CHAPTER 8

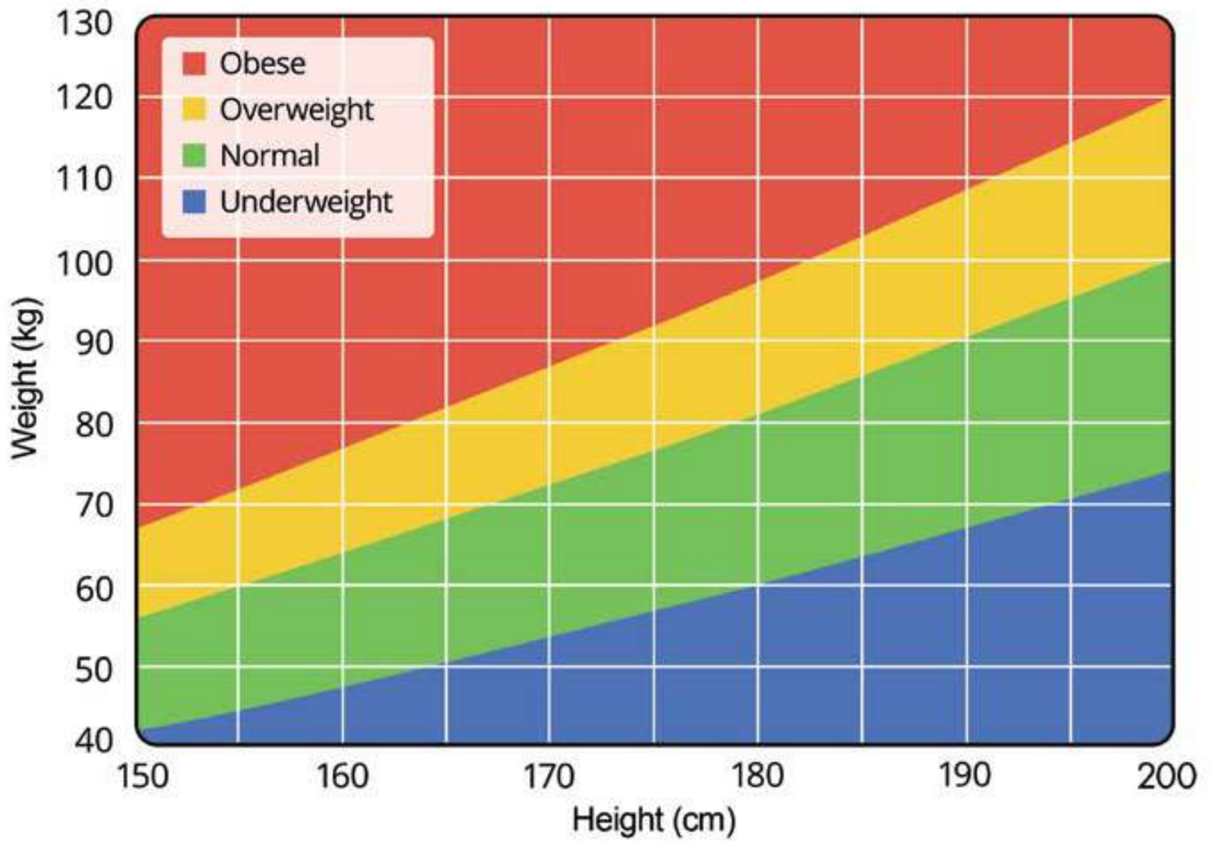
Graphs of Functions

WHAT WILL YOU LEARN?

WORD LINK

• Graph of function	• *Graf fungsi*
• Function	• *Fungsi*
• Variable	• *Pemboleh ubah*
• Relation	• *Hubungan*
• Linear equation	• *Persamaan linear*
• Table of value	• *Jadual nilai*
• Linear function	• *Fungsi linear*
• Non-linear function	• *Fungsi bukan linear*
• Scale	• *Skala*
• Reciprocal function	• *Fungsi salingan*
• Cubic function	• *Fungsi kubik*
• Quadratic function	• *Fungsi kuadratik*

Body mass index (BMI) is a measurement of body fat based on height and weight. A higher measurement of BMI indicates a lot of fat content.



WALKING THROUGH TIME

René Descartes (1596-1650), stated that function is a mathematical relationship between two variables. The term 'function' was introduced by Gottfried Wilhelm Leibniz (1646- 1716) in his book. The concept of function was further studied by Leonhard Euler (1707-1783) and he introduced the notation of function, that is $y = f(x)$.

For more information:

http://rimbunanilmu.my/mat_t2e/ms145

WHY STUDY THIS CHAPTER?

- Function is applied in the fields of economy, technology, science, engineering, banking and mathematics. Among the careers that need knowledge on functions are engineers, economist, auditors, lecturers and bankers.
- The concept of function helps in predicting the best time to trade shares in the stock market.

CREATIVE ACTIVITY

Aim: Knowing relationship between two quantities

Material: Worksheet

Steps:

1. The advertisement below shows the entry rates to a water theme park according to categories. Based on the advertisement, complete the table.

Family	Category		
	Adults	Children	Elderly/Disabled
1			
2			
3			
Total			

2.

Category	Number	Cost	Total
Adults	2	2×30	60
Children			
Elderly/ Disabled			
Total			

3. From the table above, what is the relationship between the total cost of the tickets for each family with the category of family members?

From the table above, we know that the total cost of the tickets depends on the number and category of family members.

8.1 Functions

8.1.1 Definition of functions

LEARNING STANDARD

Explain the meaning of functions.

COGNITIVE STIMULATION Group

Aim: Identifying functions

Materials: Worksheets and calculator

Steps:

1. Use the symbol $\sqrt[3]{\ }$ (cube root) on your calculator to determine the output number of some input numbers and complete Table *A*.
2. Use x^3 (power of cube) on your calculator to determine the output number of some input numbers and complete Table *B*.

Input	$\sqrt[3]{\ }$	Output
64	$\sqrt[3]{64}$	4
27	$\sqrt[3]{27}$	3
0		
$\frac{1}{8}$		
$\frac{1}{125}$		

Table *A*

Input	x^3	Output
2	2^3	8
3	3^3	27
5		
7		
10		

Table *B*

Discussion:

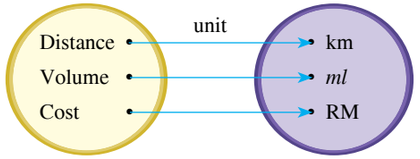
If the input is a domain while the output is a range, specify the range for set $A = \{64, 27, 0, \frac{1}{8}, \frac{1}{125}\}$ and set $B = \{2, 3, 5, 7, 10\}$.

From the activity above, function is a relation where each input has only one output.

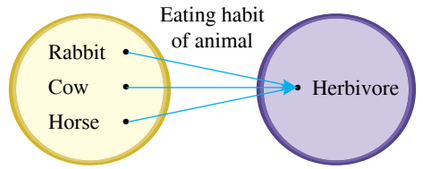
Identifying functions

Relations that are functions

(a) One-to-one relation

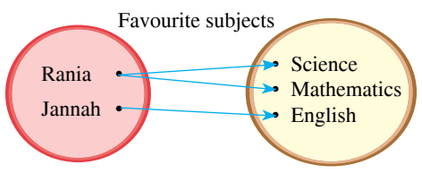


(b) Many-to-one relation

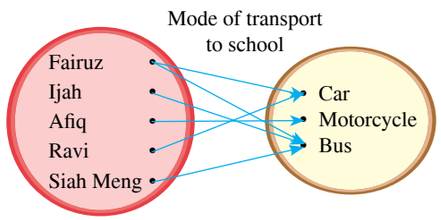


Relations that are not functions

(a) One-to-many relation



(b) Many-to-many relation



Relation is the matching of items from set *A* to set *B*. Relations can be represented by using the

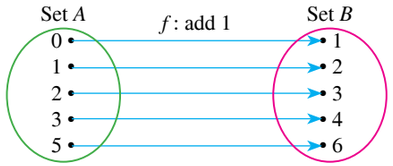
(a) arrow diagram

(b) graph

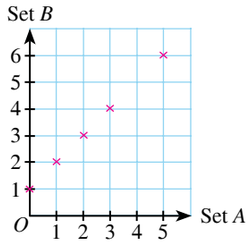
(c) ordered pair

EXAMPLE 1

(a) Arrow diagram



(b) Graph



(c) Ordered pair

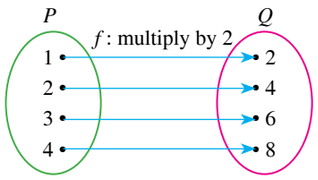
$P = \{(0, 1), (1, 2), (2, 3), (3, 4), (5, 6)\}$

EXAMPLE 2

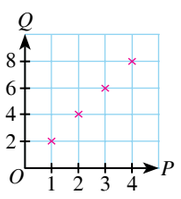
(a) **One-to-one functions**

Relation where the object in the domain has only one image.

(i)



(ii)

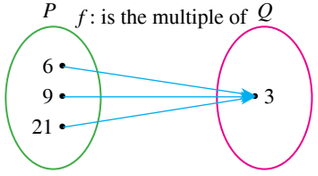


(iii) Ordered pair, $A = \{(1, 2), (2, 4), (3, 6), (4, 8)\}$

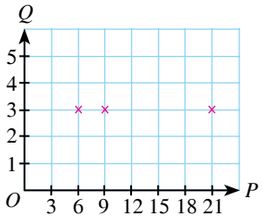
(b) **Many-to-one functions**

Relation where more than one object is matched to the same image.

(i)



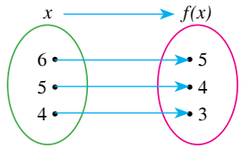
(ii)



(iii) Ordered pair, $B = \{(6, 3), (9, 3), (21, 3)\}$

DO YOU KNOW ?

x is mapped to $f(x)$



The function above can be written with notations as follows:

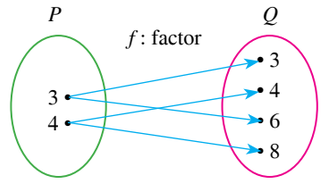
$f : x \to x - 1$ or

$f(x) = x - 1$

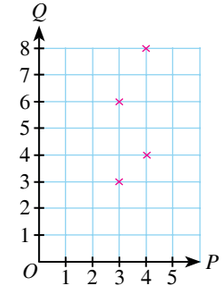
(c) **One-to-many relations**

Relation where the object in the domain has more than one image.

(i)



(ii)

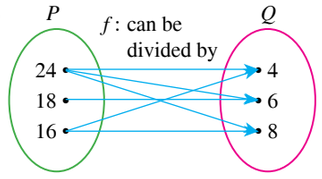


(iii) Ordered pair, $R = \{(3, 3), (3, 6), (4, 4), (4, 8)\}$

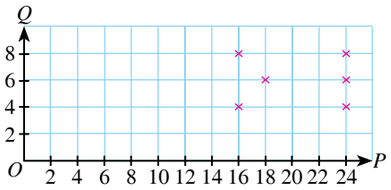
(d) **Many-to-many relations**

Relation where at least one object has more than one image, and more than one object has the same image.

(i)



(ii)



(iii) Ordered pair, $S = \{(24, 4), (24, 6), (24, 8), (18, 6), (16, 4), (16, 8)\}$

Provide justification based on the observation of the relation represented by a graph in the example above.

FLASHBACK

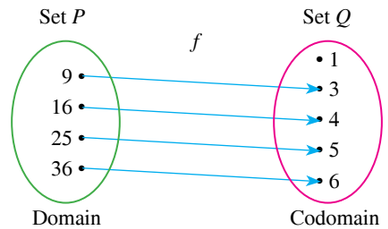
A straight line graph is obtained when all ordered pairs for linear equations are plotted and connected.

8.1.2 Function representation

LEARNING STANDARD

Identify functions and provide justifications based on function representations in the form of ordered pairs, tables, graphs and equations.

The diagram below shows the function f that maps x to $\sqrt{x}$ which is represented by $f(x) = \sqrt{x}$.



Set $P = \{9, 16, 25, 36\}$ is the domain and the element is the object. Set $Q = \{1, 3, 4, 5, 6\}$ is the codomain. The elements in set Q that is matched to the object in set P is the image. Set $\{3, 4, 5, 6\}$ is the range of the function.

EXAMPLE 3

Given set $P = \{1, 2, 3\}$ and set $Q = \{4, 5, 6\}$, the function f maps P to Q by adding 3. Represent this function using

(a) ordered pair (b) table (c) graph (d) equation

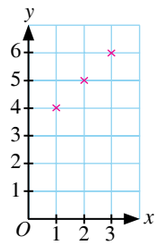
Solution:

(a) $\{(1, 4), (2, 5), (3, 6)\}$

(b)

P	1	2	3
Q	4	5	6

(c)



(d) $4 = 1 + 3$

$5 = 2 + 3$

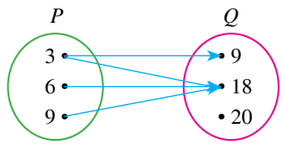
$6 = 3 + 3$

$y = x + 3$ or $f(x) = x + 3$

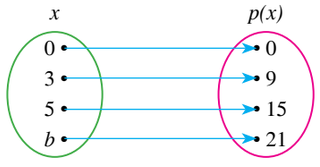
The function that maps x to y can be written using $f(x)$. Therefore, this function can be written as $f(x) = x + 3$.

SELF PRACTICE 8.1

1. The diagram shows the relation between set P and set Q.
   State
   (a) the type of relation.
   (b) the range of the relation.

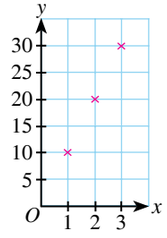


2. The diagram shows a function.
   State the value b.

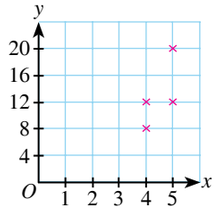


3. Determine whether the set of ordered pair is a function.
   (a) $P = \{(1, 2), (2, 3), (3, 4), (4, 5)\}$
   (b) $Q = \{(1, 3), (0, 3), (2, 1), (4, 2)\}$
   (c) $R = \{(1, 6), (2, 5), (1, 9), (4, 3)\}$

4. Determine whether the relation in the following graph is a function or not a function.
   (a)

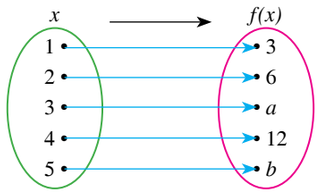


   (b)

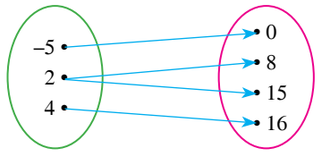


5. Given set $S = \{10, 12, 18, 20\}$ and set $R = \{2, 4, 10, 12\}$, set S is mapped to set R by subtracting 8. Represent the function using
   (a) ordered pair (b) table (c) graph (d) equation

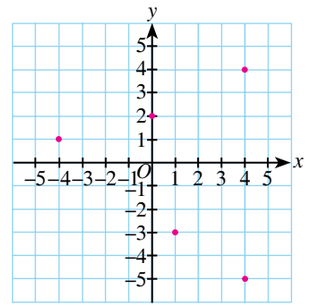
6. The following diagram shows the function $f(x) = 3x$ for the domain $1 \leqslant x \leqslant 5$.
   Determine the values a and b.



7. State the domain and range the following relation.
   (a)

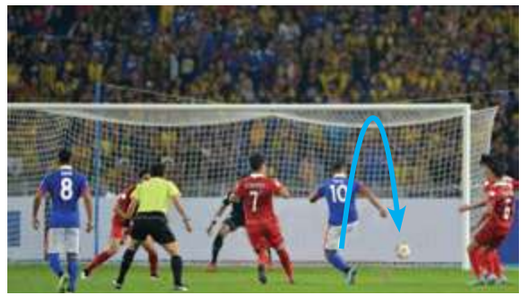


   (b)



8.2 Graphs of Functions

We have learned that the representation of a function can be done in the form of a graph. A graph of function is the representation of a function on a Cartesian plane. By drawing a graph, we can explain the relationship between variables in the function. This graph also helps us identify information to solve problems.



The diagram shows a player kicking a ball, making it bounce into the goalmouth. The bouncing action forms a curve.

If the curve represents the function $s = 25t - 2.5t^2$, t is the time in seconds and s is the height in metre. The relationship between s and t can be represented in the form of a graph. Some information can be obtained from the graph, such as the maximum height of the ball, the time the ball takes to hit the ground again and the distance from where it was kicked.

DO YOU KNOW?

Malaysian football star from Penang Mohd Faiz Subri received the FIFA Puskas Award for the best goal in 2016.

8.2.1 Constructing a table of values

LEARNING STANDARD

Construct tables of values for linear and non-linear functions, and hence draw the graphs using the scale given.

From the given function, a **table of values** can be constructed to determine the corresponding value of the **ordered pair** (x, y) before the graph is drawn.

EXAMPLE 4

(a) Construct a table of values for the function $y = 5 - x$, given $x = -2, -1, 0, 1$.

(b) Construct a table of values for the function $y = 2x^2 - 1$, given $x = -1, 0, 1, 2$.

Solution:

(a) When $x = -2$
$y = 5 - x$
$y = 5 - (-2)$
$y = 5 + 2$
$y = 7$

When $x = -1$
$y = 5 - x$
$y = 5 - (-1)$
$y = 5 + 1$
$y = 6$

When $x = 0$
$y = 5 - x$
$y = 5 - 0$
$y = 5$

When $x = 1$
$y = 5 - x$
$y = 5 - 1$
$y = 4$

Therefore, the table of values for the function $y = 5 - x$ is

x	–2	–1	0	1
y	7	6	5	4

(b) When $x = -1$
$y = 2x^2 - 1$
$y = 2(-1)^2 - 1$
$y = 2 - 1$
$y = 1$

When $x = 0$
$y = 2x^2 - 1$
$y = 2(0)^2 - 1$
$y = 0 - 1$
$y = -1$

When $x = 1$
$y = 2x^2 - 1$
$y = 2(1)^2 - 1$
$y = 2 - 1$
$y = 1$

When $x = 2$
$y = 2x^2 - 1$
$y = 2(2)^2 - 1$
$y = 8 - 1$
$y = 7$

Therefore, the table of values for the function $y = 2x^2 - 1$ is

x	–1	0	1	2
y	1	–1	1	7

TIPS

Use a calculator to calculate the values of $y = 2x^2 - 1$
Press

2 ALPHA) ^ 2 - 1

For

$x = -1$,
press CALC -1 =
answer = **1**

$x = 0$,
press CALC 0 =
answer = **–1**

$x = 1$,
press CALC 1 =
answer = **1**

$x = 2$,
press CALC 2 =
answer = **7**

Drawing a graph

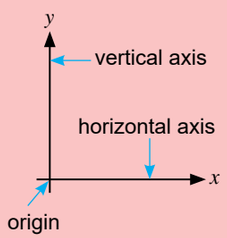
The ordered pairs *(x, y)* can be plotted on a Cartesian plane using the scale given. Next, the points are joined to form a graph. To make it easier in constructing the graph, we can use the following steps.

Steps to draw a graph:

1. Construct a table of values for the given function.
2. Draw and label each axis with the given scale or any suitable scale.
3. Plot the point (x, y) for the ordered pairs from the table.
4. Join the points to form a straight line or a smooth curve.

FLASHBACK

- (0,0) is also known as origin.
- x-axis is known as horizontal axis
- y-axis is known as vertical axis.



EXAMPLE 5

(a) Complete the table of values below for the function $y = 2x + 4$.

x	–2	–1	0	1	2	3
y	0			6		10

(b) Using a scale of 2 cm to 1 unit on the x-axis and 2 cm to 2 units on the y-axis, draw a graph of the function for values of x from -2 to 3.

This set of data can be written in the following form $-2 \leqslant x \leqslant 3$.

Solution:

(a) $y = 2x + 4$

When $x = -1$
$y = 2(-1) + 4$
$= -2 + 4$
$= 2$

When $x = 0$
$y = 2(0) + 4$
$= 0 + 4$
$= 4$

When $x = 2$
$y = 2(2) + 4$
$= 4 + 4$
$= 8$

Therefore, the table is

x	–2	–1	0	1	2	3
y	0	2	4	6	8	10

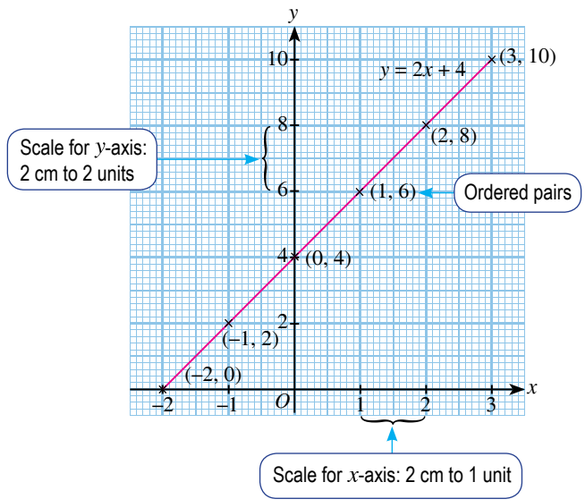
(b) Draw axes using the scales given.

Scale for x-axis : 2 cm to 1 unit.

Scale for y-axis : 2 cm to 2 units.

Plot the points according to ordered pairs from the table of values, (–2, 0), (–1, 2), (0, 4), (1, 6), (2, 8) and (3, 10).

Join the points with a straight line.



TIPS

This graph is also known as linear function graph, the highest power of a variable x is 1.

DO YOU KNOW ?

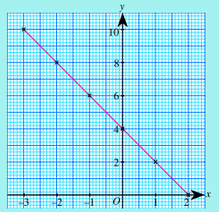
Equation for straight line $y = mx + c$, with m as the gradient and c is the y-intercept.

TIPS

Use a ruler to draw a straight line graph.

THINK SMART

What type of graph is shown below? State the function.



EXAMPLE 6

(a) Complete the table of values below for the function $y = x^2 - 2x - 3$.

x	–2	–1	0	1	2	3	4
y	5		–3			0	5

(b) Using a scale of 2 cm to 1 unit on the x-axis and 2 cm to 1 unit on the y-axis, draw a graph of the function for values of x from $-2 \leqslant x \leqslant 4$.

Solution:

(a) $y = x^2 - 2x - 3$.

When $x = -1$
$y = (-1)^2 - 2(-1) - 3$
$= 1 + 2 - 3$
$= 0$

When $x = 1$
$y = 1^2 - 2(1) - 3$
$= 1 - 2 - 3$
$= -4$

When $x = 2$
$y = 2^2 - 2(2) - 3$
$= 4 - 4 - 3$
$= -3$

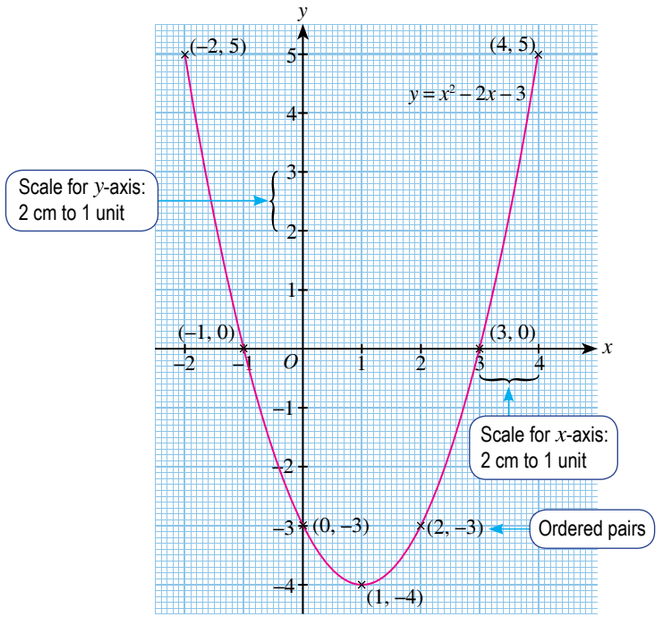
Therefore, the table is

x	–2	–1	0	1	2	3	4
y	5	0	–3	–4	–3	0	5

(b) Draw axes using the scale given. Plot the points using the table of values and join the points.

Scale for x-axis : 2 cm to 1 unit.

Scale for y-axis : 2 cm to 1 unit.



TIPS

Quadratic function
$f(x) = ax^2 + bx + c$,
The highest power for the variable in a quadratic function is 2, and $a \neq 0$.

TIPS

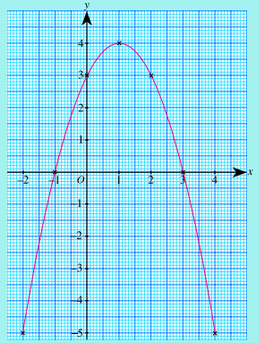
- A sharp pencil can help a student draw a line or curve smoothly.
- Students are allowed to use a flexible ruler to draw a curve.

DO YOU KNOW ?

This shape of graph is called parabola.

THINK SMART

What type of graph is shown below?
State the function.



EXAMPLE 7

(a) Complete the table of values below for the function $y = 12 - x^3$.

x	–3	–2	–1	0	1	2	3
y	39		13	12			–15

(b) Using a scale of 2 cm to 1 unit on the x-axis and 2 cm to 5 units on the y-axis, draw a graph of the function for $-3 \leqslant x \leqslant 3$.

Solution:

(a) $y = 12 - x^3$

When $x = -2$
$y = 12 - (-2)^3$
$= 12 + 8$
$= 20$

When $x = 1$
$y = 12 - (1)^3$
$= 12 - 1$
$= 11$

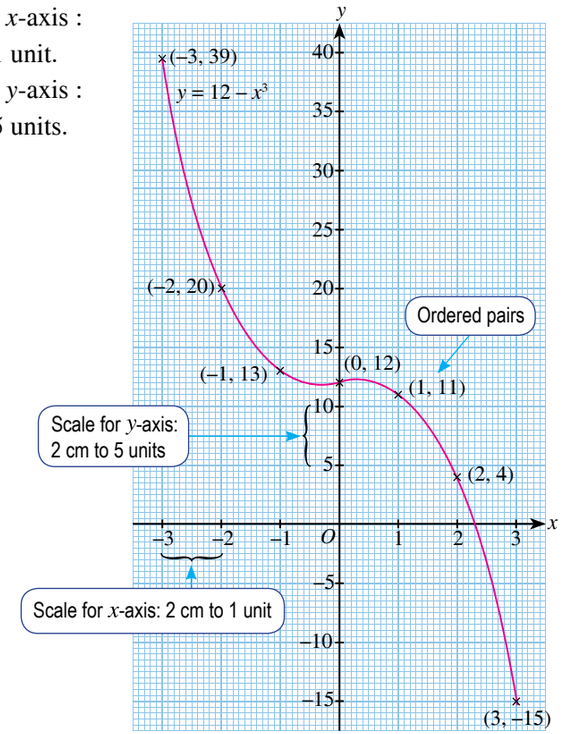
When = 2
$y = 12 - (2)^3$
$= 12 - 8$
$= 4$

Therefore, the table is

x	–3	–2	–1	0	1	2	3
y	39	20	13	12	11	4	–15

(b) Draw the axes using the scale given. Plot the points using the above table of values and join the points

Scale for x-axis :
2 cm to 1 unit.
Scale for y-axis :
2 cm to 5 units.

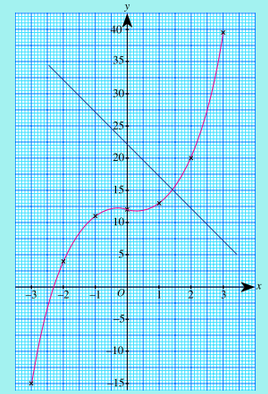


TIPS

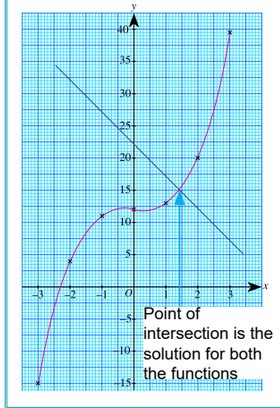
For the cubic function $ax^3 + c$, the highest power for the variable is 3.

THINK SMART

Determine the type of graph. State the function.



DO YOU KNOW ?



EXAMPLE 8

(a) Complete the table of values for the function $y = \frac{24}{x}$.

x	−4	−3	−2	−1	1	2	3	4
y	−6		−12	−24		12	8	

(b) Using a scale of 2 cm to 1 unit on the x-axis and 2 cm to 5 units on the y-axis, draw a graph of the function for $-4 \leqslant x \leqslant 4$.

Solution:

(a) $y = \frac{24}{x}$

When $x = -3$: $y = \frac{24}{-3} = -8$

When $x = 1$: $y = \frac{24}{1} = 24$

When $x = 4$: $y = \frac{24}{4} = 6$

Therefore, the table is

x	−4	−3	−2	−1	1	2	3	4
y	−6	−8	−12	−24	24	12	8	6

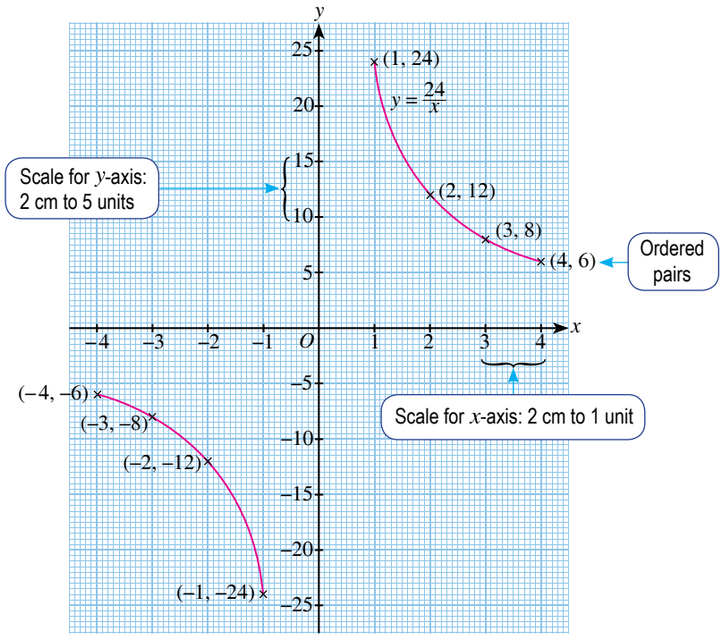
DO YOU KNOW ?

The reciprocal function $y = \frac{a}{x}$ is undefined if $x = 0$. The reciprocal function can also be written as $y = ax^{-1}$. This shape of graph is called hyperbola.

(b) Draw the axes using the scale given. Plot the points using the above table of values and join the points.

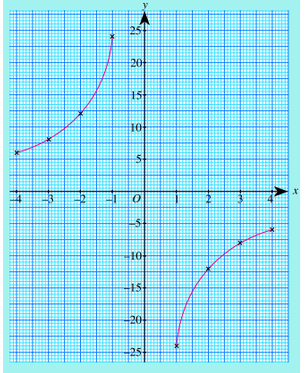
Scale for x-axis : 2 cm to 1 unit.

Scale for y-axis : 2 cm to 5 units.



THINK SMART

Determine the type of graph. State the function.



EXAMPLE 9

(a) Complete the table of values for the function $y = x^{-2}$

x	−4	−3	−2	−1	−0.5	0.5	1	2	3	4
y	0.06		0.25		4		1	0.25	0.11	0.06

(b) Using a scale of 2 cm to 1 unit on the x-axis and 2 cm to 0.5 unit on the y-axis, draw a graph of the function for $-4 \leqslant x \leqslant 4$.

Solution:

(a) $y = x^{-2}$

When $x = -3$: $y = (-3)^{-2} = 0.11$

When $x = -1$: $y = (-1)^{-2} = 1$

When $x = 0.5$: $y = (0.5)^{-2} = 4$

Therefore, the table is

x	−4	−3	−2	−1	−0.5	0.5	1	2	3	4
y	0.06	0.11	0.25	1	4	4	1	0.25	0.11	0.06

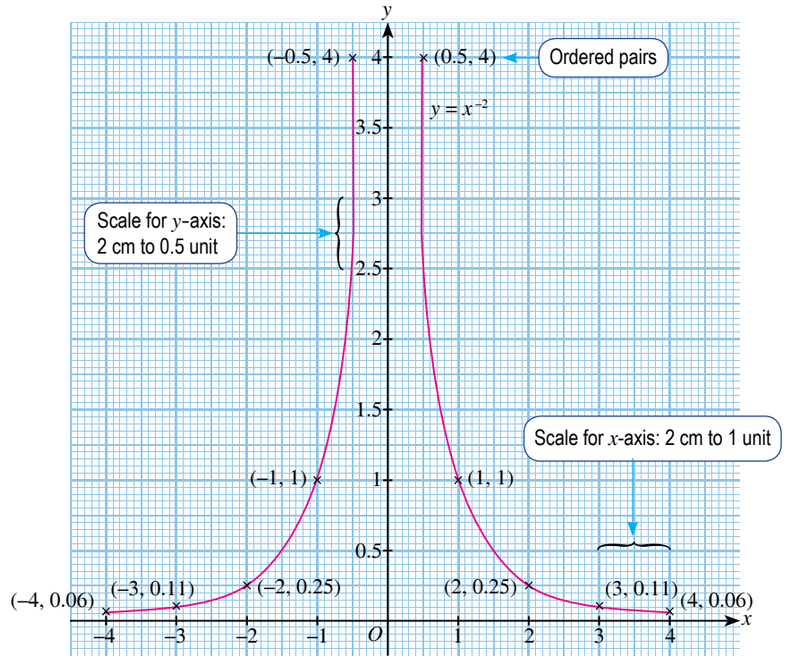
TIPS

$y = ax^n$ when $n = -1, -2$ is a reciprocal function.

(b) Draw the axes using the scale given. Plot the points using the above table of values and join the points.

Scale for x-axis: 2 cm to 1 unit

Scale for y-axis: 2 cm to 0.5 unit



This shape of graph is called hyperbola.

8.2.2 Interpreting graphs of functions

LEARNING STANDARD

Interpret graphs of functions.

Interpreting graphs of functions is like studying trends and making predictions according to the relations derived from the variables.

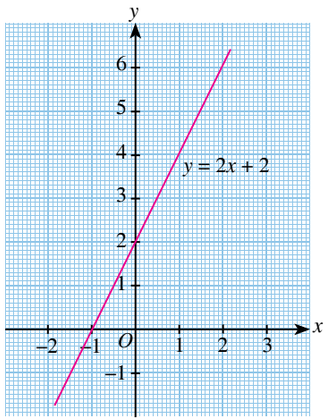
EXAMPLE 10

The diagram shows a graph of the function for $y = 2x + 2$.

From the graph, determine

(a) value of y when $x = 2$

(b) value of x when $y = 4$

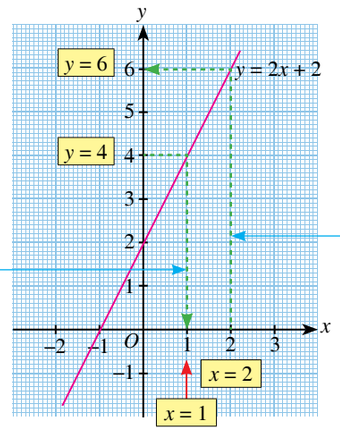


Solution:

From the graph:

(a) when $x = 2$, then $y = 6$

(b) when $y = 4$, then $x = 1$



When $y = 4$, draw a horizontal line to the right until it meets the function $y = 2x + 2$. Then, construct a vertical line downwards until it meets x-axis. State value x.

When $x = 2$, draw a vertical line upwards until it meets the function $y = 2x + 2$. Then, construct a horizontal line to the left until it meets y-axis. State value y.

EXAMPLE 11

The diagram shows a graph of the function for $y = x^2 - 2$. Based on the graph, determine

(a) value of y when $x = 2$

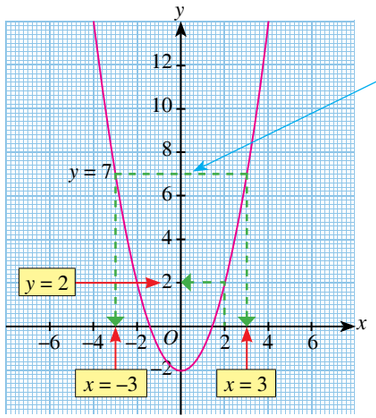
(b) value of x when $y = 7$

Solution:

From the graph:

(a) when $x = 2$, then $y = 2$

(b) when $y = 7$, then $x = 3$ or -3



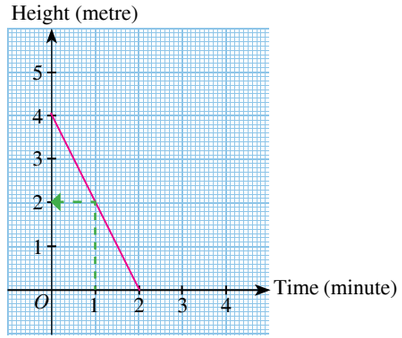
Construct a horizontal line $y = 7$, until it meets both sides of the function $y = x^2 - 2$. Then, construct a vertical line downwards towards x-axis.

EXAMPLE 12

The graph of function shows the movement of a ball that was dropped from a height of 4 metres. Based on the graph, determine

(a) the distance of the ball from the ground at the first minute.

(b) time the ball touches the ground.



Solution:

From the graph:

(a) when $x = 1$, $y = 2$

Therefore, the distance of the ball from the ground is 2 metres.

(b) when the ball touches the ground, the height is zero.

When $y = 0$, $x = 2$

Therefore, ball touches the ground at the second minute.

EXAMPLE 13

During the entrepreneurial expo, Anis sold fried rice at the Consumer Club's stall. The graph shows the number of fried rice packets sold with the profit that Anis gained.

From the graph,

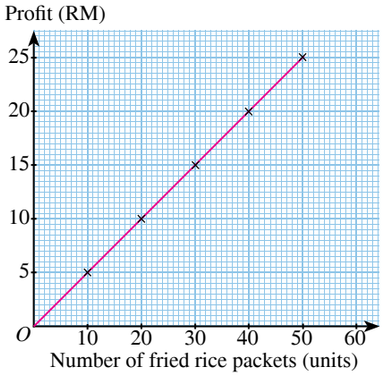
(a) what is the profit earned by Anis if she sold 20 packets of fried rice?

(b) if Anis earned a profit of RM20, how many packets of fried rice did she sell?

(c) state the profit made by Anis if she sold 26 packets of fried rice.

(d) state a suitable inference.

(e) predict the profit Anis would have made if 60 packets of fried rice were sold.



Solution:

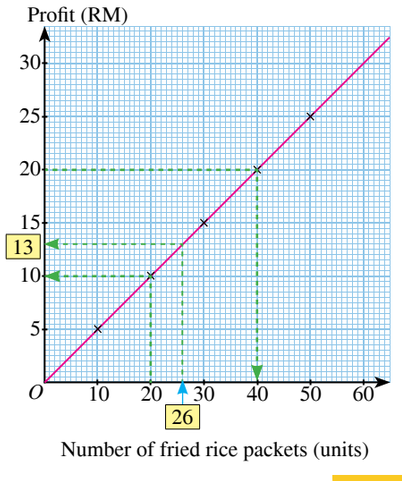
(a) RM10

(b) 40 packets

(c) Profit = RM13

(d) The more number of fried rice packets sold, the higher the profit gained.

(e) RM30



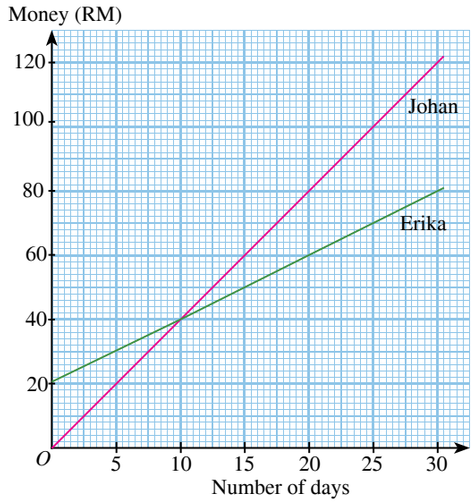
8.2.3 Solving problems

LEARNING STANDARD

Solve problems involving graphs of functions.

EXAMPLE 14

Every day Johan and Erika receive pocket money from their father. They keep part of the money in their saving boxes. The graph below shows the total amount of money saved (RM) according to the number of days.



(a) How much money is saved at the end of the 20th day by
   (i) Johan (ii) Erika

(b) When will the amount of money saved by Johan and Erika become equal?

(c) When will the amount of money between Johan and Erika have a difference of RM30?

(d) Their father promised to give a present to the person who saves the most at the end of one month. Who will get the present? Justify your answer.

Solution:

Understanding the problem

(a) Determine how much money is saved at the end of the 20th day by Johan and Erika.
(b) Determine when the amount of money saved by Johan and Erika become equal.
(c) Calculate when the amount of money between Johan and Erika will differ by RM30.
(d) Determine the person who would saves the most at the end of one month and justify it.

Planning the strategy

(a) State the value of y when $x = 20$ for Johan and Erika's graph.
(b) State the value of x at the point of intersection of the two lines.
(c) Calculate the difference of RM30 of the two lines.
(d) State the value of y when the number of days is 30.

Implementing the strategy

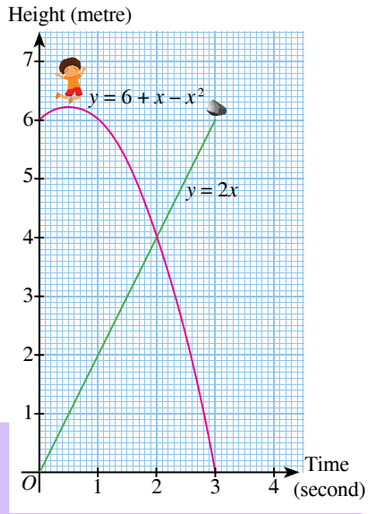
(a) From the graph, on the 20th day
   (i) Johan's savings = RM80 (ii) Erika's savings = RM60
(b) Value of x at the point of intersection in Johan and Erika's graph. Point of intersection (10, 40). Therefore on the 10th day the total savings for Johan and Erika is the same, which is RM40.
(c) RM100 – RM70 = RM30, on the 25th day.
(d) When $x = 30$; in Erika's graph, y = RM80; in Johan's graph, y = RM120. Hence Johan has more savings. Therefore, Johan will receive the present from his father.

Conclusion

(a) Total amount of money saved on the 20th day by
   (i) Johan = RM80 (ii) Erika = RM60
(b) Johan and Erika have the same amount of money saved on the 10th day.
(c) The difference between their savings is RM30 on the 25th day.
(d) Johan will get the present as on the 30th day, he will have RM120, whereas Erika's savings will be only RM80.

EXAMPLE 15

In an animated game while the cartoon character Jibam jumps from a rock, a pebble is thrown at him to make him fall. Graph of the function $y = 6 + x - x^2$, represents Jibam's movements and graph of the function $y = 2x$ represents the movement of the pebble. Y represents height in metre and x represents time in seconds.



(a) What is the maximum height of Jibam's jump?

(b) When does the pebble touch Jibam?

(c) When does Jibam touch the ground?

Solution:

Understanding the problem

- The function $y = 6 + x - x^2$ represents Jibam's jump.
  The function $y = 2x$ represents the movement of the pebble.
- Identify the maximum height of Jibam's jump, the time the pebble touches Jibam and the time Jibam touches the ground.

Planning the strategy

(a) Determine maximum height from the graph $y = 6 + x - x^2$
(b) Determine the value of x at the intersection of both graphs.
(c) Determine the value of x when $y = 0$

Implementing the strategy

From the graph,
(a) the highest point is (0.5, 6.25), therefore the maximum height is 6.25 m.
(b) the intersection point of both graphs is (2, 4), the value of x is 2. Therefore, the pebble touches Jibam at the 2nd second.
(c) when $y = 0$, $x = 3$. Therefore, Jibam touches the ground at the 3rd second.

Conclusion

Therefore,
(a) the maximum height of Jibam's jump is 6.25 metres.
(b) the pebble touches Jibam at the 2nd second.
(c) Jibam touches the ground at the 3rd second.

SELF PRACTICE 8.2

1. Copy and complete the following table of values for the functions given.

(a) $y = 3x + 2$

x	0	1	2	3	4
y		5			14

(b) $y = 2x^2$

x	0	1	2	3	4
y			8	18	

(c) $y = x^3 + 2$

x	–2	–1	0	1	2	3
y	–6		2			

2. Construct a table of values for each of the following using the given value of x.

(a) $y = 2x - 2$ for $-3 \leqslant x \leqslant 3$.

(b) $y = 2x^2 + x - 5$ for $-1 \leqslant x \leqslant 3$.

(c) $y = 3x^3 - 6$ for $-2 \leqslant x \leqslant 4$.

3. Copy and complete the following tables of values for the functions given, and draw the graph using the given scale.

(a) $y = 5 + x$

x	–3	–2	–1	0	1	2	3	4
y	2		4	5			8	9

Using a scale of 2 cm to 1 unit on the x-axis and 2 cm to 1 unit on the y-axis, draw the graph of function $y = 5 + x$ for $-3 \leqslant x \leqslant 4$.

(b) $y = 4 - x^2$

x	–3	–2	–1	0	1	2	3
y		0			3	0	–5

Using a scale of 2 cm to 1 unit on the x-axis and 2 cm to 1 unit on the y-axis, draw the graph of function $y = 4 - x^2$ for $-3 \leqslant x \leqslant 3$.

(c) $y = 8 - x^3$

x	–3	–2	–1	0	1	2	3
y	35		9	8			–19

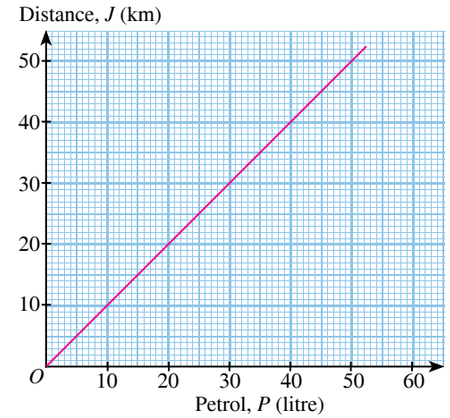
Using a scale of 2 cm to 1 unit on the x-axis and 2 cm to 10 units on the y-axis, draw the graph of function $y = 8 - x^3$ for $-3 \leqslant x \leqslant 3$.

(d) $y = \frac{4}{x}$

x	–4	–3	–2	–1	–0.5	0.5	1	2	3	4
y	–1	–1.33		–4	–8		4		1.33	

Using a scale of 2 cm to 1 unit on the x-axis and 2 cm to 2 units on the y-axis, draw the graph of function $y = \frac{4}{x}$ for $-4 \leqslant x \leqslant 4$.

4. The graph shows petrol P (litre) used by a taxi for a distance of J km.



From the graph,

(a) calculate how far will the taxi travel if its tank is filled with

(i) 30 litres of petrol

(ii) 42 litres of petrol

(b) calculate the cost of petrol for the taxi to travel 36 km if 1 litre of petrol costs RM 2.30.

5. Given a function $y = 5x^2 - 9x - 5$.

(a) Complete the table of values for the function above for $-2 \leqslant x \leqslant 3$.

x	–2	–1	0	1	2	3
y	33		–5	–9		

(b) Using a scale of 2 cm to 1 unit on the x-axis and 2 cm to 5 units on the y-axis, plot all the points.

(c) Construct the graph of function.

(d) From the graph, determine the value of x when $y = 0$.

GENERATING EXCELLENCE

1. Determine if each of the relation is a function.

(a) {(0, 0), (1, 4), (2, 8), (3, 12)} (b) {(25, 5), (25, –5), (9, 3), (9, –3)}

2. Represent the relation of the set given in the form of ordered pairs, tables, graphs and equations.

(a) Set for integers, $B = \{1, 2, 3, 4, 5\}$
Set for multiples 11, $A = \{11, 22, 33, 44, 55\}$

(b) Set for integers, $I = \{1, 2, 3, 4, 5\}$
Set for perfect squares, $S = \{1, 4, 9, 16, 25\}$

3. The surface area of a ball L in the shape of a sphere is the product of 4π with the square of its radius, r.

(a) State

(i) the dependent variable.

(ii) the independent variable.

(b) Write the relation between L and r.

4. Given $T = \{1, 2, 3 ,4\}$ and $U = \{1, 8, 27, 64\}$. The relation from set T to set U is to the power of three. Represent the following functions in the form of

(a) ordered pair (b) table (c) graph (d) equation

5. Amira's father gave her RM100 as her spending money.

(a) If she spends RM2 every day, calculate the balance after

(i) 2 days (ii) 5 days (iii) 10 days

(b) Given RMy represents the balance after x days, complete the table of values below.

x	5	10	15	20	25	30	35	40	45	50
y			70	60		40		20	10	

(c) Draw the graph for the function $y = 100 - 2x$ for $5 \leqslant x \leqslant 50$. Use the scale 2 cm to 10 units on the x-axis and 2 cm to 10 units on the y-axis.

(d) From the graph, calculate

(i) when Amira will spend all her money.

(ii) when Amira will have a balance of RM44.

6. Wilson wants to build a rectangular rabbit cage with the width p metres and the length $3p$ metres. Given A is the area of the cage, then $A = 3p^2$.

(a) Complete the table of values below for $0 \leqslant p \leqslant 6$

p	0	1	2	3	4	5	6
A				27		75	108

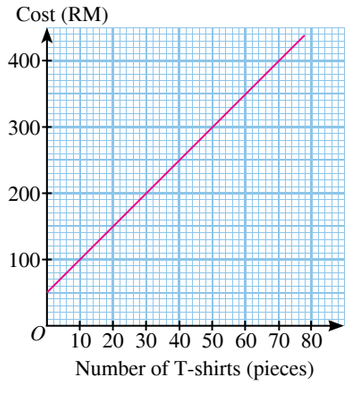
(b) Draw the graph of the function A for $0 \leqslant p \leqslant 6$. Use the scale 2 cm to 1 unit on the x-axis and 2 cm to 10 units on the y-axis.

(c) Based on the graph, calculate

(i) the area of the rabbit cage when the width is 5.2 metres.

(ii) the area of the rabbit cage if Wilson has 40 metres of wire mesh.

7. Raj is the chairman of the Computer Club. He wants to order T-shirts for his club members from Puan Aini, the school cooperative teacher. Puan Aini has prepared a graph to show the cost in RM, with the number of T-shirts.



(a) Complete the table below based on the graph.

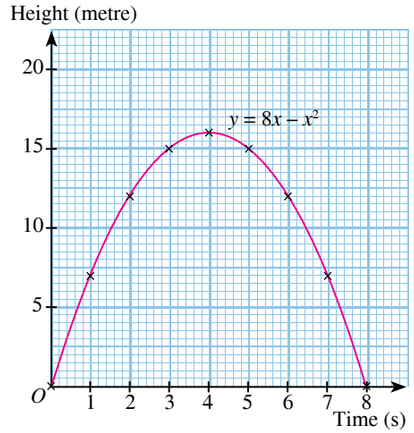
Number of T-shirts (pieces)	10	30	50	70
Cost (RM)				

(b) After Raj studied the graph, he was surprised that 0 pieces of T-shirts cost RM50. If you were Puan Aini, what would your explanation be?

(c) Calculate the total cost Raj needs to pay for 68 pieces of T-shirts.

(d) If Raj has a budget of RM410, state the total number of T-shirts that he can order.

8. Nizam hits a golf ball. The height of the ball, y metre from the surface of the ground after x seconds is $y = 8x - x^2$. The graph shows the movement of the golf ball after being hit.



From the graph,

(a) what is the height of the ball at the third second?

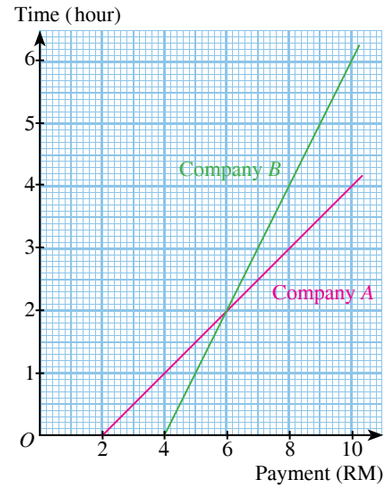
(b) calculate the time when the ball is at the height of 10 m.

(c) at which second will the ball fall on the ground?

(d) what is the maximum height achieved by the ball?

(e) what is the trend in the movement of the ball?

9. Zarul wants to rent a bicycle to go sightseeing at the recreational park. There are two shops offering bicycle rental services, Company A and Company B. The graph shows the hourly rate charged by each company.



(a) How much does Company A charge to rent a bicycle for 3 hours?

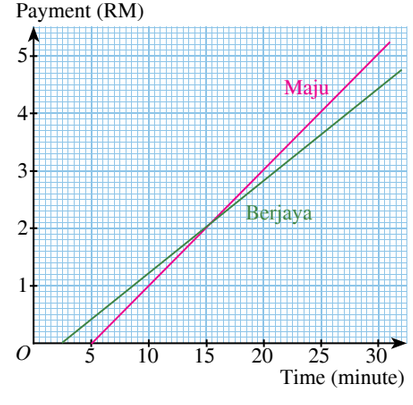
(b) If Zarul wants to rent a bicycle for only one hour, which company offers a cheaper rate? Explain.

(c) Zarul has RM7. From which company should he rent the bicycle? Explain.

(d) After which hour the payment charged at both bicycle companies will be the same.

(e) If Zarul rents a bicycle for 6 hours from Company B, how much must he pay?

10. Maju and Berjaya are two telecommunication companies offering packages for a mobile phone prepaid plan. The graph shows the payment rate and talk time offered by both companies.



(a) How much is the payment for 20 minutes talk time charged by

(i) Maju

(ii) Berjaya

(b) If Erin uses more than 30 minutes of talk time in a month, which company offers a cheaper rate? Explain.

(c) Umai wants to spend only RM4 for the service. Which company should Umai choose? Explain.

CHAPTER SUMMARY

Functions

It is the relation in which each object in the domain has only one image in the codomain.

One-to-one functions
Many-to-one functions

Representation of functions

- Ordered Pairs
- Tables
- Graphs
- Equations

Graphs of functions

Representation of a function with a straight line or curve on a Cartesian plane.

Table of values is a table that shows values for independent and dependent variables and it is constructed before a graph is drawn.

Steps to draw a graph.

1. Construct a table of values for the selected range.
2. Draw and label axes with the scale given.
3. Plot points from the table of values.
4. Join the points to construct the graph.

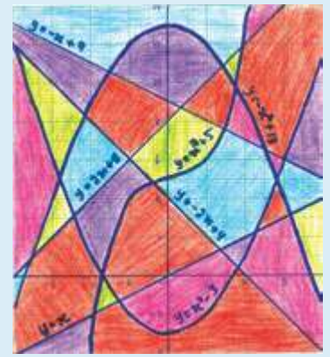
SELF REFLECTION

At the end of the chapter, I am able to:

1. Explain the meaning of functions.	
2. Identify function and provide justifications based on function representations in the form of ordered pairs, tables, graphs and equations.	
3. Construct tables of values for linear and non-linear functions and draw the graphs using the scale given.	
4. Interpret graphs of functions.	

MINI PROJECT

You are required to design a greeting card by using the function given. Complete the table of values. Draw seven graphs of the functions below by using a scale of 2 cm to 1 unit on the x-axis and 2 cm to 2 units on the y-axis. Label each graph and colour each region with your favourite colours. Then, cut out the graph paper according to your chosen size and make sure that the coloured design covers the whole surface area. Paste it on a manila card and decorate it creatively into a greeting card.



Example of design from several graphs of functions

Number	Function (y)	x						
		–3	–2	–1	0	1	2	3
1	$y = x$							
2	$y = -x + 9$							
3	$y = 2x + 8$							
4	$y = -2x + 4$							
5	$y = x^2 - 3$							
6	$y = -x^2 + 13$							
7	$y = -x^3 + 5$							
8	$y = 2x^{-2}$							