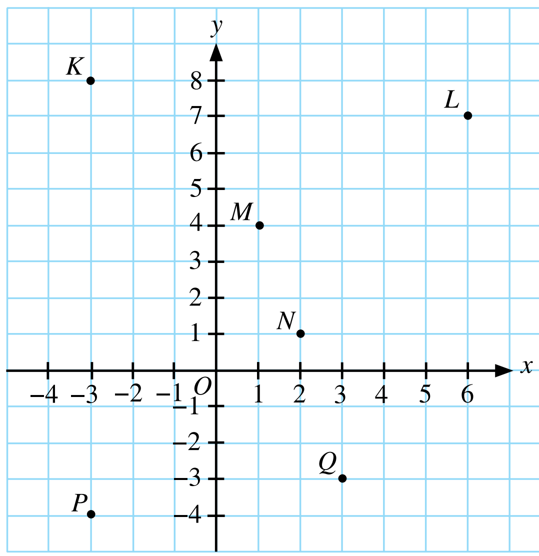
(c)



(d)



5. The diagram shows the points K, L, M, N, P and Q on the Cartesian plane.
   Calculate the distance between the points.
   (a) KM
   (b) ML
   (c) PN
   (d) KQ

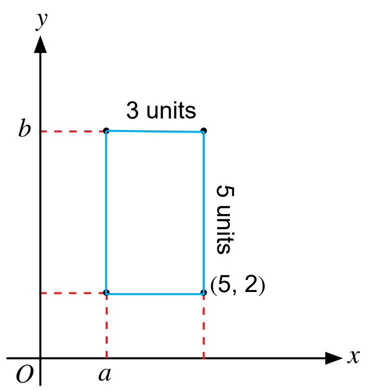


6. Determine the distance of the points KL given K (2, 2) and L is on the x-axis with a distance of 7 units to the right from the y-axis.

7. Determine the distance of AB if each of them is located on the y-axis with a distance of 5 units upwards and 2 units downwards from the x-axis.

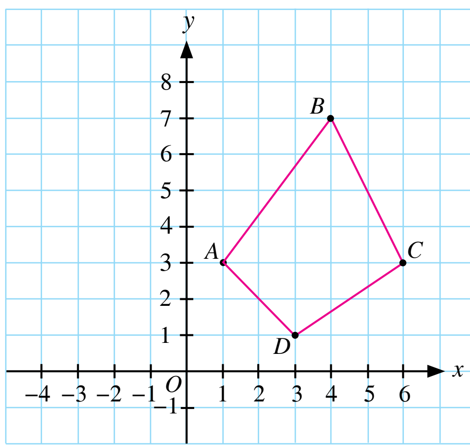
8. Calculate the distance between the points KL if L is located on the origin and K is 3 units to the left of y-axis and 5 units upwards from the x-axis.

9. Determine the values of a and b based on the information in the diagram below.



10. The vertical distance of point V is 4 units to the north of point W. Determine the coordinates of W if the coordinates of V are
(a) (4, –3)
(b) (2, –5)
(c) (5, –2)
(d) (0, –4)

11. Based on the diagram, calculate the perimeter for $ABCD$.



12. The triangle ABC has vertices A (-2, –1), B (–2, 5) and C (1, –1). Calculate the perimeter for the triangle.

7.2 Midpoint in The Cartesian Coordinate System

7.2.1 Midpoint between two points

You have learned how to determine a radius for a certain diameter in a circle. Do you understand the concept of midpoint? Discuss this concept with your friends.

LEARNING STANDARD

Explain the meaning of midpoint between two points on the Cartesian plane.

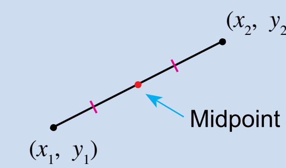
COGNITIVE STIMULATION (Group)

Aim: Identifying the midpoint on a line
Materials: Grid paper, compasses and ruler
Steps:
1. Student A constructs a Cartesian plane on grid paper.
2. Student B chooses two coordinate points and draws a line that joins the points.
3. Student C contructs a perpendicular bisector on the line.

Discussion:
What do you understand when you construct the perpendicular bisector on the line?

Midpoint is a point that divide a line segment equally.

TIPS

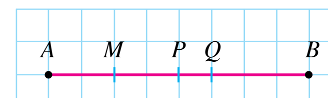


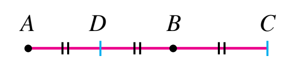
QR CODE

Scan the QR Code or visit http://rimbunanilmu.my/mat_t2e/ms133 to watch an animated video on determining midpoints.

EXAMPLE 8

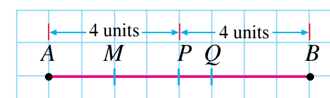
Determine the midpoint of the straight line AB.

(a)


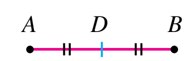
(b)


Solution:

(a) Midpoint of the straight line AB is P.

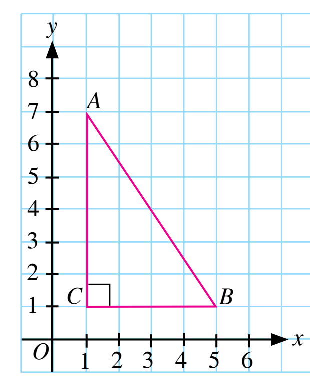


(b) Midpoint of the straight line AB is D.



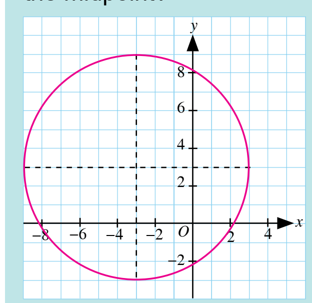
EXAMPLE 9

P is the midpoint of the straight line AB. Determine coordinates P.



THINK SMART

State the coordinates of the centre of the circle for the diagram below. What is the connection between the centre of a circle and the midpoint?



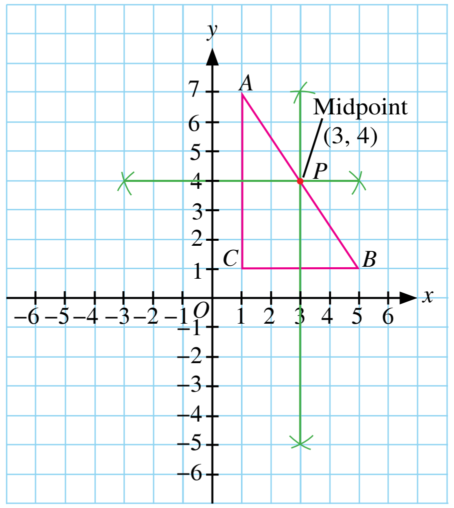
Solution:

Step 1: Determine the midpoint of *AC* and *BC*.

Step 2: Construct a perpendicular bisector of *AC* and *BC*.

Step 3: Intersection between the perpendicular bisector of *AC* and *BC* is the midpoint of the line *AB*.

Step 4: Therefore, point *P* is (3, 4).



7.2.2 The midpoint formula

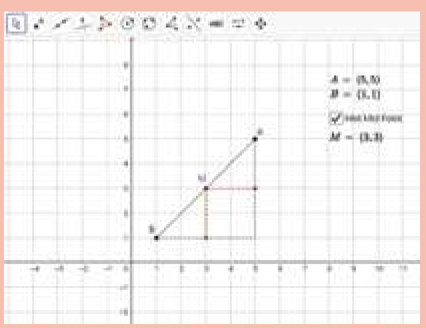
LEARNING STANDARD

Derive the formula of the midpoint between two points on the Cartesian plane.

COGNITIVE STIMULATION Group

Aim: Deriving the midpoint formula
Material: Dynamic geometry software
Steps:

1. Open the file MS134.



2. Identify point *A* and point *B*.
3. Change the location of the points as in the table provided.
4. Identify horizontal distance and vertical distance.
5. Open file MS135 and complete the table given.
6. Calculate the midpoint *M*.

QR CODE

Scan the QR Code or visit http://rimbunanilmu.my/mat_t2e/ms134 to identify the midpoint.

Point		Midpoint of:		Midpoint $\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$
A	*B*	Horizontal distance	Vertical distance	
(4, 5)	(2, 1)			
(–1, 5)	(3, 1)			
(1, 3)	(7, 1)			
(3, 4)	(–5, –1)			
(1, 2)	(–5, 2)			

Discussion:

(i) Is the midpoint of the straight line *AB* the result of the intersection for the midpoint of horizontal distance and vertical distance?

(ii) Derive the formula for midpoint.

QR CODE

Scan the QR Code or visit http://rimbunanilmu.my/mat_t2e/ms135 to get the worksheet.

Midpoint for a slanting line can be determined by identifying the horizontal distance and vertical distance which are both divided by two.

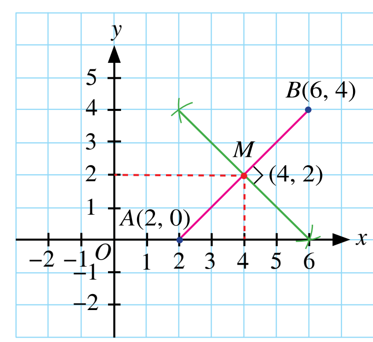
$$\text{Midpoint} = \left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$$

7.2.3 Midpoint coordinates between two points

LEARNING STANDARD

Determine the coordinates of midpoint between two points on the Cartesian plane.

The location of a midpoint can be shown by constructing a perpendicular bisector. The intersection between the perpendicular bisector with the line segment will determine the coordinates of the midpoint on a Cartesian plane.



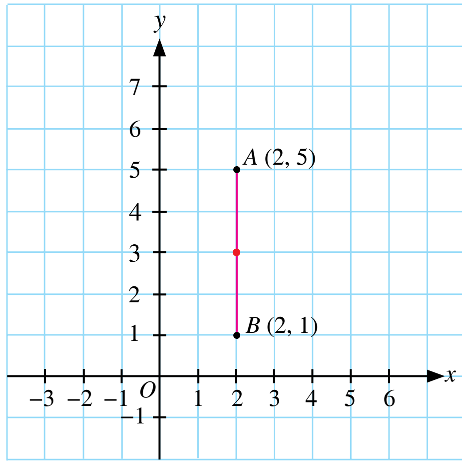
$$M = \left(\frac{6+2}{2}, \frac{4+0}{2}\right)$$

$$M = (4, 2)$$

EXAMPLE 10

Calculate the coordinate of the midpoint on the straight line AB given A (2,5) and B (2,1).

Solution:



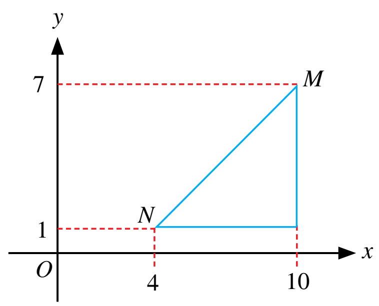
A (2, 5) is (x_1, y_1) and B (2, 1) is (x_2, y_2)

$$\text{Midpoint } AB = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$
$$= \left(\frac{2+2}{2}, \frac{5+1}{2}\right)$$
$$= \left(\frac{4}{2}, \frac{6}{2}\right)$$
$$= (2, 3)$$

Therefore, the midpoint of AB is (2, 3).

EXAMPLE 11

Calculate the coordinate of the midpoint on the straight line MN.



Solution:

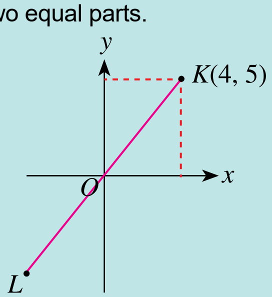
M (10, 7) is (x_1, y_1) and N (4, 1) is (x_2, y_2)

$$\text{Midpoint } MN = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$
$$= \left(\frac{10+4}{2}, \frac{7+1}{2}\right)$$
$$= \left(\frac{14}{2}, \frac{8}{2}\right)$$
$$= (7, 4)$$

Therefore, the midpoint of MN is (7, 4).

THINK SMART

Midpoint is a point on a line segment that divides it into two equal parts.



If the origin is the midpoint for the line KL, can you determine the coordinates of L?

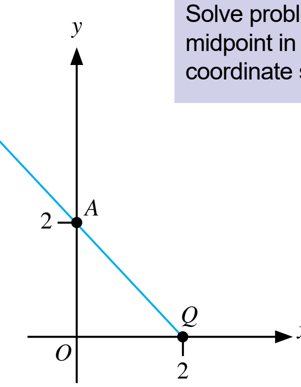
7.2.4 Solving problems

LEARNING STANDARD

Solve problems involving midpoint in the Cartesian coordinate system.

EXAMPLE 12

The diagram shows the line PAQ on a Cartesian plane. A is the midpoint for the line PQ. Determine the coordinates of P.



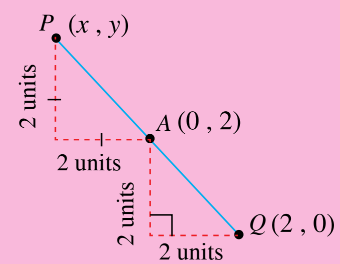
Solution:

Understanding the problem

Distance of $AP = AQ$.
Let $P = (x, y)$.

Planning the strategy

Distance of $AP = AQ$



Implementing the strategy

Calculate the horizontal and vertical distance starting from point A which is 2 units respectively.

Midpoint, A (0, 2)

$P(x, y)$
$Q(2, 0)$

$$\frac{x+2}{2} = 0 \quad , \quad \frac{y+0}{2} = 2$$
$$x + 2 = 0 \quad , \quad y = 4$$
$$x = -2$$

Conclusion

Therefore, coordinates of P are (–2, 4).

EXAMPLE 13

Point P is the midpoint on the straight line KL. Given coordinates of K (–3, 12) and coordinates of P (2, 9), calculate coordinates of L.

Solution:

K (–3, 12) is (x_1, y_1) and L (x_2, y_2)

$$\text{Midpoint, } P = \left(\frac{-3 + x_2}{2}, \frac{12 + y_2}{2}\right)$$
$$(2, 9) = \left(\frac{-3 + x_2}{2}, \frac{12 + y_2}{2}\right)$$
$$\frac{-3 + x_2}{2} = 2, \quad \frac{12 + y_2}{2} = 9$$
$$-3 + x_2 = 4, \quad 12 + y_2 = 18$$
$$x_2 \quad = 7 y_2 = 6$$

Therefore, coordinates of L are (7, 6).

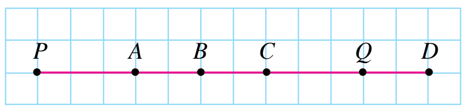
THINK SMART

The KLCC Tower has 88 floors. The distance which is the most suitable to build a skybridge is at the 42nd and the 43rd floors. Why?

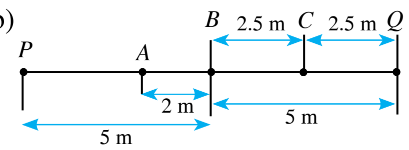
SELF PRACTICE 7.2

1. In each of the diagrams below, determine the midpoint of the straight line PQ.

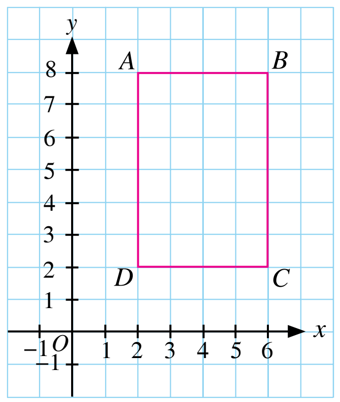
(a)



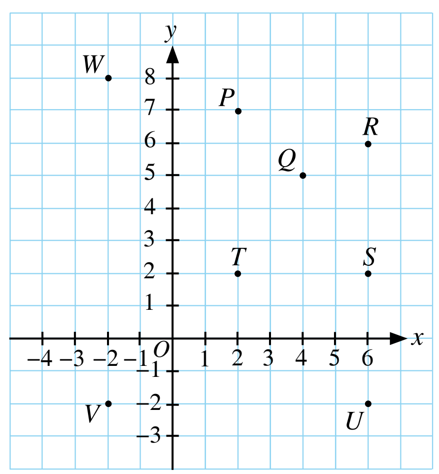
(b)



2. Based on the diagram below, state the coordinates of the midpoint of
   (a) AB
   (b) CD
   (c) AD

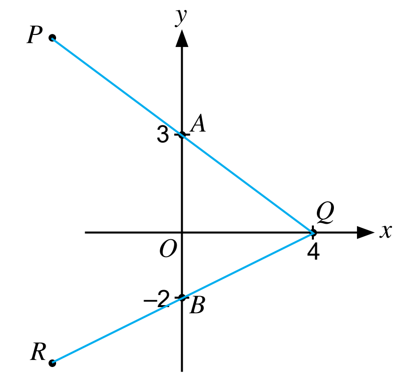


3. Determine the midpoint of the straight line
   (a) PQ
   (b) RS
   (c) TU
   (d) WV

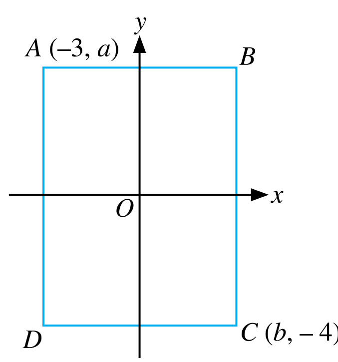


4. Determine the midpoint for the following coordinates
   (a) $P(-1, 7)$ and $Q(-1, 1)$.
   (b) $R(3, -6)$ and $S(3, 2)$.
   (c) $A(3, 1)$ and $B(5, 1)$.
   (d) $C(5, 0)$ and $D(1, 0)$.

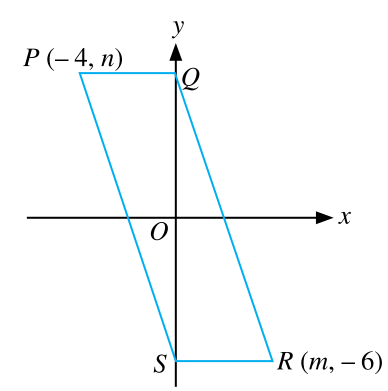
5. Referring to the diagram, A is the midpoint of PQ and B is the midpoint of RQ. Determine the coordinates of P and R.



6. Midpoint for the square in the diagram below is at the origin. Determine
   (a) the values of a and b.
   (b) the distance of line BC.
   (c) the coordinates of B.



7. The origin is the midpoint for the height of the parallelogram. Calculate
   (a) the values of m and n.
   (b) the midpoint of PQ.
   (c) the midpoint of SR.



8. Given the straight line $AB = BD$ with D (–1, 3) and B (1, 1), calculate the coordinates of point A.

9. A straight line that joins points (–8, 3) and (s, 3) has the midpoint (0, u). Calculate the values of s and u.

10. The line AB is parallel to x-axis with point A(3, a) and midpoint of AB is (5, 1). Calculate
(a) the value of a.
(b) the coordinates of B.

7.3 The Cartesian Coordinate System

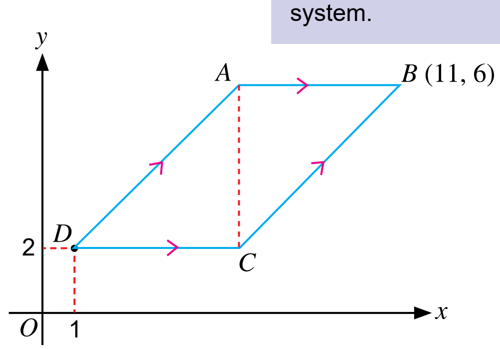
7.3.1 Solving problems

LEARNING STANDARD

Solve problems involving the Cartesian coordinate system.

EXAMPLE 14

The diagram shows a rhombus. Given the distance between the point A and B is 5 units. Calculate
(a) the coordinates of A.
(b) the midpoint of the straight line AC.



Solution:

(a)

Understanding the problem

Determine point A where AB is parallel to DC.

Planning the strategy

Straight line AB is parallel to the x-axis. y-coordinate for point A is 6.

Implementing the strategy

Distance of AB = 5 units.

x-coordinate = 11 – 5
= 6

Conclusion

Therefore, coordinates of A are (6, 6).

(b)

Understanding the problem

The line AC is parallel to the y-axis, with a common x-coordinate, which is 6.

Planning the strategy

Formula for midpoint $\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$.

Implementing the strategy

A (6 , 6) C (6 , 2)
x_1 , y_1 x_2 , y_2

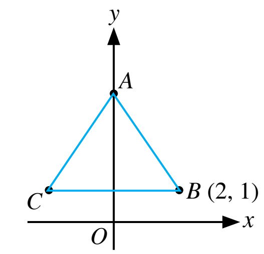
$\left(\frac{6 + 6}{2}, \frac{6 + 2}{2}\right) = (6, 4)$

Conclusion

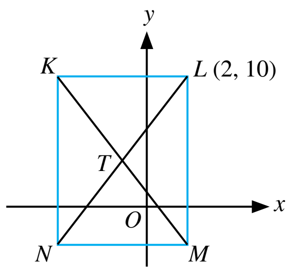
Therefore, midpoint of AC is (6, 4).

SELF PRACTICE 7.3

1. The diagram on the right is an isosceles triangle where the height is 4 units. Calculate
(a) the coordinates of C.
(b) the coordinates of A.
(c) the coordinates of midpoint of the line AB.
(d) the distance of the line AC.



2. The diagram on the right is a rectangle. The distance of KL is 8 units and KN is 12 units. Calculate
(a) the distance of LN.
(b) the coordinates of midpoint of line MN.
(c) the coordinates of T.



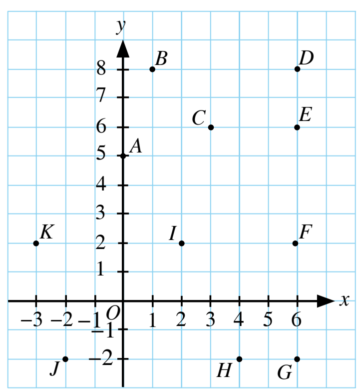
3. If the line PQ is parallel to the y-axis and the midpoint M (4, 0) with a distance of MP is 3 units, calculate
(a) the coordinates of P. (b) the coordinates of Q. (c) the distance of PQ.

4. The distance $AB = KL$, that is 8 units and each is parallel with the y-axis. If midpoint AB is (0, 3) and the distance from midpoint of AB to midpoint of KL is 2 units downwards, calculate
(a) the coordinates of K and L.
(b) the coordinates of the midpoint of KL.

5. Given that P(4, 0) and Q is located on the y-axis with 6 units upwards from the x-axis, calculate
(a) the midpoint of PQ.
(b) the distance between the point P and midpoint of PQ.

GENERATING EXCELLENCE

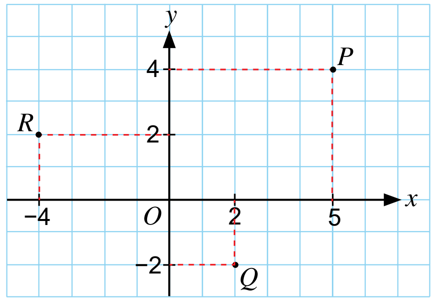
1. Which of the following points represent
(a) (–3, 2)
(b) (0, 5)
(c) (4, –2)
(d) (6, 8)



CHAPTER 7

2. If point K is located on the x-axis and 4 units to the left of the y-axis, determine coordinates of L which is 5 units upwards from point K.

3. If points P, Q and R each move 2 units to the south and 1 unit to the left respectively, state the new locations for the points. Calculate the distance for each new location from PQ to RQ.



4. $ABCD$ is a square with A as the origin and B (−5, 0). Calculate the perimeter of the square.

5. KLM is a right angled triangle with points K (1, 0) and L (5, 0) as the base and ML is the height for the triangle. If the distance from M to L is 5 units, calculate the area of the triangle.

6. The midpoint of the diagonal of a square is 2 units from the vertex of the square. Calculate the area of the square.

CHAPTER SUMMARY

Coordinates

x-axis
Axis which is horizontal and perpendicular to the y-axis in the Cartesian coordinate system.

y-axis
Axis which is vertical and perpendicular to the x-axis in the Cartesian coordinate system.

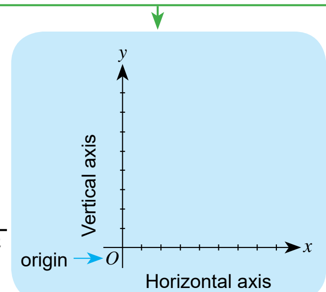
Origin
Point of intersection between horizontal and vertical axes. Coordinates of the origin are (0, 0).

Cartesian Plane
Cartesian plane comprises one horizontal line and one vertical line that intersect at a right angle.

Distance between two points
Measurement of distance or length between two points.

Common axes
$(y_2 - y_1)$ and $(x_2 - x_1)$

$\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$



Midpoint
Point that bisects a line segment into two equal parts.

$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$

SELF REFLECTION

At the end of this chapter, I am able to:

	😊	😐	☹
1. Explain the meaning of distance between two points on the Cartesian plane.			
2. Derive the formula of the distance between two points on a Cartesian plane.			
3. Determine the distance between two points on the Cartesian plane.			
4. Solve problems involving the distance between two points in the Cartesian coordinate system.			
5. Explain the meaning of midpoint between two points on the Cartesian plane.			
6. Derive the formula of the midpoint between two points on the Cartesian plane.			
7. Determine the coordinates for midpoint between two points on the Cartesian plane.			
8. Solve problems involving midpoint in the Cartesian coordinate system.			
9. Solve problems involving the Cartesian coordinate system.			

MINI PROJECT

Draw a plan of your classroom seating position on a grid paper with a scale of 1 cm to 2 metres on the horizontal axis and 1 cm to 2 metres on the vertical axis. You may change the scale. Determine the coordinates of your friends' seats. Paste the plan in front of your class for reference.